4MM013/UM1: **Computational Mathematics**

Welcome to Week 12

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Lecture 12

Revision of Quizzes

In the following slides, I have selected some of the quiz questions which seem to be difficult.

Week 2 Quiz

1 Matching 10 points Expressing set in different forms

A set represented as $\mathcal{A} = \{4, 5, 6, 7, 8, 9\}$ is called Roster form while the same set represented as $\mathcal{A} = \{x : x \text{ is a natural number and } x < 3 < 10\}$ is called Set-builder form.
Match each of the set on the left represented in the Roster form with the same set on the right represented in Set-builder form:

{P, R, I, N, C, A, L}	{ x: x is a letter of the word PRINCIPAL }
{0}	{x: x is an integer and x + 1 = 1}
{1, 2, 3, 6,9,18}	{x: x is a positive integer and is a divisor of 18}
{3, -3}	{x: x is an integer and x*x - 9 = 0}

Week 2 Quiz

2 Multiple answer 5 points Set inclusion

Let $\mathcal{A} = \{0, 2, 4, 6, 8, 10, \cdots\}$ and $\mathcal{B} = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, \cdots\}$. Select all correct answers.

- [x] $\mathcal{A}$ is subset of $\mathcal{B}$
- [x] $\mathcal{A}$ is proper subset of $\mathcal{B}$
- [x] $\mathcal{A}$ is set of positive even numbers
- [ ] None of above is True

Week 2 Quiz

4 Multiple choice 20 points Function Definition

Let $\mathbb{R}$ represents a set of real-numbers and $\mathbb{R}^+$ represents a set of positive real-numbers. Which of the following is not a function?

- [x] $f : \mathbb{R}^+ \cup \{0\} \rightarrow \mathbb{R}$ given by $f(x) = \pm\sqrt{x}$
- [ ] $f : \mathbb{R} \rightarrow \mathbb{R}$ given by $f(x) = x^2$
- [ ] $f : \mathbb{R} \rightarrow \mathbb{R}$ given by $f(x) = e^x$
- [ ] $f : \mathbb{R}^+ \rightarrow \mathbb{R}$ given by $f(x) = \log_e(x)$

Week 2 Quiz

5 Multiple choice 20 points Type of Functions

Let $\mathbb{R}$ represents a set of real-numbers. The function $f : \mathbb{R} \to \mathbb{R}$ given by $f(x) = x^3$ is of type:

- ◉ One-to-one (injective) and Onto (surjective)
- ○ Many-to-one and Onto
- ○ Onto
- ○ One-to-one

Week 2 Quiz

6 Multiple choice 5 points Invertible Function

A function is invertible if it is

- ○ Many-to-one
- ○ Onto
- ◉ One-to-one and Onto
- ○ One-to-one

Week 2 Quiz

7 Multiple choice 20 points Invertible Function

Let $\mathbb{R}$ represents a set of real-numbers.
Which one of the following is the inverse of the function $f : (2, \infty) \to \mathbb{R}$ given by $f(x) = \log_e(x-2)$?
Note the representations:

$(a, b) = \{x : x \in \mathbb{R}, a < x < b\}$

$(a, b] = \{x : x \in \mathbb{R}, a < x \leq b\}$

$[a, b] = \{x : x \in \mathbb{R}, a \leq x \leq b\}$

$[a, b) = \{x : x \in \mathbb{R}, a \leq x < b\}$

- (selected) $g : \mathbb{R} \to (2, \infty)$ given by $g(x) = e^x + 2$
- $g : \mathbb{R} \to (-2, \infty)$ given by $g(x) = e^x - 2$
- $g : \mathbb{R} \to (-\infty, 2)$ given by $g(x) = 2 - e^x$
- $g : [0, \infty) \to [2, \infty)$ given by $g(x) = x^e + 2$

