P3A Intermediate Report
Multi-scale tests of gravity

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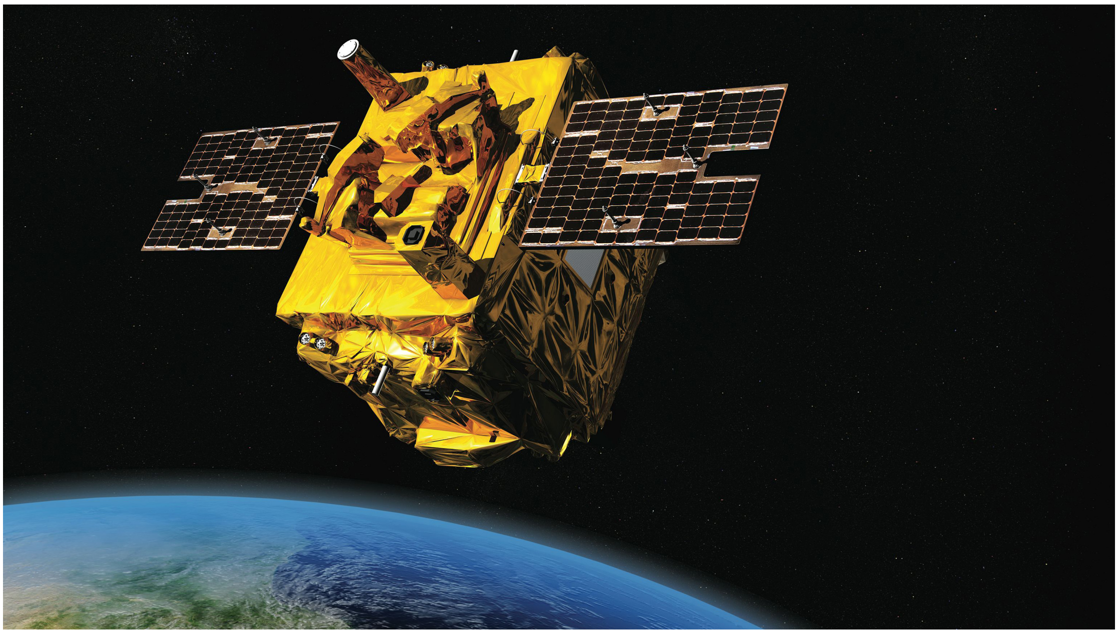

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Introduction

Gravity is one of the most important phenomena in cosmology and astrophysics. Yet, it is the least well understood fundamental force, with general relativity failing to explain some observations at the galactic scale [2][4].

In particular, there is a myriad of different models to explain this force and its deviations to general relativity : from the introduction of non-standard particles such as the weakly interacting massive particle [6] or the chameleon particle [10], to modified Newtonian dynamics [9]. All of these theories are designed to explain the observations that differ from the predictions of general relativity. But no other prediction from any of these theories has yet been confirmed or refuted by experience.

The Voyage 2050 mission would be a new experiment with hopefully the precision needed to refute some of the models. The method would be to launch a probe to the outer solar system and monitor its trajectory to eliminate the models that predicted a significantly different trajectory.

There are several major obstacles to overcome. First, the distribution of mass in the solar system is not well known, especially the in the Kuiper belt. Hence the trajectory expected by each of the models comes with uncertainties. Second, there are other forces that are non gravitational, such as solar radiation pressure, solar winds, or thermo-propulsion. Hence the trajectory observed needs to be corrected from these effects before it can be analyzed.

This is done thanks to an unbiased electrostatic accelerometer developed at ONERA. Since an accelerometer cannot detect gravitational accelerations, it will measure all non gravitational accelerations to a precision of 10^{-12} ms^{-2}. The purely gravitational trajectory can then be deduced by subtracting the non-gravitational accelerations from the observed movement.

