Assignment 2: setup and Question 1-2

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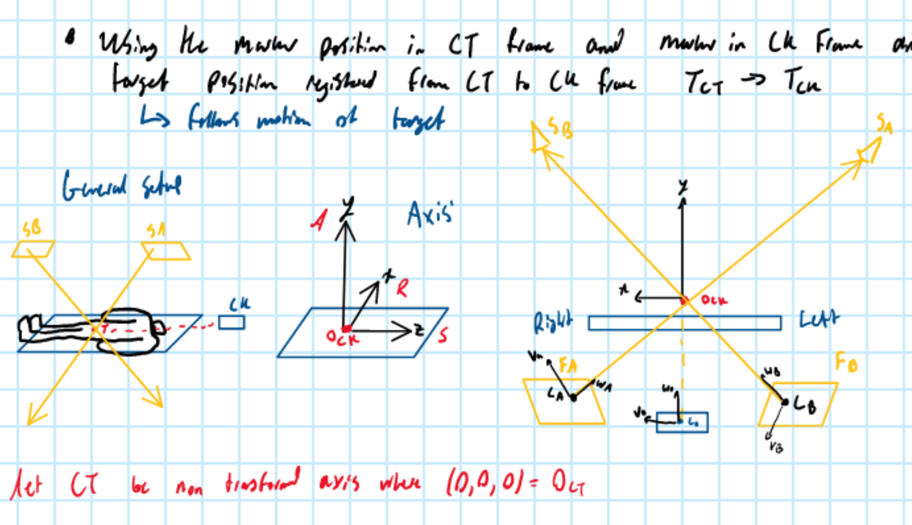
Setup Notes:

- Treatment Planning is Conducted in the Computed tomography (CT) frame, F_{CT} in metric Scale
- 3 implanted markers in CT frame $M1_{CT}, M2_{CT}, M3_{CT}$

Goal to accurately Compute any target position in Cyber Knife Frame
Target (tumor) a Sphere radi R Centered C_{CT} point in CT frame

- During treatment markers are Localized by X-ray System in Cyber Knife Frame ($M1_{CK}, M2_{CK}, M3_{CK}$)
- Using the marker position in CT frame and marker in CK Frame any target position registered from CT to CK frame $T_{CT} \to T_{CK}$
  - ↳ follows motion of target

General setup



let CT be non transformed axis where $(0,0,0) = O_{CT}$

- Use 3 Markers to detect target with Sensors

$M1_{CT} = (0,-40,-10)$

$M2_{CT} = (-60,-10,20)$

$M3_{CT} = (-30,-40,50)$

$TC_{CT} = (-30,20,20)$ ← target in CT frame

$R = 20$, radial distance from target

in CK
$M1_{CK} = (30,-50,0)$
$M2_{CK} = (-30,0,30)$
$M3_{CK} = (0,-30,60)$
$TC_{CK} = (0,30,30)$
$R = 20$

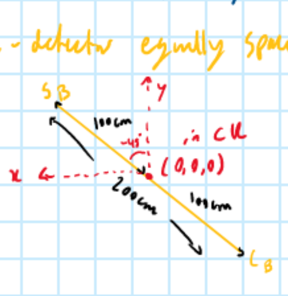
Translation

$(0,30,30) - (-30,20,20) = (30,10,10)$

$T_{CK} \leftarrow T_{CT}$

1. Frame Transform:

- Develop a module to generate frame transformations from CK frame to A, B detector frames
- Source-detector equally spaced



in CK

$S_B = (100\cos(45°), 100\sin 45°, 0)$

$S_B = (\frac{100}{\sqrt2}, \frac{100}{\sqrt2}, 0)$

$L_B = (-\frac{100}{\sqrt2}, -\frac{100}{\sqrt2}, 0)$

$\therefore\ S_A = (-\frac{100}{\sqrt2}, \frac{100}{\sqrt2}, 0)$

$L_A = (\frac{100}{\sqrt2}, -\frac{100}{\sqrt2}, 0)$

Rotation in Z

$$R_z(\theta) = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R = \begin{bmatrix} \cos(45°), & -\sin(45°), & 0 \\ \sin(45°), & \cos(45°), & 0 \\ 0, & 0, & 1 \end{bmatrix}$$

where
$V_{1A} = (\cos(45°), \sin(45°), 0) = (\frac{1}{\sqrt2}, \frac{1}{\sqrt2}, 0)$
$V_{2A} = (-\sin(45°), \cos(45°), 0) = (\frac{1}{\sqrt2}, \frac{1}{\sqrt2}, 0)$
$V_{3A} = (0,0,1)$

