

Syntax, Symmetry, and Systematizing of Rotational Arrays in Stravinsky's Late Music

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Abstract

The totality of Stravinsky's musical works reveals dualisms, equilibrium between tonality and atonality, centricity and equality, and abstract and concrete. Throughout Stravinsky's life, he was guided and influenced by many composers and theorists, such as Craft, Schoenberg, Webern, Boulez, Babbitt, and most notably in this paper, Krenek, who directly influenced Stravinsky's operations with the rotational array. Krenek and Stravinsky's precompositional charts can be broadly examined and compared to perceive commonalities between their pitch-class collections, intervallic contents, and divergence of style characteristics. Melodic instances of rotational arrays with relation to transpositional procedures will be discussed to declare norms that frequently occur throughout Stravinsky's late serial music. Additionally, harmonic occurrences, along with applicable terms for categorizing Stravinsky's late serial music, will be presented to recognize the diverse and innovative ways Stravinsky frequently states rotational harmonic rows. In the midst of considering harmonic rotational events, some note sets from Stravinsky's pieces will be systematized into set classes and examined for transpositional and inversional symmetry. To culminate the expansive, thorough, and in-depth analysis of rotational arrays' syntax and applications, symmetry focusing around a basis of interval-class and pitch-class duplication will form a mathematical methodology pertinent to Stravinsky's other rotational sets throughout his canonic period, in addition to other composer's serial applications.

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Chapter I: Introduction

Although much of Igor Stravinsky's music was innovative and in the neoclassic style, employing pitch centricity and tonal reference, his later works of atonality from 1951-1966 sought to experiment with new compositional methods and develop additional insights into areas of theoretical scholarship and research. When examining Stravinsky's later theoretical compositional techniques, it is important to provide an historical account of his influences and timeline of transitional style that influenced Stravinsky's late compositions.

Much of Stravinsky's early music was written during his years in Europe, until he migrated to the United States in 1939 to escape Hitler's reign. During his time in Europe and his early period in the United States until 1951, Stravinsky continued to indulge in his neoclassic style; his compositions revealed the contrasting techniques used in serialism and twelve-tone works, such as music from members of the Second Viennese School, among other serial composers. Before 1952, when Stravinsky composed *The Rake's Progress*, reestablishing his iconic stature in America, he endured financial and social hardship, as he vigorously worked in America to regain the same prominent reputation he had previously obtained in Europe. Following the successful premiere of *The Rake's Progress* and experiencing a performance of Schoenberg's *Violin Concerto*, Stravinsky began moving towards a new compositional style, engaging in diverse aspects of serialism and atonality.

He managed to transform his compositional style, leaving his current neoclassic perspective he had adhered to for decades. Hoping to appeal to the idiosyncratic avant-

garde supporters, Stravinsky's change of style insured an increase in his prominent reputation among various generations. From *The Rake's Progress* to the end of his compositional period, 1952-1966, Stravinsky's compositions contained distinct new approaches to serialism and atonality, expanding his own theoretical and compositional methodology, in addition to leading to advances in musical scholarship. Joseph Straus gives an account of Stravinsky's compositions during this period:

From a technical, music-constructivist point of view, his works after *The Rake's Progress* describe a remarkable succession of firsts, including his first works to use a series (*Cantata* [1952], *Septet* [1953], "Three Songs" from *William Shakespeare* [1954]); his first fully serial work (*In Memoriam Dylan Thomas* [1954]); his first work to use a twelve-tone series (*Agon* [1954-7]); his first work to include a complete twelve-tone movement ("Surge, aquilo," from *Canticum Sacrum* [1956]); his first completely twelve-tone work (*Threni* [1958]); his first work to make use of twelve-tone arrays based on hexachordal rotation (*Movements* [1959]); his first work to use the verticals of his rotational arrays (*A Sermon, a Narrative, and a Prayer* [1961]); his first tetrachords of the series (*Introitus* [1965]); his first work to use two difference series in conjunction (*Requiem Canticles* [1966] – his last major work).¹

Before Stravinsky experimented with serialism, he met a young, aspiring composer, Robert Craft, an advocate of Schoenberg and Webern's music, who was influential in introducing Stravinsky to the music of the Second Viennese School. This association was the catalyst to Stravinsky reinventing his style. Being a prominent inspiration to Stravinsky, Craft also remarked on his notable influence on Stravinsky's music: "I say in all candor that I provided the path and that I do not believe Stravinsky would ever have taken the direction he did without me. The music that he would otherwise have written is impossible to imagine."² Since Craft did not attain the same theoretical and compositional prominence as Stravinsky in his lifetime, Craft only

¹ Straus, Joseph N. *Stravinsky's Late Music*. New York: Cambridge University Press, 2001. 4.

² "A Centenary View, Plus Ten," in *Glimpses of a Life*. 16-17.

contributed to Stravinsky's initial compositional transition but was limited in the guidance and theoretical insight he could provide in expanding Stravinsky's serial style:

Craft was in a position to introduce Stravinsky to this music, to impart some of his own enthusiasm, and to explain, in a rudimentary way, how it was put together. But Craft had no particular interest in theoretical abstractions. He apparently understood little of twelve-tone composition and twelve-tone theory beyond the basic facts of writing and manipulating a series.³

Arnold Schoenberg, as a prominent figure of the Second Viennese School, was another influential character of Stravinsky's late, atonal innovations. It is evident, through studying their music, that Stravinsky's early works, before 1951, utilized different approaches than Schoenberg's, which made their hostile relationship apparent to the public. This antipathy towards one another was displayed in the music and writings of both composers and was apparent on numerous occasions to the public. Stravinsky expressed his disdain to a journalist for, "the gentlemen who work with formulas instead of ideas", referring to Schoenberg, among others.⁴ Schoenberg responded with articles in 1925 and 1926, neither of which specified a composer's name but insulted the neoclassical style of music. Ironically, in 1940 when Stravinsky moved to Hollywood, where Schoenberg had been living since 1934, they were virtually next-door neighbors, while they maintained a mutual animosity for each other.

It is ironic to note that immediately after Schoenberg's death, Stravinsky began engaging in the music of Schoenberg. The connection between Stravinsky's new musical style in 1951 and Schoenberg's death is perplexing to say the least, considering the extended hostility towards one another. An explanation of the concurrent timing of Stravinsky's association with Schoenberg's music can be justified; until Schoenberg's

³ Straus, Joseph N. *Stravinsky's Late Music*. New York: Cambridge University Press, 2001. 8.

⁴ Messing, Scott. "Neoclassicism in Music: From the Genesis of the Concept Through the Schoenberg/Stravinsky Polemic." Ann Arbor, Mich: UMI Research Press, 1988. 141.

death, his music was basically unavailable in America, explaining Stravinsky's lack of familiarity with serialism and Schoenberg's music until the 1950s.⁵ Before discussing Stravinsky's other musical inspirations, it is important to note that his late works were profoundly influenced by Schoenberg, given that he significantly impacted atonality and serialism. Oddly, despite Schoenberg's influence on Stravinsky, Stravinsky greatly challenged the style presented in Schoenberg's works, as illustrated by Joseph Straus:

Stravinsky began almost immediately to adopt the essential Schoenbergian principle of serial ordering. Only in portions of *Canticum Sacrum* (1956) and *Agon* (1957), and then in the entire vast expanse of *Threni* (1958), did Stravinsky begin to rely exclusively on twelve-tone series. But while Stravinsky adopted Schoenberg's points of departure, he moved immediately in very different musical directions. From the very outset, Stravinsky simultaneously invokes and remakes Schoenberg.⁶

Although Schoenberg most likely influenced Stravinsky's music to a far greater extent than discussed in this historical account, the quotation above explains the most notable styles from Schoenberg's music prevailing in Stravinsky's serial style.

Anton Webern was another composer who had a major impact on Stravinsky's music. Stravinsky's initial familiarity with Webern's music dates from 1952. Interestingly, this encounter between Stravinsky and Webern occurred around the time of Schoenberg's death. Being a student of Schoenberg, Webern emulated many aspects of Schoenberg's music, but he additionally developed his own musical language very distinct from Schoenberg's, affecting Stravinsky's underlying compositional style. Primarily, Stravinsky had a tendency to employ canons in his later works as in his *Double Canon: Raoul Duffy in memoriam*, composed in 1959, just as Webern had a propensity to do in his works such as *Five Canons, Op. 16*, composed in 1923-24 and published in

⁵ Straus, Joseph N. *Stravinsky's Late Music*. New York: Cambridge University Press, 2001. 15.

⁶ Ibid., 10-11.

1928. Subsequently, Stravinsky employed a somewhat regular technique by presenting the prime form of a row, while also displaying the row in its inversional state, regularly displacing it rhythmically and organizing it in a canon. Schoenberg and Webern's compositional aspects notably influenced Stravinsky's music, but as Stravinsky continued composing, he gradually created his own unique style, distinct from his earlier influences.⁷

In the midst of Stravinsky's compositional style change, he encountered a young composer by the name of Pierre Boulez. Around the same time of contact between Webern and Schoenberg, Stravinsky began communicating with Boulez. As Craft observed, Stravinsky attended rehearsals of *Polyphonie X* and, "made an analysis of the score."⁸ Besides being a serialist composer and the constant contact between Stravinsky and Boulez, there is no concrete evidence proving that Boulez heavily influenced Stravinsky's music.

Despite the influential composers discussed so far in relation to Stravinsky's music, there has been little content discussed pertaining to theoretical writings and composers' academic ideas. Since many of Stravinsky's influences thus far have primarily been composers, it is important to mention one of Stravinsky's friends and theoretical critics of his music, Milton Babbitt. Milton Babbitt was credited with the earliest writings about Stravinsky's compositional and theoretical output. Additionally, independent from Stravinsky, he maintained a prominent successful career as a theorist and composer throughout his lifetime. Milton Babbitt presented many scholarly articles related to Stravinsky's late music; specifically, the rotational array and other applications

⁷ Ibid., 26.

⁸ Smyth, David H. "Stravinsky as Serialist: The Sketches for "Threni"" *Music Theory Spectrum* 22, no. 2 (2000): 205-24.

to his new approaches of serialism. Babbitt gives a vivid portrayal, describing the pinnacle and impact of Stravinsky's new compositional style:

For the shared compositional modes of the major works beginning with *Movements* and ending with the *Requiem Canticles* not only affected and engaged all the dimensions of these compositions in a manner unprecedented for Stravinsky personally, but remarkable define a different position for Stravinsky vis-à-vis the music which preceded these works, both his own and that of others.⁹

As shown above, Babbitt often remarked on how Stravinsky's late style impacted theoretical and academic scholarship for future generations. Babbitt's academic writings associated with Stravinsky's music extended through many years of the 20th century, representing his attention dedicated to Stravinsky's music. During Stravinsky's early style, Babbitt started thoroughly examining Stravinsky's music and followed it through his late period. After Stravinsky's death, Babbitt continued to present new discoveries and theoretical breakthroughs, expressing the commitment Babbitt illustrated towards Stravinsky's music.

