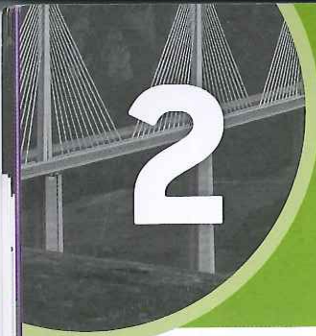


2 FOUNDATIONS OF PHYSICS

2.1 Quantities and units

Specification reference: 2.1.1, 2.1.2

Learning outcomes

Demonstrate knowledge, understanding, and application of:

→ units for physical quantities

→ SI base quantities and units, their symbols and prefixes.

▲ **Figure 1** *The correct use of units would have prevented the destruction of the Mars Climate Orbiter*

Measurements

Measurements are very important in physics. Not only must they be recorded accurately, they must also be communicated clearly. In 1998 NASA launched the Mars Climate Orbiter, a mission costing almost £195 million. When the probe arrived at Mars a few months later, it disintegrated in the planet's upper atmosphere instead of going into orbit. The disaster had a simple cause: one of NASA's teams worked in feet and pounds, whilst the other team worked in metres and kilograms. Each team assumed that the other was using the same units.

In A Level Physics, failure to use units correctly may not cost millions of pounds but it will cost you valuable marks in the examination.

Quantities

A physical **quantity** is a property of an object or of a phenomenon that can be measured. Some quantities are just numbers. For example, proton number, efficiency, and magnification are numbers. They have a numerical magnitude or size, but no units. Many other quantities consist of numbers *and* units. For example, length is a quantity that has units. It has many different units, including metres, inches, and miles. To avoid problems like the one NASA experienced with the Mars Climate Orbiter, scientists use a standard system of units called the *Système International d'Unités* (International System of Units), abbreviated to **SI**.

SI base units

SI is built around seven **base units**, six of which are shown in Table 1. The seventh unit, the unit for luminous intensity (the candela, cd), is not assessed in the A Level Physics course.

▼ **Table 1** *SI base units*

Quantity	Base unit	Unit symbol
length	metre	m
mass	kilogram	kg
time	second	s
electric current	ampère	A
temperature	kelvin	K
amount of substance	mole	mol

Symbols

A unit symbol is written in lower case, for example, m rather than M for metres, unless the unit is named after a person. In that situation, its name still begins with a lower-case letter but its symbol has a capital letter. The unit of electric current is named after André-Marie Ampère, so its name is the ampère (often just amp) and its symbol is A.

Prefixes

SI uses prefixes to show multiples and fractions of units (Table 2). For example, km stands for kilometre. The **prefix** is the 'kilo', and the unit is the 'metre'.

Notice that, apart from k for kilo, the prefixes for multiples all have initial capitals. Similarly, the prefixes for fractions are all lower case (μ is the lower-case Greek letter mu).

Worked example: Using prefixes

a Convert 1.25 kA into A.

$1.25\,\text{kA} = 1.25 \times 10^3\,\text{A}$ (or 1250 A)

b Convert 234 μm into m.

$234\,\mu\text{m} = 234 \times 10^{-6}\,\text{m} = 2.34 \times 10^{-4}\,\text{m}$

c Convert 0.567 s into ms.

There are 10^3 ms in 1 s. To change from seconds to milliseconds, you have to *multiply* by a factor of 10^3.

Therefore, $0.567\,\text{s} = 0.567 \times 10^3 = 567\,\text{ms}$

Summary questions

1 A student records the following figures in his notes: 60 cm and 40 ms.
- **a** Name the two quantities being measured. *(2 marks)*
- **b** Change these measurements into their base units. *(2 marks)*

2 **a** A collision between two molecules lasts for about 100 picoseconds. Write this time in seconds. *(1 mark)*
- **b** A chemical bond is approximately 0.15 nanometres long. Write this length in metres. *(1 mark)*
- **c** The Sun's core has a temperature of approximately 16 megakelvin. Write this temperature in kelvin. *(1 mark)*

3 Convert the following measurements to their base units. Write your answers in standard form.
a 200 pm; **b** 0.40 Mm; **c** 35 μs; **d** 0.25 mA; **e** 756 ns. *(5 marks)*

4 There are 86 400 s in a day. Alternatively you could say there are 86.4 ks in a day.
- **a** The distance by train from London to Edinburgh is 5.34×10^5 m. What is this distance in km?
- **b** The diameter of the Earth is 1.274×10^7 m. What is this diameter in Mm?
- **c** The thickness of a human hair is about 7.5×10^{-5} m. What is this thickness in μm?
- **d** The electric current in a nerve cell is about 1.4×10^{-7} A. What is this current in nA? *(4 marks)*

