Nathan Phipps

Electronics Lab

Lab 2: Sallen-Key Bandpass Filter

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Objectives:

In this lab we were tasked to, “design build and test an active band pass filter, using Sallen-Key topology. Within these objectives we made sure to assume a 10mV to 2 V (peak to peak) input signal, a quality factor of 10, and a center frequency of 2500 Hz as specified by our lab manual.

Conclusion:

Due to the length of this lab, this conclusion will be divided as follows:

1.) Analysis of Preliminary Part,
2.) Analysis of Part A Pspice Simulations,
3.) Analysis of Part B Pspice Simulations,
4.) Analysis of Part C Pspice Simulations,
5.) Analysis of Oscilloscope Images,
6.) Concluding Statement

Conclusion: Analysis of Part A Pspice Simulations:

In the preliminary part of this lab we derived a transfer function based on the lab rubric’s specified parameters, using nodal analysis (Appendix A). Our model was based on a 2nd order Sallen-Key bandpass filter. After deriving the functions, we symbolically substituted our impedance values back into our transfer functions; this was done as an alternative instead of substituting admittance values back in (admittance is typically 1/Z). Our transfer function fit the general form provided in the lab guide, thus we concluded our values and the function itself to be valid. Again we substituted several values into the function for R, and C, while receiving a valid general form as our output. In our calculations for numerous components, our values were used and calculated to be as follows: C = 0.001 microFarad, Resistors R1, R2 = 64k Ohms, Resistor R3 = 128k Ohms, Resistor R4 = 1.9k Ohms, and Resistor R5 = 1k ohms. Our center frequency was provided by the instructor, as 2500 Hz. Our pole values were w1 = 14942.2 rad/s = 2378.13 Hz, and w2 = 16512.9 rad/s = 2628.11 Hz. Lastly, our zero for the system was 0 rad/s. Regarding our calculations, as well as function derivations for this section, each of our numbers seemed reasonable, and the final forms of our transfer functions correctly adhered to the lab guide’s provisional general forms.

Conclusion: Analysis of Part A Pspice Simulations:

The part A Pspice simulations from the *“4.A. OrCAD/PSpice Circuit Simulation,”* section, of the lab guide, correctly reflected our center frequency of 2500 Hz, with a gain of 29 db at the resonant center frequency. In addition to this, the corner frequencies of the pspice magnitude plot reflected the about the same values as our theoretical/ideal calculations whereby the very slight difference was due to human error in tracing an approximate 3 db roll of versus an exact 3db rolloff. Our theoretical, hand calculated values were: w1 = 2378.13 Hz, w2 = 2628.11 Hz, while our Pspice values reflected 3 db rolloffs at w1 = 2339kHz, w2 = 2586.2kHz. Our phase plot reflected about a 90 degree phase difference between corner frequencies caused by our frequency cutoffs, which agrees with our expectations for the phase plots. Additionally our calculated quality factor (Q = 9.43) ended up being -5.7% different from the Q value of 10. Overall, our phase and magnitude plots for this section appeared to be valid and in agreement with our data.

Conclusion: Analysis of Part B Pspice Simulations:

In part B from the *“4.B. OrCAD/PSpice Circuit Simulation,”* section, of the lab guide, we observed Pspice images pertaining to voltage versus time for 3 different frequencies (one decade below center frequency, center frequency, and one decade above center frequency) using our Sallen-Key Bandpass circuit. At one decade below the center frequency (with 1 V input) we observed a voltage output of 265.6 mV. Here, Vin lagged Vout by 90 degrees. The output voltage amplitude makes sense since the frequency is below the left cutoff by one decade, so the voltage should be lower than the value found at center frequency. At center frequency (with 1 V input) we observed a voltage output of 17.8 V. Here, the voltage input appeared to be in phase with the voltage output. The voltage at the center frequency makes sense being the highest value of the three frequency sweeps because this voltage represents the resonant peak. The output voltage value of 17.8 makes sense as the value approaches 18 V, but the voltage waveform saturates the closer it gets to the DC input voltage of 18 V. At one decade above the center frequency (with 1 V input) we observed a voltage output of 517 mV. Here Vin led Vout by 90 degrees. This last output voltage also makes sense, as the frequency for this voltage is above the right cutoff and well above the center frequency. Thus the voltage should be lower than the voltage at center frequency, while the voltage decreases past the right cutoff. Overall, our data/images from this section appeared to agree with our calculations and expectations.

Conclusion: Analysis of Part C Pspice Simulations,

Again, in part C from the *"4.C. OrCAD/PSpice Circuit Simulation,"* section, of the lab guide, we observed Pspice images pertaining to voltage versus time, using our Sallen-Key Bandpass circuit. Interestingly our voltage input for our 10 mV sine wave input circuit, at center frequency, reflected a slight phase difference from our expectations for this voltage versus time plot. Here, Vin was found to be slightly leading Vout. This may have been due to some of the components (op amp, resistors, values, capacitors, Q value), or due to the nature of 2nd order filters. The resulting gain was 24 V/V which seemed reasonable. With regards to our voltage versus time plot, for a 2 V Sine wave input, at center frequency, we found our voltage input, as well as voltage output phases to be almost, if not, completely in phase, which agreed with our expectations for Vin, Vout and gain. The gain itself appeared to be 9 V/V, which seemed reasonable. Throughout this part in the lab the output of our Pspice simulations appeared to agree with our calculations and data.

Analysis of Oscilloscope Images:

In this part we performed the oscilloscope and hardware trials of the *"Laboratory"* section from the lab rubric. In this part we designed a bandpass filter with a Q value of 10 and a center frequency of 2500 Hz. The input signal value we used was 1 V, which fit our rubric's designation of using a value between 10mV and 2 V. At one decade below the center frequency we observed a voltage output of 302mV, with Vout leading Vin by 87.7 degrees. Here we saw the voltage output dip because the corresponding frequency was below the left cutoff. At center frequency we observed a voltage output of 26.9, with Vout leading Vin by 24.6 degrees. Here we observed the resonant peak of value for the output voltage, as the center frequency contains the max value for voltage output in a bandpass filter. Again, as was the case with our Pspice simulations, our hardware simulation reflected a slight phase shift for the output voltage of our circuit, despite being at center frequency. This may have been due to some of the components (op amp, resistors, values, capacitors, Q value), or due to the nature of 2nd order filters. At one decade above center frequency we observed a voltage output of 314mV, Vin leading Vout by 94.6 degrees. Here we saw the voltage output dip because the corresponding frequency was above the right cutoff. Overall, all of the amplitudes, observed in these hardware trials, appear to be valid and in alignment with our calculations.

Concluding Statement:

Overall, the Pspice simulations we conducted appeared to be in alignment with our data, calculations and expectations. Our hardware simulations from the "*Laboratory*" section of the lab reflected similar notions that we observed in the Pspice simulations for Part C, whereby a slight phase difference for output voltage to input voltage was observed in our center frequency, despite the expectation for a 0 degrees difference in phase. However, in the Pspice simulations, the Voltage input appeared to be leading by about degrees or so, while the lab simulation yielded the output as leading by 24 degrees. Again, the reason for this may have been based on components (resistors, capacitors, values, etc.), or maybe even human error. Beyond this, our observations in Part B and D appeared as expected as well. Whereby our center frequency yielded the highest voltage and the two other frequencies (one above, one below), yielded lowering voltages, as they were past the corner frequencies. Our preliminary calculations, Part A calculations, also conformed to our expectations and calculated values, and yielded no visible differences from our expectations. From this lab it is safe to conclude that we fulfilled our objectives to design build and test a Sallen-Key bandpass filter, while gaining a better understanding of bandpass filters in general.