Although four influential people in Stravinsky's life have been briefly observed, showing their contribution to musical scholarship or compositional style, Krenek is a composer that is partially responsible for the development of Stravinsky's procedures with the rotational array. Though many differences appear between Krenek and Stravinsky's approach with rotational arrays, Stravinsky often examined Krenek's musical characteristics and writings containing detailed arrays, similar to Stravinsky's array charts, which will be further explored in subsequent chapters. Stravinsky commented on Krenek's *Studies in Counterpoint* (1940) – a beginner manual of twelve-

⁹ Babbitt, Milton. "Order, Symmetry, and Centricity in Late Stravinsky," in *Confronting Stravinsky*, ed. Jann Pasler (Berkeley: University of California Press, 1986), 247-48.

tone composition to affirm, “the first work I read on that subject.”¹⁰ As clarified by Joseph Straus, Krenek’s arrays distinctly differ from Stravinsky’s rotational arrays. Krenek’s arrays contain mostly diatonic collections, whereas Stravinsky appears to employ chromaticism with mostly disjunct motion, in hopes of avoiding scalar action and a close reference to tonal music, showing a clear departure from his earlier music. Additionally, Krenek only used his arrays in melodic settings, employing canons and producing renaissance polyphony. Stravinsky’s arrays are used regularly in melodic and harmonic instances, commonly employing both of them somewhat habitually.¹¹

Before exploring rotational arrays, it is important to establish a basis for the way Stravinsky conceptualizes his music and other composers’ works. Comparisons between Stravinsky and Schoenberg offer a strong source of Stravinsky’s musical positions, and show a clear antithesis of their style, explaining the contrast of their musical output in relation to one another. “When Craft, in one of the conversation books, asked Stravinsky to ‘comment on the popular notion of Schoenberg and Stravinsky as thesis and anti-thesis,’ Stravinsky responded by constructing a revealing set of contrasts.”¹² Two comparisons provided below show their dissimilarities, inspired by Joseph Straus, although Stravinsky and Robert Craft first presented the two respective composers’ ideas:¹³

Stravinsky:	Schoenberg:
Music is powerless to express anything at all	Music expresses all that dwells in us
Metronomic strictness, no <i>rubato</i> . Ideal is of mechanical regularity (<i>Octuor</i> , Piano Concerto, etc.)	Much use of rubato

¹⁰ Stravinsky, Igor and Robert Craft. *Dialogues*. Berkeley: University of California Press, 1982.

¹¹ Straus, Joseph N. *Stravinsky’s Late Music*. New York: Cambridge University Press, 2001. 32.

¹² *Ibid.*, 18.

¹³ Stravinsky, Igor and Robert Craft. *Dialogues*. Berkeley: University of California Press, 1982.

Moving beyond Stravinsky and Schoenberg's musical contrasts, Stravinsky was also highly influenced by the neoclassical style of tonal centers and ideas of centricity prevalent in many of his earlier works, continually shaping his new style of the 1950s and 1960s. Since his late style employed atonality, without an evident tonal reference, one might erroneously infer that his rotational arrays lack any sense of centricity. However, this supposition would be inaccurate as Stravinsky manages to often begin his rotational arrays on the same first note through transposing each rotation, as declared by Joseph Straus:

Stravinsky's neoclassical music had been deeply concerned with issues of musical centricity, of channeling and focusing the musical discourse around certain referential notes. In the "rotational" arrays, comparable centricity is assured, in part, by the simple fact that each row of the array begins on the same note.¹⁴

These musical ideas and concepts greatly impacted Stravinsky's later compositional works, showing a drastic departure from his contemporaries and predecessors.

Furthermore, as Stravinsky experimented with procedures using rotational arrays, he gradually began to move away from the influence of Krenek and his other predecessors' music, surpassing his contemporaries and musical authorities. His journey of serial writing in the 1950s led him to his new theoretical basis in the 1960s, drifting away from the impact of previous composers, and creating his own unique, innovative style by developing what is appropriately referred to now as the rotational array.¹⁵

¹⁴ Straus, Joseph N. *Stravinsky's Late Music*. New York: Cambridge University Press, 2001. 38.

¹⁵ Babbitt, Milton. "Stravinsky's Verticals and Schoenberg's Diagonals: A Twist of Fate," in Stravinsky Retrospectives, 18. Lincoln and London: University of Nebraska Press, 1987.

Chapter II: Rotational Arrays

Stravinsky began using rotational arrays with serialism, due to Krenek's rotational charts and musical examples; however, Stravinsky's approach differed greatly from Krenek's process. Although the apparent differences were already discussed in the historical introduction, the rotational arrays of both Krenek and Stravinsky have not been considered and explained thoroughly. A limited amount of information in this chapter will briefly explore Krenek's ideas, looking at what can be considered as his modal rotational array, providing a solid structural background for Stravinsky's rotational array. Specifically, the majority of this chapter will focus on Stravinsky's late serial music, observing his writings, charts, and musical examples, providing a coherent understanding of Stravinsky's twelve-tone music from 1960-66, employing what is now coined as the rotational array.

The musical examples and emphasis in this chapter will be entirely melodic and linear examples, since melodic rotational arrays appear in the earliest of Stravinsky's music from 1960-66. Subsequent chapters will include instances that introduce and discuss different horizontal, harmonic phenomena, explaining the theoretical basis behind Stravinsky's notions of what he appropriately called verticals. Musical examples throughout this chapter will be observed in select pieces from 1960-66 to show the clearest examples of various melodic rotational arrays, as well as prominent melodic techniques which are common in Stravinsky's twelve-tone works and relate to rotational arrays.

Before observing the rotational array of Krenek, it is important to present a general definition of how a rotational array works. Clare Hogan provides a suitable definition, explaining the concept of rotational arrays:

By rotation we understand a procedure in which the elements of a given series systematically and progressively change their relative positions according to a plan, which in itself is serially conceived in that the changes occur in regular phases. [A twelve-note row] is then treated as two hexachords and the notes are progressively rotated within each of the two sets of hexachords by moving the first note to the end until each group has rotated once.¹⁶

Krenek's approach in creating rotational rows or arrays, in contrast to Stravinsky, was trying to exhibit modal scales from the Renaissance era in a serial, twelve-tone setting, hence the diatonic or tonal collections Krenek mostly used in his works. In discussing Krenek and Stravinsky, the differences between the two respective arrays were clarified in the historical introduction; the similarities are also prevalent, making a convincing argument for Krenek acting as a crucial influence for Stravinsky in his late works. Joseph Straus explains Krenek's impact on Stravinsky's late works and style traits he adopted from Krenek:

Although rotation is also used in Schoenberg's early twelve-tone music, including the works apparently best known to Stravinsky, Krenek is the unmistakable source for what became the distinctive Stravinskian rotational array, including its independent treatment of the hexachords of the series, their systematic rotation, and the transposition of each of the rotations to begin on the same first note.¹⁷

Each of these similarities between Krenek and Stravinsky's arrays listed by Joseph Straus will be thoroughly elaborated later in musical examples.

In addition, Stravinsky's twelve-tone compositions contain other impacts from Krenek's compositional style. Particularly, Stravinsky practices the typical prime (P), inversion (I), and retrograde (R) forms, commonly employed by early twelve-tone

¹⁶ Hogan, Clare. "Threni: Stravinsky's Debt to Krenek." *Tempo* 141 (June 1982): 23

¹⁷ Straus, Joseph N. *Stravinsky's Late Music*. New York: Cambridge University Press, 2001. 29.

composers, such as Schoenberg; but instead of using the typical retrograde of the inversion (RI), Stravinsky more frequently utilizes the practice of the inversion of the retrograde (IR).¹⁸ This decision by Stravinsky to alter this specific form, in which the row is manipulated, is said to be directly influenced by Krenek, who frequently employed inversion of the retrograde (IR) on his serial rows, instead of the more standard Schoenbergian alternative technique.¹⁹

Both Krenek and Stravinsky supplied their compositions with row charts, so the listener would be able to clearly conceive of the theoretical perspectives behind their works. Example 2.1 is from the opening pages of *Lamentatio Jeremiae Prophetae, Op. 93*, which was composed in 1941 for chorus. It is Krenek's first rotational work, showing the two scalar rows Krenek employed, along with all the rotations for each scale to clarify his theoretical process. As shown, the collections on the far left contain the ordering of notes rotated without any transposition; the collections on the right are what Stravinsky most often writes in his music, which transpose each collection to begin on the first note of the transformational row, while changing the intervallic pattern. In this case, the collections on the right always begin on F for the first hexachord and always begin on B for the second hexachord.

Compare Example 2.1, rows created by Krenek, to Example 2.2, showing a chart of the row collections used in Stravinsky's setting of his work, *Movements*, containing the first occurrence of rotational arrays appearing in Stravinsky's compositions. Although much of Stravinsky's rotational array influential procedures are derived from Krenek, the

¹⁸ Robert Sivy presents a chart to clearly explain the different transformational forms, especially (RI) and (IR). See Sivy, Robert, "An Explanation of Anomalous Hexachords in Four Serial Works by Igor Stravinsky." Master's thesis, University of Tennessee, 2011.

¹⁹ Straus, Joseph N. *Stravinsky's Late Music*. New York: Cambridge University Press, 2001. 28.

Example 2.1
 Krenek's Row Chart for *Lamentatio Jeremiae Prophetae*, Op. 93



Example 2.2
 Stravinsky's Row Chart for *Movements: for piano and orchestra*

1 2 3 4 5 6

A musical row chart with five horizontal staves labeled I, II, III, IV, and V from top to bottom. Above the staves, numbers 1 through 6 are aligned with specific notes. Below the staves, Greek letters α, β, γ, δ, 3, and 4 are placed above certain notes. The notes are represented by small circles with stems, and accidentals like flats and sharps are indicated above or below the notes. The chart shows how different notes from the twelve-tone row are distributed across the five staves.

differences between their row charts are apparent. Krenek's collections contain mostly scalar and diatonic rows, while Stravinsky's rows comprise large amounts of chromaticism and leaps between contiguous pairs of notes. Additionally, Stravinsky provides valuable analytical insights by marking on his sketches, scores, and array charts, as seen in Example 2.2, labeling the four hexachords with Greek letters, the notes of his arrays with numbers, and drawing a line for the transposed rotational hexachords to display what process is occurring with each hexachord, which are also relevant to the hexachords in Example 2.1. Furthermore, because rotational arrays have been explained and defined, along with a comparison of Krenek and Stravinsky's rows with arrays, Stravinsky's music can be more meticulously examined to observe how his array is treated in various melodic settings. Since *Movements* was Stravinsky's first rotational work, it offers a concrete basis to observe how his beginning style progressed from 1960-66, gradually continuing through his works, providing multiple examples to see various rotational array applications.

Example 2.3 is the first line from *Movements*, which is significant since it is the first documented occurrence of rotational procedures in Stravinsky's compositions.²⁰ Although Example 2.3 is the first initial rotational design in Stravinsky's music, it is not illustrated as much as others of Stravinsky's examples from *Movements*.²¹ This exchange in theoretical interest from the opening line of *Movements* might be in part to the fact that the rows are rarely stated in their distinct melodic entirety, making them challenging to track. Even though *Movements* provides limited opportunities to explore Stravinsky's use of rotational arrays, the piece can still be partially studied to show how Stravinsky

²⁰ Ibid.

²¹ Ibid.

initially began using his arrays, given by Example 2.3. Originally, *Movements* was for solo piano and orchestra, but for clarification of the rows, a piano reduction is supplied in Example 2.3, using the piano 2 in place of the orchestra. In Example 2.3, the first row, P3, in measures 1 and 2, reveals a traceable, definable complete aggregate, and then another presentation of the first hexachord of a second row beginning in measure 3, exhibiting an instance in which Stravinsky demonstrates his earlier experimentation with rotational arrays.

The piano 1 presents the aggregate using two hexachords, divided between measures 1 and 2, exercising mostly disjunct motion and changing between the left and right hand of the piano 1. While the piano 1 produces the opening row, the piano 2 begins playing the first two notes of the first hexachord from Example 2.2, Eb and E, and then withdraws from the row statement, leaving it incomplete. Afterwards, measure 3 begins employing the rotational array in the piano 1 by beginning with a tritone, the second

Example 2.3
Opening Rows from *Movements: for piano and orchestra*

IGOR STRAVINSKY
1958-59

Piano SOLO

Piano II ORCHESTRA

I

110

1 3 4 6 2 9 10

4 8 2 5 7 3 8 11 12 1 3 6 5

3 2 4 3 3 32

interval from the opening row, instead of the ascending minor second, which previously occurred as the first interval of the P row.²²

Instead of beginning the rotated hexachord with E or pitch-class 4, just as the original opening row from measure 3, the collection is transposed by seven semi-tones to begin on B or pitch-class 11. The transposing to begin the second row on B is not a practice habitually employed by Stravinsky, since this pitch-class 11 appears to represent an arbitrary transpositional procedure, in opposition to his other transpositional procedures in subsequent later works, making it an anomaly and prime point of discussion when observing his style. For example, his arrays are typically transposed to begin on the opening row's first note, Eb in the case of the first hexachord, or not employing the transposition operation at all, leaving the pitch-classes invariant. This row alters the intervallic property to begin one interval later than the previous occurrence of the row, as well as transposing to begin the row on a distinct pitch-class. As mentioned previously, the second hexachord of the second row is split up into various parts and disjunct, making it difficult to track. The splitting up of the second hexachord of the second row demonstrates Stravinsky's innovative diversity with the rotational hexachordal applications. Representing the first occurrence of rotational properties in Stravinsky's music, this excerpt above exemplifies a monumental moment in theoretical discovery and Stravinsky's late compositional style with rotational arrays.