During this P3A, we will first design a model for the trajectory of objects in the outer solar system, under different gravitational models. We will then use this model to quantify the ability of l'ONERA's accelerometer to discriminate between the standard model of gravitation and other models from the literature, especially the tensor-vector-scalar theory of gravity [9].

1 Scientific objectives of the Voyage 2050 mission, and scope of the P3A

In general relativity, gravity is equivalent to the curvature of space-time. And the geometry of space-time is linked to the distribution of matter and energy by Einstein Field Equations (EFE) [1] :

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}$$

$G_{\mu\nu}$ is Einstein's tensor. It is linked by the first equation to $T_{\mu\nu}$ the energy-impulsion tensor that represents the distribution of matter. $G_{\mu\nu}$ is also linked to the geometry of space-time by the second equation, with $R_{\mu\nu}$ being Ricci's curvature tensor and $g_{\mu\nu}$ the metric.

This equation is the part of general relativity that generalize Newton's law of gravitation. It explains incredibly well several observations such as the period of Mercury's orbit, gravitational lensing or gravitational waves. Yet there are observational limits to this theory:

- H. Van de Hulst and M. Schmidt observed in 1957 that the M31 rotation curve (that is, the curve of the average radial velocity of the stars in galaxy M31, depending of distance to the center of the galaxy) does not match the curve that would be predicted from the mass distribution that is observed in M31. This observation has been reproduced for several other galaxies since then [2].

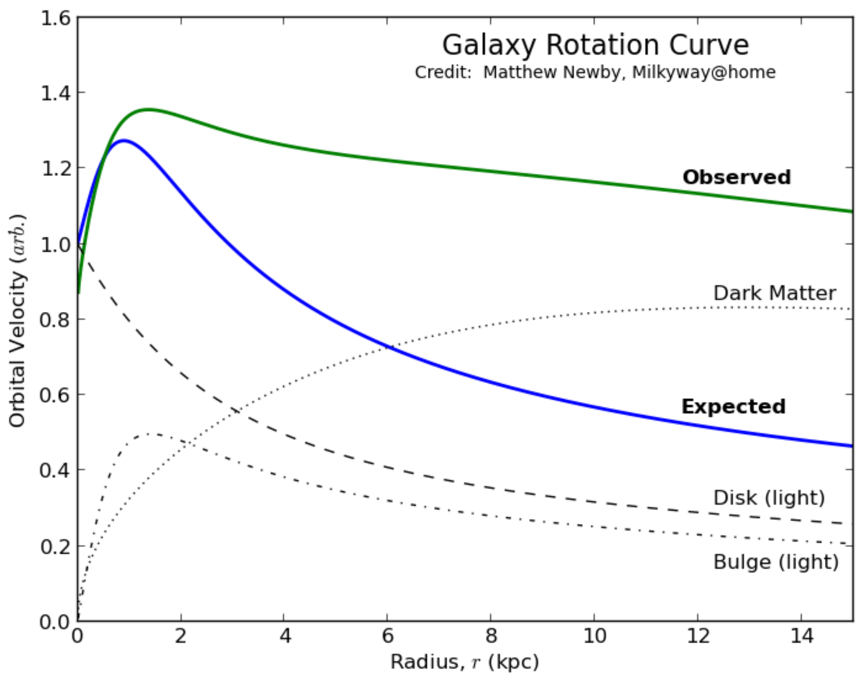

Figure 1: Rotation curve observed vs. expected from the observed mass distribution [3]

- F. Zwicky used in 1933 the virial theorem to calculate the total mass of the galaxies in the Coma cluster, and observed a large discrepancy with the mass expected from the light received during observations [4].

