Homework Two

Nathan Rose

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1. A system is described by
$$\dot{x} = \begin{bmatrix} -2 & 1 \\ -1 & 0 \end{bmatrix} x + \begin{bmatrix} 3 \\ 1 \end{bmatrix} u \tag{1}$$

   Obtain the STM of the uncontrolled system using the following methods:

   (a) Via taking the laplace inverse of $(sI - A)^{-1}$
   $$\mathscr{L}^{-1}\left((sI - A)^{-1}\right) = \begin{bmatrix} (1-t)\, e^{-t}\theta(t) & te^{-t}\theta(t) \\ -te^{-t}\theta(t) & (t+1)\, e^{-t}\theta(t) \end{bmatrix} \tag{2}$$

   The theta in the equation is the step function as described here

   This is accomplished with the following code

```
import numpy as np
import sympy
from sympy import eye, shape, simplify, inverse_laplace_transform


def STM_laplace_inverse(A: np.matrix) -> None:
    s, t = sympy.symbols('s, t')
    s_I = eye(shape(A)[0])*s
    return simplify(
        inverse_laplace_transform((s_I - A).inv(), s, t)
    )
```

   (b) Via model decomposition of Matrix A

   (c) Via the Cayley-Hamilton theorem

2. In the system in Problem 1,

   (a) Obtain the zero input solution $x_{Z1}(t)$ for the initial condition $\bar{x}(0) = \begin{bmatrix} 10 \\ 1 \end{bmatrix}$

(b) Obtain the zero state solution $x_{ZS}(t)$ for input $u(t) = e^{2t}$ for $t > 0$

(c) Obtain the total solution $\bar{x}(t)$ for the initial conditions and input in 2a and 2b

3. Given

$$A = \begin{bmatrix} -5 & -6 & 0 \\ 2 & 2 & 0 \\ 0 & 0 & -3 \end{bmatrix} \tag{3}$$

(a) Find A^{-1} using the Cayley-Hamilton theorem

(b) Obtain e^{AT} using one of the three mentioned methods in Problem 1

4. Let the STM of the system $\dot{\bar{x}}(t) = A\bar{x}(t)$, where A is a constant matrix, be $\phi(t, t_0)$. Also, let the STM of the system $\dot{\bar{z}}(t) = -A^T \bar{Z}(t)$, where A^T is the transpose of A, be $\Phi(t, t_0)$. Use the properties of the STM on Slide 39 to show that $\Phi(t, t_0) = \phi^T(t, t_0)$.

5. Given a system in state space

$$\dot{\bar{x}}(t) = A\bar{x}(t) + B\bar{u}(t) \tag{4}$$

$$\bar{y}(t) = C\bar{x}(t) + D\bar{u}(t) \tag{5}$$

prove that the transfer function matrix is invariant to any similarity transformation of the state i.e. $\bar{x} = T\bar{z}$, where T is a constant invertible matrix.

6. Give the algebraic and geometric multiplicities of the repeated eigenvalue and find e^{3t} for the matrices below.

(a) $J = \begin{bmatrix} \lambda & 1 \\ 0 & \lambda \end{bmatrix}$

(b) $J = \begin{bmatrix} \lambda & 1 & 0 \\ 0 & \lambda & 1 \\ 0 & 0 & \lambda \end{bmatrix}$

(c) $J = \begin{bmatrix} \lambda & 1 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}$