After viewing the first row and first rotation from *Movements*, further examples can be scrutinized more comprehensively to see Stravinsky's common applications for rotational rows. Beyond *Movements*, after Stravinsky became more experimental with the

²² Observe Example 2.2 for further clarification.

rotational array, Stravinsky began composing his second rotational composition, *A Sermon, a Narrative, and a Prayer*, which was published in 1961. As before, the entire work is comprised of four different transformational forms mostly used in twelve-tone works, but (IR) is replaced with (RI).²³ Example 2.4 provides a vocal score with a piano reduction in place of the orchestra to offer clarity of rotational examples. Example 2.4 displays the opening row from “A Sermon”, showing the first page of the piano part before the vocal lines commences. The point of presenting this example is for two purposes; showing the notes of the opening row of *A Sermon, a Narrative, and a Prayer*, in addition to emphasizing Stravinsky’s treatment of pairing a row with another transformational form, which will appear in later examples exploring melodic rotational rows, signifying the importance of this compositional technique.

The row, P3, is stated in measure 1 until measure 7, with some notes scattered throughout the row presentation, and the row is labeled on the score. Then, P3 is repeated again in measure 7, changing registers and rhythmic elements the second time through. In contrast to Example 2.3, Example 2.4 illustrate shows how Stravinsky tends to usually express a row or array across multiple measures by utilizing temporal space with large note values and rests, instead of compressing it into short, faster occurrences. The expansion of this twelve-note row can possibly affect the listener’s perception of the row, factoring in the length of the row and the listener’s note and interval retention, as opposed to the listener being cognitively conscious of each note of the row if the row was condensed into a smaller temporal space. In measure 7, when the opening row is restated,

²³ Eric Walter White presents a full chart of the different transformational rows for *A Sermon, a Narrative, and a Prayer*. See White, Eric Walter. *Stravinsky: The Composer and His Works*, 2nd edition. London: Faber and Faber, 1979. 511.

Example 2.4
Opening Page of “A Sermon” from *A Sermon, a Narrative, and a Prayer*

I

A Sermon
(from St. Paul)

IGOR STRAVINSKY
1960-61

PIANO

Fl. alto

VI. I, II

Cl.

Fl. gr.

(f)

Vl. Solo

p

(f)

Fl. alto

Vl. Solo

P3:1

R3:1

Cl. bas.

sub p

3/4

ffp(=)f

3

4

5

10

6

7

8

9

11

12

3

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10

11

12

3/4

3

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its retrograde is paired with it, generating a palindromic effect and creating contrast by rhythmically displacing each row. Throughout all three movements, the palindromic result or row pairing is the underlying compositional technique exercised during this work. Although this paper is meant to illustrate primarily rotational aspects, it appears the row pairings are a melodic technique presented in the majority of Stravinsky's late compositions, so it is necessary to state the importance of them, in addition to the rotational aspects which will occur later in subsequent works viewed. As in *Movements*, the melodic hexachords are disjunct and difficult to locate due to the amount of detail represented in each part.²⁴

The Flood, composed in 1962, presents other designs of how Stravinsky employed his hexachordal arrays. An important distinction to be made with this piece is the addition of two transformational rows which are not typically employed in Stravinsky's other late works. He adds the retrograde inversion (RI) and also the retrograde of the inversion of the retrograde (RIR).²⁵ These forms will not be thoroughly analyzed for instances throughout, but it is important to make known of their importance and occurrences in this piece. The opening row, [C#, B, C, F#, D#, F, E, D, Bb, A, G, G#], is laid out in large harmonic entities multiple times on the first page and throughout this work, encompassing all twelve pitch classes, but it is unnecessary to provide examples of these instances within this section of the paper, due to the lacking of melodic importance.²⁶ Though many segments in this piece are impertinent with regards to melodic importance, other portions of the piece contain many cases in which rotational

²⁴ Robert Sivy provides a more thorough representation of anomalous hexachords and Stravinsky's nonconformity. See Sivy, Robert, "An Explanation of Anomalous Hexachords in Four Serial Works by Igor Stravinsky." Master's thesis, University of Tennessee, 2011.

²⁵ Straus, Joseph N. *Stravinsky's Late Music*. New York: Cambridge University Press, 2001. 103.

²⁶ Ibid., 149.

arrays are employed in a melodic setting. A chart of the different forms for reference is listed below in Example 2.5.

Example 2.5
Transformational Rows in *The Flood*

P: C# B C F# D# F E D A# A G G#

I: C# D# D G# B A A# C E F G F#

R: G# G A A# D E F D# F# C B C#

IR: G# A G F# D C B C# A# E F D#

RI: F# G F E C A# A B G# D D# C#

RIR: D# F E A# C# B C D F# G A G#

Example 2.6 from two pages of *The Flood* provides a multitude of rich theoretical applications Stravinsky used. This Illustration comes from the end of *The Flood* with only the orchestra playing, expressing large impactful, melodic, and harmonic moments. Although the rows are presented in an orchestral setting, a piano reduction is afforded. Before reviewing the score for rotational rows, a defining of terminology is necessary. Since this section is largely employing a splitting up of the hexachords from the rows in *The Flood*, specific hexachords of the various transformational rows need to be assigned a symbol to differentiate them from one another. From Example 2.5, each hexachord of the transformational forms can be labeled by which type they are with an upper-case letter of the beginning letter of each row (P, I, RIR, etc.). Additionally, it is necessary to provide each hexachord of the given rows, either with “a” or “b”, depending on if the notes are in the first hexachord, first through sixth note, or second hexachord, seventh through twelfth note. For example, Pa would be used to represent the first hexachord of a prime form of the row, and Pb would be used to indicate the second hexachord of a prime form of the row. Furthermore, pitch-class ordering within each row is important to

mention; each hexachord is rotated inside the twelve-tone row as separate units, so the individual twelve-tone row ceases to maintain its original pitch-class ordering from Example 2.5, due to the fact that each twelve-tone row is not being rotated as a complete entity, but instead, as individual musical hexachords, separating contiguous notes from one another.

Now that a basis for terminology is established, the piano 1 will be the beginning point of discussion in *The Flood*. In measure 428 of Example 2.6, the top piano 1 part continues a rhythmically fast, ostinato pattern, mostly maintaining an invariant ordering of pitches through measure 433. The pattern is comprised of a row being rotated. Throughout *The Flood*, Stravinsky does not transpose the majority of his rows, keeping the pitch classes invariant, but the intervallic orderings of the rotated row are varied.

This aggregate row at measure 428 begins with RIb, beginning on A. Because the second hexachord begins this row aggregate, instead of the first hexachord, the hexachords swap places, making the last six notes of the row, RIa, and the first six notes of the row, RIb.²⁷ Interestingly enough, although the rhythmic property is maintained through measure 433, the row starts to gradually contract in the amount of notes, losing its hexachordal authenticity, as well as its twelve-tone integrity. Beginning at measure 429 to the end of the ostinato pattern, the time signature changes each measure to one less sixteenth note than the measure before, inevitably condensing each row. Each time the row condenses, RIb remains integral, but RIa loses one note each measure, thus, the seventh note of the row ceases to reappear in the next measure. The first appearance of the row is labeled in measure 428 and 429, and the notes are pointed to with arrows on

²⁷ Refer to Example 2.5 for further clarification.

Example 2.6

The Flood Rotation

Fl.

I

8
16 f marc.

12
16

p come sopra

II

8
16 f marc.

12
16 p come sopra

Vc.
C.B.
Cl. cb.

Rlb

1 2 3 4 5 6

7 8 9 10 11 12

Rla

11
16

11
16

Cl. eb.

430

11
16

10
16

9
16

Cor. Trb. Cor.

II

11
16 ,

10
16 ,

9
16 ,

Sheet music for two staves (I and II) in 12/16 time. The top staff (I) has a treble clef and the bottom staff (II) has a bass clef. The key signature is A major (no sharps or flats). The music consists of six measures. Red arrows point to the first three measures of staff I, highlighting specific notes. Red numbers below the staff indicate note values: 3, 4, 5, 6, 7, 8, 9, 10, 11, and 12.

Sheet music for two staves (I and II) at measure 435. The top staff (I) has a treble clef and the bottom staff (II) has a bass clef. The key signature changes to G major (one sharp). Measure 435 starts with a dynamic of f marc. in 16/16 time, followed by p in 2/4 time, then $3/4$, and finally $6/16$. The bottom staff (II) also starts with f marc. in 16/16 time, followed by $2/4$ with mf , then $3/4$, and finally $6/16$. Red numbers 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, and 12 are placed above the notes in staff II. The page number 22 is at the bottom.

the row is labeled in measure 428 and 429, and the notes are pointed to with arrows on the score to show which notes disappear in each subsequent measure. The subtle evaporation of this rhythmic and pitched motive provides contrast and interest for the audience without altering anything considerably significant, and at the same time, creating a difficult challenge for the performer. Since the notes are at a rapid tempo, only varying one note from the previous measure, it makes this section visually and conceptually problematic for the performer.

Moreover, the bottom staff of the piano 1 presents additional rotations of the row, while the top staff of the piano 1 sustains the tempo and maintains the rhythmic stability. In measure 431, the row is stated as R1, but rotated, beginning on the third pitch of this particular form from Example 2.5. Since multiple ideas are being stated in Example 2.6, the row is labeled to provide a clearer observing of the musical idea and notes of the row. Strangely enough, the row does not continue through the G#, which would be the entire presentation of the row and also complete the aggregate, but instead, the row ends with the tenth note on the C#. In the measure before 431, there is a G# and G stated in the piano 1, which are the first notes of the R1 form that has been rotated; these two notes are among the previous row's statement, in addition to being a part of the previous sections musical and theoretical idea, revealing the previous two notes are not contained in the row at measure 431.

Another illustration of rotational rows, representing a prime illustration in Example 2.6, is in measure 435. Momentarily after the rhythmic phenomena in the piano 1 ends, measure 435 begins a new motivic idea in the piano 1, not encompassing the row presentation, as well as another thematic idea in the piano 2 that comprises the row. In

measure 435 of the piano 2, the row begins with a B in the bass. Rather than separating the row with temporal space or other melodic or harmonic ideas, as Stravinsky often employs in his music, the notes emerge as contiguous pairs of notes and the register is altered regularly. This row, beginning on B and continuing through G# in measure 437, is the appearance of the original P form rotated to begin on the second note of the original row. As in the previous example of measure 431, the row does not finish the complete aggregate, but in this case, ending on the eleventh note of the aggregate, formulating it as an incomplete row and aggregate; as before, the C#, or last note from this particularly rotational row, begins a measure earlier in 436, but the note is incorporated in a separate musical idea as I perceived, thus, not solidifying it as a structural first note of the prime form, displaying it as the last note of the row from the previous measure.

Lastly, the final rotational work composed by Stravinsky in 1966, *Requiem Canticles*, is Stravinsky's first and only late work that included two unrelated series, not a part of the same row class.²⁸ Stravinsky mostly employs only one row class at a time in a single movement, not considering rows related by some transformational procedure. In *Requiem Canticles*, with the exception of two movements, "Interlude" and "Postlude", both series are juxtaposed simultaneously by using various operations throughout. For the primary purpose to not deviate from rotational designs, only one of the movements and one of the rows from *Requiem Canticles* will be observed in this chapter. The piece of music is orchestrated for chorus, four vocalists, and orchestra. Example 2.7 provides the series used for the musical example from *Requiem Canticles*, which will be expounded on in Example 2.8.

²⁸Straus, Joseph N. *Stravinsky's Late Music*. New York: Cambridge University Press, 2001. 4.