To explain this observational limits to general relativity, we know of 2 approaches:

- The first solution is to modify the right-hand side of the first EFE, that is to modify $T_{\mu\nu}$ and the distribution of matter in the universe. This is done by introducing a dark matter that would be a massive matter that does not interact with electromagnetic radiation [5]. To obtain the observed abundance of such particles via thermal production, a specific value for the self-annihilation cross-section of those particles is needed: $\langle \sigma v \rangle = 3\ 10^{-26} cm^3 s^{-1}$ [6]. A very good candidate used to be the Weakly Interacting Massive Particle (WIMP) that had been already been predicted by super-symmetric extensions of the standard model. But those super-symmetric theories have been found not to match observations at LHC.
  This first approach also includes the introduction of a dark energy, an unknown form of energy that would explain the universe's expansion rate [7]. On the whole, across the entire universe, the energy distribution presented in figure 2 is expected:

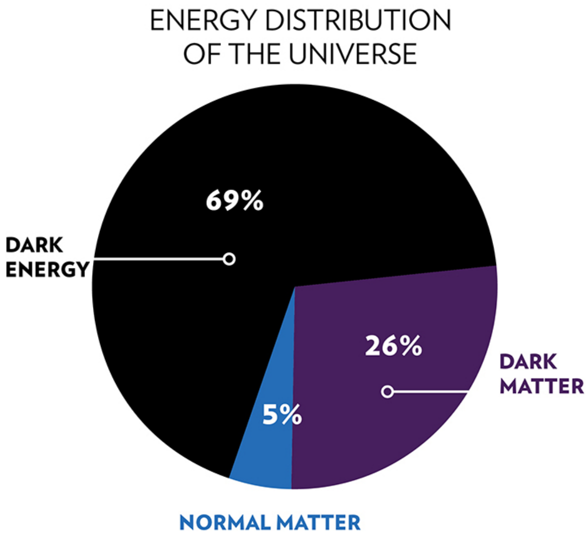

Figure 2: Expected distribution of total energy of the universe into baryonic matter, dark matter, and dark energy [8]

- The second approach is to modify the definition of $G_{\mu\nu}$, that is to modify the right-hand side of the second EFE. The MOdified Newtonian Dynamics (MOND) theory can be generalized to be coherent with special relativity through the tensor-vector-scalar gravity theory [9], which also englobes Brans & Dick theory. In this theory, a Yukawa potential is added to Newton's gravitational potential :

  $$V(r) = -\frac{GM_AM_B}{r}(1 + \alpha e^{-r/\lambda})$$

  Since Newton's potential makes excellent prediction at the scale of the solar system, this theory is often coupled with a theory that explain a screening effect that makes parameters α and λ very weak in stellar systems but noticeable on the galactic scale. On such theory is the Chameleon Particle theory [10]. Notably, this theory is also a theory for Dark Energy. The 2 approaches are not incompatible.

The objective of the Voyage 2050 mission would be to reach the outer solar system and, once there, to measure the force of gravity to deduce information either on the distribution of dark matter in the solar system, or on the acceptable values for α and μ. There has already been such inquiries, whose results are given in figure 3. But they could not find a correct value for the parameters and only could exclude some values:

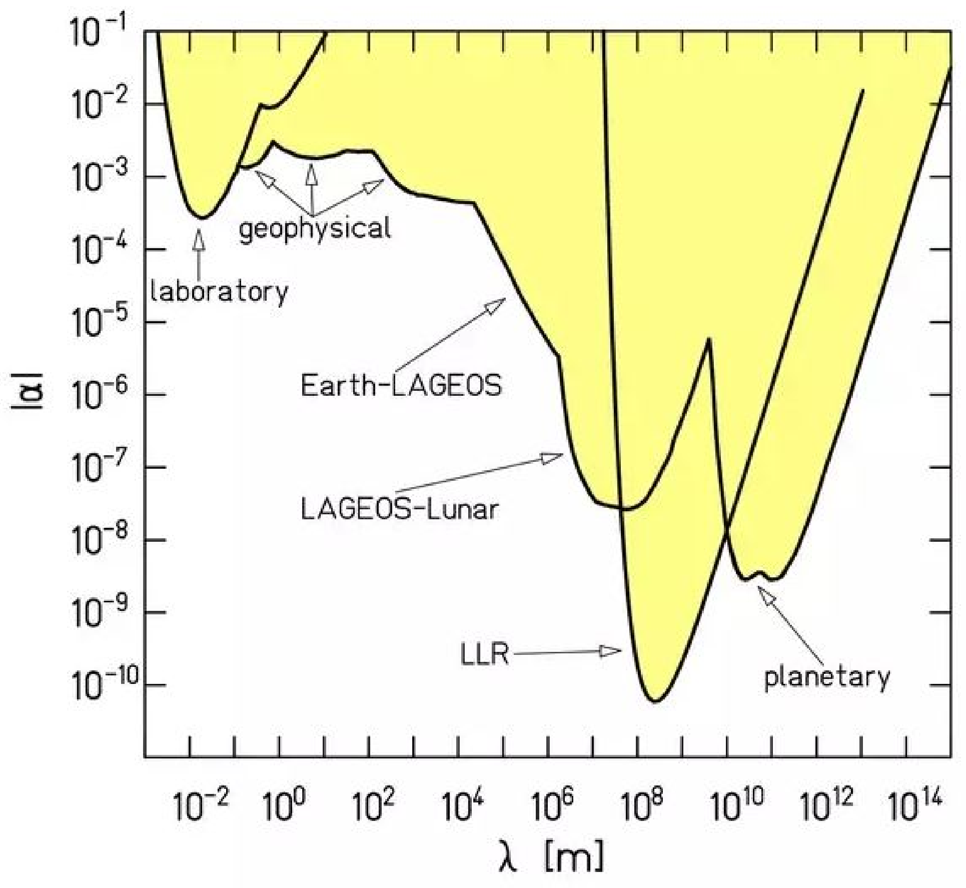

Figure 3: α - λ couple exclusion area [12]

The method to measure the gravitational force relies on the weak equivalency principle that states that the inertial mass and the gravitational mass are equal to each other [11]. Hence the acceleration $\vec{a}$ and the gravitational field $\vec{g}$ are equal for an ideal mass. Since an accelerometer measures the change in distance between a cage and a test mass, and since both the test mass and the cage feel the same gravitational field, an accelerometer cannot measures gravitational accelerations.
Hence, Voyage 2050's solution to measure the gravitational force is to send a probe in the outer solar system, to measure its position thanks to a radar and its non-gravitational accelerations thanks to an accelerometer. With that data, one can compute the position the probe would have had without any non-gravitational acceleration. And then deduce the equations for gravitation from the corrected trajectory of the probe.

This P3A is a preliminary study whose goal is to find how much information one can expect to gain from the mission. If we had perfect knowledge of the corrected trajectory, we would deduce perfectly the law of gravitation by a double derivation. But the accelerometer and the radar produce uncertainties. With that in mind, how much will we be able to restrain the set of acceptable values for α and μ ?
To answer this question, we have to model numerically the trajectory of the probe in Newton's potential and in Yukawa's potential. Then we have to find a quantity that gives, for two given trajectories with uncertainties, the statistical significance of the differences between these trajectories. Then, we must compute this quantity over a grid (α, λ) to find the set of values of these parameters for which we will been able to discriminate between a Newtonian trajectory and a Yukawa trajectory.
Additionally, if time permits, we intend to model numerically the non-gravitational accelerations that will be measured during the mission. Because it will allow to research the ability of the mission to provide more information on the outer solar system environment. And it will make sure there will not be any non-linear effects such as saturation of the accelerometer.

2 Modeling the outer solar system environment

Our first goal is to simulate the trajectory of a probe in the usual Newtonian gravitational potential, but also in Yukawa's potential. These potentials dominate the large-scale shape of the trajectory, but there are also many perturbations that have to be taken into account in order to obtain a precise trajectory. In this section, we describe these effects and the models we use to simulate them in the outer solar system.