▼ **Table 2** *Prefixes for SI units*

Prefix name	Prefix symbol	Factor
peta	P	10^{15}
tera	T	10^{12}
giga	G	10^{9}
mega	M	10^{6}
kilo	k	10^{3}
deci	d	10^{-1}
centi	c	10^{-2}
milli	m	10^{-3}
micro	μ	10^{-6}
nano	n	10^{-9}
pico	p	10^{-12}
femto	f	10^{-15}

Study tip

Standard form is used to display very small or very large numbers in a scientific way. For scientific notation it is ideally expressed in the form $n \times 10^m$, where $1 < n < 10$, and m is an integer.

Standard form

You can show small and large numbers in **standard form.**

For example, instead of writing 230 km or 230×10^3 m, we could express this distance as 2.3×10^5 m.

Write 45 ns (45×10^{-9} s) in standard form.

Study tip

Take care when you are writing prefixes and units. For example, ms means milliseconds, but Ms means megaseconds.

2.2 Derived units

Specification reference: 2.1.2

Learning outcomes

Demonstrate knowledge, understanding, and application of:

→ derived units of SI base units and the quantities that use them.

▼ **Table 1** *Some derived units*

Derived quantity	Derived unit
area	m^2
volume	m^3
acceleration	$m\,s^{-2}$
density	$kg\,m^{-3}$

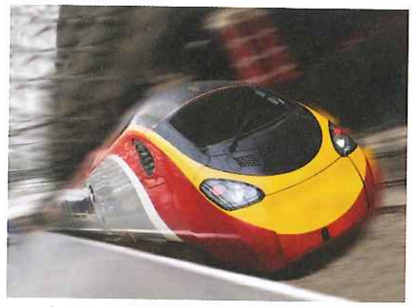
Study tip

You can determine derived units from the equation for the derived quantity. For example, for density, you need the equation that links density, mass, and length:

$$\text{density} = \frac{\text{mass}}{\text{volume}}$$

(where volume = length^3)

The derived unit for density is therefore the unit for mass (kg) divided by the unit for volume (m^3): $kg\,m^{-3}$.



▲ **Figure 1** *Speed is measured in $m\,s^{-1}$, a derived unit in SI*

Beyond base units

The seven base units are used to measure the base quantities that they represent. However, there are many more quantities to measure than just mass, length, electric current, time, and the other three base quantities. For example, what are the units for speed and force? Quantities like these are called **derived quantities**. They use **derived units**, which can be worked out from the base units and the equations relating derived quantities to the base quantities. With derived units any quantity can be communicated.

Names and symbols

Derived units without special names

You already know some derived units. For example, the unit for speed is $m\,s^{-1}$. It comes from the equation that links average speed with two base quantities – distance and time.

$$\text{average speed} = \frac{\text{distance travelled}}{\text{time taken}}$$

Since m is the unit for distance, s is the unit for time, and we are *dividing* m by s, the derived unit for speed is m/s, written $m\,s^{-1}$ at A Level ($s^{-1} = \frac{1}{s}$). We write derived units like this because it is better for more complex units, such as the unit for specific heat capacity, $J\,kg^{-1}\,K^{-1}$, which is much clearer than J/(kg K).

Table 1 shows some derived units without any special names.

Derived units with special names

Some derived quantities are used so often that they have special names. SI has 22 derived units with special names and symbols, but you will not need to know them all for your physics course. Table 2 shows a small selection of these units.

▼ **Table 2** *Some named derived units*

Derived quantity	Unit name	Unit symbol	Unit expressed in other SI units
force	newton	N	$kg\,m\,s^{-2}$
pressure	pascal	Pa	$N\,m^{-2}$
energy or work done	joule	J	N m
power	watt	W	$J\,s^{-1}$
electric potential difference	volt	V	$J\,C^{-1}$
electric resistance	ohm	Ω	$V\,A^{-1}$
electric charge	coulomb	C	A s
frequency	hertz	Hz	s^{-1}

SI units can be combined to form a huge range of other derived units. You may be familiar with some of these already. For example, the moment of a force is measured in newton metres, N m.

Temperature

The SI base unit for temperature is the kelvin, K. In everyday life you are likely to use a different unit for temperature, a derived unit called the degree Celsius, °C. To convert from °C to K you add 273, so 20°C is 293 K and 100°C is 373 K.

