

Assignment 2: Question 5

October 29, 2023 3:39 PM

Target Registration

- Module to register tumor target point from CT \rightarrow Ch frame

Input: TP_{CT} , $M1_{CT}$, $M2_{CT}$, $M3_{CT}$, $M1_{Ch}$, $M2_{Ch}$, $M3_{Ch}$

Output: TP_{Ch}

Testing: Sketch $F_{Ch \leftarrow CT}$, prove $M_{1,2,3 CT} \rightarrow M_{1,2,3 Ch}$

Strategy:

- Make Frames of $M_{1,2,3 CT}$ using generate Ortho Frame Function with $M_{1,2,3}$ as A, B, C
- Make Frames of $M_{1,2,3 Ch}$ " "
- Find the Transformation Matrix using Frame Transform to Home with O_C, e_1, e_2, e_3 from new ortho frames
- Take the Transformation matrices and multiply to find $F_{Ch \leftarrow CT}$

Solution:

We can see that a trivial linear transformation exists:

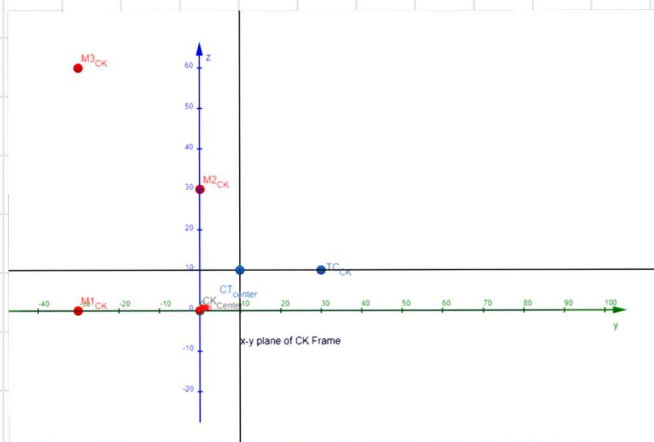
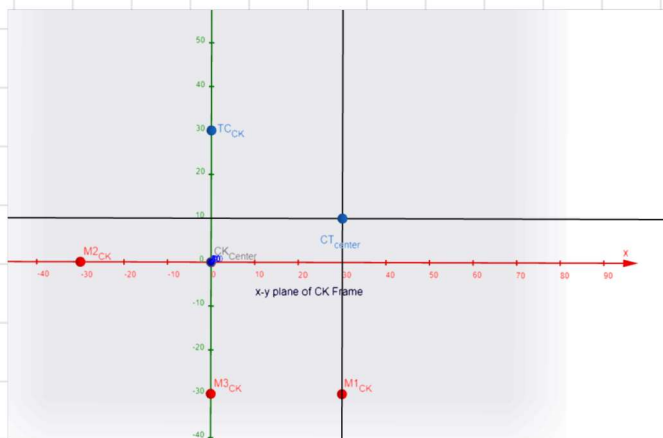
From markers it is clear

that the transformation is

$$M_{1,Ch} - M_{1,CT} = (30-0, -30+40, 0+10) = \begin{matrix} x & y & z \\ (30, 10, 10) \end{matrix}$$

$$\begin{matrix} F_{CT \rightarrow Ch} \\ \begin{bmatrix} 1 & 0 & 0 & 30 \\ 0 & 1 & 0 & 10 \\ 0 & 0 & 1 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{matrix} \begin{matrix} TP_{CT} \\ \begin{bmatrix} -30 \\ 20 \\ 20 \\ 1 \end{bmatrix} \end{matrix} = \begin{matrix} \begin{bmatrix} -30+30 \\ 20+10 \\ 20+10 \\ 1 \end{bmatrix} \\ \begin{matrix} TP_{Ch} \\ \begin{bmatrix} 0 \\ 30 \\ 30 \\ 1 \end{bmatrix} \end{matrix}$$

If we did not jump to a transformation matrix from the observation that each point is linearly transformed (30, 10, 10) in x,y,z then we could generate planes using the markers as input :



Orthonormal Vector



Given any three unique points:

(or for orthonormal basis i, j, k)

$i = \text{normalized}(B-A)$ (vector from A to B)

$k = \text{normalized}(i \times (L-A))$ // vector orthogonal to AB & AC

$j = i \times k$ (orthogonal to i and k)

Complete basis i, j, k

where $i = e_1, j = e_2, k = e_3$

Center of gravity point between A, B, L

$$O_c = \frac{A+B+L}{3}$$

We can see how to generate an orthonormal frame given any 3 points on the left:

We can then use the orthonormal frames from the MCK and MCT points to populate a transformation matrix

For MCK

$$O_c = [-30, -30, 20]$$

$$e_1 = [-0.9165, 0.4082, 0.4082]$$

$$e_2 = [0.4192, -0.4364, 0.8724]$$

$$e_3 = [0.5345, 0.8918, 0.1673]$$

For MCT

$$O_c = [0, -20, 30]$$

$$e_1 = [-0.9165, 0.4082, 0.4082]$$

$$e_2 = [0.2192, -0.4044, 0.877]$$

$$e_3 = [0.53, 0.80, 0.27]$$