Week 4 Quiz

4 Matching 15 points Conic Sections

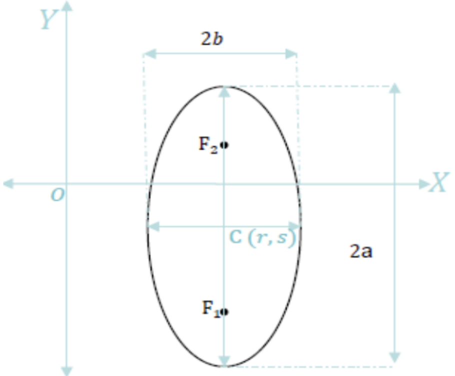
Match correctly the following parabola equations (with variables x and y) with the correct descriptions on the right-hand-side.
Equation 1: $(y-k)^2 = 4a(x-h)$
Equation 2: $(y-k)^2 = -4a(x-h)$
Equation 3: $(x-h)^2 = 4a(y-k)$
Equation 4: $(x-k)^2 = -4a(y-h)$
where $a, h,$ k are positive numbers.

Equation 1	a parabola that opens toward the right side
Equation 2	a parabola that opens toward the left side
Equation 3	a parabola that opens upward
Equation 4	a parabola that opens downward

Week 4 Quiz

5 Multiple choice 10 points Conic Section

Which one of the following equations correctly represent the ellipse shown in the figure below?



O represent origin, $C(r, s)$ represents the centre, and F1 and F2 represent foci.

- $\frac{(x-r)^2}{b^2} + \frac{(y-s)^2}{a^2} = 1$
- $\frac{(x-r)^2}{a^2} + \frac{(y-s)^2}{b^2} = 1$
- $\frac{(y-r)^2}{b^2} + \frac{(x-s)^2}{a^2} = 1$
- $\frac{(x-r)^2}{a} + \frac{(y-s)^2}{b} = 1$

Week 4 Quiz

6 Multiple choice 10 points Inequalities

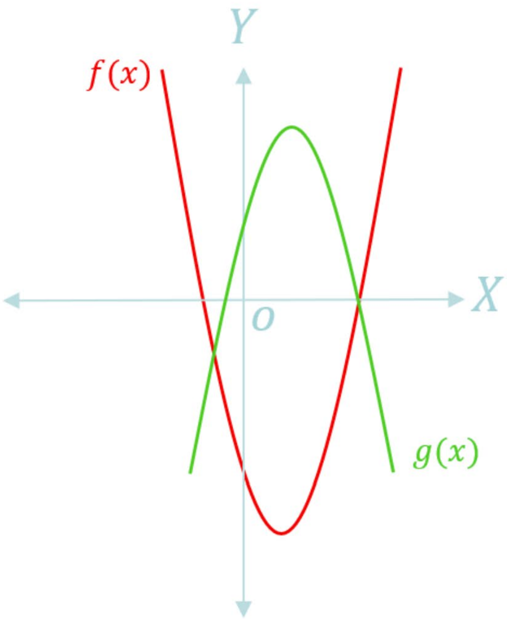
Choose the correct set of value of x satisfying the inequality:
$3x - 5 < x + 8$ and $5x > x - 8$.

- $-2 < x < 6.5$
- $-2 > x > 6.5$
- $-2 \leq x \leq 6.5$
- $-2 \geq x \geq 6.5$

Week 4 Quiz

8 Multiple choice 20 points Inequalities

Choose the correct set of values of x, which satisfy $f(x) < g(x)$, where $f(x) = x^2 - 4x - 12$ and $g(x) = 6 + 5x - x^2$.
Hint: The graphs of $f(x)$ and $g(x)$ given below can help.

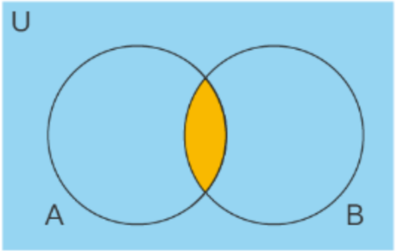


- $-\frac{3}{2} < x < 6$
- $-\frac{3}{2} > x > 6$
- $-3 \geq x \geq 6$
- $-2 < x < 5$

Week 5 Quiz

1 Multiple answer 15 points Sets

Let $\mathcal{A}$ and $\mathcal{B}$ be two set in the universe $\mathcal{U}$. $\mathcal{A}^C$ represent the Complement set of $\mathcal{A}$. Select the correct statements.
Hint: See the figure to visualise it.