A difference of 1°C is the same as a difference of 1 K, so temperature *differences* do not need conversion. For example, if you warm some water from 20°C to 100°C its temperature increases by 80°C, which is also 80 K.

1. Converting from K to °C is equally simple. Convert 298 K to °C.
2. The degree Fahrenheit, °F, is a non-SI unit for temperature. To convert from °F to °C you subtract 32, multiply by 5 then divide by 9. For example, $68°F = (68 - 32) \times \frac{5}{9} = 20°C$. Deduce the temperature that has the same value, whether given in °F or in °C.

Summary questions

1. The unit of mass is the kg. Acceleration has the derived unit $m\,s^{-2}$. The force acting on an object can be determined using the equation force = mass × acceleration. Determine the derived unit for force in base units. *(2 marks)*
2. Use the equations given to determine the derived unit of each quantity in base units.
   a. $\text{force constant} = \frac{\text{force}}{\text{extension}}$
      Extension is the change in length. Determine the derived unit for force constant. *(2 marks)*
   b. work done = force × distance moved in direction of force
      Determine the derived unit for work done. *(2 marks)*
   c. $\text{pressure} = \frac{\text{force}}{\text{cross-sectional area}}$
      Determine the derived unit for pressure. *(2 marks)*
3. State the difference between 1 N m, 1 nm, 1 mN and 1 MN. *(3 marks)*
4. In electrical work, it is useful to define a quantity known as *number density* of free electrons. Number density of free electrons is the number of electrons per unit volume. What is the unit for number density in base units? *(2 marks)*

2.3 Scalar and vector quantities

Specification reference: 2.3.1

Learning outcomes

Demonstrate knowledge, understanding, and application of:

→ scalar and vector quantities.

▲ **Figure 1** *Flyboarders can hover up to 15 m above the water*

▼ **Table 1** *Some scalar quantities and units*

Scalar quantity	SI unit
length	m
mass	kg
time	s
speed	$m\,s^{-1}$
temperature	K, °C
volume	m^3
energy	J
potential difference	V
power	W

Going up

Flyboarding is a sport in which the rider stands on a board with a long hose attached that hangs into a lake. Water from the lake is forced through the hose and into jets under the board. The water rushes out of the jet nozzles, pushing the rider into the air. Skilled flyboarders can perform all sorts of aerial acrobatics, thanks to practice in judging scalar and vector quantities.

Scalar quantities

A **scalar quantity** has magnitude (size) but no direction. For example, the *distance* between a flyboarder and the surface of the water is a scalar quantity, and so is his *mass* and the *time* he can stay in the air. Table 1 shows some examples of scalar quantities with their SI units.

Adding and subtracting scalar quantities

Scalar quantities can be added together or subtracted from one another in the usual way. For example, if your mass is 55 kg and you pick up a 5 kg bag, your new total mass is (55 + 5) = 60 kg. If you sharpen a 16 cm pencil and remove 1 cm as you do so, the new length of the pencil is (16 – 1) = 15 cm.

Scalar quantities must have the same units when you add or subtract them. If you time something in an experiment you cannot add together 1 *minute* and 30 *seconds* as (1 + 30). Instead, you would convert the time from minutes into seconds and then add the times: (60 + 30) = 90 s. Alternatively, you could work in minutes to get a time of (1 + 0.5) = 1.5 minutes.

Multiplying and dividing scalar quantities

Scalar quantities can also be multiplied together or divided by one another. However, in this case the units can be the same or different, unlike adding and subtracting. It is important that you work out the final units correctly.

Worked example: Lighter than air

A balloon is inflated with $6.1 \times 10^{-3}\,m^3$ of helium. Its mass increases by 0.98 g. Calculate the density of helium.

Step 1: The equation for density is

$$\text{density} = \frac{\text{mass}}{\text{volume}}$$

Step 2: Consider the units of the equation.

You are dividing together two scalar quantities. The SI base unit for mass is the kg. Volume has the unit m^3. The mass must be converted into kg before substitution; mass = 9.8×10^{-4} kg.

Step 3: Substitute the values into the equation and calculate the density.

