Assignment 3 Navigation Transformations

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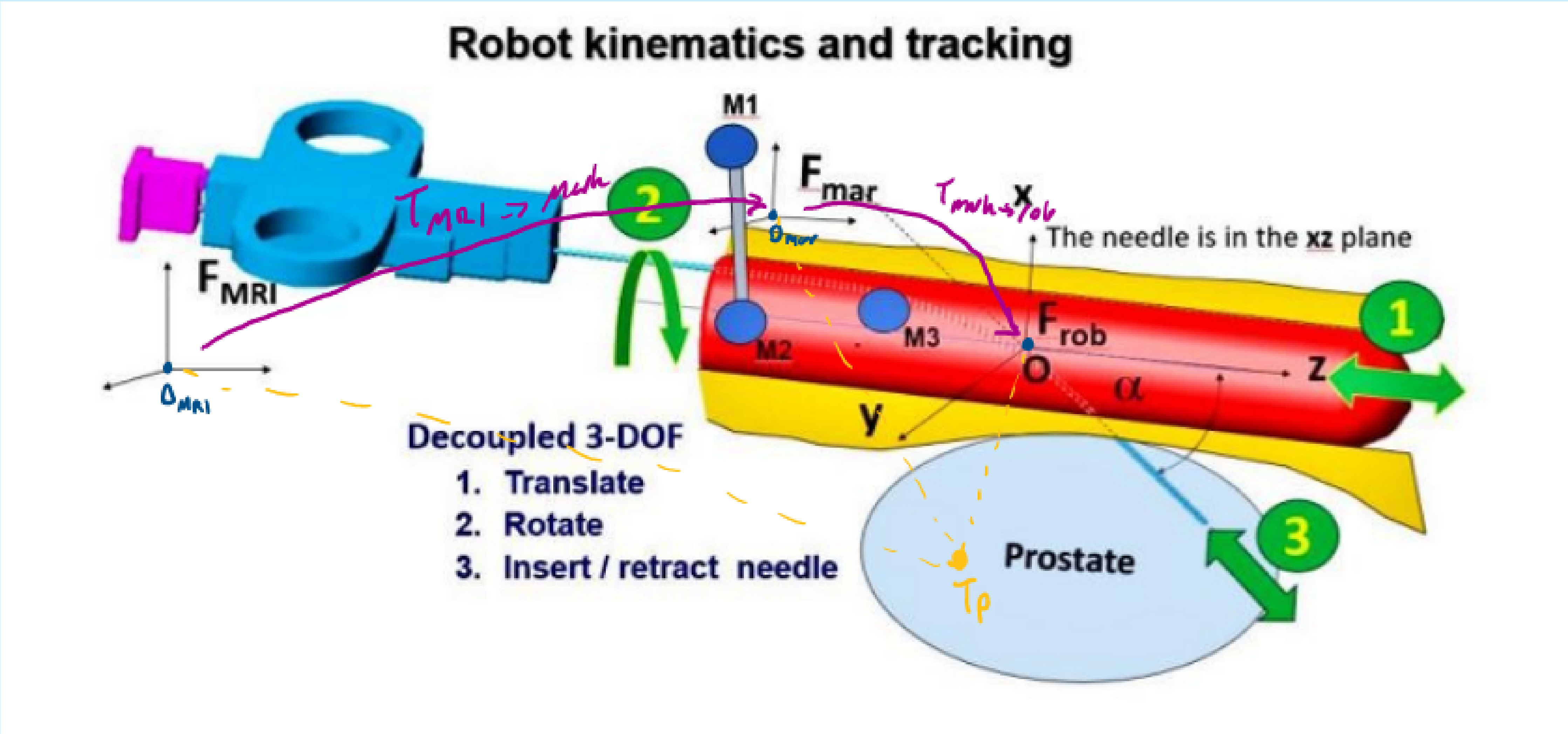
Task:
- to mark the relevant coordination frame transformations in Give Diagram
- Write formula for transformation of biopsy target point from F_{MRI} Scanner to F_{rob} robot frame through F_{mar} marker frame.



Formula for the transformation of Target from $T_{MRI \to mark}$ then $T_{mark \to rob}$

let T_p be some arbitrary target point

P_{MRI} is the target point in the MRI Frame

P_{rob} is the target point in rob Frame

then:

$$P_{rob} = P_{MRI} \underbrace{\left(T_{MRI \to mark} \right)}_{\text{transformation of MRI Frame to Mark}} \cdot \underbrace{\left(T_{mark \to rob} \right)}_{\text{transformation of Mark Frame to rob frame}}$$