PART A:

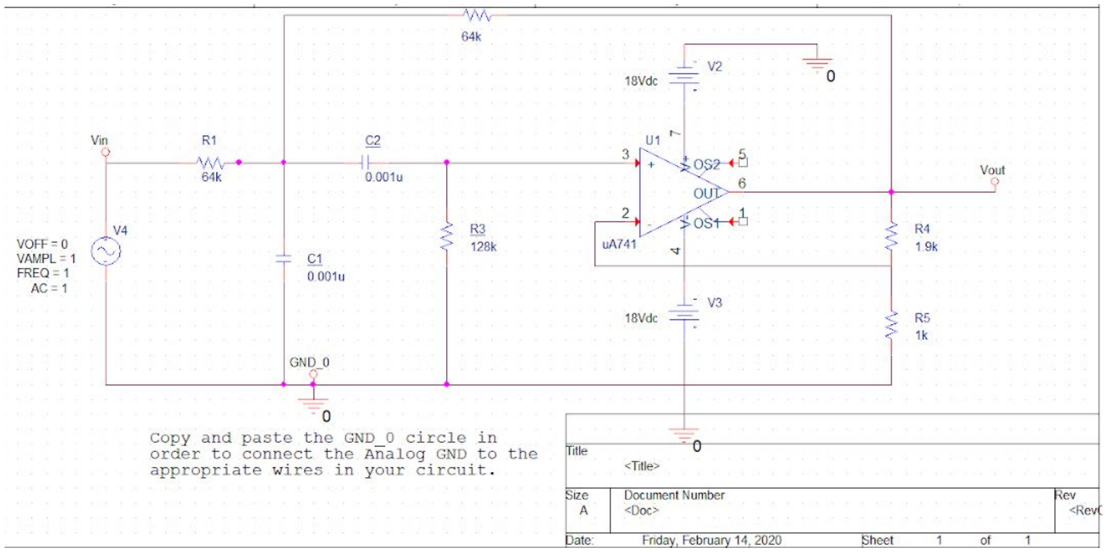


Figure 1: Part A: Schematic:

In figure 1 we see the schematic interpreted from our lab rubric to Pspice.

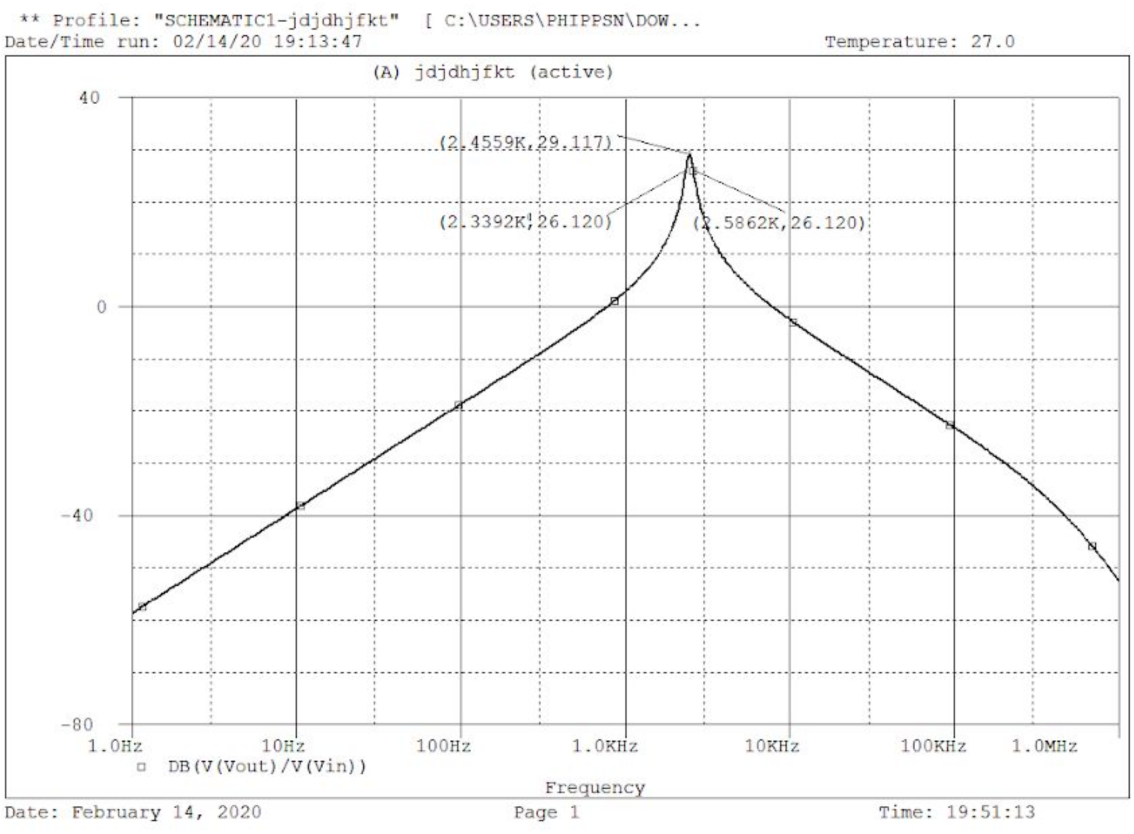


Figure 2: Part A: Magnitude bode plot:

In figure 2 a bandpass filter can be observed. Here we see a resonant like peak (29.117 dB) at the center frequency (2455.9Hz), with 3dB rolloffs at 2339.2 Hz and 2586.2 Hz. Here our center frequency is really 2500 Hz, but due to hand tracing with the cursor functions in Pspice it shows up as 2455 Hz.