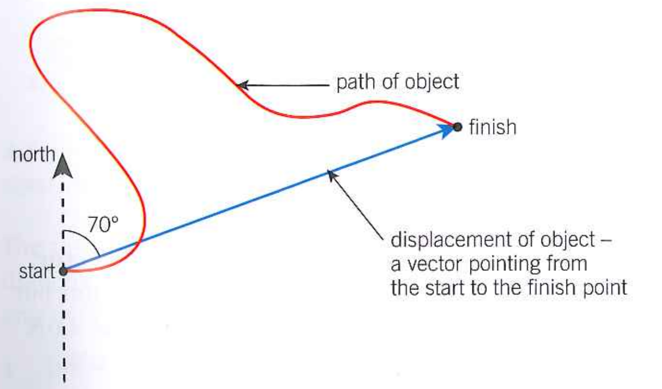
$$\text{density} = \frac{9.8 \times 10^{-4}}{6.1 \times 10^{-3}} = 0.16\,kg\,m^{-3}$$

Vector quantities

A **vector quantity** has magnitude *and* direction. For example, the weight of a flyboarder is a vector quantity, and so is the force from the rushing water from the jet nozzles. Table 2 shows some examples of vector quantities and their SI units.

Distance and displacement

Distance and displacement are both measured in m, but distance is a scalar quantity and displacement is a vector quantity. This is illustrated in Figure 2.



▲ **Figure 2** *Distance travelled is the length of the red path, whereas the magnitude of the displacement is the length of the blue arrow and the direction of the displacement is 70° off due north*

▼ **Table 2** *Some vector quantities and units*

Vector quantity	SI unit
displacement	m
velocity	$m\,s^{-1}$
acceleration	$m\,s^{-2}$
force	N ($kg\,m\,s^{-2}$)
momentum	$kg\,m\,s^{-1}$

Synoptic link

You find out more about vector quantities when studying motion, forces, and momentum in Chapters 3, 4, and 7 of this book.

Synoptic link

In Chapter 3, you will come across two important vector quantities – velocity and acceleration.

Summary questions

1 Explain what is wrong with the following calculation:
mass$_1$ = 150 g, mass$_2$ = 0.500 kg; total mass = 150 + 0.500 = 150.5 g *(2 marks)*

2 Compare and contrast distance and displacement. *(2 marks)*

3 You can calculate power by dividing energy by time. Explain whether power is a scalar or a vector quantity. *(2 marks)*

4 Figure 2 shows the path of a beetle that takes 20 s to travel from the start to the finish. The diagram is drawn to 1:1 scale. Determine:

a the distance travelled, using a length of string; *(1 mark)*
b the magnitude of the displacement; *(1 mark)*
c the average speed of the beetle. *(2 marks)*

5 Explain why the magnitude of the displacement of an object can never be greater than the distance travelled by the object. *(1 mark)*

2.4 Adding vectors

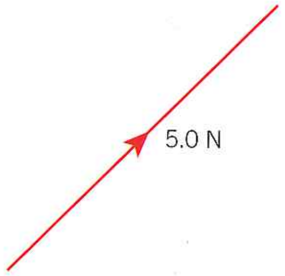
Specification reference: 2.3.1

Learning outcomes

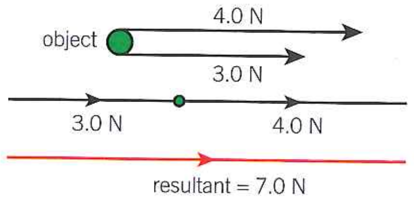
Demonstrate knowledge, understanding, and application of:

→ addition of two vectors with scale drawings and with calculations.

▲ **Figure 1** *What effect will the flowing water have on the dog's progress across the river?*



▲ **Figure 2** *Representing a vector quantity, in this example a force of 5.0 N*



▲ **Figure 3** *Two parallel forces acting on an object are shown at the top, with the corresponding vector diagrams below*

Going against the flow

Many dogs love to jump into rivers to fetch sticks thrown for them. When a dog swims back to a point on the river bank, it has to swim against the current. The velocity of the flowing water and the velocity of the dog's paddling are vector quantities, so it is possible to work out the overall or **resultant** velocity of the dog by adding the two vectors together.

Vectors in one dimension

As you have already seen with displacement in Topic 2.3, a vector quantity is represented by a line with a single arrowhead:

- the length of the line represents the magnitude of the vector, drawn to scale
- the direction in which the arrowhead points represents the direction of the vector.

For example, Figure 2 shows a line representing a single vector. It is drawn to a scale of 1.0 cm ≡ 1.0 N, so a line 5.0 cm long represents a force of 5.0 N.

Parallel vectors

Where two vectors are **parallel** (they act in the same line and direction), you just add them together to find the **resultant vector**. The direction of the resultant is the same as the individual vectors but its magnitude is greater. For example, if two forces of 3.0 N and 4.0 N act in the same direction on an object, the resultant force is 7.0 N.

