HW5, Due: April 12, 2022 **Spring 2022**

Homework 5

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Instructions: *i) Paper size "ANSI A" (8.5×11 in) is preferred; ii) Write your answers in order; iii) Show all details for credit.*

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1. (15pts) Consider a LTI SISO system with matrices

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Assuming all state variables can be measured, find a state feedback control law such that the eigenvalues of the closed-loop system become $\lambda_{d_1} = \lambda_{d_2} = \lambda_{d_3} = -1$.

2. (30pts) Consider a LTI MIMO system with matrices

$$A = \begin{bmatrix} -2 & -2 & 0 \\ 0 & 0 & 1 \\ 0 & -3 & -4 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}$$

Assuming all state variables can be measured, find a state feedback control law such that the eigenvalues of the closed-loop system become $\lambda_{d_1} = \lambda_{d_2} = -3$ and $\lambda_{d_3} = -4$. For this purpose, use:

   a) Method 1 in Slide#106 and verify the eigenvalues of closed-loop system.
   b) Method 2 in Slide#107 and verify the eigenvalues of closed-loop system.

3. (15pts) For the system in Problem 2, design an output feedback control law such that two of the eigenvalues of the closed-loop system become -3 and -4. Verify the eigenvalues of the closed-loop system.

4. (40pts) Consider the nonlinear control system in which it is desired to stabilize the origin:

$$\begin{aligned} \dot{x}_1 &= -2x_1 + x_2 + \sin x_1 \\ \dot{x}_2 &= -x_2 \cos x_1 + \cos(2x_1)\, u \end{aligned}$$

   a) (5pts) Verify that $(x_1^*, x_2^*, u^*) = (0, 0, 0)$ is an equilibrium. [Side note: This system has several other equilibria such as $(x_1^*, x_2^*, u^*) = (\frac{\pi}{2}, \pi - 1, 0)$.]
   b) (5pts) Linearize the system about that equilibrium.
   c) (5pts) Study controllability of the linearized system.
   d) (10pts) Assuming both state variables can be measured, find a state feedback control law such that both eigenvalues of the linearized closed-loop system become -4.
   e) (15pts) Apply the state feedback control law obtained in part (d) to the <u>nonlinear</u> system above. Simulate and plot the state response of the closed-loop system using the following initial conditions: $\bar{x}_0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $\bar{x}_0 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, and $\bar{x}_0 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$. Briefly interpret the results.