Example 2.7
Transformational Forms in *Requiem Canticles*

P: F G D# E F# C# B C D A G# A#

I: F D# G F# E A B A# G# C# D C

R: A# G# A D C B C# F# E D# G F

IR: A# C B F# G# A G D E E# C# D#

Example 2.8 displays a movement, “Rex Tremendae”, from *Requiem Canticles*.

In Stravinsky’s early and late compositions, he tended to favor canons, which was already discussed in the historical introduction with relation to being influenced by Webern. “Rex Tremendae” is a central example of an occasion in which Stravinsky employs rotational rows with canons. Beginning in measure 203, at the start of “Rex Tremendae”, Stravinsky begins his row with the R transformational form. One measure later, at measure 204, a rotation occurs in a canonic setting, beginning on D#. The row has been transposed and rotated two times to begin with an ascending perfect fourth, but deceptively, the A# from the alto line actually intersects with the R row of the alto and the rotated version of the R of the soprano, making two different rotational rows beginning on A#. To better illustrate this procedure, the A from the fourth note of R is transposed up by one semi-tone to transform it into A#, and then turning the first interval into a perfect fourth from A# to D# after the rotational procedure occurs, marked with an arrow to show the procedure. At each additional occurrence in this canonic situation, each voice entrance is representing a different definable row, with the exception of the tenor voice, which, based on my analysis, does not contain any distinct row presentation from this section. The other rotational procedures, previously witnessed, present different ways a rotational row is treated either by transposing it to an arbitrary pitch-class, as in

Example 2.8
Rotational Canon in “Rex Tremendae” from *Requiem Canticles*

REX TREMENDAE

SOPRANI f 2 **205**

ALTI 1 2 *Rex,* 3 4 *rex* 5 6 *tre - men - dae* 7 8

CORO *Rex,* 2 2 *TENORI f* > *Rex* 5 6 *tre - men - dae* 3 2 2

BASSI f > *Rex* *tre - men - dae*

205

nonf 2 2 *mf marc.* 3 2 2

S. 10 11 **210**

A. 12

CORO 2 2 *ma - jes - ta - tis,* 9 *Qui - sal - van -*

T. *ma - jes - ta - tis,* *Rex tre - men - dae*

B. *ma - jes - ta - tis,* *Qui sal -*

210

Example 2.3, or not employing the transpositional operation at all, leaving the pitch-classes invariant, as in Example 2.6; Example 2.8 provides yet another technique, transposing each rotational row to begin on the starting note from the original, unrotated transformational form. It is unnecessary to list and define every rotation in these two measures from the beginning of the movement, but it is sufficient to declare them as notable rotational examples, in addition to clarifying that the entrances of each canonic voice, with the exception of the tenor voice, contains some form of one of the four rows being rotated. Before the voices present the entirety of the row or aggregate, harmonic entities occur in the form of block chords from measure 206 through 207. At first glance, it is perplexing that this harmonic section is interspersed in the middle of multiple melodic rows, but it is evident that the variety of rows are mixed in these block chords, changing voices regularly, so the rows are unable to track clearly, which is not the prime focus of this chapter. The full presentation of the original R row in the tenor is marked through all twelve-pitches, but the other row presentation that begins with the A# is marked on the score until measure 206, when the row becomes less distinct.

Now that many of Stravinsky's melodic and linear applications of rotational arrays have been discussed in detail with many examples and charts, a comprehensive structural background has been provided of how an array of this nature works. Many stylistic characteristics of rotational arrays were considered in Stravinsky's music, including the three types of transformations applied to rotational rows; transforming each row to an arbitrary pitch-class occurs in Example 2.3; a rotational row without any transpositional operation occurs in Example 2.6; transposing a rotational row to begin on

the first note of the original transformational row occurs in Example 2.8. These rotational procedures are all important when looking at Stravinsky's music, but Stravinsky most often transposed his rotational row to begin a musical row on the first note of an unrotated transformational row, as in Example 2.8, classifying this as a noteworthy attribute of Stravinsky's rotational array. Stravinsky presents many other characteristic styles, such as presenting two forms simultaneously in Example 2.4 and 2.6, the compression of a given row in Example 2.6, and the treatment of a canonic setting in Example 2.8. These style characteristics summarized above in his rotational works were provided for a distinct purpose, indicating they are all representative of what most frequently occurs throughout Stravinsky's works.

The theoretical insight into the linear approaches of rotational arrays requires an intellectual vision and innovative perspective, as realized by the previous illustrations. Moreover, although much of Stravinsky's works and theoretical notes represent linear methodologies of his rotations, there are many examples that provide a comprehensive, widespread study of how he also employs harmonic instances or aptly termed verticals. Verticals, among others of his practices, indicate Stravinsky's far-reaching influence with his theoretical and compositional procedures, supported by the scholastic research and posthumous interest in Stravinsky today.

Chapter III: Harmonies

Stravinsky's twelve-tone works are mostly comprised of contrapuntal rows and harmonic occurrences, which appear as random or unsystematic when their theoretical notions are considered, hence the necessity to differentiate between these instances and the purposes behind verticals. According to Joseph Straus, when Stravinsky began composing his twelve-tone works, he used his verticals for three purposes; the first purpose was to verticalize a presentation of the row; the second intent shows the verticals from *The Flood* in Example 3.1, where the aggregate is divided into two hexachords, rotated and then transposed to begin each hexachord on the first note of the tranpositional row; the third goal is “a layering of four series from which twelve chords are created as vertical slices through the array.”²⁹ Example 3.1’s verticals, shown with pitch-class integers, are labeled with numbers from 1-6 to show the ordering of the Ra verticals through rotation.

Example 3.1
Ra Hexachordal Verticals from *The Flood* (C = 0)

1 2 3 4 5 6

8 7 9 t 2 4

8 t e 3 5 9

8 9 1 3 7 6

8 0 2 6 5 7

8 t e 1 3 4

²⁹ Straus, Joseph N. "Stravinsky's 'Construction of Twelve Verticals': An Aspect of Harmony in the Serial Music." *Music Theory Spectrum* 21, no. 1 (Spring 1999): 45.

For the purposes of this paper, the first intention of verticals will not be explored since it is not associated with rotated verticals; regardless, this chapter will move away from contrapuntal instances from chapter two and will provide an in depth analysis of harmonic, vertical events with rotational arrays.³⁰ Furthermore, it is important to mention verticals are employed on a rare basis, in contrast to the linear examples, which occur on a more frequent basis, further confirming the importance behind Stravinsky's intent for the uses of vertical rotational arrays.

³⁰ Although his first intention will not be discussed, Joseph Straus provides a chart of all the instances that uses verticals in his early twelve-tone works. For examples, see *Ibid.*, 43-45.

Part I: Verticals

For clarification, when I refer to verticals in this section, I am not referring to random appearances of harmonies throughout Stravinsky's work. Conversely, I am indicating horizontal rows, as in the top row of Example 3.1 that are rotated and transposed to create verticals that are formed from an original transformational row. When the two hexachords of a twelve-tone row are divided, the first vertical from each hexachord of the twelve-tone row creates a one-note chord, since each vertical row is transposed to begin on the first note of the original row. Additionally, since each hexachord is being rotated as its own entity, the twelve-tone integrity of the row evaporates, making the twelve-tone aspect secondary to the hexachordal structure. Joseph Straus gives an account and definition of the procedure of creating verticals; I have altered the note he refers to in this case to relate back to Example 3.1: "The first hexachord of the retrograde form of the series occupies the top row of the array; the remaining rows of the array rotate this hexachord systematically and transpose the rotations to begin on [G#], creating the six verticals."³¹

In Example 3.3, the verticals begin the last beat of measure 75 and present all twelve verticals from the Ra and Rb hexachords. Ra and Rb are shown in Example 3.2 for reference in comparing the verticals. In Stravinsky's works, it is rare that a complete aggregate is vertically presented as in this case with Ra and Rb. Because the Ra and Rb rotated verticals are created with the original, unrotated row, there is an aggregate formed linearly, while the rotated rows are creating the verticals. Since each hexachordal vertical

³¹ Ibid., 45.

chord is used as its own separate unit by rotating and transposing, there is not an aggregate produced with the verticals, only with the horizontal line. The verticals are labeled on the score with numbers 1-6. Ra begins on the last beat of measure 75 and lasts until the end of 81. Rb begins in measure 82 and lasts through 85 on the score, excluding the last vertical in my example. Usually, the first linear hexachord is played throughout and then Rb begins with another one note chord, starting on the first note of Rb; this procedure does not happen exactly as illustrated in Example 3.3.

In measure 82, when Rb begins, there is supposed to be an A# one note chord to begin Rb. It is apparent that there is no A# in this measure to begin the hexachord. Instead, there is a B that is sounded on its own. Joseph Straus often refers to Stravinsky's "mistakes" throughout his twelve-tone compositions. Occasionally in Stravinsky's compositions, a note or multiple notes in a row is altered, thus, the defining of the term "mistakes", as classified as theorists today might think of them. Basically, since there is a

Example 3.2
Ra and Rb Verticals from *A Sermon, a Narrative, and a Prayer*

Ra	Rb
<u>1 2 3 4 5 6</u>	<u>1 2 3 4 5 6</u>
5 6 4 3 8 2	t 1 2 0 4 3
5 3 2 7 1 4	t e 9 2 0 7
5 4 9 3 6 7	t 8 0 e 6 9
5 t 4 7 8 6	t 2 1 8 e 0
5 e 2 3 1 0	t 9 4 7 8 6
5 8 9 7 6 e	t 5 8 9 7 e

Example 3.3
A Sermon, a Narrative, and a Prayer Verticals

The musical score consists of three main sections, each enclosed in a red box:

- A Sermon (Section 1):** Measures 75-78. Instruments: Flute (Fl. gr., Fl. alto), Horn (Coro), Cello/Bass (C.B.). Dynamics: *p*, *mf*. Measure 75: Flute and Horn play eighth-note chords. Measure 76: Horn continues eighth-note chords. Measure 77: Flute enters with eighth-note chords. Measure 78: Horn and Cello/Bass play eighth-note chords.
- A Narrative (Section 2):** Measures 79-82. Instruments: Clarinet (Cl.), Flute (Fl. gr., Fl. alto), Horn (Corni). Dynamics: *mf*, *p*. Measure 79: Clarinet and Flute play eighth-note chords. Measure 80: Horn enters with eighth-note chords. Measure 81: Clarinet and Flute play eighth-note chords. Measure 82: Horn and Cello/Bass play eighth-note chords.
- A Prayer (Section 3):** Measures 83-86. Instruments: Trombones (Trb. II, III). Dynamics: *p*. Measure 83: Trombones play eighth-note chords. Measure 84: Trombones play eighth-note chords. Measure 85: Trombones play eighth-note chords. Measure 86: Trombones play eighth-note chords.

ALTO SOLO:

SPEAKER:
 Then the twelve called the multitude of the disciples unto them and said: We will give

4 5 6 1 2

Trb. I 80 Cl. bas. 38 Archi

48

strict ordering of notes in a twelve-tone series, if a note was heard contrary to the theoretical concepts represented in Stravinsky's compositions, the theory was secondary to the auditory effect perceived by the listener or Stravinsky, as in this case, where the B replaces the A# a half step higher and is acting as the first vertical of Rb.³² In this example, there are other illustrations of "mistakes", which will be apparent when compared to the verticals listed in Example 3.2. Since this is the first appearance of verticals in Stravinsky's music, this excerpt illustrates his first techniques for demonstrating verticals, including expanding the verticals over ten measures by expanding the temporal space of the hexachords. His usage of rhythmic displacement adds variety to the verticals, his treatment of "mistakes" in his rows, as well as his implementation of a complete linear aggregate as a part of Ra and Rb prove the importance of this archetypal piece.