Antiparallel vectors

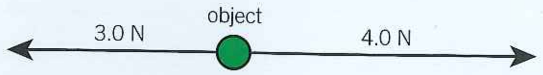
Where two vectors are **antiparallel** (they act in the same line but in opposite directions), you call one direction positive and the opposite direction negative (it does not matter which), and then add the vectors together to find the resultant. The magnitude and direction of the resultant will depend on the magnitude of the two vectors.

Worked example: Vectors in opposite directions

Two forces act in opposite directions on an object, as shown in Figure 4. Calculate the magnitude and direction of the resultant force.

Step 1: Assign positive and negative values to the vectors.

Assume that the positive direction is towards the right, so the two forces are −3.0 N and +4.0 N.



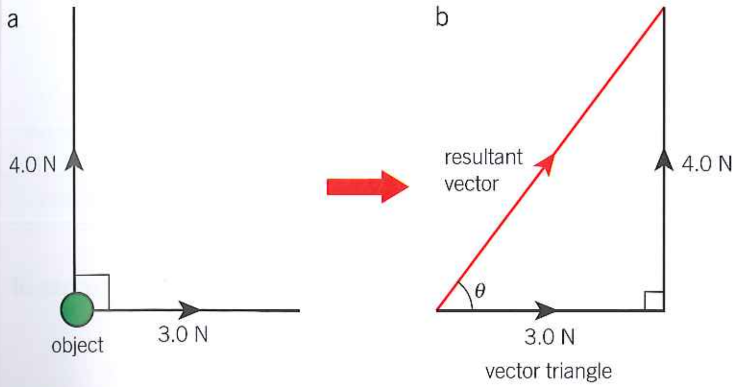
▲ **Figure 4** *Two forces acting in opposite directions*

Step 2: Calculate the resultant force.

resultant = −3.0 + 4.0 = +1.0 N towards the right

Two perpendicular vectors

Perpendicular vectors act at right angles to each other. Figure 5a represents two perpendicular forces of magnitudes 4.0 N and 3.0 N acting on an object.



▲ **Figure 5** *Two perpendicular forces: (a) the two forces acting on the object; (b) the vector triangle used to determine the resultant vector*

The resultant vector can be found either by calculation or by a scale drawing of a **vector triangle**. Follow the rules below when adding any two vectors.

1. Draw a line to represent the first vector.
2. Draw a line to represent the second vector, starting from the *end* of the first vector.
3. To find the resultant vector, join the start to the finish. You have created a vector triangle (Figure 5b).

The method can be used to determine the resultant vector for any two vectors – displacements, velocities, accelerations, and so on. The angle between the vectors need not be 90°; any triangle works.

In this case, since the angle is 90°, you can also determine the magnitude of the resultant force F using **Pythagoras' theorem**.

$$F^2 = 4.0^2 + 3.0^2$$

$$F = \sqrt{4.0^2 + 3.0^2} = \sqrt{25}$$

$$F = 5.0\,\text{N}$$

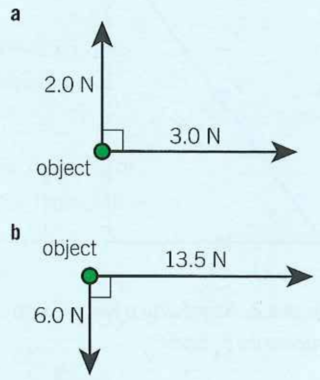
To find the direction of the resultant force, you can calculate the angle θ made with the 3.0 N force.

$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{4.0}{3.0} = 1.333$$

$$\theta = 53°$$

Summary questions

1. The steps on an escalator move upwards at 0.5 m s^{-1}. Calculate the resultant vertical velocity of a person:
   a. standing still on the escalator; *(1 mark)*
   b. walking upwards at 2.0 m s^{-1}; *(1 mark)*
   c. walking downwards at 1.0 m s^{-1}. *(1 mark)*
2. The diagrams in Figure 6 represent forces acting on an object. For each one, draw a vector triangle and therefore determine the magnitude and direction of the resultant force. *(10 marks)*



▲ **Figure 6**

3. A river flows due north at 0.90 m s^{-1}. A dog swims at 0.30 m s^{-1}. Calculate the magnitude and direction of the resultant velocity when the dog swims:
   a. due north; *(2 marks)*
   b. due south; *(2 marks)*
   c. due east. *(3 marks)*

2.5 Resolving vectors

Specification reference: 2.3.1

Learning outcomes

Demonstrate knowledge, understanding, and application of:

→ resolution of a vector into two perpendicular component vectors.