2.1 Gravitational potentials

As introduced in the *Principia Mathematica*, the Newtonian model of gravity is characterised by a decay of the force in r^2. The force exerted from a body A on a body B is:

$$\vec{F_{A/B}} = -\frac{GM_AM_B}{r_{AB}^2}\hat{r_{AB}}$$

By defining $\mu = G \times M_A$ we get the acceleration of the Newtonian gravitational force:

$$\vec{a} = -\frac{\mu}{r_{AB}^2}\hat{r_{AB}}$$

Yukawa's model of gravity is named as such because it is based on the addition of a Yukawa potential [12] to the Newtonian potential. This potential is characterised by an additional exponential decay defined using two parameters, α and λ. And since Yukawa's model is seen as a correction of Newton's, α and λ are very small:

$$V(r) = -\frac{GM_AM_B}{r_{AB}} \times (1 + \alpha e^{-\frac{r_{AB}}{\lambda}})$$

Taking the gradient of this potential and using the earlier definition of μ, we get an acceleration given by:

$$\vec{a} = -\frac{\mu}{r_{AB}^2} \times (1 + \alpha e^{-\frac{r}{\lambda}}(1 + \frac{r_{AB}}{\lambda}))\hat{r_{AB}}$$

2.2 Solar radiations

The sun emits a large number of photons which transmit linear momentum to the probe via absorption and reflection on its surface. Thus a force is exerted on the probe, impacting its trajectory. The impulse of a single photon is given by:

$$P_\nu = \frac{E_\nu}{c}$$

By defining $\phi = \frac{dE}{dt \times dA}$ the flux of energy through a surface dA, we get that the impulse of the photon flow at the satellite's position:

$$dI = \frac{\Delta p}{\Delta t} = \frac{\phi}{c}dA$$

Given that the solar flux amounts to $\phi = 1367 W.m^{-2}$ at a distance of 1AU and is isotropic, we can get the impulse of the photon flow at any distance needed trough an inverse square law. To find the force exerted on the satellite, we now have to take into account the orientation of the satellite's surface and its absorption rate (see figure 4).

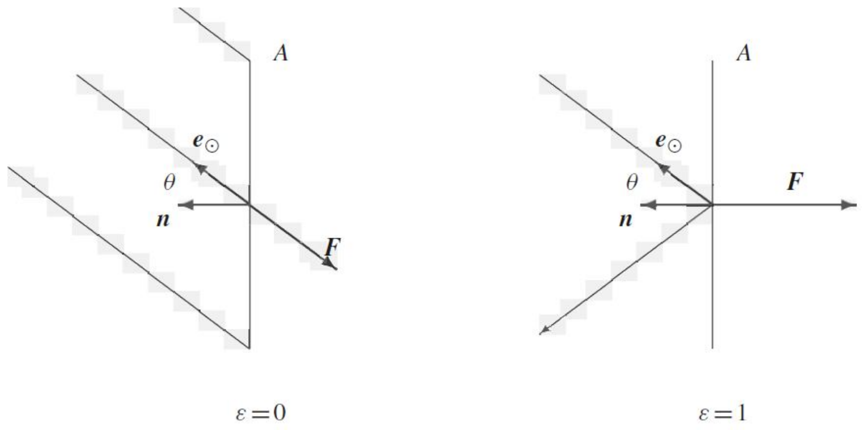

Figure 4: A photon that is reflected transmits twice the momentum in the direction normal to the surface [16]

Taking this effect into account, we get for an absorption rate of $1-\epsilon$ (ϵ being the reflectivity of the surface) the following force :

$$d\vec{F_{gen}} = -dI\cos\theta \times ((1-\epsilon)\hat{e_\theta} + 2\epsilon\cos\theta\hat{e_n})$$

Finally, by integrating this force on our spherical satellite model, we get:

$$\vec{F_{real}} = (1+\epsilon)\pi r^2 \frac{\phi}{c}\hat{e_r}$$

2.3 Solar winds

The solar wind is a stream of charged particles released from the upper atmosphere of the sun. This stream is mainly composed of electrons, protons and alpha particles, with traces of other heavy ions. This wind exits isotropicaly, creating the heliosphere. We can distinguish two types of solar winds depending on their velocity [13]:

- The slow solar wind with a speed of 300-500 km/s in the near-earth space.
- The fast solar wind with a typical speed of 750 km/s in the near-earth space but with a lower density.