To further emphasize Stravinsky distributing his verticals across multiple measures, Example 3.4 from *The Flood* also represents this notion. Most of Stravinsky's instances of verticals are ten measures or less, but *The Flood* is an unusual example

Straus, Joseph N. "Stravinsky's Serial 'Mistakes.'" *The Journal of Musicology* 4, no. 2 (Spring 1999): 231–271.

because it represents an anomaly of length in which Stravinsky uses verticals. Measures 180-247 is the longest example of Stravinsky's verticals, partially represented in Example 3.4. Since the full presentation lasts over 40 measures, I have only provided measures 180-199 to prove the importance of this model. For clarification, refer to Example 3.1 to observe the different Ra hexachordal chords that are represented in *The Flood*. The complete Ra hexachordal verticals are stated and then repeated in reverse order, finishing in measure 190 but excluding the first vertical which would usually be a one-note chord on G#. Additionally, the chords are labeled on the score as part of Ra to make the verticals easy to locate. Although the excerpt continues through 199, to simplify, I have labeled the verticals through measure 190, when Ra appears to break off from the previous pattern. Throughout the rest of the verticals in this section, through measure 247, additional hexachordal rotated verticals can be sought out and identified as part of various transformational forms, although they are not labeled in my example; specifically, the hexachords alternate between Pa, Pb, IRb, IRa, and Rb.³³

Firstly, it is significant that the top bass line is presenting the original Ra form as its own individual melodic line, while the other parts help to complete the verticals; therefore, the parts create two simultaneous rows, one created with the verticals and the other exercised in a linear line. Observe how the top two voices provide melodic lines of one note each, and the bottom two voices jump around registers with various notes to exhibit an accompaniment part. Furthermore, the parts regularly repeat notes multiple times, which is a technique Stravinsky used to expand the row across temporal space. By mixing up the voices and repeating notes in different registers, Stravinsky is able to remain on the same single vertical for multiple measures without rotating to the next one,

³³ Ibid., 48.

extending the length of the statement of all twelve verticals.

Additionally, it is important to note that there seems to be an emphasis Stravinsky puts on voice leading throughout this particular illustration, unlike the last model shown. As observed, the notes that move between corresponding chords usually employ small intervals or step-wise motion, which seems to be a facet that separates Stravinsky from Schoenberg, Webern, and others of his contemporaries, who regularly employ more disjunct motion than conjunct. In instances where an interval of a sixth or seventh is exercised, it appears to act as a part of changing registers for a part in the score, providing a specific purpose to the interval.

Interestingly, it appears Stravinsky most often deviates from employing a particularly highly dissonant interval in harmonic settings, the minor second. The minor second is used in rare occurrences, making it a crucial feature to comment on when discussing this excerpt. Considering Stravinsky's voice leading with minor seconds, notice that Stravinsky uses his intervals very delicately to resist causing extreme

Example 3.4
The Flood Verticals

**GOD
GOTT** { 2 Bassi Soli

180
♩ = 80-86

2 4 **p** G.C.

1 **2** **3** **4**

I, Ich,
God, Gott,
that der all ich
the world have schuf die - ses

Arpa, Piano sempre col voci
Vl. Vle. p trem. sul pont.

5 6 5 4 3 2

2 Bassi
Soli

185

2

2 Bassi
Soli

190

Arpa, Piano.

2 Bassi
Soli

195

dissonance between notes. For example, in measure 185, the bottom staff of the accompaniment part contains a D and then jumps up to an F#. Above these notes, there is an Eb and then a G, creating minor seconds between both the D and the F#, but they are both over an octave above the lower notes to resist causing an abrupt shock to the audience if these notes were sounded close together. Although Stravinsky exhibits this methodology with minor seconds, its inversion, the major seventh does not appear to follow the same tactics, as seen by the abundances of major sevenths in this example. Intriguingly, this approach is not necessarily adhered to with major seconds and minor sevenths either, since these intervals are less dissonant and do not generate the same perceptual reaction as the sharp dissonant intervals.

Example 3.5 is taken from an article by Robert Morris, where he discusses generalizing rotational arrays.³⁴ His perspective and outlook is thought-provoking, particularly because Stravinsky rarely or never employed canonic settings with his verticals, although he used canons quite frequently in melodic rotational rows. Robert Morris remarks on the procedure and theoretical perspectives of this generated canonic rotational sample:

From the example we see that a rotational array is literally a linear canon linked beginning to end - wrapped around - into a cyclic structure. As a result, we can regard the arrays as temporal analogues to the crucial relation between the cyclic pitch-space of pitch-classes and its articulation in the linear space of literal pitches. This balance (or contrast, depending on the compositional occasion) between cyclic and linear categories which underlies the rhythmic and formal structures in so much Western music might well have stimulated and satisfied Stravinsky's musical appetites.³⁵

Robert Morris created the vertical chart from his Example 0.1 and subsequently, turned it into a canonic setting by starting with a one-note vertical on the F#. He then expanded it

³⁴ Morris, Robert D. "Generalizing Rotational Arrays." *Journal of Music Theory* 32, no. 1 (1988): 77.

³⁵ Ibid., 76.

Example 3.5
Robert Morris's Six-Part Canon Verticals

0	A	7	9	2	8
0	9	B	4	6	2
0	2	7	1	5	3
0	5	B	3	1	A
0	6	A	8	5	7
0	4	2	B	1	6

Example 0.1

0.2a. Six-part canon

0.2b. The canon of 0.2a written as pcs (D = pc 0)

	0	A	7	9	2	8	0	A	7	9	2	8
2	0	9	B	4	A	2	0	9	B	4	A	
5	3	0	2	7	1	5	3	0	2	7	1	
3	1	A	0	5	B	3	1	A	0	5	B	
A	8	5	7	0	6	A	8	5	7	0	6	
4	2	B	1	6	0	4	2	B	1	6	0	

Example 0.2

until the quantity of notes reached the full hexachordal verticals, beginning on the octave Ds at the second downbeat and continuing until the octave Ds at the third downbeat. It would have provided a different perspective if Stravinsky had employed his verticals in a manner similar to Robert Morris's example. Based on what I have perceived from Stravinsky's examples of verticals, he never approached his verticals in the same way as this complex canonic example. As noted in the other examples, he exercised them where the chords sound at the same time, occasionally displacing them rhythmically, but never developing them in such a way as Robert Morris had because they were used irregularly in his late twelve-tone works. Although Stravinsky used his rotated verticals rarely, Joseph Straus designated multi-part arrays as another harmonic instance that is also related to his rotated verticals. This practice is discussed in part two of this chapter, representing it as a prominent component of theoretical innovations in Stravinsky's late serial literature.³⁶

³⁶ Straus, Joseph N. "Stravinsky's 'Construction of Twelve Verticals': An Aspect of Harmony in the Serial Music." *Music Theory Spectrum* 21, no. 1 (Spring 1999): 73.

Part II: Multi-Part Arrays

Although verticals discussed in part one were employed as rare entities, multi-part arrays occurred just as much or even less frequent in Stravinsky's literature.³⁷ Stravinsky employed multi-part arrays by rotation and also without rotation. The four-part array is most definitely discussed more often in theoretical writings with multi-part arrays, but Stravinsky also regularly used the two-part array, especially in his earlier twelve-tone works. Multi-part arrays are the employment of verticals by using multi-transformational forms to create harmonies, or as Joseph Straus defines the four-part array: "a layering of four series from which twelve chords are created as vertical slices through the array."³⁸ Stravinsky began using the two-part array with different various transformational forms in his early twelve-tone works, and then the first piece to employ the four-part array was *Movements*.³⁹

This section will show two examples, one of a four-part array and the other of a two-part array that exercises a rotational aspect. The examples will be scrutinized and examined to notice important aspects of them, including voicing, voice leading, compositional techniques, and also how the prime forms of the verticals relate to each other. The first Example is 3.7 of "Postlude" from *Requiem Canticles*. As mentioned previously, *Requiem Canticles* utilizes two twelve-tone series. The illustration from *Requiem Canticles* is a prime sample because it is different than all of Stravinsky's other multi-part array cases; it is the only one of his four-part array examples that alternates

³⁷ Ibid., 49.

³⁸ Ibid. 45.

³⁹ Ibid. 49.

between two series with his verticals. Refer to Example 3.6 for the transformational forms of the first and second series. The series and the vertical numbers from Example 3.8 are labeled on the score in Example 3.7 to provide more clarity.

This opening musical example is used to provide a thorough understanding of the four-part array, and the other musical example is of a two-part array that represents Stravinsky's only application with arrays that creates the verticals through drawing from the linear hexachords of part one's discussion, instead of the habitual practice with twelve-tone series. Example 3.7 begins with a multi-note block chord, and unexpectedly, it does not represent any structural relationship to multi-part arrays. The employment of the four-part array does not begin until the next measure in measure 290, which starts with the first series, and it changes to the second series in measure 295, and then back to the first series at measure 300. Before the start of a changing of series in this movement,

Example 3.6
First and Second Series from *Requiem Canticles*

1st Series:

P: F G D# E F# C# B C D A G# A#

I: F D# G F# E A B A# G# C# D C

R: A# G# A D C B C# F# E D# G F

IR: A# C B F# G# A G D E E# C# D#

2nd Series:

P: F C B A A# D C# D# G# F# E G

I: F Bb B C# C G# A G D E F# D#

R: G E F# G# D# C# D A# A B C F

IR: G Bb G# F# B C# C E F D# D A

Example 3.7
Verticals in “Postlude” from *Requiem Canticles*

1st Series

1st Series

2nd Series

1st Series

2nd Series

1st Series

1st Series

there is a block chord at measure 289, 294, and 299 that does not contribute to the multi-part array, making them stand out as misleading and odd harmonic occurrences.

When I was observing the four-part arrays, I noticed that the arrays contained many “mistakes”. Usually, there is at least one note that is altered in each vertical, typically by half step or whole-step. For example, in measure 290, the second vertical row from Example 3.7 contains all the notes of the C, G, D#, A# vertical, except for the A# which is altered to be a G#. Additionally, there are only eleven verticals employed within each series presentation, except the last appearance that begins at measure 300. Starting at measure 300, there are only nine verticals exhibited, with the fourth vertical being stated twice. The vertical numbers I provided from measures 300 through 302 contain many more “mistakes” than the previous two vertical statements, providing that my markings are educated guesses given the verticals provided in Example 3.8 and the pattern of the other two vertical presentations in Example 3.7.

Due to the block chords continually stated throughout this movement, the harmonies are easily trackable, making the theoretical concepts of the verticals easier to determine, since all the harmonies are rhythmically aligned. Throughout this example, it is apparent that Stravinsky is not particularly concerned with the voice leading of his chords. He makes large disjunct leaps and changes voicing and registers of the parts relatively frequently. For example, looking at the top part and second part of the top staff (soprano and alto) of measure 290, the notes are A#, B, F#, A#, G#, and B of the soprano part and D#, G, A, E, A, C# of the alto part. Both of these patterns of notes show no semblance of typical or parsimonious voice leading with conjunct part writing; the

Example 3.8
Four-Part Array of 1st Series and 2nd Series from *Requiem Canticles*

1 st Series	2 nd Series
1. [F,A#] = (05)	1. [F, G] = (02)
2. [G,G#,C,D#] = (0158)	2. [Bb, C, E] = (026)
3. [G,A,B,D#] = (0248)	3. [F#, G#, B] = (025)
4. [D,E,F#] = (024)	4. [F#, G#, A#, C#] = (0247)
5. [C,E,F#,G#] = (0248)	5. [B, C, D, D#] = (0134)
6. [A,B,C#] = (024)	6. [G#, C#, D] = (016)
7. [G,B,C#] = (026)	7. [A, C, C#, D] = (0125)
8. [A#,C,D,F#] = (0248)	8. [D, D#, E, G] = (0125)
9. [D,E,G#] = (026)	9. [D, F, G#, A] = (0147)
10. [C#,D#,E#,A] = (0248)	10. [B, D#, E, F#] = (0248)
11. [C#,D,G,G#] = (0167)	11. [C, D, E, F#] = (0246)
12. [A#,C,D#,F] = (0257)	12. [D#, F, G, A] = (0246)

intervals continually leap a fourth or more, occasionally also engaging in odd intervals, as in the D# to G (diminished fourth) of the alto part.

Example 3.8 shows a prime form classification of each of the verticals from both series of *Requiem Canticles*.⁴⁰ It is interesting that the majority of verticals seem to contain a close relation to the various prime forms. In particular, 3 through 10, can all be deduced to be comprised of prime form [024] or [026], both subsets of the whole tone collection. Additionally, the 3, 5, 8, and 10 verticals contain the prime forms of not just

⁴⁰ The first series' forms are taken from Joseph Straus's theoretical ideas on four part arrays. *Ibid.*, 63.

one of [024] and [026], but both of them. 1, 2, and 12 verticals are all subsets of the diatonic collection, while 11 is a subset of the octatonic 01 collection.

