Intro to DSP for ML - HW 0

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January 2024

1. Let us define the sequence of functions $f_n(x) : (0,1) \to \mathbb{R}$ by

$$f_n(x) = \begin{cases} 1, & x \in (0, \frac{1}{n}) \\ 0, & \text{otherwise} \end{cases}$$

   Show that it converges pointwise but not uniformly.

2. Check whether the following series converge. Do they converge uniformly as well?

   (a)
$$\sum_{n=1}^{\infty} \frac{\log(nx)}{n^3} \quad x \in [1,2]$$

   (b)
$$\sum_{n=1}^{\infty} \frac{\cos(n^3 x)}{n^2} \quad x \in [0,1]$$

   (c)
$$\sum_{n=1}^{\infty} x^n \quad x \in [0,1)$$

3. Find a parameter α such that the following functions will have a continuous continuation to a 2π-periodic function from $[-\pi, \pi]$ to the entire real line $\mathbb{R}$

   (a)
$$f(x) = \alpha x + x^3$$

   (b)
$$f(x) = \sin(\alpha x) + \sin(x)$$

   (c)
$$f(x) = \alpha x + e^x$$

4. Show that if $f(x)$ is 2π-periodic then

$$\int_{-\pi}^{\pi} f(x)dx = \int_{-\pi+\alpha}^{\pi+\alpha} f(x)dx$$

   for any $0 < \alpha \in \mathbb{R}$