This flow of high speed particles exerts a pressure on any object it encounters proportional to the relative speed and the density of the jet. The pressure is given by $P = m_p \times n \times V^2$ where P is the pressure, m_p the mass of a proton, n the density of protons, and V the velocity of the protons.

This pressure can greatly affect the trajectories of satellites, especially close to the sun where the velocity is the highest. Further from the star, this wind will decelerate, supposedly creating a termination shock where the heliosphere ends, at around 84 AU from the sun [14][15].

The force applied on a spherical probe of radius r from the solar wind is given by [16]:

$$\vec{F} = 2\pi r^2 \times d_{sun}^2 \times \phi\hat{e_r}$$

Where d_{sun} is the distance from the Sun to the probe, and ϕ is the flux of mass:

$$\phi = m_p \times V \times \frac{n_0}{4\pi r_{sun}^2}$$

m_p is the mass of protons, V the relative velocity of the solar wind to the satellite, n_0 the number of protons ejected by the sun per second and r_{sun} the radius of the sun.

2.4 Thermal recoil

The probe can be considered as a black body. Once heated such a body emits radiations proportional to its temperature. As before, the radiations emitted have an impulse equals to:

$$P_\nu = \frac{E_\nu}{c}$$

Thus this radiation will generate a force on the satellite, this is what caused Pioneer's drift. In an ideal case, the temperature of the satellite would be homogeneous, leading to isotropic emission hence no force. However in reality neither internal heating nor exposition to the sun is the same for all surfaces of the probe, resulting in faces warmer than other. Those faces will emit more radiations, hence generating more recoil. This will cause the satellite to undergo a non-zero thermal recoil force.

We expect that the surface exposed to the sun will be the warmest, thus the propulsion would be a radial propulsion from the sun towards outer space. However, in case the satellite rotates, we will observe a Yarkovsky effect where the side that was just exposed to the sun is hotter than the side that is currently exposed, thus generating angular propulsion as well.

2.5 Interstellar dust

Interstellar space is filled with dust. Even though the density of the dust is very low, it still affects the mechanical energy of a satellite, and reduces its speed. A crude model is to that of a drag:

$$F = c \times nm \times v^2$$

If we assume a drag coefficient $c = 1$, a dust density of $n = 10^{-6}$ part./m^3, an average mass $m = 10^{-17}$ g for the dust particle and a stationary dust ($v = v_{sat}$) [19], we find a force 9 orders of magnitude below that of gravity. Hence we considered this effect to be negligible and we did not model it.

3 Computational aspects of trajectory modelling

The main part of our work to this point has been to develop a Python library with the functions necessary to compute the trajectory of a probe in the outer solar system, with the numerical precision required by the P3A's objectives, and taking into account all the forces described in section 2.

Our solution is the most straightforward: we consider the Solar System Barycenter frame to be inertial, and calculate the forces exerted on the probe in this frame. This gives us the equations of motion, which we solve numerically through a Runge-Kutta method. There are several other methods, for example a perturbative approach of the Keplerian problem. But we consider some non-standard forces for which the Lie developments are not known. Hence we favor the equations of motion as it is the simplest representation. Still, we had to answer several questions of computational efficiency.

The first thing is to represent time efficiently. To do that, we use the Julian day [17]. This choice, common both in astronomy and in software development, allows for calculating the time between two events easily. And it makes Lorentz transformations simple.