$O_{V_A} = (50\sqrt2, -50\sqrt2, 0)$

Since General Translation:

$$X_{hov} = \begin{bmatrix} V_{1x} & V_{2x} & V_{3x} & O_{vx} \\ V_{1y} & V_{2y} & V_{3y} & O_{vy} \\ V_{1z} & V_{2z} & V_{3z} & O_{vz} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

the Transformation $T_{A\to CK} = \begin{bmatrix} \frac{1}{\sqrt2} & -\frac{1}{\sqrt2} & 0 & 50\sqrt2 \\ \frac{1}{\sqrt2} & \frac{1}{\sqrt2} & 0 & -50\sqrt2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$T_{A\leftarrow CK} = T_{A\to CK}^{-1} = \begin{bmatrix} \frac{1}{\sqrt2} & \frac{1}{\sqrt2} & 0 & 0 \\ -\frac{1}{\sqrt2} & \frac{1}{\sqrt2} & 0 & 100 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

For $CK \leftarrow B$

$O_V = (-50\sqrt2, -50\sqrt2, 0)$
$V_{1B} = (\cos(-45°), -\sin(-45°), 0) = (\frac{1}{\sqrt2}, -\frac{1}{\sqrt2}, 0)$
$V_{2B} = (\sin(45°), \cos(45°), 0) = (\frac{1}{\sqrt2}, \frac{1}{\sqrt2}, 0)$
$V_{3B} = (0,0,1)$

Transformation $T_{B\to CK} = \begin{bmatrix} \frac{1}{\sqrt2} & \frac{1}{\sqrt2} & 0 & -50\sqrt2 \\ -\frac{1}{\sqrt2} & \frac{1}{\sqrt2} & 0 & -50\sqrt2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$T_{B\leftarrow CK} = T_{B\to CK}^{-1} = \begin{bmatrix} \frac{1}{\sqrt2} & -\frac{1}{\sqrt2} & 0 & 0 \\ \frac{1}{\sqrt2} & \frac{1}{\sqrt2} & 0 & 100 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Testing:

Points in A

Ground Truth: in A Frame

Can use Calculated Center of A

$P_{OA(CK)} = (\frac{100}{\sqrt2}, -\frac{100}{\sqrt2}, 0)$

$P_{OA(A)} = (0,0,0)$

$$P_{OA(A)}' = \begin{bmatrix} \frac{1}{\sqrt2} & -\frac{1}{\sqrt2} & 0 & \frac{100}{\sqrt2} \\ \frac{1}{\sqrt2} & \frac{1}{\sqrt2} & 0 & -\frac{100}{\sqrt2} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} \frac{100}{\sqrt2} \\ -\frac{100}{\sqrt2} \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{\sqrt2} & \frac{1}{\sqrt2} & 0 & 0 \\ -\frac{1}{\sqrt2} & \frac{1}{\sqrt2} & 0 & 100 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \frac{100}{\sqrt2} \\ -\frac{100}{\sqrt2} \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ -100+100 \\ 0 \\ 1 \end{bmatrix} = (0,0,0)$$

and Source A

$SA_{CK} = (-\frac{100}{\sqrt2}, \frac{100}{\sqrt2}, 0)$
↳ $SA_A = (0,200,0)$

$$= \begin{bmatrix} \frac{1}{\sqrt2} & \frac{1}{\sqrt2} & 0 & 0 \\ -\frac{1}{\sqrt2} & \frac{1}{\sqrt2} & 0 & 100 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} -\frac{100}{\sqrt2} \\ \frac{100}{\sqrt2} \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 200 \\ 0 \\ 1 \end{bmatrix} \to (0,200,0)$$