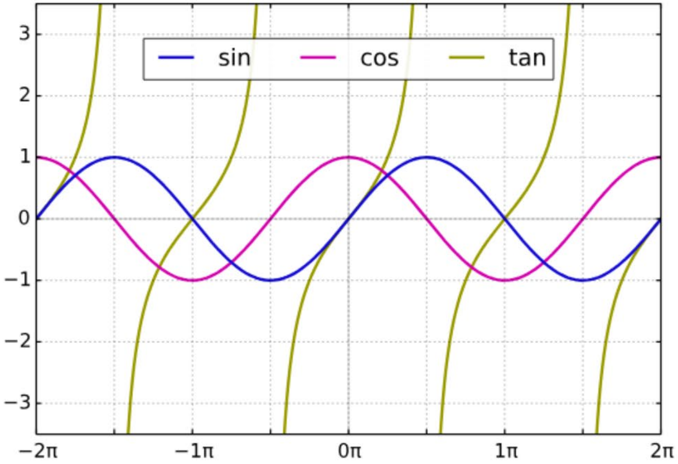


- [x] $(\mathcal{A} \cup \mathcal{B})^C = \mathcal{A}^C \cap \mathcal{B}^C$
- [x] $(\mathcal{A} \cap \mathcal{B})^C = \mathcal{A}^C \cup \mathcal{B}^C$
- [ ] $\mathcal{A} \cup \mathcal{B} = \mathcal{A} \cup \mathcal{U}$
- [ ] $(\mathcal{A} \cup \mathcal{B})^C = A \backslash B$

Week 5 Quiz

2 Multiple choice 10 points Functions

A function f is called periodic function with period $T > 0$ if $f(x) = f(nT + x)$ where n is an integer. What is the period of the trigonometric functions: $sin(x)$ and $tan(x)$, respectively?
Hint: See the graph

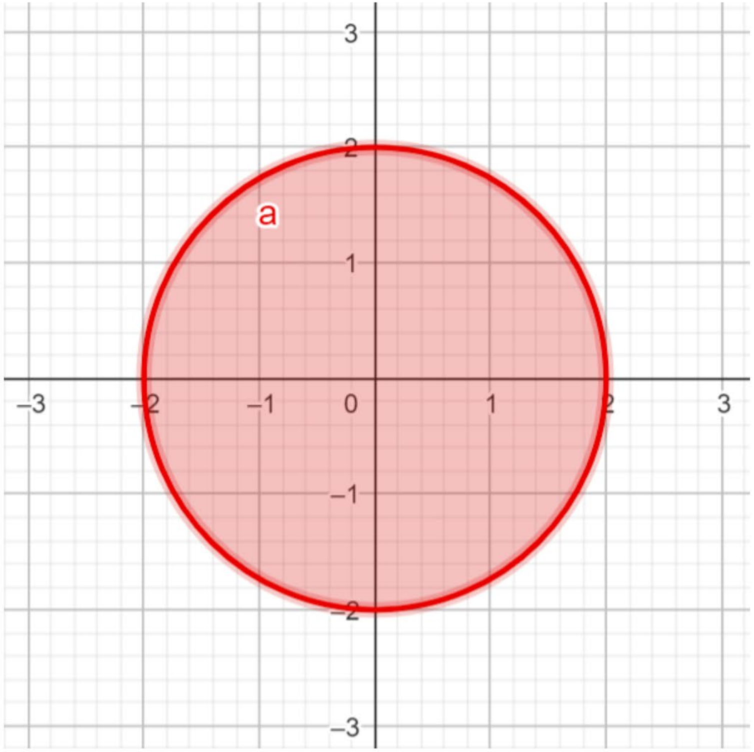


- 2π and π
- 2π and 2π
- $\frac{\pi}{2}$ and π
- $\frac{\pi}{2}$ and $\frac{\pi}{2}$

Week 5 Quiz

3 Multiple choice 10 points Inequalities

Which one of the graphs correctly depict set of values of pairs (x, y) for which $x^2 + y^2 \leq 4$? The red shaded region represents the inequality. The horizontal bold line represents the X-axis and vertical bold line represents Y-axis.
Hint: Select a few arbitrary points in the shaded region and check if those points satisfy the inequality.