Since the Newtonian gravitational field depends on the positions of the sun and the planets, the next thing is to find an algorithmically efficient way to know these positions at any given time. It would be incredibly inefficient to calculate them at each time step. It would be less but still highly inefficient to update them at each time step concurrently with the probe's position. Ultimately, the most efficient way is to calculate the positions at all times once, save them in a document, and read this document at each time step. This is essentially our strategy: we use the jplephem Python library from the Jet Propulsion Laboratory [18] which gives such a document, with a high precision on the positions.

Finally we had to find an way to compute the forces exerted on the probe. The naive approach is to calculate the accelerations from the formulas at each time step. But this makes the code much less modular and readable. We preferred to use object-oriented programming, hence we defined a class representing each kind of forces. We only have to instantiate one object from each class at the beginning of the algorithm and ask these objects for the forces at each time step. This does not make our algorithm any faster or memory-efficient, but it makes testing, debugging and usage much simpler.

Indeed, since our objective is to discriminate between models of gravity that have the same {low speed & low mass} limit – that is the Newtonian gravity – we expect the differences in trajectories between models to be subtle. We must not drown these differences in the uncertainties and bias of our numerical integration. Hence we had to test the precision of our Runge-Kutta method.
To do so, we first computed the trajectories with a Newtonian potential and a with Yukawa potential. This gave us very similar trajectories that differed by only 15 m after 1 month. This is the precision our numerical integration needs to yield.
Then we computed the trajectory for a Keplerian problem both from the theoretical formula and from our library. We found that our numerical integration produces a drift of about 2 m per month. This is lower the order of magnitude we aimed for, so our integration method is acceptable.

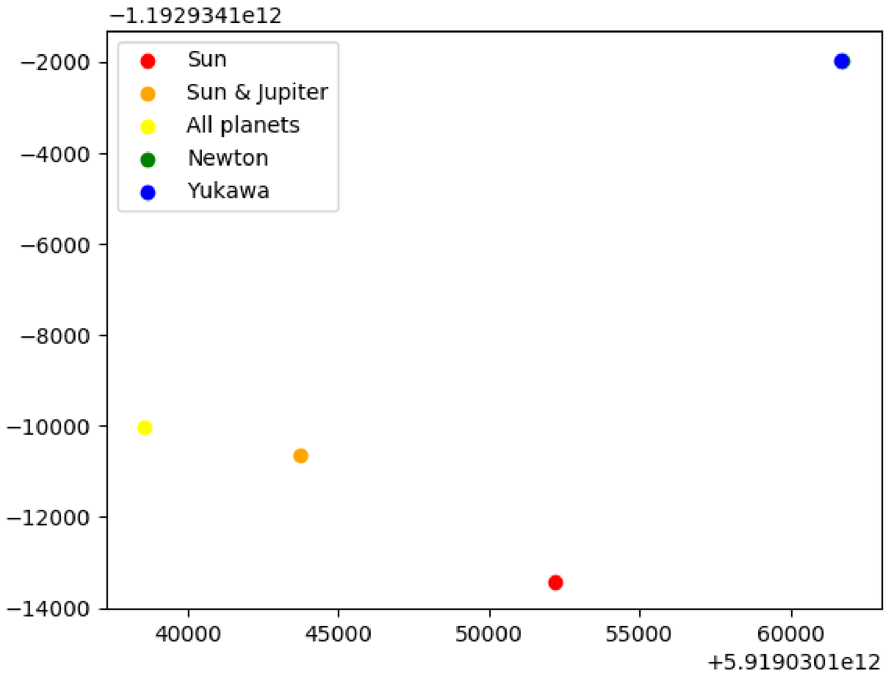

Figure 5: Position of the probe after 1 month of evolution, with the same initial position and velocity but under the influence of different forces.
In red only the influence of the Sun is considered. In orange we add the influence of Jupiter and in yellow that of all the other planets. In blue and green we also add the solar radiation pressure and the solar wind effects.
It appears that the Newton's and Yukawa's potentials give very similar trajectories. It also appears that none of the other forces considered is negligible.