Ground Truth in B Frame

Can use Center of B in CK
$P_{B(CK)} = (-\frac{100}{\sqrt2}, -\frac{100}{\sqrt2}, 0)$

$$P_{O(B)} = \begin{bmatrix} \frac{1}{\sqrt2} & -\frac{1}{\sqrt2} & 0 & 0 \\ \frac{1}{\sqrt2} & \frac{1}{\sqrt2} & 0 & 100 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -\frac{100}{\sqrt2} \\ -\frac{100}{\sqrt2} \\ 0 \\ 1 \end{bmatrix}$$

$$P_B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = (0,0,0)$$

Can use $SB_{(in\ CK)}$
$P_{SB\,CK} = (\frac{100}{\sqrt2}, \frac{100}{\sqrt2}, 0)$

$$P_{SB_B} = \begin{bmatrix} \frac{1}{\sqrt2} & -\frac{1}{\sqrt2} & 0 & 0 \\ \frac{1}{\sqrt2} & \frac{1}{\sqrt2} & 0 & 100 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{100}{\sqrt2} \\ \frac{100}{\sqrt2} \\ 0 \\ 1 \end{bmatrix}$$

$$P_{SB_B} = \begin{bmatrix} 0 \\ 200 \\ 0 \\ 1 \end{bmatrix} = (0,200,0)$$

$\therefore$ we can see the Center and Source in the A and B Frame are as expected

2. X-ray Projection

- develop a digital X-ray projector to create ground truth X-ray images:
  - ↳ Project a point in CK space onto 2-imaging detectors
    - ↳ report the coordinates in detector frame

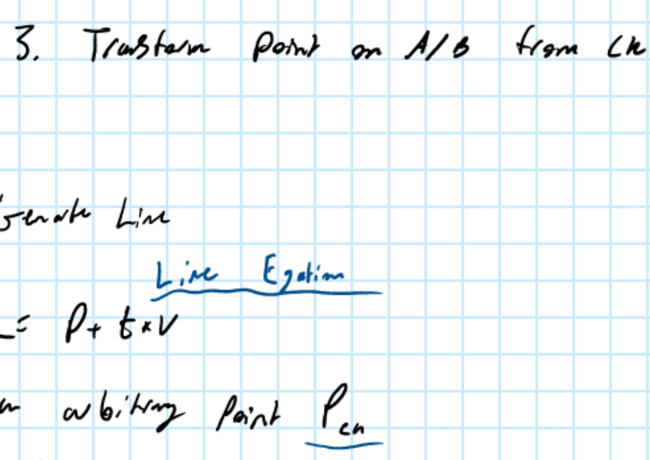
Input: point in CK

Output: Points in A, B detector frames

Test: 3 Suitable points, Sketch, run

Solution

Use Source A or B as Second Point



1. Generate Beam from Source A or B
2. Use intersect line and plane to find point projection on A/B
3. Transform point on A/B from CK → A/B

1. Generate Line

Line Equation

$L = P + t \cdot V$

Given arbitrary point P_{CK}
and SX_{CK} (SA_{CK} or SB_{CK})

$V = \mathrm{norm}(P_{CK} - SX_{CK})$

$L = P_{CK} + t\,(\mathrm{normalized}(P_{CK} - SX_{CK}))$

2. Plane Equation

let A be the origin of the plane
either $C_A = (\frac{100}{\sqrt2}, -\frac{100}{\sqrt2}, 0)$ or $C_B = (-\frac{100}{\sqrt2}, -\frac{100}{\sqrt2}, 0)$

Points in A
P_{Ax}, P_{Az}, C_A let $C_A = A$ and $C_B = B$

Points in B
P_{Bx}, P_{Bz}, C_B

In A

then $n = \mathrm{normalized}((P_{Az} - A) \times (P_{Ax} - A))$

$n = \mathrm{normalized}\,(((70.71, -70.71, 1) - (70.71, -70.71, 0)) \times ((71.42, -70, 0) - (70.71, -70.71, 0)))$
$n = (0,0,1) \times (0.71, 0.71, 0)$