Week 6 Quiz

7 Multiple answer 10 points Vector Inequalities

Given two vectors $\vec{a}$ and $\vec{b}$, which of these following identities are true?

- [x] $|\vec{a}+\vec{b}| \leq |\vec{a}| + |\vec{b}|$
- [x] $|\vec{a}\cdot\vec{b}| \leq |\vec{a}||\vec{b}|$
- [ ] $|\vec{a}+\vec{b}| \geq |\vec{a}| + |\vec{b}|$
- [ ] $|\vec{a}\cdot\vec{b}| \geq |\vec{a}||\vec{b}|$

Week 7 Quiz

4 Multiple choice 10 points Vector Projection

Consider Scalar-Valued Function $f : \mathbb{R}^n \times \mathbb{R}^n \rightarrow \mathbb{R}$ given by $f(\mathbf{u}, \mathbf{v}) = \sum_{i=1}^{n} u_i v_i$, where u_i represents ith component of $\mathbf{u}$. What is the output of the function for the input vectors: $\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{bmatrix}$ and $\begin{bmatrix} 6 \\ 5 \\ 4 \\ 3 \\ 2 \\ 1 \end{bmatrix}$?

- (selected) 56
- $\begin{bmatrix} 6 \\ 10 \\ 12 \\ 12 \\ 10 \\ 6 \end{bmatrix}$
- 81
- 21

Week 7 Quiz

7 Multiple answer 15 points Symmetric Matrix

For any square matrix $\mathbf{A}$, which of the following are Symmetric matrices?

Hint: An example of symmetric matrix is $\begin{bmatrix} a & b \\ b & c \end{bmatrix}$ or $\begin{bmatrix} a & b & c \\ b & d & e \\ c & e & f \end{bmatrix}$.

- [x] $\mathbf{A} + \mathbf{A}^T$
- [x] $\mathbf{A}^T\mathbf{A}$
- [x] $\mathbf{A}\mathbf{A}^T$
- [ ] $\mathbf{A} - \mathbf{A}^T$

Week 7 Quiz

8 Multiple choice 10 points Matrix

A matrix of order $m \times n$ are usually represented as $\mathbf{A} = [a_{ij}]_{m\times n}$ where a_{ij} represents the element at ith row and jth column. Which of the following matrices is given by $a_{ij} = \frac{ij}{i+j}$?

- [x] $\begin{bmatrix} \frac{1}{2} & \frac{2}{3} & \frac{3}{4} \\ \frac{2}{3} & 1 & \frac{6}{5} \\ \frac{3}{4} & \frac{6}{5} & \frac{3}{2} \end{bmatrix}$
- [ ] $\begin{bmatrix} \frac{1}{2} & \frac{2}{3} & \frac{3}{4} \\ \frac{3}{2} & 1 & \frac{6}{5} \\ \frac{3}{4} & \frac{6}{5} & \frac{2}{3} \end{bmatrix}$
- [ ] $\begin{bmatrix} \frac{1}{2} & \frac{2}{3} & \frac{3}{4} \\ \frac{2}{3} & \frac{1}{2} & \frac{6}{5} \\ \frac{4}{3} & \frac{6}{5} & \frac{3}{2} \end{bmatrix}$
- [ ] Not possible to find

Week 7 Quiz

9 Multiple choice 20 points Symmetric and Skew Symmetric Matrices

A matrix for form $\begin{bmatrix} a & b \\ b & c \end{bmatrix}$ or $\begin{bmatrix} a & b & c \\ b & d & e \\ c & e & f \end{bmatrix}$ is called symmetric matrix, and a matrix of form $\begin{bmatrix} 0 & -b \\ b & 0 \end{bmatrix}$ or $\begin{bmatrix} 0 & -b & c \\ b & 0 & -e \\ -c & e & 0 \end{bmatrix}$ is called Skew symmetric.