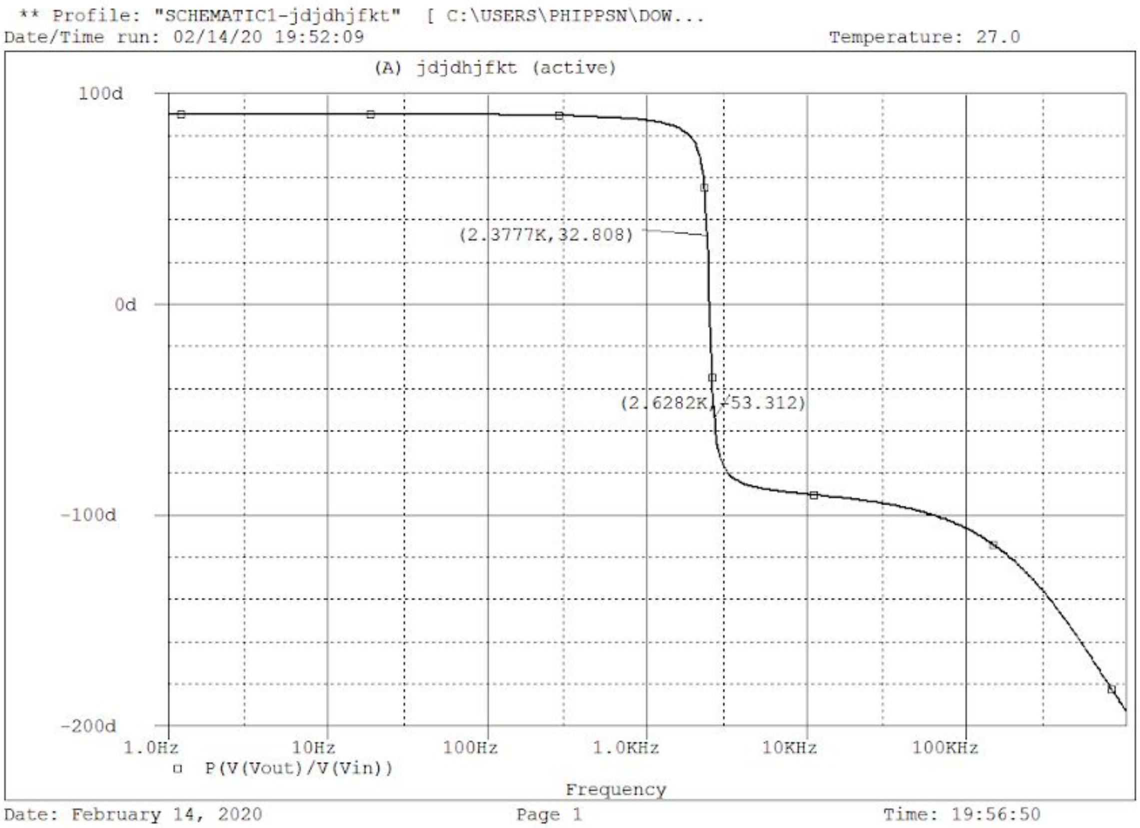
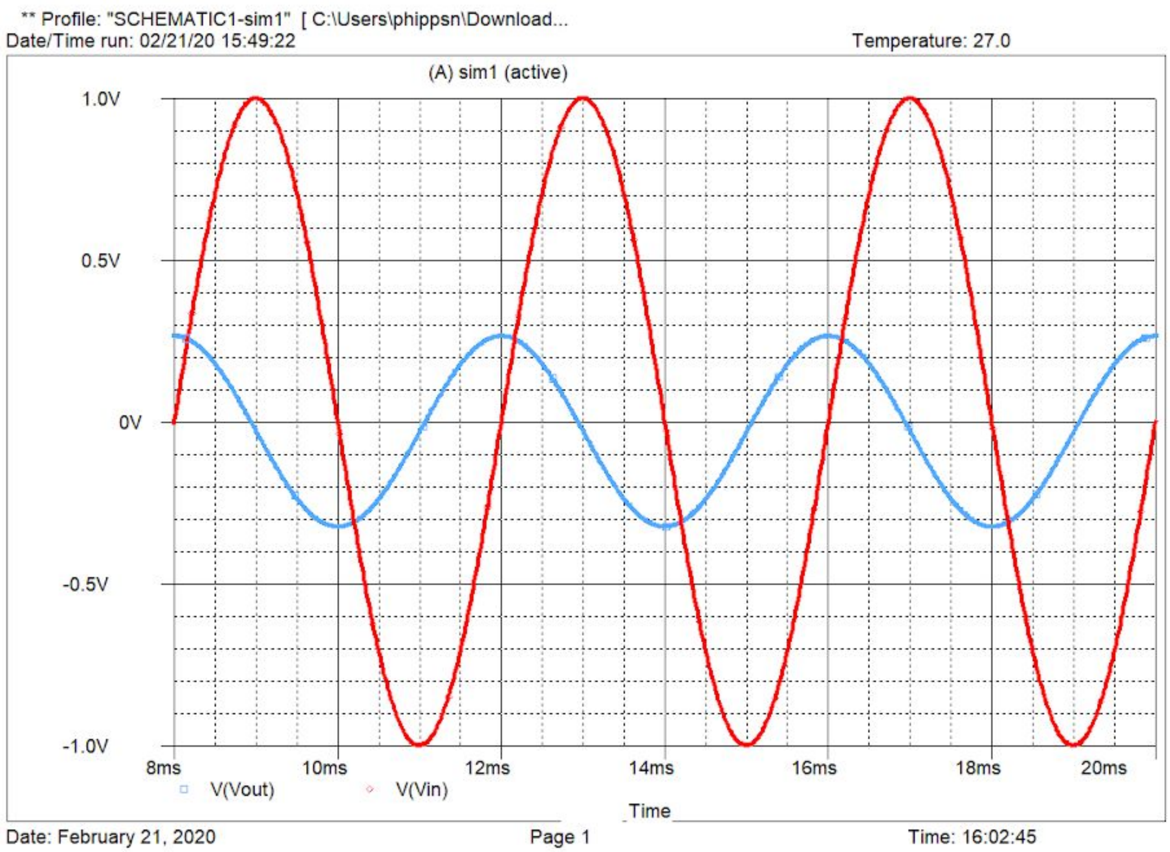


Figure 3: Part A: Phase bode plot:

In figure 3 our Pspice simulated results matched our expectations for the phase plot of this bandpass filter. From this graph we can see a difference of about 90 degrees from our left cutoff frequency, our right cutoff frequency. We can see our center frequency reflected as the asymptotic center between the two frequency cutoffs (2377 Hz and 2628 Hz) in this graph.

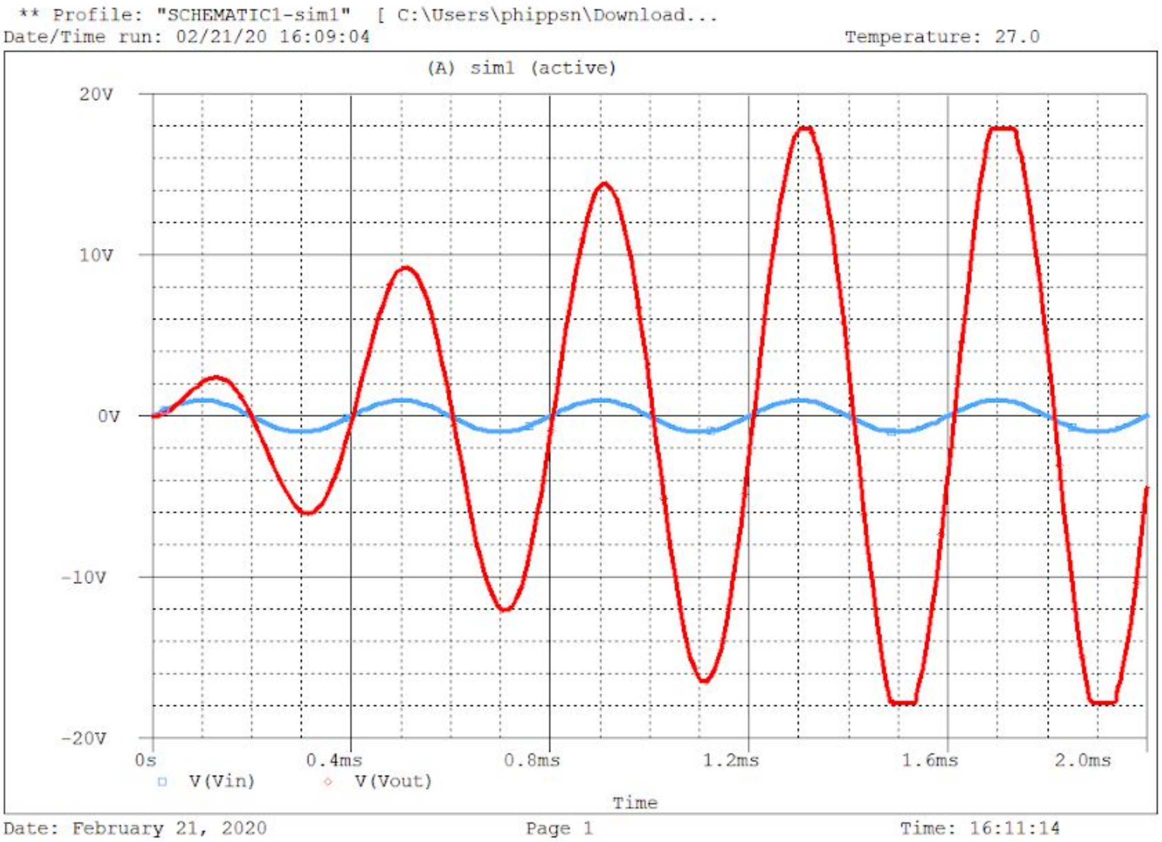
Part B:



	Evaluate	Measurement	Value
	☑	Max(V(Vout))	265.64568m
	☑	Min(V(Vout))	-323.92510m
	☑	ZeroCross(V(Vout))	8.94283m
	☑	ZeroCross(V(Vin))	10.00000m

Figure 4: Part B, Graph and Table: Gain Voltage Versus Time: At 250 Hz, 1 Decade below cutoff:

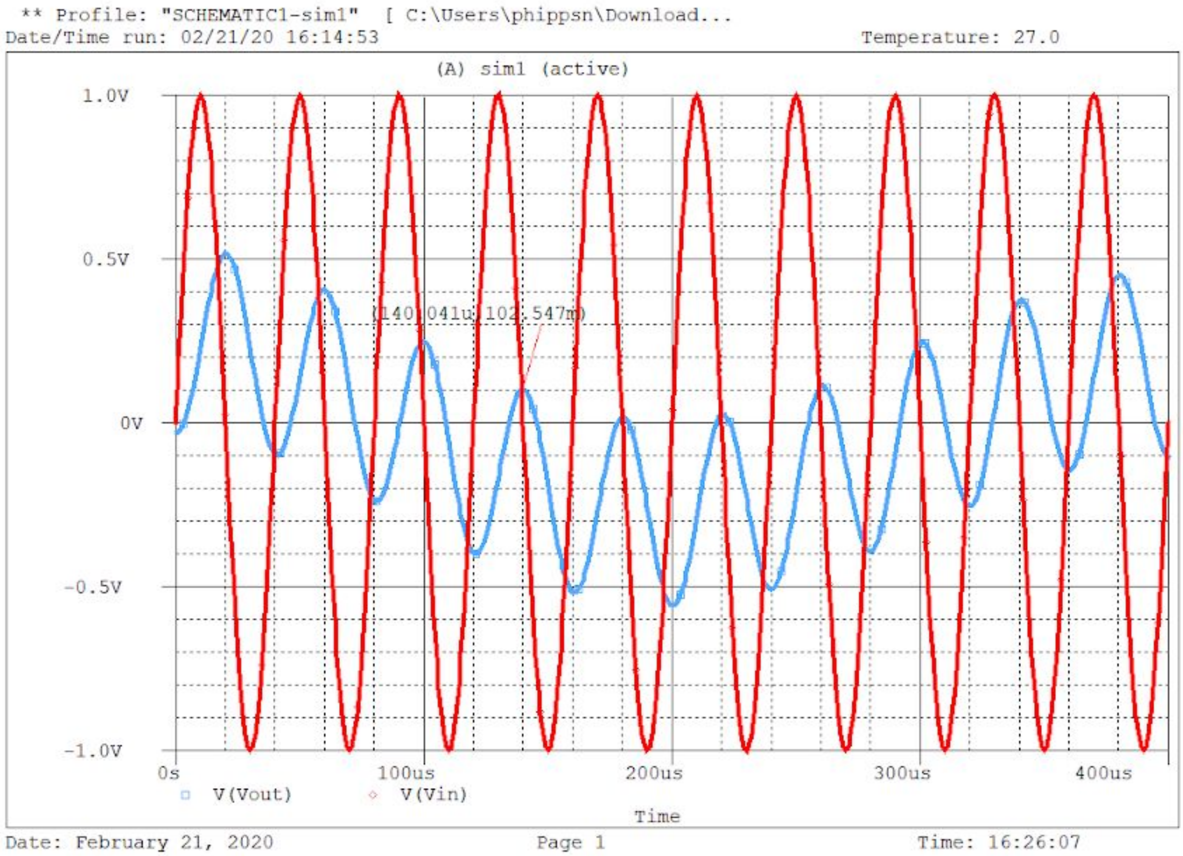
In figure 4 we observe a voltage versus time graph. Here, Vin lags Vout by 90 degrees, which makes sense for a bandpass filter. According to the table and graph, the max values for Vin are 1 V. The max value for vout was 266 mV, while the minimum value was -324 mV. Here, our max Vout value is less than the maximum value we would see at center frequency or in-between the two cutoffs. This lowered voltage is due to the frequency being below the left or lower cutoff frequency.



	Evaluate	Measurement	Value
	☑	Max(V(Vout))	17.81189
	☑	Min(V(Vout))	-17.81189
	☑	ZeroCross(V(Vout))	9.14956u
	☑	ZeroCross(V(Vin))	199.99624u

Figure 5: Part B, Graph and Table: Gain Voltage Versus Time: At 2500 Hz, at cutoff:

In figure 5 we observe a voltage versus time graph. Here, Vin is in phase with Vout. According to the table and graph, the max values for Vin are 1 V. The max value for vout was 17.8 V, while the minimum value was -17.8 V. The Vin appears to oscillate between consistent values, while the initial Vout appears to grow over a short time period; this behavior, in appearance, seems almost transient like. In observing the Pspice max value here, it can be seen that the waveform saturates at close to 18V because the DC input was 18V. Here we are observing the resonant peak, where Vout is maximum



	Evaluate	Measurement	Value
	☑	Max(V(Vout))	517.10668m
	☑	Min(V(Vout))	-556.70637m
	☐	ZeroCross(V(Vout))	
	☐	ZeroCross(V(Vin))	

Figure 6: Part B, Graph and Table: Gain Voltage Versus Time: At 25000 Hz, Above cutoff:

In figure 6 we observe a voltage versus time graph. Here the Vout values follow an oscillating pattern, but the oscillations themselves also appear to be following an oscillation curve as well. Again the table displays the max and min values for Vout, while the voltage input levels follow a typical oscillation scheme (as opposed to Vout's scheme), moving from +1 V to -1 V. Here the phase difference is 90 degrees, with Vin leading. Here, our max Vout value is less than the maximum value we would see at center frequency or in-between the two cutoffs. This lowered voltage is due to the frequency being below the right or higher cutoff frequency.

Part C:

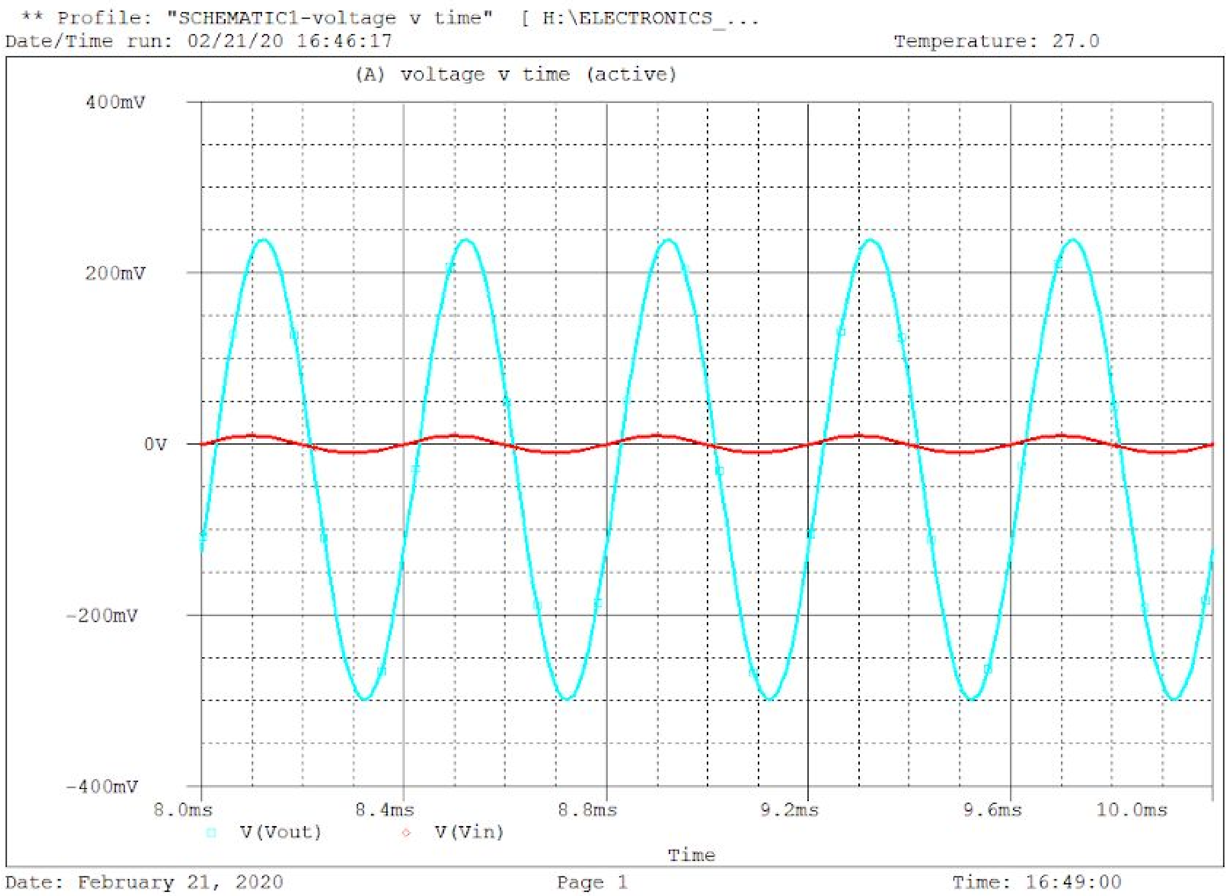
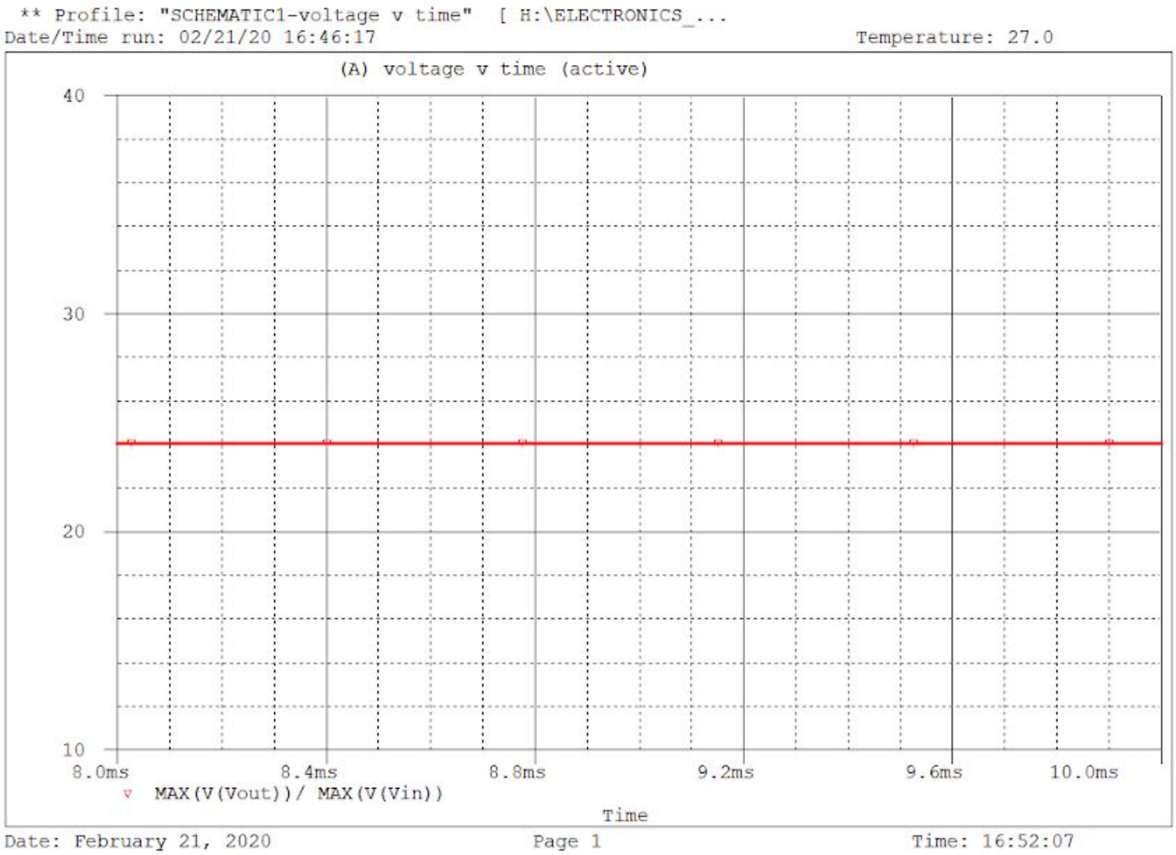


*Figure 7: Part C: V*oltage Versus Time Simulation for 10mV, at center frequency 2500 Hz

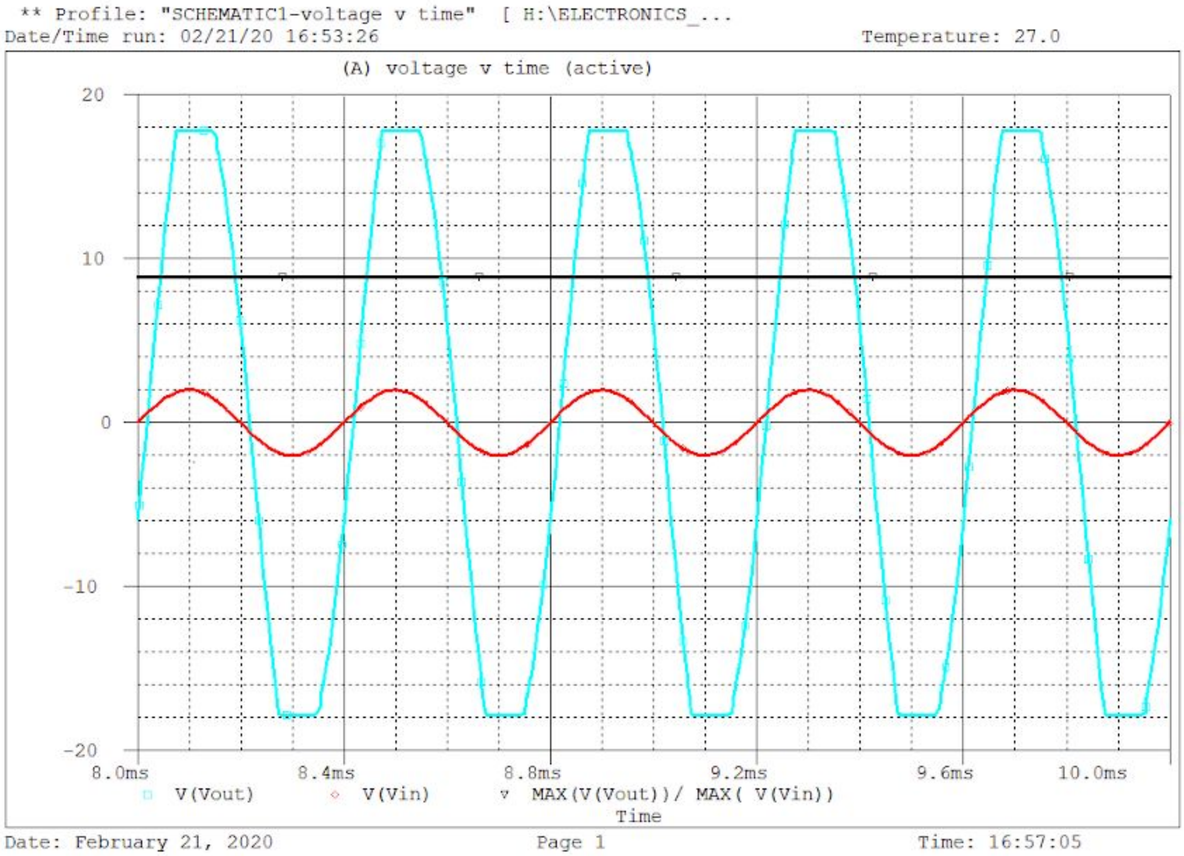
Analysis of Figure 7: See Figure 8 Analysis



	Evaluate	Measurement	Value
	☑	Max(V(Vout))	239.68710m
	☑	Max(V(Vin))	9.97043m
	☑	(MAX(V(Vout)))/(MAX(V(Vin)))	24.03980

Figure 8: Part C: Gain 10mV (this is the gain for Figure 7), at center frequency 2500 Hz:

In figures 7 and 8 we observe the Vin, Vout and gain (Vout/Vin). Specifically this is a voltage versus time plot. The graphs of Vin and Vout appear to be almost in phase, however, the input voltage appears to be leading by a slight amount. The slight phase difference of the input voltage to the output voltage is interesting, as, typically, the two voltages should be in phase at center frequency. This may have been due to some of the components (op amp, resistors, values, capacitors, Q value), or due to the nature of 2nd order filters. The gain level is 24 V/V. The max Vout level is 239 mV, while the input voltage was 10 mV.