4 Objectives for the second semester

As we have seen, Yukawa's potential is defined through two parameters α and λ, and the objective of our project is to find the values of those parameters that will yield trajectories that are significantly different from the trajectory in a Newtonian potential. We have spent the first semester developing the tools to simulate a trajectory. In this section, we outline how we will use these tools to meet our objective.

4.1 $\alpha - \lambda$ graph

In the actual mission, we would compute the trajectory of the satellite for all points on a grid (α, λ) and then estimate the divergence of the simulated trajectory to the observed trajectory.

For our study, we also have to simulate the "observation". The comparison between the "simulation" and the "observation" gives us the capacity of the accelerometer to discriminate between 2 models from their trajectory. Our "observation" will be with the Newtonian potential and we will "simulate" all the pairs $(\alpha,\ \lambda)$ on a grid. This will allow us to draw a graph in the $(\alpha,\ \lambda)$ domain similar to that of figure 3.

But in order to do this, we need to find a method to analyze whether the differences between 2 trajectories are sufficient to outweigh the uncertainties from the radar and the accelerometer's measurements:

- Error due to the radar's measures: $\sigma_{radar} = 1$ m
- Error due to the accelerometer's measures: $\sigma_{acc} = 10^{-12}$ m/s^{-2}

The first thing is to define the corrected observed trajectory y_{obs}, that is the trajectory the probe would have had in the absence of any non-gravitational effect. For that, we deduce from the observed trajectory x_{obs} the non-gravitational accelerations measured and we find the corrected trajectory by double time integration:

$$\vec{y_{obs}} = \vec{x_{obs}} - \iint \vec{x''_{NG}} dt_1 dt_2$$

Afterwards, we quantify the difference between the corrected observed trajectory and the simulation thanks to the χ^2 observable:

$$\chi^2 = \sum_t \frac{|\vec{y_{obs}} - \vec{y_{model}}|^2}{\sigma_{total}^2}$$

The χ^2 is defined as a comparison between the measured differences in the trajectories and the uncertainties on the measures. It allows us to easily deduce the probability p that the 2 trajectories are the same:

$$\chi^2 - \chi^2_{min} = -2\ln(1-p)$$

χ_{min} is the lowest value for χ that any model we simulated could yield. In our case, since our "observation" is a already a simulation, we will always have $\chi_{min} = 0$.

4.2 Calculation methods

The simulation of a trajectory is very time-consuming (4 hours for a simulation over 1 month). Since we have a large number of models to test (a whole grid (α, λ)), our naive solution will necessitate either a high computational power or a parallelized multi-processors approach . Alternatively, we consider finding the exclusion area in the (α, λ) domain by a Markov Chain Monte-Carlo(MCMC) method.

Conclusion

The Voyage 2050 mission's goal would be to investigate on several alternatives and corrections to general relativity that have been proposed in the literature to explain the observations that contradict general relativity. This P3A can be seen as a preliminary feasibility study that focuses on the tensor-scalar-vector theories, with the goal of finding the capability of ONERA's accelerometer to improve the current state of knowledge on these theories.

As of now, we have already established a library in order to simulate the gravitational and the main non-gravitational forces, and compute the trajectory of the probe. We were also able to assess the impact of the different forces. Also, we are able to compute the χ^2 between two trajectories to quantify their differences.

The next step will be to create the (α, λ) graph, either with a grid approach or with a MCMC approach. Later on, we hope we will have the time to improve our simulation of the environment both by including the Kuiper belt (gravitational effect) and other, more subtle non-gravitational effects. We also hope to we will have the time to research the influence of the initial conditions on the ability of the accelerometer to discriminate between different values of α and λ.

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