▲ **Figure 1** *Pilots must compensate for the effect of crosswinds during take-off and landing*

Crosswinds

The wind can be helpful to aircraft. A tailwind, blowing in the same direction as the aircraft is travelling, reduces the journey time and saves fuel. On the other hand, a headwind can increase the journey time and waste fuel. Crosswinds can blow an aircraft off course unless the pilot takes them into account. An understanding of vectors is helpful in situations like these.

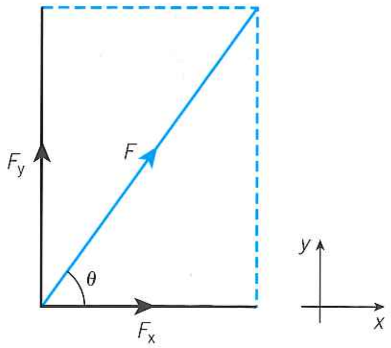
Resolving a vector into two components

You already know how to add together two perpendicular vectors to find a resultant vector. You can reverse this procedure to split a vector into two perpendicular components. This is called **resolving the vector**. It can be done using a scale drawing, but more often vectors are resolved by calculation.

To resolve a force F into the x and y directions, the two **components** of the force are

- $F_x = F\cos\theta$
- $F_y = F\sin\theta$

where θ is the angle made with the x direction. These equations can be used with any vector in the place of x.



▲ **Figure 2** *Resolving a force F into components F_x and F_y*

Worked example: A crosswind

At an airport, a horizontal wind is blowing at $15\,\text{m}\,\text{s}^{-1}$ at an angle of 60° north of east (Figure 3). Calculate the components of the wind velocity in the north and east directions.

Step 1: Select the equations for resolving vectors.

- $v_x = v\cos\theta$
- $v_y = v\sin\theta$

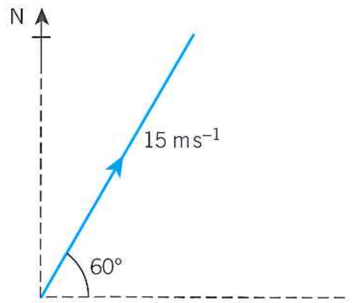
Step 2: Substitute the values into the equations and calculate the components.

velocity component due east $= v_x = 15 \times \cos 60° = 7.5\,\text{m}\,\text{s}^{-1}$

velocity component due north $= v_y = 15 \times \sin 60° = 13\,\text{m}\,\text{s}^{-1}$

You can quickly check your answer using Pythagoras' theorem.

$$v^2 = v_x^2 + v_y^2 = 7.5^2 + 13^2 = 56.25 + 169$$
$$v = 15\,\text{m}\,\text{s}^{-1}$$



▲ **Figure 3**

Worked example: Going down

A freely falling object has a vertical acceleration of $9.81\,\text{m}\,\text{s}^{-2}$. The object is placed on a smooth ramp that makes an angle of 30° to the horizontal (Figure 4). Calculate the component of the acceleration a down the ramp.

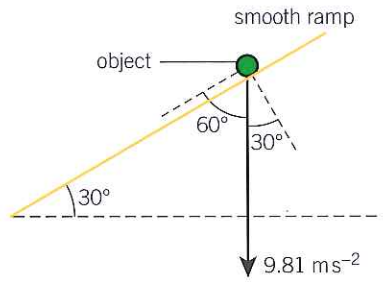
Step 1: Select the equation.

acceleration component down the ramp $= a\cos\theta$ where θ is the angle a makes to the slope.

Step 2: Substitute the values into the equations and calculate the component.

component $= 9.81 \times \cos 60° = 4.91\,\text{m}\,\text{s}^{-2}$

You could have used $9.81 \times \sin 30°$ instead. The answer will be the same because $\sin 30°$ is the same as $\cos 60°$.



▲ **Figure 4**

Study tip

Always check that your calculator is in the correct mode – in this case degrees – when you resolve vectors.