Assignment 3 Workspace

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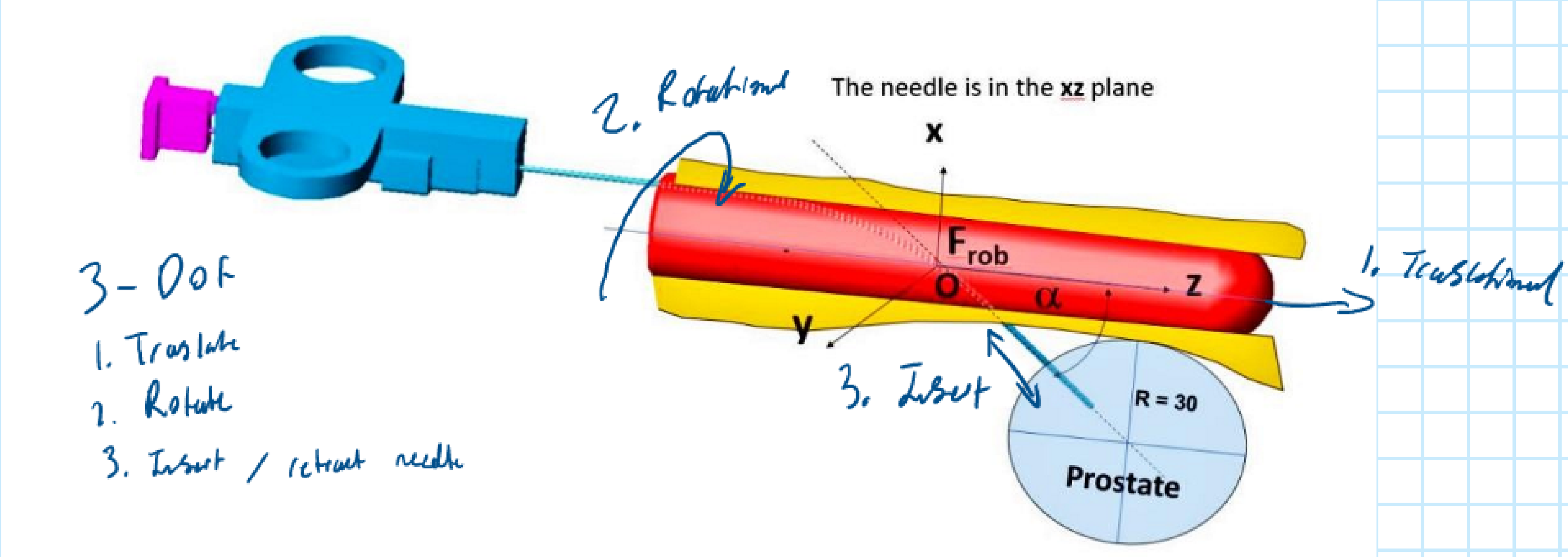
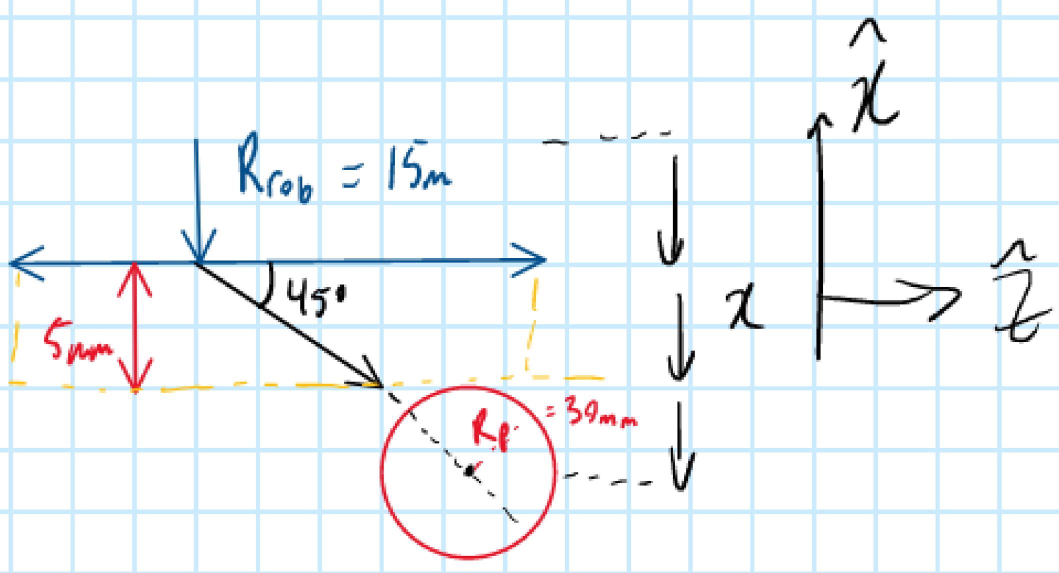


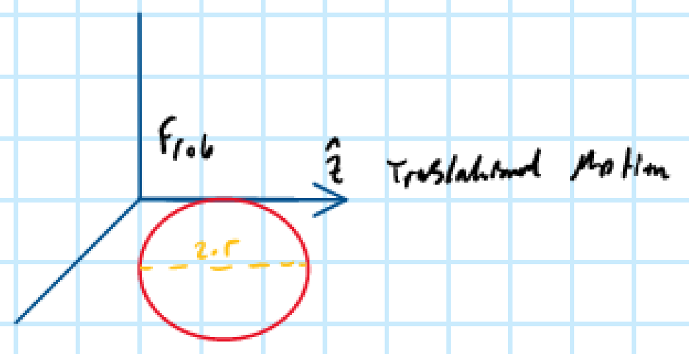
- Need to compute the minimum range of motion for each degree of freedom

Given:
- Diameter of robot's cylindrical end effector 30 mm
- The needles exit angle is 45°
- Prostate is spherical, 30 mm radii
- The thickness of the rectum wall is uniformly 5 mm



Task: Derive formulas to compute the minimum required range of motion for each DOF to be able to sample the largest prostate.

1. Translation:

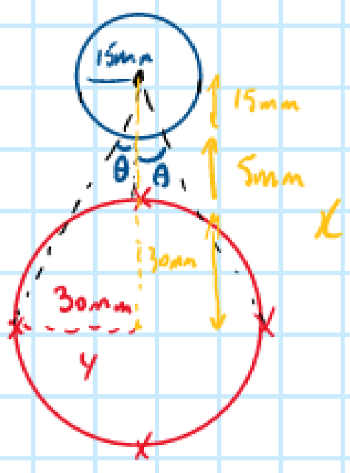


The minimum translation to sample the largest prostate is equivalent to the diameter of the prostate or 2 times the given radius:

$$Min_{Translation} = 2 \cdot r_p = 2 \cdot (30mm) = 60mm$$

∴ the minimum Translational Range of Motion is 60mm

2. Rotation



$$\tan\theta = y/x$$

full range of rotation is $2 \cdot \theta = \beta$

∴ full range of rotation: $\beta = 2\tan^{-1}(y/x)$

$$\beta = 2 \cdot \tan^{-1}\left(\frac{30mm}{15mm + 5mm + 30mm}\right)$$

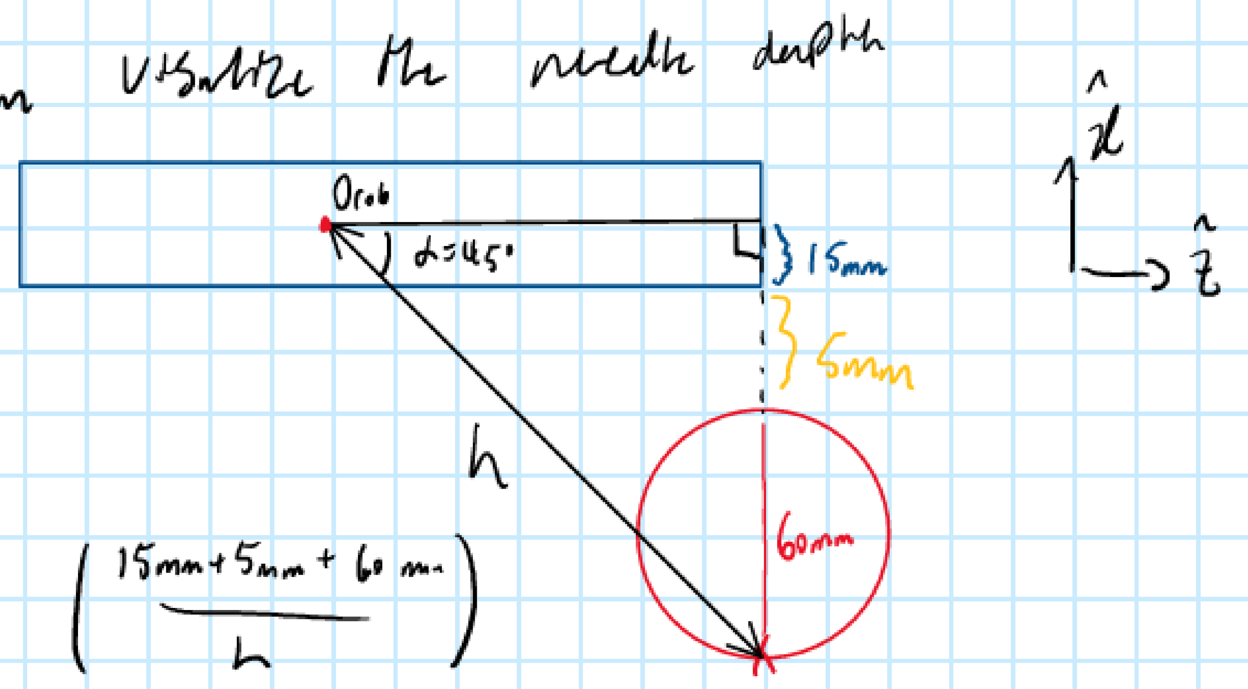
$$\beta = 2 \cdot \tan^{-1}(30/50) \simeq 2 \times 30.964°$$

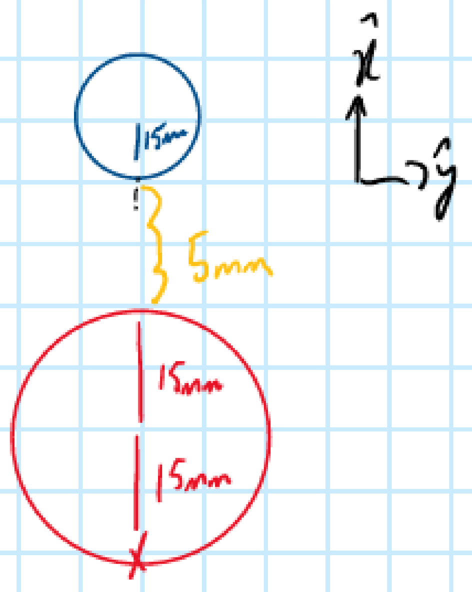
$$\therefore \beta \simeq 61.93°$$

∴ the minimum full range of motion for rotation is about 62°

3. Needle Depth:

Can visualize the needle depth





$$\therefore \sin\alpha = \left(\frac{15mm + 5mm + 60mm}{h}\right)$$

$$h = 80mm / \sin(45°)$$

$$h \cong 113.14mm$$

∴ the minimum full range of motion for needle depth is about 113mm

Assignment 3 Calibration

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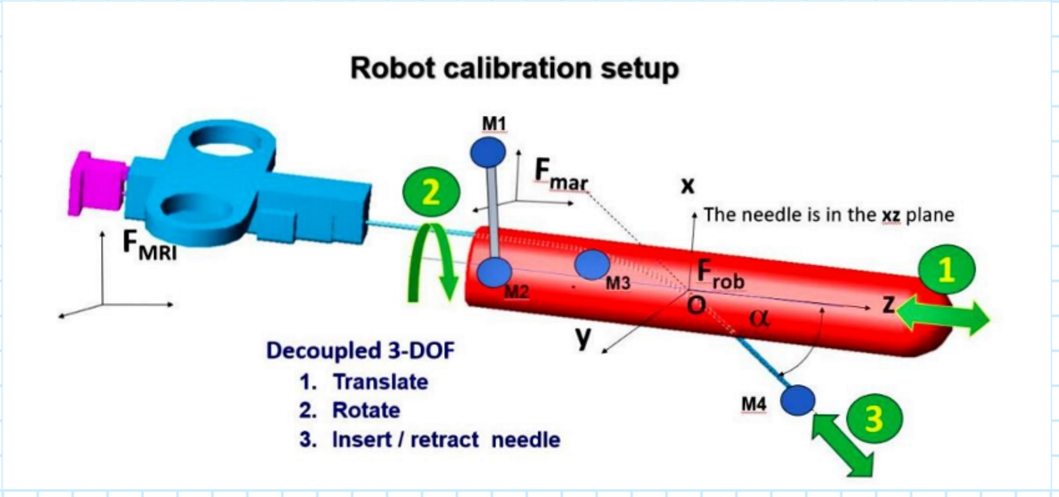


1. Needle Calibration:

Can use Sin law to evaluate d Given θ and ϕ

Diagram shows the deformation of the needle; due to symmetry and assuming the length of the needle (L) stays the same the distance from the target can be represented by an isosceles triangle

$$\frac{\sin\theta}{d} = \frac{\sin\phi}{L}$$

$$\therefore \; d = \frac{L\sin\theta}{\sin\phi}$$

Since isosceles triangles have angular symmetry $180° = \theta + 2\phi$

$$\therefore \; \phi = \frac{180° - \theta}{2}$$

$$\therefore \; d = \frac{L\sin\theta}{\sin\left(\frac{180°-\theta}{2}\right)}$$

Since $\cos\theta = \sin(180° - \theta)$

Since $\sin(2A) = 2\sin(A)\cos(A)$

$$d = \frac{L\sin\theta}{\cos(\theta/2)} \quad \text{if } A = \theta/2 \Rightarrow d = \frac{L\sin(2A)}{\cos(A)} = \frac{L(2\sin A\,\cancel{\cos A})}{\cancel{\cos A}}$$

$\therefore$ the targeting error, $d = \frac{L\sin\theta}{\cos(\theta/2)} = 2L\cdot\sin(\theta/2)$

if we want to ensure that no needle exceeds 1mm error

and assuming the max length of the needle corresponds to the max needle depth

then sub in $L \cong 113\,mm$ $d < 1mm$

$$\therefore \; \theta = 2\cdot\sin^{-1}\left(\frac{d_{max}}{2L}\right)$$

$$\theta \cong 2\cdot\sin^{-1}\left(\frac{1mm}{2(113mm)}\right)$$

$$\theta \simeq 0.5064°$$

$\therefore$ to ensure the needle is safe to use ie, does not cause an error distance from target point under the assumptions mentioned should have a deflection angle less than 0.5° any angle above 0.5° could cause targeting error greater than 1mm

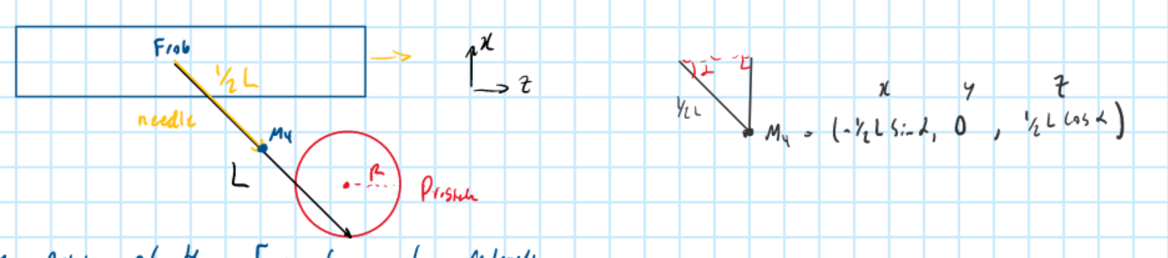
2. Parameter Calibration

- Next figure out where F_{rob} robot frame is w.r.t F_{mar}
- need x, y, z, O and α

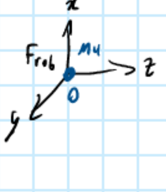
Steps
1. Bring robot to home position
2. execute predefined motion sequences
3. Observe positions of 4 tracking markers in F_{MRI}
4. Use observation to compute the unknown parameters of F_{rob} frame (x, y, z, O, α)
5. Bring Robot back to home observe markers in F_{MRI} and transform them to F_{rob} frame

To compute F_{rob} parameters the following steps can be taken:

1. Move robot to home rest position, let this be the center of its range of motion in all degrees of freedom
This would mean the needle is ½ its full length, rotation about z alligns needle with z-x plane (y=0)
Translation is 30 mm in z, all in robot frame



2. Can find the origin of the F_{rob} frame by retracting the needle fully and then measuring M4 in the MRI frame

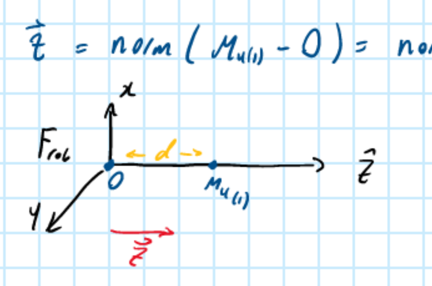


$O = M_4 = (x_0, y_0, z_0)$

3. With the needle fully contracted, translate the robot some small (must just be smaller than L) distance (d) in the $+\hat{z}$ direction in the robot frame then measure M_4

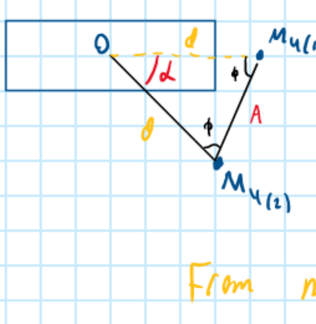
Using the previously found origin we can now compute the z basis vector of F_{rob} by taking the difference of the M4 observations and normalizing the results

$\vec{z} = norm(M_{4(1)} - O) = norm((x_1, y_1, z_1) - (x_0, y_0, z_0))$, $\vec{z} = avg\left(norm(M_{4(1)} - M_4 + M_{3(1)} - M_3 + M_{2(1)} - M_2 + M_{1(1)} - M_1)\right)$



★ To make this more accurate take average of new M1, M2, M3, M4 to find $\vec{z}$, reducing dependance on one marker

4. Noting that the needle should still be contracted, translate back to origin, so $M_4 = O$, extend the needle the distance d and measure M4 again.



We now have an isosceles triangle with O, $M_{4(1)}$ and, $M_{4(2)}$, where 2 sides are d
Side A length $= |M_{4(2)} - M_{4(1)}|^2$

From needle calibration

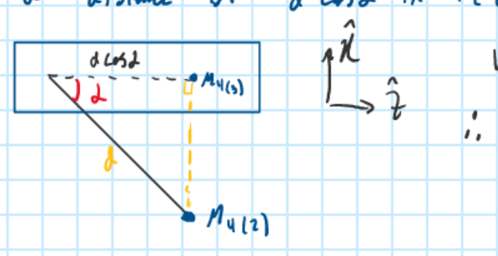
$$A = 2d\sin(\alpha/2)$$

by symmetry of isosceles

$$\therefore \; \alpha = 2\sin^{-1}\left(\frac{A}{2d}\right)$$

given same d as in step 3, and origin O, as in step 2.

5. Now retract the needle fully and move (Translate) the robot a distance of $d\cos\alpha$ in $+\hat{z}$ (rob frame)



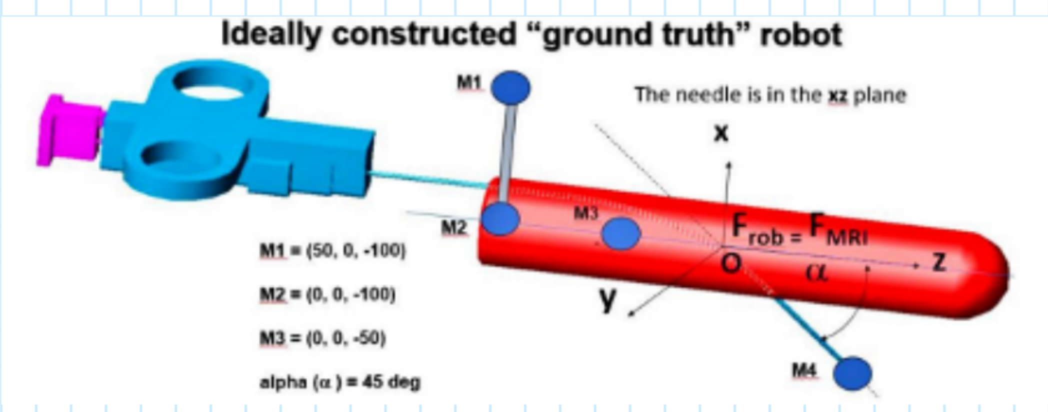
We can see that $M_{4(3)} - M_{3(2)}$ points in $+\hat{x}$
$\therefore \; \vec{x} = norm(M_{4(3)} - M_{4(2)})$

6. Can now use $\vec{x}$ and $\vec{z}$ to find $\vec{y}$ by taking cross product:

$$\vec{x} \times \vec{z} = \vec{y}$$

Now can move robot back to rest position since we have (α, x, y, z, O)

Need to implement this in software using Ground Truth Setup



Solution: Calibration function that takes M1, M2, M3, α, error and distance d
→ computes the frame parameters using the outlined steps

Test Case:

No Error

use given marker positions to generate basis vectors proving $F_{rob} = F_{MRI}$

With Error

→ Introduce a 1mm error randomly in M1, M2, M3
↳ 1mm translated to vector coordinates normally distributed

- Use generate unit vector from previous assignment
↳ ensure 1mm magnitude
- Apply to markers and complete steps
- Observe delta from ground truth

Then compare results

Assignment 3 Kinematics

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- First move robot to home position. Pick Desired Target Point in F_{MRI}

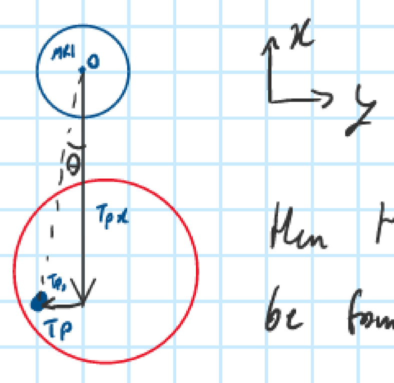
Inverse Kinematics:

- Need to compute values of translation, rotation, needle Insertion that will hit target point

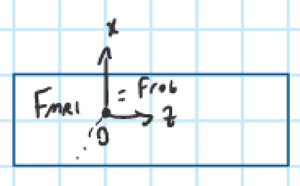
Need to develop Method of Deriving a rotation translation from TP_{MRI} to translations:

1. Find rotation angle of target

Assuming the MRI Frame is still located at the robot frame



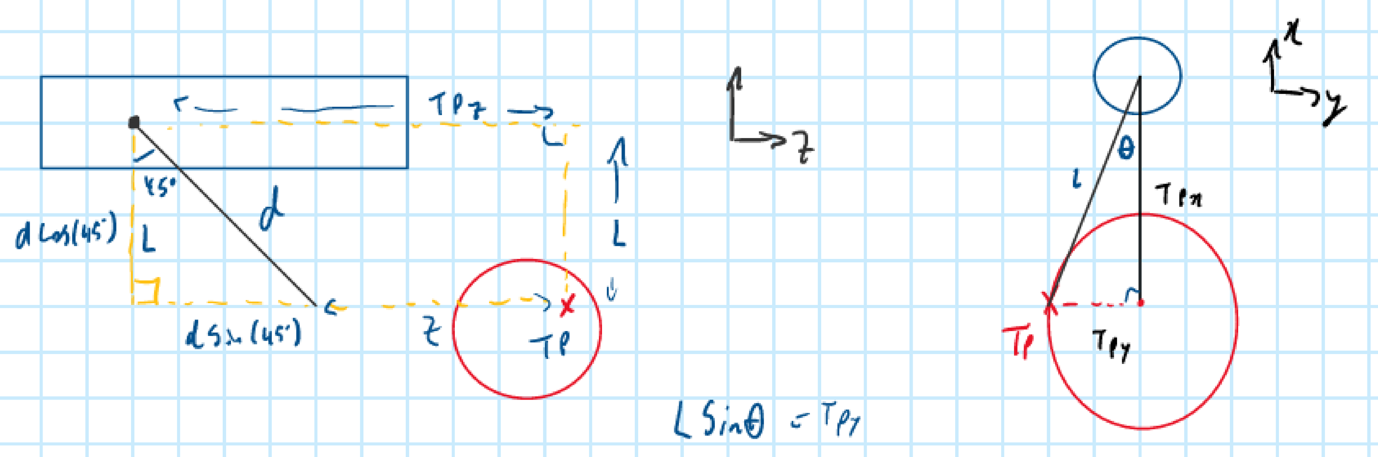
then the angle can be found using the x, y, coordinates



$$\tan\theta = \frac{Tp_y}{Tp_x} \quad \text{where} \quad Tp_{MRI} = (Tp_x, Tp_y, Tp_z)$$

$$\theta = \tan^{-1}\left(\frac{Tp_y}{Tp_x}\right)$$

2. Find the Insertion depth
   - know that the total distance to the point is the magnitude in the frame



$L \sin\theta = Tp_y$

$$L = \sqrt{Tp_y^2 + Tp_x^2} = \frac{\sin\theta}{Tp_y}$$

$$\therefore \quad d\cos(45^\circ) = L = \sqrt{Tp_y^2 + Tp_x^2}$$

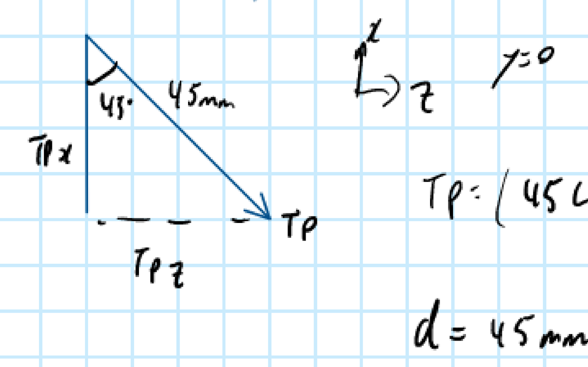
$$d = \frac{\sqrt{Tp_y^2 + Tp_x^2}}{\cos(45^\circ)}$$

3. Use L to find Z translation

$$z = Tp_z - d\sin(45^\circ)$$

Note: d should be less than 50mm
z should be less than 50mm

Test 1. 45mm exactly from 0, No Translation or rotation

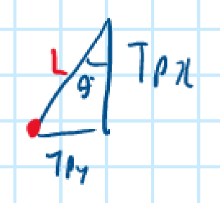


$TP = (45\cos(45), 0, 45\sin(45))$

$d = 45mm$
$\theta = 0°$
$z = 0$

Test 2. now add some rotation about z, No translation

$\tan\theta = Tp_y / Tp_x$



$\theta = 30°$
$d = 45mm$
$z = 0$

$L = 45\cos(45°)$

$Tp_x = L \cdot \cos\theta = 27.56mm$

$Tp_y = L\sin\theta = 15.91mm$

Since $z = 0 \rightarrow Tp_z = 45\sin(45°)$

$Tp_z = 31.82mm$

Test 3. Now shift back from z to add Translation

Shift the target point ahead 10mm

$TP = (27.56, 15.91, 45\sin(45°) + 10)$

$d = 45mm \quad z = 10mm, \theta = 30°$

Forward kinematics

Need to develop a method to compute resulting location of needle tip from translations

From needle depth, rotation and Translation Determine TP (or current point)

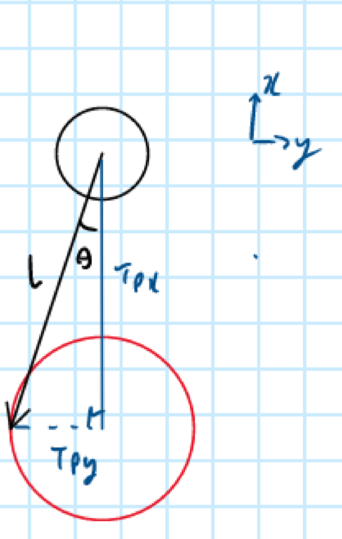
Forward:

Give d, z, θ find Tp_x, Tp_z, Tp_y

1. Tp_z

$$Tp_z = d\cos(45°) + z$$

2. Tp_x



$Tp_x = L\cos\theta$

$$Tp_x = d\sin(45°)\cos\theta$$

3. Tp_y

$Tp_y = L\sin\theta$

$$Tp_y = d\sin(45°)\sin\theta$$

Testing: Use Point $(30, -20, -50)$
→ Generate translations using Inverse
→ Feed Translation into Forward → Point $(30, -20, -50)$

Also use same Test's from Inverse

Assignment 3 Kinematics Test

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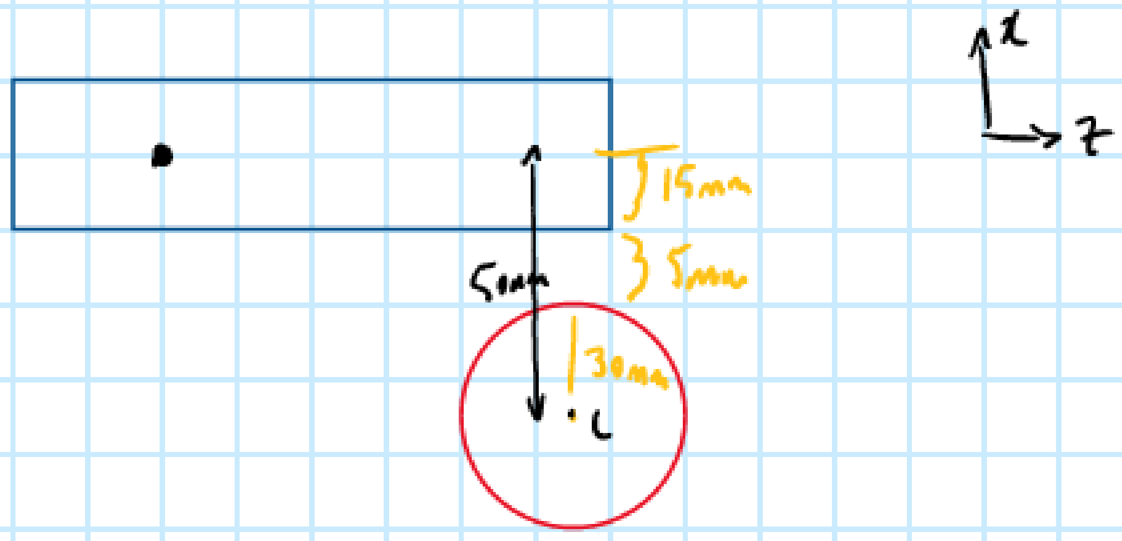
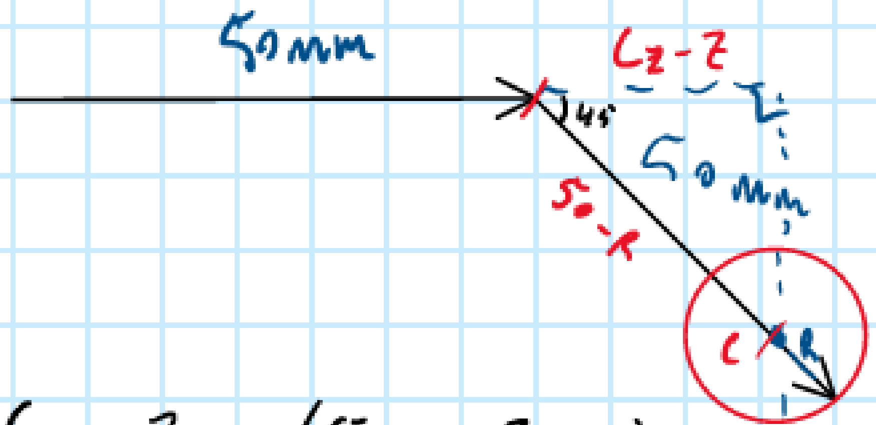
Need to test Forward / Inverse

Using Generated Points:



Worst Case Scenario $z = 50mm$

$d = 50mm$



$C_z - z = (50mm - 30mm)\cos 45°$

$C_z = (20)\cos(45°) + 50$

$C_z = 64.142$

∴ Prostate Center worst case scenario

$C = (50mm, 0, 20\cos(45°) + 50mm)$

To Generate test points

Supply Center and add Random Unit vector scaled $0 \rightarrow R$ (0 to 30mm)