It is interesting that a series in a piece such as *Requiem Canticles* can be reduced to prime forms in a systematic way as this, designating all the prime forms with a specific collection classification. It is unclear if Stravinsky meant to write the notes with this implication, but based on his musical intellect and ear, there is a strong possibility that he employed this notion intentionally. The peculiar aspect of “Postlude” is that Stravinsky altered specific notes throughout his verticals in this instance, as already observed, so the prime forms would be changed in the verticals of this composition, ruling out this prime form and collection classification, and leaving Stravinsky’s mindset unclear as to what he was trying to exhibit.

The second series is a little more unusual and more difficult to figure out than the first. Each vertical from the second series can basically be categorized into three supersets, with the exception of two vertical prime forms. Additionally, the first series similarities were more noticeable between sets, but series two appears on the surface much different with prime forms. Verticals, 1, 2, 10, 11, and 12 are all subsets of the even whole tone collection. The 3 and 4 verticals are subsets of the diatonic scale. 5, 6, and 9 are subsets of octatonic 01. Oddly, 8 and 9 do not clearly fit in a common category except obviously, the chromatic collection. Although this series was not as clearly distinguishable on the surface as the first series, Stravinsky appears to have still constructed his prime forms in such a way that they associate with each other, making them uniquely interesting to analyze.

Example 3.9

Transformational Rows and Rotated Linear Hexachords in *Abraham and Isaac*

		1 2	
P:	F, F#, E, D, D#, B, A, G, G#, A#, C, C#	Original Ia:	F E F# G# G B
I:	F, E, F#, G#, G, B, C#, D#, D, C, A#, A	1 st Rotation:	F G A G# B# F# 3 4
R:	C#, C, A#, G#, G, A, B, D#, D, E, F#, F	2 nd Rotation:	F G F# A# E D# 5 6
IR:	C#, D, E, F#, G, F, D#, B, C, A#, G#, A	3 rd Rotation:	F E G# D C# D# 5 6
RI:	A, A#, C, D, D#, C#, B, G, G#, F#, E, F	4 th Rotation:	F A D# D E F# 5 6
		5 th Rotation:	F B A# B# D C# 5 6

Example 3.10 is from *Abraham and Isaac*, composed in 1962-63, which employs the multi-part array differently than in *Requiem Canticles*, whereas, it employs the linear hexachords from rotational rows to use as a two-part array, instead of the full twelve-tone series. Since this example of verticals uses a collection of small two note harmonies each, it is unnecessary and meaningless to deduce the prime forms used since they do not afford any beneficial theoretical observations in this passage.

Throughout this piece of music, Stravinsky used his four preferred rows, P, I, R, and IR, but he also added the RI row.⁴¹ Example 3.9 is for reference with all five transformational rows and the rotated linear hexachords in *Abraham and Isaac*. Stravinsky most often used four-part arrays in his late works and gradually retired the two-part array towards the end of his compositional period; however, this employment of the two-part array is unique and presents new theoretical advancements since it is his only multi-part array that implemented a rotational aspect, conveying it as an imperative theoretical concept for this paper.

⁴¹ Sivy, Robert. "An Explanation of Anomalous Hexachords in Four Serial Works by Igor Stravinsky." Master's thesis, University of Tennessee, 2011.

The passage is from measures 91-92, in which Stravinsky uses Ia to create segments of verticals from the rotational linear hexachords. This portion of the piece presents part of the orchestra at a fast tempo, varying the orchestration regularly. The verticals are marked in Example 3.9 and Example 3.10 with numbers to clarify the notes that belong to each vertical. As observed with the four-part array, to be classified as a four-part, two-part, or another classification of multi-part array, the vertical must contain the amount of pitch classes that the multi-part array is named. It is interesting that there

Example 3.10
Abraham and Isaac Verticals

1 2 3 4 5 6

I

Fag.

II

3/4

ten.

Trbu.

bas.

Tuba

marc. imp

marc. imp

marc. imp

appears to be a repeating pattern in Stravinsky's compositional process in the way he derived his two-part verticals from this passage. The employment of the verticals is already odd and different than his other instances, but now, there might be an underlying way he meant to present these verticals. As in this instance, there are two pitch classes comprised in these harmonies. Notice how the first two verticals are the first two notes of the first two columns of the rotated linear hexachords, the next two chords are the third and fourth notes from the third and fourth columns, and lastly, the last two chords are the fifth and sixth notes from the fifth and sixth columns, thus, completing the rotation through the linear hexachords. As previously mentioned, this procedure of the two-part array is marked in Example 3.9. To summarize the pattern, Stravinsky uses two notes from each column of the rotated linear hexachords, and after two completed verticals, he transitions to the next two notes of the next column (i.e. notes 1 and 2 to notes 3 and 4) to repeat the progression again. It is unclear why Stravinsky derived these two-part verticals from part of the linear hexachords since it is the only passage that performs this design, but it presents insight into Stravinsky's compositional originality with this innovative theoretical process.⁴²

Nevertheless, this design with rotational linear hexachords further confirms Stravinsky's necessity to incorporate this into his harmonic structures, as well as his contrapuntal procedures, verifying the changeability that comes from the practices with rotational arrays. This chapter has shown various procedures that harmonic instances exhibit with rotational arrays in Stravinsky's music. Examples of voice leading, compositional techniques, register components, and rhythmic elements have all been

⁴² For further examining and perceiving of examples of various procedures with rotational arrays, refer to Straus, Joseph N. *Stravinsky's Late Music*. New York: Cambridge University Press, 2001.

observed in Stravinsky's compositions, as well as the theoretical understanding of classifying his verticals into similar categories based on their prime forms. The procedure behind creating his rotational arrays provides a thorough understanding of comprehending different aspects of a rotational array and it's applicable approaches to serial music.

Chapter IV: Symmetry

Based on the nature that atonal or twelve-tone compositions are designed, they suggest the resisting of centricity or focusing around an underlying pitch-class; the majority of Stravinsky's rotational instances contradict this implication. Unlike his contemporaries, Stravinsky rarely exercised his atonal compositional techniques where each note receives the same level of authority. Instead, he continued engaging in properties from his neoclassical period, employing a strong symmetrical property and pitch-class centricity, not fully adhering to the problems he perceived in the music of serial music. He categorized the usage of each pitch-class equally as problematic in the nature of atonal music, notably represented in his late music and precompositional designs.⁴³ Since a characteristic of Stravinsky's rotational arrays is transposing the opening pitch-class of each hexachord to begin on the same first note, this directly expressed an aspect of centricity in his rotational designs.

This chapter will explore and list some symmetrical properties of Stravinsky's late music precompositional designs, considering various of his applications of rotational arrays and how they provide a symmetrical relationship, most often with pitch-class or interval-class duplication to exercise centricity. Specifically, a brief definition of symmetry in music will be explained and how symmetry is most often achieved throughout Stravinsky's rotational arrays, the nth order of intervals will be examined to generate a list of interval-classes to show how a row chart is created with symmetrical

⁴³ Paul Lombardi presents variations on these charts for *Movements* as part of his article. See Lombardi, Paul. "A Symmetrical Property of Rotational Arrays in Stravinsky's Late Music." *Indiana Theory Review* 25 (2004): 78.

aspects, and IC (interval-class) vectors and index vectors will be exhibited in one of Stravinsky's hexachordal sets to show symmetrical transpositional and inversional properties with different degrees of symmetry.

Symmetrical associations with rotational arrays present a unique, mathematical approach different than what was already talked about previously in this paper. Davorin Kempf generally defines and elaborates the concept of symmetry in music, interestingly, focusing on temporal relations:

Symmetry is a specific aspect of repetition. There are various ways of its realization in musical structure and form, as well as presuppositions of its application, concerning different compositional systems and styles. Regarding the dimension of time, there are two basic ways in which symmetry is realized. The first is in the domain of succession of formal parts or sections and appears as their symmetrical arrangement within a compositional wholeness. The second is the so-called mirror symmetry, that may also be applied to a micro and macroformal structure.... As regards such a kind of bilateral symmetry in musical structure and form that appears as a result of mirror reflection around an axis, first of all the mirror symmetry around a vertical axis should be discussed. This aspect is most interesting regarding the dimension of time and musical consequences.⁴⁴

It is apparent that time relations in Stravinsky's music will not be the focus of symmetry with rotational arrays. Although time will not be explored, his first temporal approach with symmetry will not be examined further, but his second approach, which presents a definition applicable to this symmetrical discussion in the quote above, can still be applicable to the symmetrical schemes of this chapter, explaining the significance of this quote. In fact, a mirroring factor of rotational arrays, relating to centricity, will appear as an underlying symmetrical aspect of rotational arrays throughout this chapter, as will be observed. Additionally, to clarify, when I refer to symmetry throughout this paper, I am specifically using the term in relation to pitch-class or interval-class

⁴⁴ Kempf, Davorin. "What Is Symmetry in Music?" *International Review of the Aesthetics and Sociology of Music* 27, no. 2 (1996): 155-59.

Example 4.1
The Eight Rotational Arrays from *Movements*

Pa	<table border="1" style="width: 100%; border-collapse: collapse;"> <tbody> <tr><td>3</td><td>4</td><td>0</td><td>2</td><td>1</td><td>t</td><td>e</td><td>6</td><td>7</td><td>9</td><td>8</td><td>5</td></tr> <tr><td>3</td><td>e</td><td>1</td><td>0</td><td>9</td><td>2</td><td>e</td><td>0</td><td>2</td><td>1</td><td>t</td><td>5</td></tr> <tr><td>3</td><td>5</td><td>4</td><td>1</td><td>6</td><td>7</td><td>e</td><td>1</td><td>0</td><td>9</td><td>2</td><td>8</td></tr> <tr><td>3</td><td>2</td><td>e</td><td>4</td><td>5</td><td>1</td><td>e</td><td>t</td><td>7</td><td>0</td><td>6</td><td>9</td></tr> <tr><td>3</td><td>0</td><td>5</td><td>6</td><td>2</td><td>4</td><td>e</td><td>8</td><td>1</td><td>7</td><td>t</td><td>0</td></tr> <tr><td>3</td><td>8</td><td>9</td><td>5</td><td>7</td><td>6</td><td>e</td><td>4</td><td>t</td><td>1</td><td>3</td><td>2</td></tr> </tbody> </table>	3	4	0	2	1	t	e	6	7	9	8	5	3	e	1	0	9	2	e	0	2	1	t	5	3	5	4	1	6	7	e	1	0	9	2	8	3	2	e	4	5	1	e	t	7	0	6	9	3	0	5	6	2	4	e	8	1	7	t	0	3	8	9	5	7	6	e	4	t	1	3	2	Pb
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duplication and how Stravinsky derives his rotational occurrences with invariant pitches.

Before examining the symmetrical aspects, for reference, Example 4.1 presents the eight possible rotations from *Movements* that can occur with the four traditional transformational rows, identified with transpositional and inversional labels.⁴⁵ Observe Example 4.3, which illustrates the Ra linear hexachord along with its complete vertical rotational chart in pitch-class integers from *A Sermon, A Narrative, and A Prayer*. F or pitch-class 5 is the center-tone that the theoretical charts are focused around for the duration of *A Sermon, A Narrative, and A Prayer*. This specific form of symmetry can be expressed with I_F^F to indicate the pitch-class symmetry focused around F, supported by the first vertical column of all Fs from Stravinsky's precompositional designs in *A Sermon, A Narrative, and A Prayer*. Based on this inversionally related symmetrical property, column one and column five are inversionally related at F; column two and four are centered at F as well; and column three maps onto itself at F. Not inherently present, the pitch-class centricity invokes a mirroring property or relating back to the focal pitch-class, continuously relating back to the opening first vertical from his precompositional chart.