Summary questions

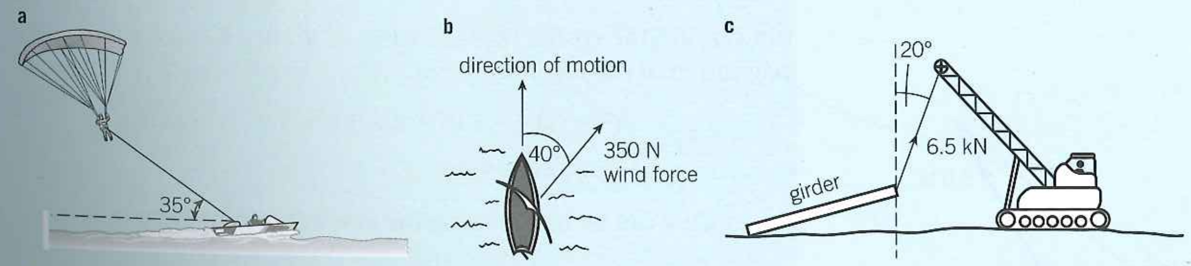
1 A force of 10 N acts on an object at an angle θ to the horizontal. Calculate the horizontal component of the force when $\theta = 0$, $\theta = 45°$, and $\theta = 90°$. Comment on your answers. *(4 marks)*

2 A parascender is attached by a rope to a boat travelling at a constant velocity (Figure 5a). The rope is angled at 35° to the surface of the sea, and the tension in the rope is 1650 N. Calculate the horizontal and vertical components of the tension in the rope. *(2 marks)*

3 A sailing boat is travelling north. It is moving because of a force due to the wind, which is 350 N blowing towards 40° east of north (Figure 5b). Calculate the components of the force from the wind:

  a towards the north (the direction in which the boat is moving); *(1 mark)*

  b towards the east (perpendicular to the direction in which the boat is moving). *(1 mark)*



▲ **Figure 5**

4 One end of a steel girder is lifted off the ground by a crane. The cable is at 20° from the vertical and the tension in the cable is 6.5 kN (Figure 5c). Calculate the vertical and horizontal components of this force. *(2 marks)*

2.6 More on vectors

Specification reference: 2.3.1

Learning outcomes

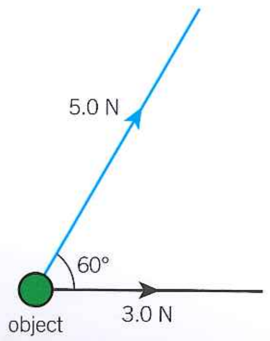
Demonstrate knowledge, understanding, and application of:

→ calculations involving vectors.

▲ **Figure 1** *Tugboats towing an oil platform*

Tugboats

A tugboat is a small but powerful boat that pushes or pulls larger vessels such as barges and tankers. Tugboats manoeuvre these large ships through crowded waterways and harbours. Larger, ocean-going tugboats can tow damaged ships to safety. Sometimes even the most powerful tugboats need to work in pairs or groups. Tugboat captains must understand the vectors involved so that the towed vessel travels in the right direction.

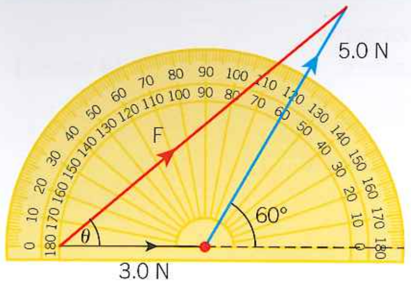


▲ **Figure 2** *Two non-perpendicular forces acting on an object*

Adding non-perpendicular vectors

There are several techniques you can use to add together two non-perpendicular vectors. They all rely on constructing a clear vector triangle. We will apply each of the techniques in turn to the following problem in order to demonstrate how to use them.

Two forces, of 5.0 N and 3.0 N, act on a single point at 60° to each other (Figure 2). What is the magnitude and direction of the resultant force?

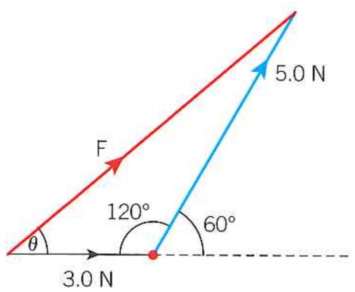


▲ **Figure 3** *A vector triangle drawn to scale*

Technique 1 – Scale diagram

Choose an appropriate scale for the drawing of your vector triangle. Use the rules outlined in Topic 2.4 to construct your vector triangle (Figure 3).

Carefully measure the length of the resultant vector: it is 7.0 cm. With 1.0 cm representing 1.0 N in the diagram, the resultant force must equal 7.0 N. The angle made by the resultant and the 4.0 N force is 38°.



▲ **Figure 4** *A vector triangle with angles and forces shown*

Technique 2 – Calculations using cosine and sine rules

Figure 4 shows a sketch of the vector triangle. The angles and magnitudes of the vectors are all shown. The resultant force is F.

