Assignment 2: Question 5

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Target Registration

- Module to register tumour target point from CT → CK frame

Input: TP_{CT}, $M1_{CT}$, $M2_{CT}$, $M3_{CT}$, $M1_{CK}$, $M2_{CK}$, $M3_{CK}$

Output: TP_{CK}

Testing: Sketch $F_{CK \leftarrow CT}$, prove $M_{1,2,3\,CT} \rightarrow M_{1,2,3\,CK}$

Strategy:

- Make Frames of $M_{1,2,3}$, CT using GenerateOrthoFrame Function with $M_{1,2,3}$ as A, B, C
- Make Frames of $M_{1,2,3}$ CK " "
- Find the Transformation Matrix using FrameTransformtoHome with O_c, e_1, e_2, e_3 from new ortho frames
- Take the Transformation matrices and Multiply to find $F_{CT \rightarrow CK}$

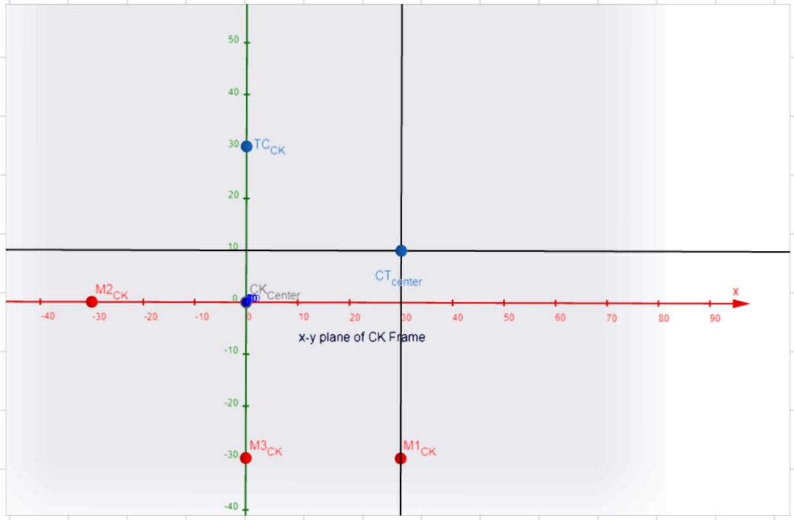
Solution: We can see that a trivial linear transformation exists:

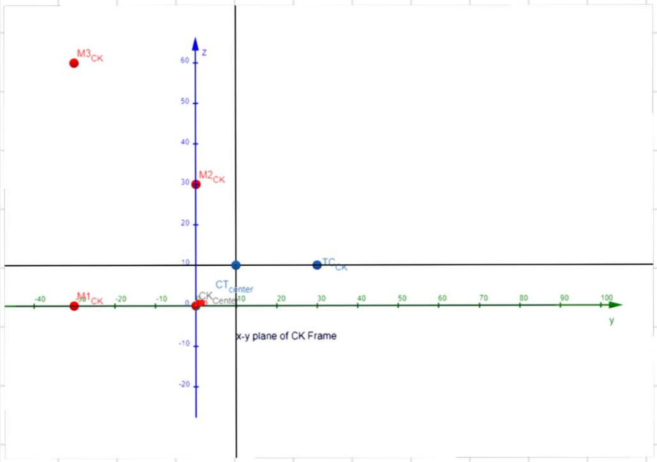
From Markers it is clear that the transformation is

$$M_{1CK} - M_{1CT} = (30-0, -30+40, 0+10) = \begin{array}{c} x \quad y \quad z \\ (30, 10, 10) \end{array}$$

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 & 30 \\ 0 & 1 & 0 & 10 \\ 0 & 0 & 1 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{F_{CT \rightarrow CK}} \overset{TP_{CT}}{\begin{bmatrix} -30 \\ 20 \\ 20 \\ 1 \end{bmatrix}} = \begin{bmatrix} -30+30 \\ 20+10 \\ 20+10 \\ 1 \end{bmatrix} = \overset{TP_{CK}}{\begin{bmatrix} 0 \\ 30 \\ 30 \\ 1 \end{bmatrix}}$$

If we did not jump to a tranformation matrix from the observation that each points is linearly transformed (30, 10, 10) in x,y,z then we could generate planes using the markers as input :





We can see how to generate an orthonormal frame given any 3 points on the left:

We can then use the orthonormal frames from the MCK and MCT points to populate a transformation matrix

Orthonormal Vector

Given any three unique points:

Can for orthonormal basis i, j, k

- i = normalized (B-A) (vector from A→B)

k = normalized $(i \times (C-A))$ // vector orthogonal to AB & AC

$j = i \times k$ (orthogonal to i and k)

Complete basis i, j, k

where $i = e_1$, $j = e_2$, $k = e_3$

Center of gravity point between A, B, C

$$O_c = \frac{A+B+C}{3}$$

For M_{CK}

$O_c = [-30, -30, 20]$

$e_1 = [-0.8165, 0.4082, 0.4082]$

$e_2 = [0.2182, -0.4364, 0.8729]$

$e_3 = [0.5345, 0.8018, 0.2673]$

For M_{CT}

$O_c = [0, -20, 30]$

$e_1 = [-0.8165, 0.4082, 0.4082]$

$e_2 = [0.2182, -0.44, 0.87]$

$e_3 = [0.53, 0.80, 0.27]$

We can see from the basis vectors that the following matricies get created:

```
T_CK =

   -0.8165    0.2182    0.5345  -30.0000
    0.4082   -0.4364    0.8018  -30.0000
    0.4082    0.8729    0.2673   20.0000
         0         0         0    1.0000


T_CT =

   -0.8165    0.2182    0.5345         0
    0.4082   -0.4364    0.8018  -20.0000
    0.4082    0.8729    0.2673   30.0000
         0         0         0    1.0000
```

which by taking the multiple of T_CT x T_CK ^-1
We find the transformation matrix as predicted:

```
T_CT_to_CK =

    1.0000         0    0.0000   30.0000
   -0.0000    1.0000   -0.0000   10.0000
    0.0000    0.0000    1.0000   10.0000
         0         0         0    1.0000
```