Since each rotated row is either the same set-class or a Z-related set, the notes are inherently different, but the IC-vector and the levels of symmetry will continue to remain invariant through the various rotations. Therefore, in *Movements*, each hexachordal rotation will alternate the axis or center note, while maintaining the same intervallic characteristics to observe an intrinsic symmetrical relationship between rotations.⁴⁶ Thus, these symmetrical characteristics point back to Stravinsky's earlier style of

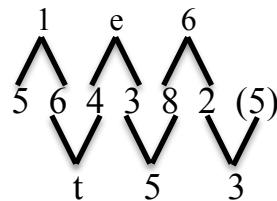
⁴⁵ Lombardi, Paul. "A Symmetrical Property of Rotational Arrays in Stravinsky's Late Music." *Indiana Theory Review* 25 (2004): 78-79.

⁴⁶ Paul Lombardi discusses this concept and presents additional charts for more information. *Ibid.*, 84-85.

Example 4.2
Generated Ra Hexachord From A Sermon, a Narrative, and a Prayer

nth-order intervals

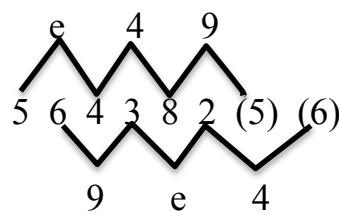
1



nth-order columns (mod 6)

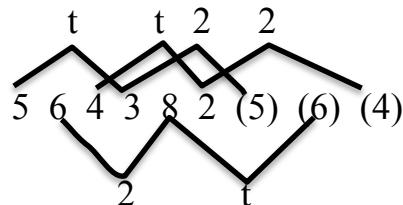
1 t e 5 6 3 1

2



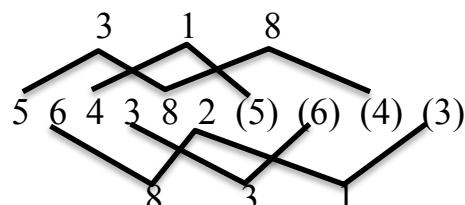
e 9 4 e 9 4 2

3



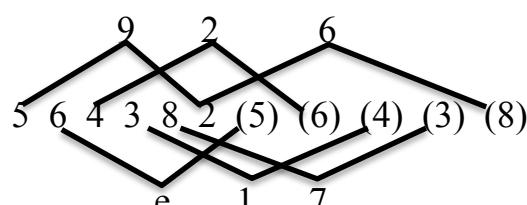
t 2 t 2 t 2 3

4



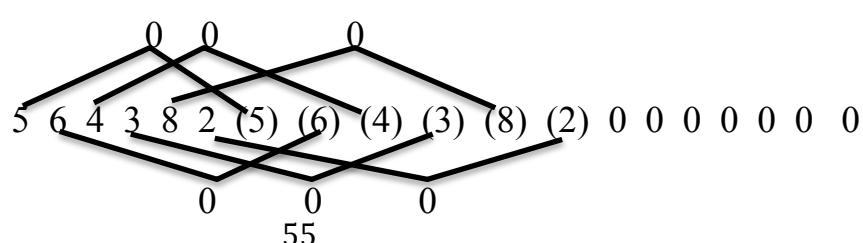
3 8 1 3 8 1 4

5



9 e 2 1 7 6 5

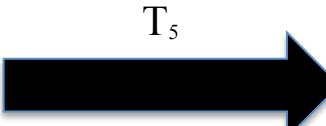
6



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Example 4.3

Ra Hexachord Original and Transposed Chart from *A Sermon, a Narrative, and a Prayer*



0 1 2 3 4 5	0 1 2 3 4 5
0 1 e t 3 9	5 6 4 3 8 2
0 t 9 2 8 e	5 3 2 7 1 4
0 e 4 t 1 2	5 4 9 3 6 7
0 5 e 2 3 1	5 t 4 7 8 6
0 6 9 t 8 7	5 e 2 3 1 0
0 3 4 2 1 6	5 8 9 7 6 e

neoclassicism, focusing around a central tonality or pitch-class, crafting his music uniquely distinctive in style from Schoenberg and Webern's atonal, serial music.⁴⁷

Based on my research and dissecting of the hexachords of Stravinsky's set classes, I noticed there is a direct correlation between the intervals in the rotated row charts and how the rotated, transposed series are generated. To understand how a rotated row is generated with a collection of intervals, observe Example 4.2, which shows the steps and process of the rotated, transposed series that uses the Ra hexachord from *A Sermon, A Narrative, and A Prayer* I used in Example 3.2.⁴⁸ Example 4.2 will be explained in more detail shortly. Example 4.3 contains the rotated, untransposed Pa hexachord from *A Sermon, A Narrative,*

⁴⁷ Joseph Straus briefly explains rotational arrays, their symmetrical properties, among others of these concepts discussed. For more detail, see Straus, Joseph N. *Introduction to Post-Tonal Theory*. Englewood Cliffs, New Jersey: Prentice-Hall, 1990. 193-96.

⁴⁸ This idea comes from Paul Lombardi, where he uses the same approach with a different hexachordal set from *Movements* to prove a symmetrical approach. See Lombardi, Paul. "A Symmetrical Property of Rotational Arrays in Stravinsky's Late Music." *Indiana Theory Review* 25 (2004): 79-81.

and A Prayer beginning on pitch-class 0, and then the transposed version that begins on pitch-class 5, which is seen throughout the composition. Although the rotated version that begins on pitch-class 0 never appears throughout this work, I have shown this untransposed chart to explain how the rotational charts are created from the information presented in Example 4.2. Before explaining Example 4.2, it is important to mention that Stravinsky rarely used the transpositional operation with his rotational designs, which is further expounded on by Paul Lombardi:

Although the inclusion of the IR and RIR labels is not typical of most twelve-tone music, they work well with Stravinsky's unique approach to twelve-tone composition because they show that the forms of the series are related by inversion and/or retrograde rather than by transposition.⁴⁹

Example 4.2 presents the original, unrotated version of the Ra hexachord six times, as labeled on the right side from 1-6 with the nth-order intervals. At each new statement of the Ra hexachord in my chart, the intervals skip a note each time, thus the nth-order intervals are stated. The intervals start with notes 1, 2, 3, etc. the first time through; the second time through in the chart, the intervals are for notes 1, 3, 5, etc., and so on. The intervals wrap around to include the notes from the beginning of the hexachord again, so there are still six interval occurrences. The wrap-around of the notes from the beginning of the hexachord is stated in parenthesis to differentiate them from the other, non-repeating notes. The intervals that I have shown are summarized to the right to accurately confirm their order. The process of generating the row charts is derived from these intervals. I have labeled the order of columns in Example 4.2 and 4.3. The procedure can be broken down into three steps. The first, as already discussed, is calculating all the nth-order of intervals as I have done. The second is simple writing the intervals down into rows as I have done to

⁴⁹ Ibid., 76.

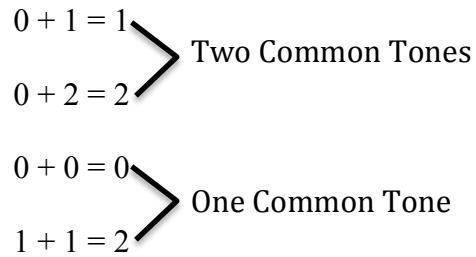
the right of the row charts, and then labeling the columns exploiting mod 6, 0-5. Notice how column 0 at the bottom of the chart in Example 4.2 corresponds to column 0 of Example 4.3, column 1 of Example 4.2 corresponds to column 1 of Example 4.3, and etc. Thus, plugging in the intervals that are laid out generates the complete untransposed Ra hexachord in Example 4.3. The last step is to accurately transpose the generated row chart to produce Stravinsky's precompositional row chart for *A Sermon, A Narrative, and A Prayer*; in this case, T₅ shown in Example 4.3.

Besides showing this simple, relatively quick process of creating a rotational array charts with the intervals, Example 4.2 also demonstrates interval duplication with the majority of columns, revealing a different level of symmetry, not in pitch-class duplication, but in interval-class duplication. Columns 1 and 5 both contain no duplication of intervals, yet the remaining columns 1, 2, 3, 4, and 0 all contain recurring interval-classes. Columns 2 and 4 are comprised of three interval-classes that repeat a second time, creating a mirroring property through each interval-class being stated twice; Columns 3 is comprised of two interval-classes that recur three times; and Column 0, the first column of the chart is comprised of all interval-class 0 that are stated six times. Initially, this example was examined for a simple way to generate a row chart from Stravinsky's rotational designs, but after further scrutinizing, it is evident there is an underlying recurring intervallic property of the nth-order of intervals.

Example 4.5 displays all the hexachordal prime form that are transpositionally symmetrical or inversionally symmetrical based on their IC and index vectors, which will be the subject of this next section. An IC vector is created by taking each interval-class in the prime form and arranging them in the vector, and since there are six interval classes and six notes in the prime form, the IC vector is comprised of fifteen numbers for a

Example 4.4
Illustration of index vector

Prime Form [012345]



Example 4.5
Hexachordal Prime Forms and Levels of Symmetry

Prime Form	Transpositional Symmetry	Inversional Symmetry
(012345)	1	1
(012348)	1	1
(012369)	1	1
(012378)	1	1
(012579)	1	1
(012678)	2	2
(013479)	1	1
(013679)	2	0
(023679)	1	1
(014589)	3	3
(023457)	1	1
(023469)	1	1
(024579)	1	1
(02468T)	6	6

hexachord. An index vector is created by adding up each note to calculate the index, including adding up the same notes, where if 1 is in the first spot, 1 and 1 have to add. Two numbers in different spots sum as two common tones, where two numbers in the same spot sum as one common tone. With index vectors, it is a part of a mod 12 system, unlike IC vectors that is part of a mod 6 system for the different interval-classes; both vectors will be included in Example 4.7 for an illustration and further examining of symmetry.

To better illustrate this common tone procedure described above with index vectors, Example 4.4 provides an illustration. “If the interval vector contains an entry equal to the number of notes in the set (or half that number in the case of the tritone), then the set [is transpositionally symmetrical].”⁵⁰ To be inversionally symmetrical, “the index vector for a

**Example 4.6
Pa and Pb from *Movements***

Pa	Pb
[3 4 t 8 9 2]	[0 e 1 6 7 5]

**Example 4.7
IC Vector and Index Vector of Prime Form [012678]**

IC Vector	Index Vector
Interval-classes: 420243 123456	Indexes: 246420246420 0123456789te

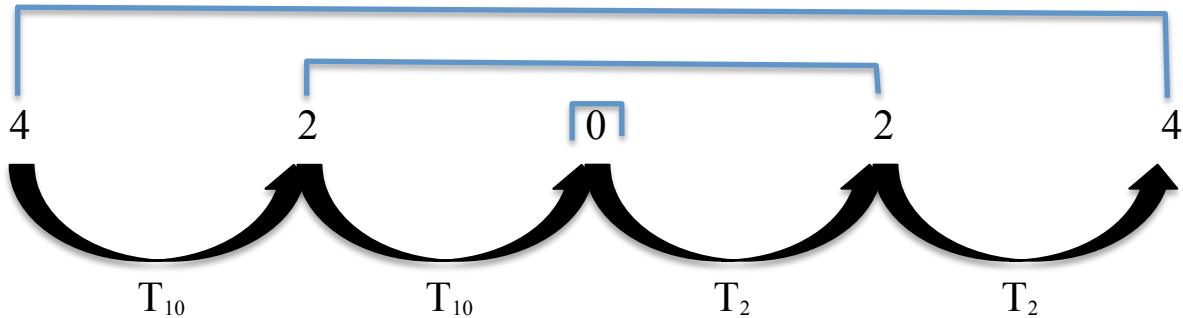
⁵⁰ Straus, Joseph N. *Introduction to Post-Tonal Theory*. Englewood Cliffs, New Jersey: Prentice-Hall, 1990. 74.

set with this property will have an entry equal to the number of notes in the set.”⁵¹ For the purposes of the discussion on symmetry with IC and index vectors, Stravinsky does not always employ a symmetrical property in his set classes, but he does in this case to make it a notable aspect in discussing symmetry. If a hexachord of a complete aggregate contains a symmetrical prime form, it is not a requirement for the other hexachord in the aggregate to be comprised of the same prime form or IC vector, unless both hexachords contain some level of symmetry, as in the case with the prime form of both hexachords of the aggregate in *Movements*. Since each prime form is always transpositionally symmetrical at T₀, every prime form, whether a hexachord or another size, is at least symmetrical on one transpositional level, but the converse is not true of inversional symmetry as seen in Example 4.5. Due to this fact, in Example 4.5, I only listed the hexachordal prime forms that encompass transpositional and inversional symmetry, with the exception of [013679] that contains two transpositional symmetrical levels and zero inversional symmetrical levels.