$n_A = (-\frac{1}{\sqrt2}, \frac{1}{\sqrt2}, 0)$

in B

$n_B = \mathrm{norm}((P_{Bz} - B) \times (P_{Bx} - B))$
$n_B = (((-70.71, -70.71, 1) - (-70.71, -70.71, 0)) \times ((-70, -71.42, 0) - (-70.71, -70.71, 0)))$
$n_B = (0,0,1) \times (0.71, -0.71, 0)$
$n_B = (\frac{1}{\sqrt2}, \frac{1}{\sqrt2}, 0)$

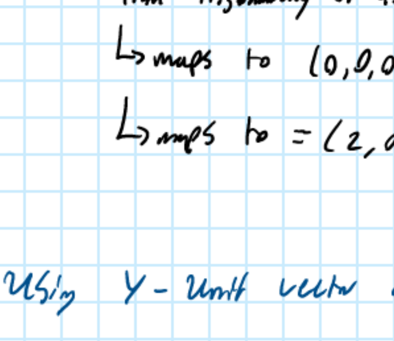
- Now can insert plane & Line into the intersect Line and Plane
- Then transform the frame

Testing

1. Using the origin:
→ Should Project to C_A and C_B

2. Z-unit vector
→ Should Project to (0,0,2) in Both A & B

Since



by Symmetry the projection Bisects the Z-unit vector and by Symmetry of right angle

3. Using the y unit Vector of A in CK coordinates
→ The y-unit vector of Frame A in CK = $(-\frac{1}{\sqrt2}, \frac{1}{\sqrt2}, 0)$
from trigonometry of 45° triangle
↳ maps to (0,0,0) in A (orthogonal to X-ray Frame)
↳ maps to = (2,0,0) (bisects SB to B of unit vector origin in B $\hat{x}$)

4. Using Y-unit vector of B in CK Coordinates
↳ y-unit vector of B in CK = $(\frac{1}{\sqrt2}, \frac{1}{\sqrt2}, 0)$
↳ maps to (0,0,0) in B (X-ray) Frame (orthogonal)
↳ maps to (2,0,0) in A by Symmetry

