HW3, Due: February 24, 2022 **Spring 2022**

Homework 3

Instructor: Morad Nazari

Instructions: *i) Paper size "ANSI A" (8.5×11 in) is preferred; ii) Write your answers in order; iii) Show all details for credit.*

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1. (20pts) In the LTI system described by $\dot{\bar{x}}(t) = A\bar{x}$ with $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -3 & -3 \end{bmatrix}$,
   a) (15pts) obtain all eigenvalues and eigenvectors of A;
   b) (5pts) use the eigenvectors in part (a) to obtain the modal matrix V and Jordan form J.

2. (20pts) In each case below, discuss BIBS stability of the LTI system $\dot{\bar{x}}(t) = A\bar{x}(t)$:

   a) $A = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}$, b) $A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -3 \end{bmatrix}$, c) $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

   d) $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -3 \end{bmatrix}$, e) $A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -3 \end{bmatrix}$

3. (35pts) The linearized equations of motion of a pendulum can be written in the form of

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -ax_1 - cx_2 \end{aligned}$$

   where $a > 0$ is a constant parameter of the system and $c > 0$ is the torsional friction coefficient.
   a) (5pts) Study BIBS stability of the system.
   b) (10pts) Consider the quadratic Lyapunov function $V = \bar{x}^T P \bar{x}$ with $P = \begin{bmatrix} \frac{a}{2} & 0 \\ 0 & \frac{1}{2} \end{bmatrix}$. What can be said about stability of the system based on this choice of Lyapunov function?
   c) (10pts) Consider the quadratic Lyapunov function $V = \bar{x}^T P \bar{x}$ and find the P matrix such that the time derivative of the Lyapunov function becomes $\dot{V} = -\bar{x}^T \bar{x}$. What can be said about stability of the system based on this choice of Lyapunov function?
   d) (5pts) What can be said about the stability of the system based on the analyses in b) and c)?
   e) (5pts) Study BIBS stability of the system when $c = 0$.

4. (25pts) For the transfer function matrix

$$H(s) = \begin{bmatrix} \frac{s}{s-2} & 0 \\ \frac{2}{s-2} & 1 \end{bmatrix}$$

   a) (10pts) obtain the controllable canonical form;
   b) (10pts) obtain the observable canonical form;
   c) (5pts) show that the realizations in (a) and (b) are dual.