If $\mathbf{A}, \mathbf{B}$ are symmetric matrices of same order, then $\mathbf{AB} - \mathbf{BA}$ is a

- (●) Skew symmetric matrix
- ( ) Symmetric matrix
- ( ) Zero matrix
- ( ) Identity matrix

Week 8 Quiz

1 Multiple choice 5 points Representing Linear System in Matrix Form

Given the System of Linear Equations:

$$a_{11}x_1 + a_{12}x_2 + a_{11}x_3 + a_{13}x_4 = b_1$$
$$a_{21}x_1 + a_{22}x_2 + a_{21}x_3 + a_{23}x_4 = b_2$$
$$a_{31}x_1 + a_{32}x_2 + a_{31}x_3 + a_{33}x_4 = b_3$$
$$a_{41}x_1 + a_{42}x_2 + a_{41}x_3 + a_{43}x_4 = b_4$$

where x_i are the unknows to be estimated, a_{ij} and b_i are known quantities. What is correct representation of this system in Matrix-vector form?

- [x] $$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \begin{bmatrix} x_1 \\ x_3 \\ x_2 \\ x_4 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_3 \\ b_2 \\ b_4 \end{bmatrix}$$
- [ ] $$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}$$
- [ ] $$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_4 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}$$
- [ ] $$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}$$

Week 8 Quiz

2 Multiple choice 5 points Linear Combination

Given the matrix $\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix}$ and the real numbers $\alpha_1, \alpha_2, \alpha_3,$ and α_4, which one of them represents the linear combination of the **rows** of the matrix?

- ( ) $\alpha_1 \begin{bmatrix} a_{11} \\ a_{21} \\ a_{31} \\ a_{41} \end{bmatrix} + \alpha_2 \begin{bmatrix} a_{12} \\ a_{22} \\ a_{32} \\ a_{42} \end{bmatrix} + \alpha_3 \begin{bmatrix} a_{13} \\ a_{23} \\ a_{33} \\ a_{43} \end{bmatrix} + \alpha_4 \begin{bmatrix} a_{14} \\ a_{24} \\ a_{34} \\ a_{44} \end{bmatrix}$
- (●) $\alpha_1 \begin{bmatrix} a_{11} \\ a_{12} \\ a_{13} \\ a_{14} \end{bmatrix} + \alpha_2 \begin{bmatrix} a_{21} \\ a_{22} \\ a_{23} \\ a_{24} \end{bmatrix} + \alpha_3 \begin{bmatrix} a_{31} \\ a_{32} \\ a_{33} \\ a_{34} \end{bmatrix} + \alpha_4 \begin{bmatrix} a_{41} \\ a_{42} \\ a_{43} \\ a_{44} \end{bmatrix}$

Week 8 Quiz

3 Multiple choice 10 points Linear Dependence and Independence

A set of vectors $\{\mathbf{v}_1, \mathbf{v}_2, \cdots, \mathbf{v}_n\} \in \mathbb{R}^m$ is said to be ***linearly independent*** if

- (selected) the linear combination of the vectors
  $\alpha_1 \mathbf{v}_1 + \alpha_2 \mathbf{v}_2 + \cdots + \alpha_n \mathbf{v}_n = \mathbf{0}$
  if and only if the real numbers $\alpha_1 = \alpha_2 = \cdots = \alpha_n = 0$.
- the linear combination of the vectors
  $\alpha_1 \mathbf{v}_1 + \alpha_2 \mathbf{v}_2 + \cdots + \alpha_n \mathbf{v}_n = \mathbf{0}$
  if there exists real numbers $\alpha_1, \alpha_2, \cdots, \alpha_n$, not all equal to zero.
- one of the vector $\mathbf{v}_i$ can be obtained by linear combination of other remaining vectors, i.e., $\mathbf{v}_i = \sum_{j=1, j\neq i}^{n-1} \alpha_j \mathbf{v_j}$ where α_j are real numbers.

Week 8 Quiz

5 Matching 15 points Linear Dependence and Independence

Given four sets of vectors: $S1 = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$, $S2 = \left\{ \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$,

$$S3 = \left\{ \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \right\}, \text{ and } S4 = \left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} -1 \\ -2 \\ -3 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix} \right\}$$

Match the left hand side items with right hand side characteristics.

S1	set of linearly Independent vectors
S2	set of linearly Independent vectors
S3	set of linearly dependent vectors
S4	set of linearly dependent vectors

Week 8 Quiz

9 Multiple choice 10 points Determinant of Matrix

What is the determinant of the matrix $\begin{bmatrix} -1 & 2 & 2 \\ 3 & -6 & 4 \\ 5 & -10 & -3 \end{bmatrix}$?