You can use the cosine rule ($a^2 = b^2 + c^2 - 2bc\cos\theta$) to determine the magnitude of the resultant force.

$$F^2 = 3.0^2 + 5.0^2 - 2 \times 3.0 \times 5.0 \times \cos 120°$$

$$F = \sqrt{49} = 7.0\,\text{N}$$

The angle θ can be found using the sine rule $\left(\dfrac{a}{\sin A} = \dfrac{b}{\sin B}\right)$.

$$\frac{5.0}{\sin\theta} = \frac{7.0}{\sin 120}$$

$$\sin\theta = \frac{5.0 \times \sin 120}{7.0} = 0.6186$$

$$\theta = 38°$$

The magnitude of the resultant force is 7.0 N at an angle of 38° relative to the 3.0 N force.

Technique 3 – Calculations using vector resolution

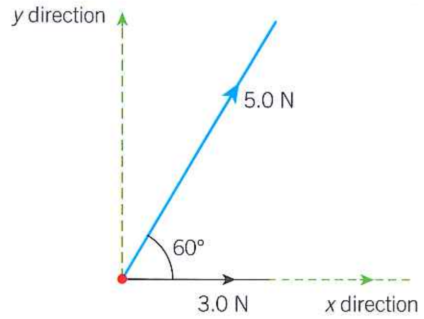
This technique relies on choosing convenient perpendicular axes. One of the vectors is resolved along each axis so that the magnitude of the resultant vector can be determined using Pythagoras' theorem (Figure 5).

total force in x direction $= 3.0 + 5.0\cos 60° = 5.5\,\text{N}$

total force in y direction $= 5.0\sin 60° = 4.33\,\text{N}$

resultant force $F = \sqrt{5.5^2 + 4.33^2} = 7.0\,\text{N}$

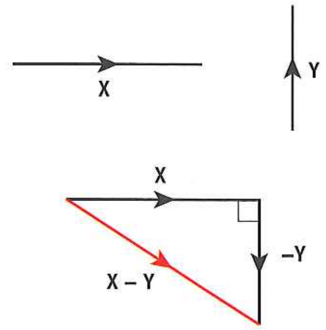
$$\theta = \tan^{-1}\left(\frac{4.33}{5.5}\right) = 38°$$



▲ **Figure 5** *Two non-perpendicular vectors shown as part of a right-angled triangle*

Subtracting vectors

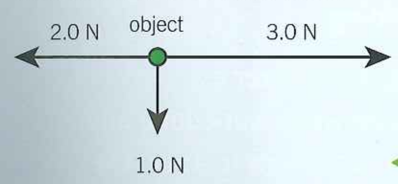
Two vectors are represented by **X** and **Y**. To subtract **Y** from **X**, you simply reverse the direction of **Y** and then add this new vector to **X** (Figure 6).



▲ **Figure 6** *Subtracting vectors*

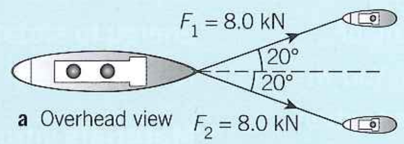
Summary questions

1. Three forces act on an object (Figure 7). Calculate the magnitude and direction of the resultant force. *(4 marks)*



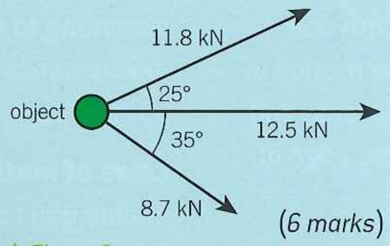
◀ **Figure 7**

2. Two tugboats are pulling a ship, each with a force of 8.0 kN, and with an angle of 40° between the cables (Figure 8). Calculate the magnitude and direction of the resultant force. *(4 marks)*



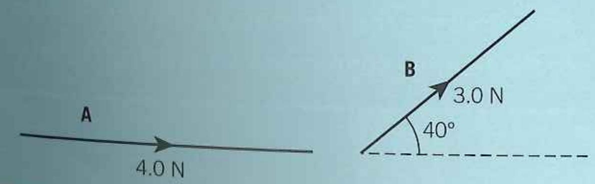
▲ **Figure 8**

3. Three tugboats are towing an object at sea. The forces and angles between the cables are shown in Figure 9. Calculate the magnitude and direction of the resultant force on the object. *(6 marks)*



▲ **Figure 9**

4. Figure 10 shows two vectors, A and B. Determine the magnitude and the direction of the resultant vector A – B. *(4 marks)*



◀ **Figure 10**