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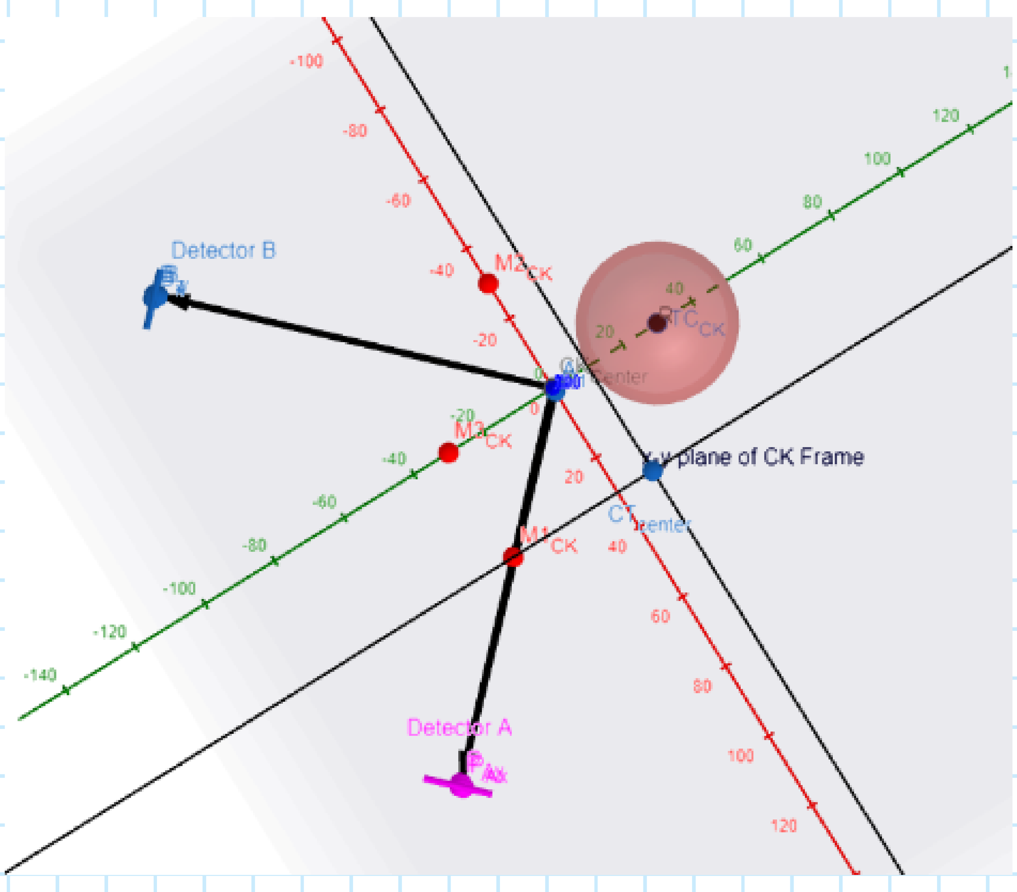
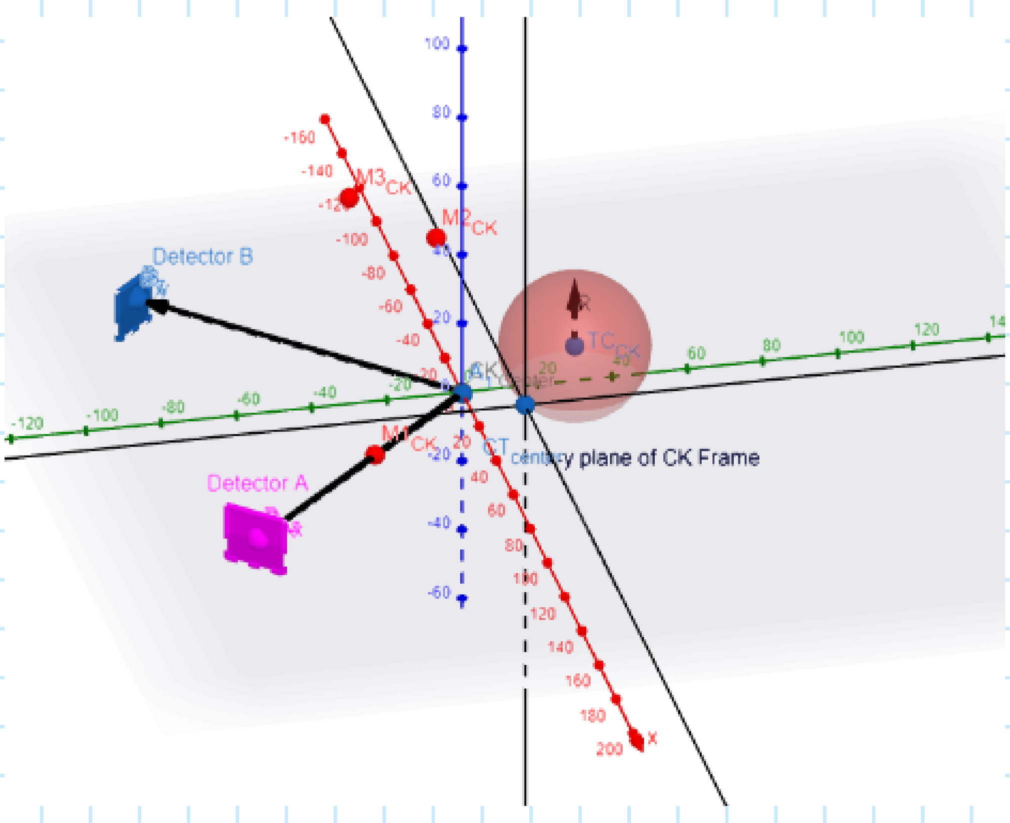
Images of the full setup can be seen below or by visiting this shared link:
https://www.geogebra.org/calculator/hymjskcs





This simulation has severial vectors used as an aid to help understand the various frames and calculations