Hint: Check if the columns or the rows are independent vectors.

- 0
- -10
- 10
- None of the other choices.

Week 8 Quiz

10 Multiple choice 20 points Inverse of Matrix

What is the inverse of the matrix $\mathbf{B} = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$?

- $\begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$
- $\begin{bmatrix} 7 & -3 & -3 \\ 1 & -1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$
- $\begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 1 & 0 \end{bmatrix}$
- $\begin{bmatrix} 7 & 3 & 3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$

Week 9 Quiz

3 Multiple answer 10 points Limit of a function

Let f and g be two function such that $\lim_{x\to a} f(x) = L_1$ and $\lim_{x\to a} g(x) = L_2$, where L_1 and L_2 are two real numbers, then which of the following statements are True (select all True statements)?

- [x] $\lim_{x\to a} (f(x) \pm g(x)) = \lim_{x\to a} f(x) \pm \lim_{x\to a} g(x) = L_1 \pm L_2$
- [x] $\lim_{x\to a} (f(x) \cdot g(x)) = \lim_{x\to a} f(x) \cdot \lim_{x\to a} g(x) = L_1 L_2$
- [x] $\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)} = \frac{L_1}{L_2}$ for $L_2 \neq 0$
- [x] All the choices are True.

Week 9 Quiz

6 Multiple choice 10 points Continuity and Differentiability of a function

Which one of the following statements is true?

- ◉ If the function $f(x)$ is differentiable at $x = a$, then $f(x)$ is always continuous at $x = a$.
- ○ If the function $f(x)$ is continuous at $x = a$, then $f(x)$ is always differentiable at $x = a$.
- ○ If $f(a)$ is defined and $lim_{x \to a} f(x) = f(a)$, then $f(x)$ is always differentiable at $x = a$.
- ○ The function $f(x) = |x| = \begin{cases} -x, & \text{when } x < 0 \\ x, & \text{when } x \geq 0 \end{cases}$, $x \in \mathbb{R}$ is differentiable at $x = 0$.

Week 9 Quiz

9 Multiple choice 20 points Application of Derivatives

Let $f(x) = 2x^3 - 15x^2 + 24x + 6$. The stationary points of the curve $y = f(x)$ are:

- (selected) Local maximum at $(1, 17)$ and local minimum at $(4, -10)$
- Local maximum at $(4, -10)$ and local minimum at $(1, 17)$
- Local maximum at $(-4, -10)$ and local minimum at $(-1, 17)$
- Local maximum at $(-1, 17)$ and local minimum at (-4, -10)

Week 10 Quiz

7 Multiple answer 15 points Indefinite Integral Properties

Let $f(x)$ and $g(x)$ be two functions. Which statements are True ? **Select all True statements.**

- [x] $\frac{d}{dx}\int f(x)dx = f(x)$ and $\int \frac{d}{dx} f(x)dx = f(x) + C$, where C is an arbitrary constant.
- [x] $\int [f(x) \pm g(x)]\, dx = \int f(x)dx \pm \int g(x)dx$
- [x] For any real number k, $\int kf(x)dx = k\int f(x)dx$
- [x] All the choices are True

Week 10 Quiz

9 Multiple answer 20 points Application of Derivatives

The equation $\frac{x^2}{2} + \frac{y^2}{3} = 1$ represents an ellipse. Which of the following equations represent the tangent lines to the ellipse at point $x = 1$ (select all correct answers)?

Hint: Equation of a tangent line to a curve $y = f(x)$ at point $x = a$ is given by: $y - f(a) = f'(a)(x - a)$, where $f'(a)$ represents derivative of $f(x)$ at $x = a$.

- [x] $y - \sqrt{\frac{3}{2}} = -\sqrt{\frac{3}{2}}(x - 1)$
- [x] $y + \sqrt{\frac{3}{2}} = \sqrt{\frac{3}{2}}(x - 1)$
- [ ] $y - \sqrt{\frac{3}{2}} = -\sqrt{\frac{2}{3}}(x - 1)$
- [ ] $y - \sqrt{\frac{2}{3}} = \sqrt{\frac{2}{3}}(x - 1)$