Evaluate	Measurement	Value
☑	Max(V(Vout))	17.81190
☑	Max(V(Vin))	2.00000
☑	(MAX(V(Vout)))/(MAX(V(Vin)))	8.90597

Figure 9: Graphs and Tables: Input voltage at 2V, at center frequency 2500 Hz, Vin, Vout, Gain:

In figure 9 we observe a voltage versus time graph for Vin, Vout and gain (Vout/Vin). Here the voltage output appears to be saturating at about 18 volts (18 V is the dc input voltage). The phases for the voltage input and output appear to again be either in phase or almost in phase, while the input voltage may lead. Additionally, gain appears to be 8.9 V/V.

LAB SIMULATIONS

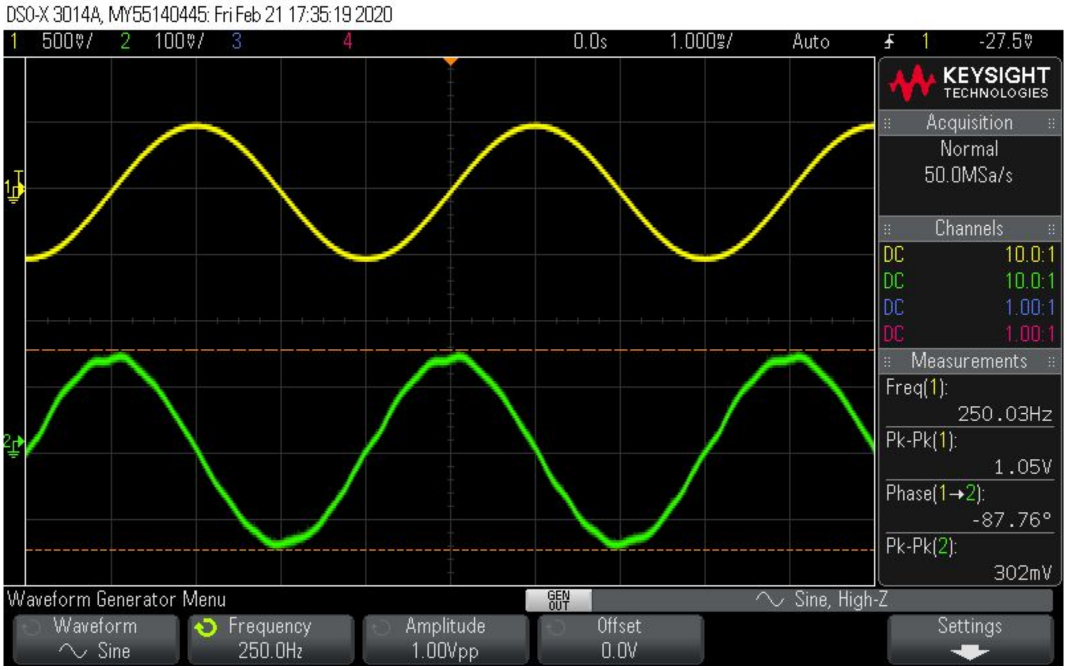


Figure 10: Hardware Simulations: 1 decade below center frequency, Vin: 1V, Vout: 302mV, Freq 250 hz:

In figure 10 we observe a phase relationship of Vout leading Vin by 87.7 degrees, one decade below the center frequency. Here, the output voltage is lower than when operating at center frequency because the frequency, here, operates below the lower cutoff.

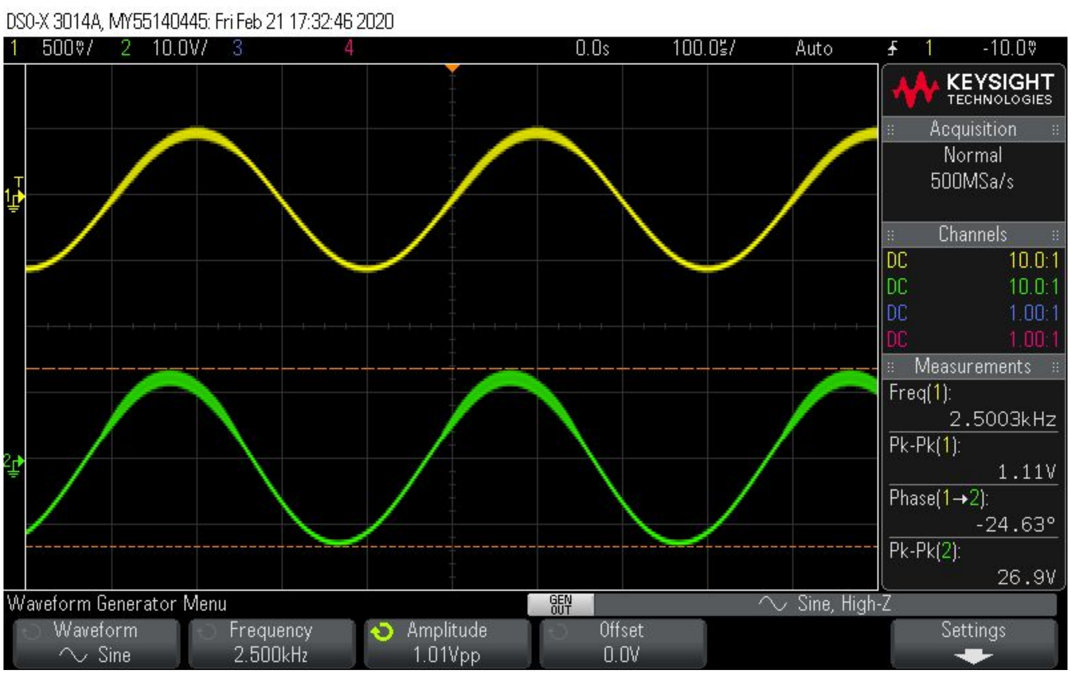


Figure 11: Hardware Simulations: At Center frequency, Vin: 1V, Vout: 26.9, Freq 2.5kHz:

In figure 11, there appears to be a slight phase shift with Vout leading Vin by 24 degrees. This phase shift is a bit interesting since the two waveforms should, in theory, be in phase. This may have been due to some of the components (op amp, resistors, values, capacitors, Q value), or due to the nature of 2nd order filters. The output voltage appears to be 26.9 V. The input voltage here is 1 V. Here the output voltage is maximum as compared to other frequencies, since this is at center frequency.