Example 4.6 exhibits a chart of the ordering of the Pa and Pb hexachords with pitch-class integers from *Movements* that shows both hexachords are prime form [012678], containing two levels of transpositional symmetry and two levels of inversional symmetry. Since *Movements* was Stravinsky’s first rotational piece, it contains his first theoretical mindset through the observing of his precompositional designs. If Example 4.5 is examined, it is apparent that the hexachordal prime form Stravinsky used in *Movements* has the third highest amount of symmetry of all the other hexachordal forms, behind [014589] and [02468T], making it particularly attention-grabbing when discussing vectors and

⁵¹ Ibid., 78.

Example 4.8
Vector Symmetrical Process



Example 4.9
Vectors Five-Note Symmetry

IC Vector Five-Note Symmetry	Index Vector Five-Note Symmetry
$\begin{matrix} 4 & 2 & 0 & 2 & 4 \\ 1 & 2 & 3 & 4 & 5 \end{matrix}$	$\begin{matrix} 2 & 4 & 6 & 4 & 2 \\ 0 & 1 & 2 & 3 & 4 \end{matrix}$ $\begin{matrix} 4 & 2 & 0 & 2 & 4 \\ 3 & 4 & 5 & 6 & 7 \end{matrix}$ $\begin{matrix} 2 & 4 & 6 & 4 & 2 \\ 6 & 7 & 8 & 9 & t \end{matrix}$

symmetry with prime forms.

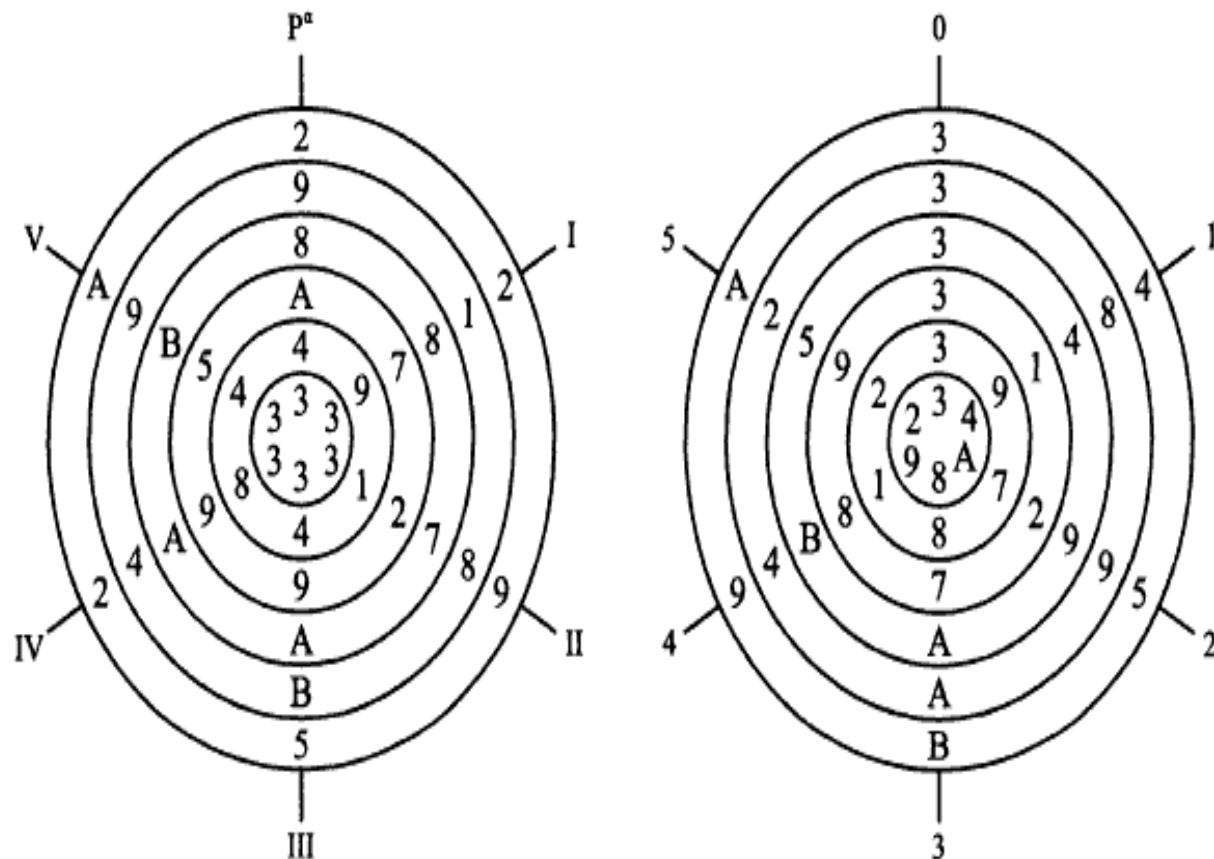
Example 4.7 presents the IC and index vector for the hexachords in *Movements* labeled with numbers below them, indicating the numbers of interval-classes or indexes. Since the intervals wrap around when a hexachord is rotated, the set-class content in pitch-classes changes, but the prime form manages to stay the same throughout each rotation. By

looking at Example 4.7, it is evident that there are two transpositional levels of symmetry in the IC vector. As already mentioned, each prime form is automatically symmetrical at T_0 , and prime form [012678] is also symmetrical at T_6 , indicated by the 3 common tones (half the number of notes for the tritone) in the sixth slot of the IC vector. The index vector, already stated, is used with a mod 12 system, starting on 0 and continuing through 11. There are six common tones under the third slot, T_{2I} , and the ninth slot, T_{8I} , designating them as symmetrical components under the inversional operation. These symmetrical properties shown in the IC and index vectors signify that under these transpositional and inversional operations, the pitch-classes of the prime form remain invariant, preserving the collection as identical.

Additionally, the IC and index vector also maintain a symmetrical property through the even numbers present in the slots of the vectors. The symmetrical property is observed by perceiving that all the slots in the IC and index vectors, with the exception of interval-class six in the IC vector, are filled with even numbers from 0-6. I noticed that the symmetrical characteristic of these numbers is declared in this way, usually of a five-note set: the middle number of the set is by itself; the inner-two numbers are the same-number and connected; and the outer-two numbers are the same-number and connected. Example 4.8 demonstrates the symmetrical relationship that was just explained by using the five-note illustration of the IC vector, while Example 4.9 shows all the symmetrical five-note relationships present in the vectors, as well as listing the numbers that they correspond to in the IC and index vectors, excluding the tritone in the IC vector and occasionally including overlapping symmetry in the index vector. There are four five-note sets total, one in the IC vector and three in the index vector, making it a noteworthy frequent phenomenon with the vectors in relation to symmetry. It is remarkable that every slot in both vectors, with the

Example 4.10 Transposed Rotations and Circular Chart for *Movements*

						0	1	2	3	4	5				
3	4	A	8	9	2	→	T ₀	→	P ^a	3	4	A	8	9	2
4	A	8	9	2	3	→	T _B	→	I	3	9	7	8	1	2
A	8	9	2	3	4	→	T ₅	→	II	3	1	2	7	8	9
8	9	2	3	4	A	→	T ₇	→	III	3	4	9	A	B	5
9	2	3	4	A	8	→	T ₆	→	IV	3	8	9	A	4	2
2	3	4	A	8	9	→	T ₁	→	V	3	4	5	B	9	A



exception of interval-class six in IC vector, is filled with one of three even numbers.

Furthermore, the ordering of intervals in the index vector can be assigned transpositional markers. The index vector begins with two common tones and from there, can be transposed by T_2 continuously until a climax is reached at six common tones. From there, T_{10} can be applied until a lower point is established at zero common tones; this procedure just described is shown in Example 4.8 on a smaller scale with four as the climax, instead of six. This process can be repeated and continued until all twelve indexes have been completed, exposing another symmetrical property of this particular prime form in *Movements*. Paul Lombardi presents a systematic illustration of a rotational array, one that has the verticals on the outside and the other one that has them on the inside, and it clearly demonstrates the differentiation between the vertical and linear rows with a circular chart in my Example 4.10, further expressing the symmetrical properties.

Many symmetrical aspects were observed, focusing around pitch-class and interval-class duplication. Inversional axis, nth-order of intervals, and the analysis of the IC vector for the set from *Movements* revealed an underlying emphasis placed on symmetry of varying degrees through different systematic approaches. Moreover, as in each symmetrical relationship observed, it is apparent that there is centricity polarized around a specific tone. This notable aspect further demonstrates a mirroring relationship, based in my opening quote from Davorin Kempf that describes mirroring as a prime symmetrical aspect in temporal relations in music, whereas the examples throughout this chapter orbit around an axis or an underlying pitch-class center, as perceived from my theoretical, calculated examples. To recapitulate, this symmetry exists on many levels, as apparent by the discussion and musical charts demonstrated. The regarding of these symmetrical

components concentrated on an inversional axis around a prevailing pitch-class, interval and pitch-class duplication through comparison of the nth-order of intervals, and the different levels of symmetry imbedded in the IC vector from the precompositional charts from *Movements*, along with the additional musical and structured charts that further illustrated the theoretical and symmetrical points considered.

Conclusion

Krenek, Schoenberg, Webern, among others, appear to have strongly influenced Stravinsky's designs of his compositional techniques, based on my in depth analysis of his works. Although Stravinsky clung to his neoclassic outlook he was immersed in for decades, it appears that he preferred and most often engaged in compositional schemes that adhered to an elevated level of intellectual and theoretical intent in his late music, in addition to mathematical approaches, as observed throughout this paper, to challenge the preceptor and auditor. His in-depth musical details, comprised of precompositional charts and notes, verify that Stravinsky firmly believed in the abstract concepts behind his works, even if he did not approach his designs as theorists' today would conceive of them.

Confirmed by the multitude of musical examples presented throughout the chapters, it is evident that Stravinsky's rotational position and mindset evolved as he became more practiced in his new theoretical innovations of his late music. When Stravinsky's entire compositional period is examined, it is pertinent to discuss that he spent a relatively short time, around fifteen years, in which he composed atonal, serial, or rotational works, compared to his more substantial time period that he was submerged in his neoclassic style, employing tonality or centricity. It is remarkable that Stravinsky achieved as much as he did throughout his short serial period, including his many innovations before rotational designs, trying to depart from his contemporaries, and embarking on his own inventive journey to ascertain new serial discoveries, while not opposing centricity as his atonal contemporaries had aptly done. Consequently, his concentration towards following his earlier outlook, particularly pitch-class and interval-class centricity, directed him to his

innovative breakthroughs with rotational arrays, expanding them in a distinctive direction from Krenek's earlier designs.

Years later, following the initial discoveries of Stravinsky's procedures with rotational arrays, his designs remain a trade mark characteristic of Stravinsky's prevalent compositional approaches, most notably in theoretical scholarship and academia. It is evident that Stravinsky experimented with a variety of methods, both linearly and vertically, to provide distinct ways that he kept his compositions unique and inventive, as clearly perceived in chapters two and three by the many musical and precompositional examples from Stravinsky's works. In addition to the musical and concrete illustrations, the intangible, abstract notions provided in regards to symmetrical features represent the imaginative mindset behind Stravinsky's procedures, whether intentionally meaning to epitomize the many symmetrical, mathematical properties of the rotational sets, or subconsciously exemplifying them throughout. Nonetheless, his late atonal works, exercising atonal, twelve-tone methods with rotational arrays, represents an intellectual and innovative accomplishment in theoretical and academic scholarship, in addition to a distinguishable compositional method, which demonstrates a scholarly, virtuosic approach to writing music.

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