Assignment 2: Question 3-4

October 28, 2023 2:28 PM

Marker Reconstruction

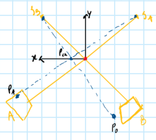
- Use Detector Image Outputs to reConstruct Point in CK

Input: Point in A Frame, Point in B Frame

Output: Point in CK Frame and REM

Testing: 1. Ground Truths from X-ray tests

2. Cam1 $M_{1,CK}$, M^2_{CK}, M_{3CK}



Strategy

1. Construct 2-Lines using Points

$P_{AA} \rightarrow P_{ACK}$ and $P_{SA\,CK}$

$P_{OB} \rightarrow P_{OCK}$ and $P_{SB\,CK}$

2. Find the intersection of the two lines and REM

using Intersect Lines Function from Assignment 1.

3. Intersection Point is Point in CK

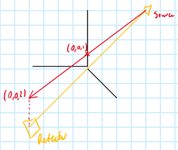
↳ Calculate REM

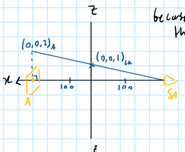
Testing

1. Z-unit Vector

→ Since the z axis of Frame A, Frame B and the Same z-axis direction as CK Frame

any Point in just z axis on CK will be the Same direction but 2x magnitude in Frame A & B





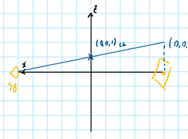
because of the Symmetry the projection will always double a point in only z

P_A P_B

$(0,0,1) \simeq (70.71, -70.71, 2)_{CK} \simeq (-70.71, -70.71, 2)$

in Frame A in Frame B

X-ray Image X-ray Image



- Find Line from P_A to S_A and P_B to S_B

$V_A = (70.71, -70.71, 2) - (-50\sqrt{2}, 50\sqrt{2}, 0)$

$= (100\sqrt{2}, -100\sqrt{2}, 0)$

$P_A = (50\sqrt{2}, -50\sqrt{2}, 2)$

$L = P + tV$

$L_A = P_A + t V_A$

$L_B = P_B + t V_B$

$V_B = (-50\sqrt{2}, -50\sqrt{2}, 2) - (50\sqrt{2}, 50\sqrt{2}, 0)$

$V_B = (-100\sqrt{2}, -100\sqrt{2}, 2)$

$P_B = (-50\sqrt{2}, -50\sqrt{2}, 2)$

Intersection at $(0,0,1)$

2. Y-unit vector in A-Frame

↳ $g_A = (-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0)_{CK}$

maps to $(0,0,0)_A$ → Since X-ray has no depth & vector originally maps to (0,0,0)

maps to $(2,0,0)_B$

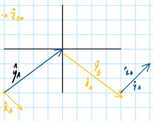
↳ By Symmetry at 2 45° slabs in $\hat{z}_{CK}$

3. Y-unit vector in Frame B

↳ $g_B = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0)_{CK}$

maps to $(2,0,0)_A$

maps to $(0,0,0)_B$



→ Testing Markers

↳ Also test M1, M2, M3, CCK

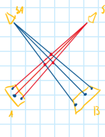
- All Should be mapped to original Points
- All Should have near zero REM

Marker Correspondences

Module to resolve Correspondence in reconstruction of 3 identical Markers

Input: Three Points in image A Frame, three points in B image Frame

Output: Correspondence Matrix



Strategy

- Find Lines between Points in A and S_A
- Find lines from Points in B to S_B
- Compute the intersection / REM of all lines
- Use REM values to fill in Correspondence Matrix

Solution & Testing:

Using $M_{1,CK} = (30, -30, 0)$ $S_{ACK} = (-50\sqrt{2}, 50\sqrt{2}, 0)$

$M_{2CK} = (-30, 0, 30)$ $S_{BCK} = (50\sqrt{2}, 50\sqrt{2}, 0)$

$M_{3CK} = (0, -30, 60)$

Project onto A, B X-ray: by finding Line

ex: M_{1CK}

$L_{A,A} = S_A + tV$

$V = M_{1CK} - S_A$

$= (30, -30, 0) - (-50\sqrt{2}, 50\sqrt{2}, 0)$

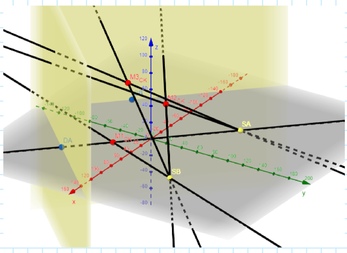
$V = (30 + 50\sqrt{2}, -30 - 50\sqrt{2}, 0)$

$L_{A,A} = (-50\sqrt{2}, 50\sqrt{2}, 0) + t\,(30 + 50\sqrt{2}, -30 - 50\sqrt{2}, 0)$

This can be done for each marker given M1,2,3 and find the corresponding lines

The lines can then be taken intersection points can be found between the detector A and B lines corresponding to the original point given

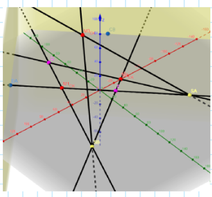
The situation can be seen below:



We can see that for each marker there are exactly two lines that intersect (always the line from SA to projection in Detector A and the line from SB to DB)

This means that there is no ambiguity since each projected point in frame A and B only correspond to one intersection (the original marker points). This should correspond to a REM matrix with exactly one (near) zero for each A and B pair.

We see that if we change M2 to lie on the x-y axis (-30, 0, 0) the reconstructed line from frame B intersects twice from SB meaning that we cannot describe which marker was originally given.



These ambiguous intersection points can be seen to the right labeled F and Q.
This will correspond to an REM matrix with two A and B pairs representing the additional intersections.

An example of what the REM Matrix and Correspondence matrix for this situation:

```
REM_Matrix =

    0.0000         0   49.4995
    0.0000    0.0000   49.4995
   51.5456   29.8573    0.0000

Correspondence Matrix:
     1    -1
     2    -1
     3     3
```