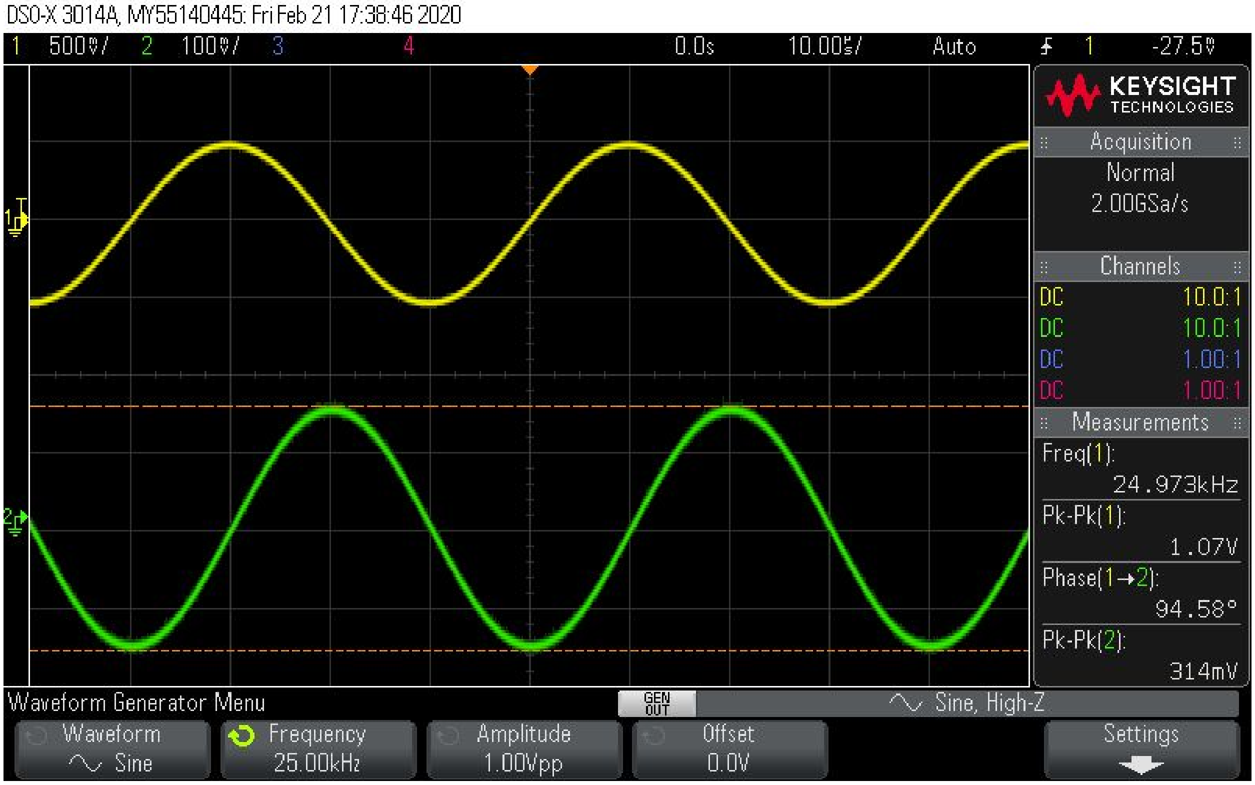
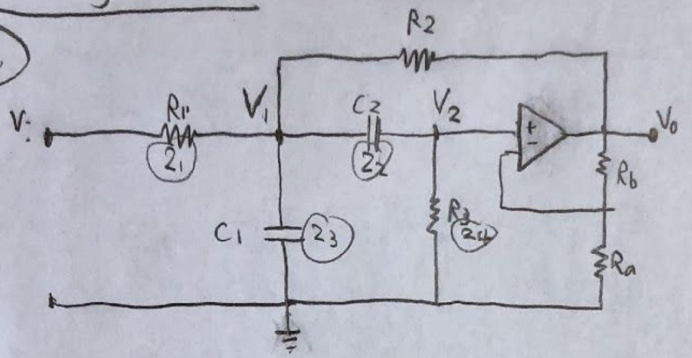


Figure 12: Hardware Simulations: 1 decade above center frequency, Vin: 1V, Vout: 318mV, Freq 25khz:
In figure 12 we observe the input voltage leading the output voltage by 94 degrees, at an input voltage of 1 V. Here the output voltage is less than it would be at center frequency since the frequency is above the right cutoff.

Appendix A: Calculations:

Preliminary Calculations

Step 1



$$\frac{1}{k} = \frac{R_a + R_b}{R_a}$$

$$\frac{1}{k} = 1 + \frac{R_b}{R_a}$$

$$V_2 = \frac{R_a}{R_a + R_b} V_{out} \qquad V_2 = kV_{out} \qquad k = \frac{R_a}{R_a + R_b}$$

$$\frac{V_1 - V_2}{Z_2} = \frac{V_2}{Z_4} \qquad V_1 = V_2\left(\frac{1}{Z_2} + \frac{1}{Z_4}\right)Z_2 = V_2\left(1 + \frac{Z_2}{Z_4}\right)$$

$$\frac{V_i - V_1}{Z_1} + \frac{V_o - V_1}{R_2} + \frac{V_2 - V_1}{Z_2} = \frac{V_1}{Z_3}$$

$$\frac{V_{in}}{Z_1} + \frac{V_{out}}{R_F} + \frac{V_2}{Z_2} = V_1\left(\frac{1}{Z_1} + \frac{1}{R_F} + \frac{1}{Z_2} + \frac{1}{Z_3}\right)$$

$$\Rightarrow V_2\left(1 + \frac{Z_2}{Z_4}\right)\left(\frac{1}{Z_1} + \frac{1}{R_2} + \frac{1}{Z_2} + \frac{1}{Z_3}\right)$$

$$V_2 = kv_{out}$$

$$\frac{V_{in}}{R_1} + \frac{V_{out}}{R_2} = kV_{out}\left[\left(1 + \frac{Z_2}{Z_4}\right)\left[\frac{1}{Z_1} + \frac{1}{R_2} + \frac{1}{Z_2} + \frac{1}{Z_3}\right] + \frac{1}{Z_2}\right)$$

$$\frac{V_{in}}{Z_1} = kv_{out}\left[k\left(\left(1 + \frac{Z_2}{Z_4}\right)\left(\frac{1}{Z_1} + \frac{1}{R_2} + \frac{1}{Z_2} + \frac{1}{Z_3}\right)\right) + \frac{1}{Z_2}\right) - \frac{1}{R_F}$$

$$H = \frac{V_{out}}{V_{in}} = \frac{1}{Z_1\left(k\left(\left(1 + \frac{Z_2}{Z_4}\right)\left(\frac{1}{Z_1} + \frac{1}{R_2} + \frac{1}{Z_2} + \frac{1}{Z_3}\right)\right) - \frac{1}{Z_2}\right) - \frac{1}{R_F}\Big)}$$

$$=$$

Calculation For Transfer Function:

$$H = \frac{1}{Z_1\left[k\left[\left(1+\frac{Z_2}{Z_4}\right)\left(\frac{1}{Z_1}+\frac{1}{R_f}+\frac{1}{Z_2}+\frac{1}{Z_3}\right)-\frac{1}{Z_2}\right]-\frac{1}{R_f}\right]}$$

$$H = \frac{1/k}{Z_1\left[\left(1+\frac{Z_2}{Z_4}\right)\left(\frac{1}{Z_1}+\frac{1}{R_f}+\frac{1}{Z_2}+\frac{1}{Z_3}\right)-\frac{1}{Z_2}\right]-\frac{Z_1}{kR_f}}$$

$$H = \frac{1/k}{1+\frac{Z_1}{R_f}+\frac{Z_1}{Z_3}+\frac{Z_2}{Z_4}+\frac{Z_1Z_2}{Z_4R_f}+\frac{Z_1}{Z_4}+\frac{Z_1Z_2}{Z_3Z_4}-\frac{Z_1}{kR_f}}$$

Step2

$$Z_1 = R_1 \qquad Z_4 = R_3 \qquad Z_2 = \frac{1}{j\omega C_2} \qquad Z_3 = \frac{1}{j\omega C_1}$$

$$H = \frac{1/k}{1+\frac{R_1}{R_f}+\frac{R_1}{R_3}+\frac{1}{j\omega C_2R_3}+\frac{R_1}{(j\omega C_2)R_fR_3}+\frac{R_1}{R_3}+\frac{R_1\,j\omega C_1}{(j\omega C_2)R_3R_3}-\frac{R_1}{kR_f}}$$

$$H = \frac{\left(1+\frac{R_b}{R_a}\right)\frac{1}{R_1C_1}j\omega}{(j\omega)^2+\left(\frac{1}{R_2C_1}+\frac{1}{R_2C_2}+\frac{1}{R_1C_1}-\frac{R_b}{R_aR_fC_1}\right)j\omega+\frac{R_1+R_2}{R_2R_1R_3C_1C_2}} \qquad Z_4 = R_3$$

Step3

$$R_1 = R_2 = \frac{R_3}{2} = R \qquad C_1 = C_2 = C \qquad s = j\omega$$

$$R_3 = 2R_1 = 2R_2$$

$$H = \frac{ks\left(\frac{1}{RC}\right)}{s^2+\left(\frac{3}{RC}-\frac{R_b}{R_aRC}\right)j\omega+\frac{2R}{2R^3C^2}}$$

$$\boxed{H = \frac{ks\left(\frac{1}{RC}\right)}{s^2+s\left(\frac{3-k}{RC}\right)+\frac{1}{R^2C^2}}}$$

Calculation For Transfer Function Continued:

4.) Calculate values for W_o, R, C, R_5, R_4

$$H(s) = \frac{H_o\left(\frac{w_o}{Q}\right)s}{s^2+\left(\frac{w_o}{Q}\right)s+w_o^2}$$

$$f_o = \frac{w_o}{2\pi} = \frac{\frac{1}{RC}}{2\pi} \qquad 2\pi f_o = \frac{1}{RC}$$

$$RC = \frac{1}{2\pi f_o}$$

Desing center frequency of 2500 HZ

RC

Choose 0.001uF $\quad R = \frac{1}{2\pi f_o C} = \frac{1}{2\pi(2500\text{Hz})(0.001\mu F)}$

$$R = 63{,}666 = 64k$$

$$R_1 = R_2 = \frac{R_3}{2} = R$$

$$\boxed{\begin{aligned} R_1 &= 64k \\ R_2 &= 64k \\ R_3 &= 128k \end{aligned}}$$

$$C_1 = C_2 = C$$

$$\boxed{C_1 = C_2 = 0.001\mu F}$$

$Q=10 \quad R_{in} > 10k$

$$\frac{W_o}{Q} = \frac{3-k}{RC} \qquad k = 1+\frac{R_F}{R} \qquad k = 1+\frac{R_4}{R_5}$$

$$\frac{1}{Q} = 3-k = 3-\left(1+\frac{R_4}{R_5}\right)$$

$$\frac{1}{Q} = 2+\frac{R_4}{R_5}$$

$Q=10$

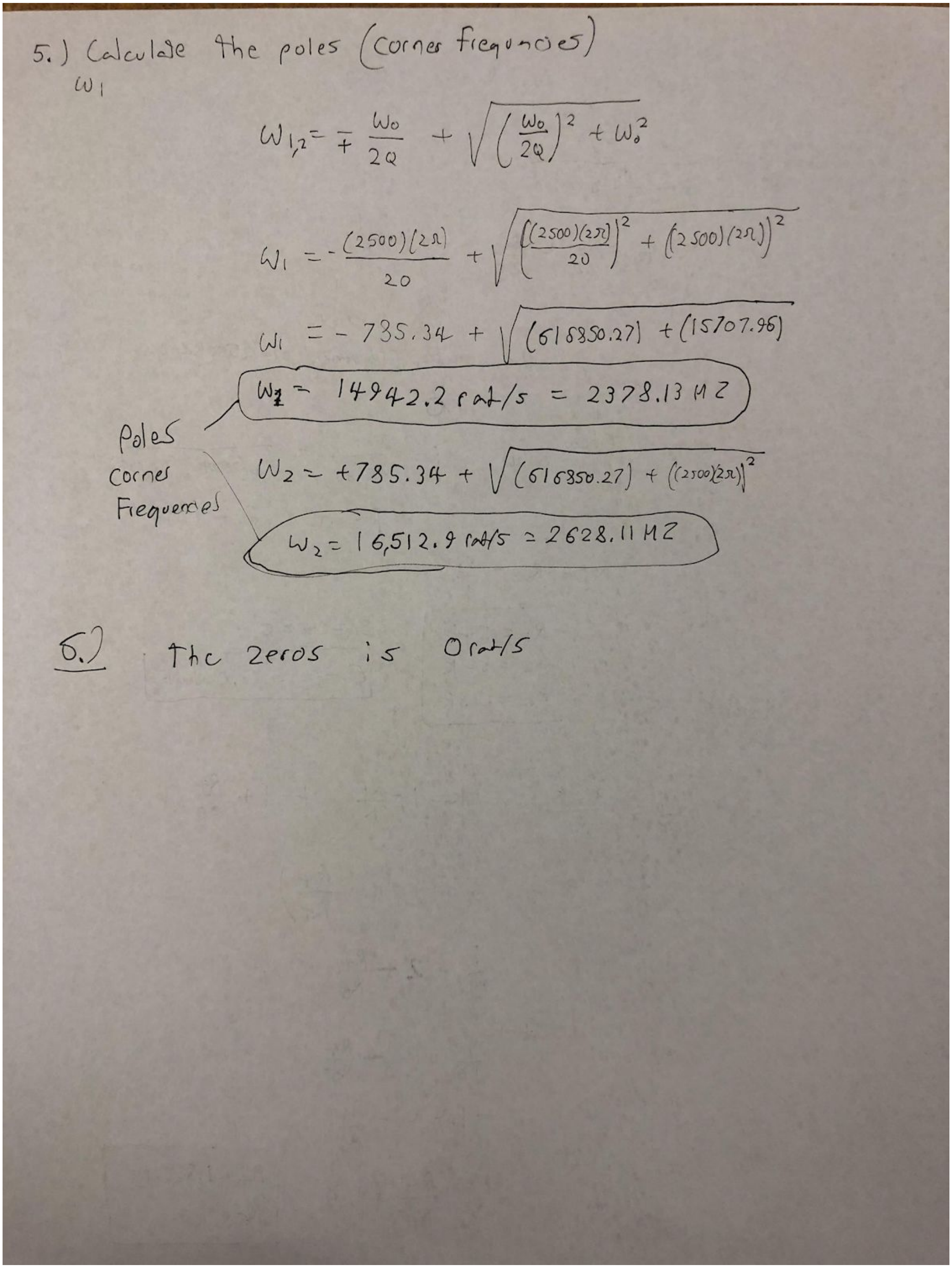
$$\frac{1}{10} = 2+\frac{R_4}{R_5}$$

$$-1.9 = -\frac{R_4}{R_5}$$

$$\frac{R_4}{R_5} = 1.9$$

$$\boxed{\begin{aligned} R_4 &= 1.875k \\ R_5 &= 1k \end{aligned}}$$

Calculations for Wo, R, C, R5, R4


5.) Calculate the poles (Corner Frequencies)

ω_1

$$\omega_{1,2} = \mp \frac{\omega_0}{2Q} + \sqrt{\left(\frac{\omega_0}{2Q}\right)^2 + \omega_0^2}$$

$$\omega_1 = -\frac{(2500)(2\pi)}{20} + \sqrt{\left(\frac{(2500)(2\pi)}{20}\right)^2 + \left((2500)(2\pi)\right)^2}$$

$$\omega_1 = -735.34 + \sqrt{(618850.27) + (15707.96)}$$

$$\boxed{W_1 = 14942.2 \text{ rad/s} = 2378.13 \text{ HZ}}$$

Poles Corner Frequencies

$$W_2 = +785.34 + \sqrt{(616850.27) + \left((2500)(2\pi)\right)^2}$$

$$\boxed{W_2 = 16{,}512.9 \text{ rad/s} \approx 2628.11 \text{ HZ}}$$

6.) The zeros is 0 rad/s

Poles and Zeros Calculations

Appendix B: Equations:

$$H(s) = \frac{\boxed{Ho\text{-}s} \; \omega_p}{s^2+\frac{\omega_p}{Q_p}s+\omega_p^2}$$ where, QP = (1)

$$BW = \frac{f_p}{Q_p}$$ (2)

$$\frac{VOUT}{VIN} \quad \frac{K(\frac{s}{C_1R_1})}{s^2+s\left(\frac{1}{?}+\frac{1}{?}+\frac{1}{?}+\frac{1-k}{?}\right)+\frac{1}{?}+\frac{1}{?}}$$ (3)

$$R_1 = R_2 = \frac{R_3}{?} = R$$ (4)

$$C_1 = C_2 = C$$ (5)

Vout

El(s) - $= \frac{K\,s\,(\frac{1}{RC})}{s^2+s\left(\frac{3-K}{RC}\right)+\frac{1}{R^2C^2}}$ (6)

$$H(s) = \frac{H_O\left(\frac{\omega_o}{Q}\right)s}{s^2+\left(\frac{\omega_o}{Q}\right)s+\omega_o^2}$$ (7)

02 = — + $\sqrt{\left(\frac{\omega_o}{2Q}\right)^2 + \omega_o^2}$ **(9)**

Equations:

Our applied Q value was 9.43, where as the ideal Q value was to be 10 as specified by the lab rubric

Percent Error For Quality Factor Q: