Homework UPDATE ME

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1. Consider the LTI SISO system with matrices:

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix} B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \tag{1}$$

Assuming all state variabels can be measures, find a state feedback control law such that the eigenvalues of the closed loop system become: $\lambda_{d_1} = \lambda_{d_2} = \lambda_{d_3} = -1$

(a) Choose the desired eigenvalues and obtain the desired characteristic equation

$$\Delta_d(s) = (s - \lambda_1)(s - \lambda_2)(s - \lambda_3) = (s+1)(s+1)(s+1) = s^3 + 3s^2 + 3s + 1 \tag{2}$$

$$\bar{\alpha} = \begin{bmatrix} 3 & 3 & 1 \end{bmatrix} \tag{3}$$

(b) Find the actual characterstic equation

$$\Delta(s) = |sI - A| = \begin{vmatrix} s-2 & -1 & 0 \\ 0 & s-2 & -1 \\ 0 & 0 & s-2 \end{vmatrix} \tag{4}$$

$$\Delta(s) = \begin{vmatrix} s-2 & -1 & 0 \\ 0 & s-2 & -1 \\ 0 & 0 & s-2 \end{vmatrix} = (s-2)^3 = s^3 - 6s^2 + 12s - 8 \tag{5}$$

$$\alpha = \begin{bmatrix} -6 & 12 & -8 \end{bmatrix} \tag{6}$$

(c) Calulate $\bar{k}$

$$\bar{k} = \bar{\alpha} - \alpha = \begin{bmatrix} 3 & 3 & 1 \end{bmatrix} - \begin{bmatrix} -6 & 12 & -8 \end{bmatrix} = \begin{bmatrix} 9 & -9 & 9 \end{bmatrix} \tag{7}$$

(d) Calculate P

$$Q = P^{-1} = \begin{bmatrix} b & AB & A^2B \end{bmatrix} \begin{bmatrix} 1 & \alpha_1 & \alpha_2 \\ 0 & 1 & \alpha_1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 4 \\ 1 & 2 & 4 \end{bmatrix} \begin{bmatrix} 1 & -6 & 12 \\ 0 & 1 & -6 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & -2 \\ 1 & -4 & 4 \end{bmatrix} \tag{8}$$

$$P = Q^{-1} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & -2 \\ 1 & -4 & 4 \end{bmatrix}^{-1} = \begin{bmatrix} 4 & 4 & 1 \\ 2 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \tag{9}$$

(e) Calculate k

$$k = \bar{k}P = \begin{bmatrix} 9 & -9 & 9 \end{bmatrix} \begin{bmatrix} 4 & 4 & 1 \\ 2 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 27 & 27 & 9 \end{bmatrix} \tag{10}$$

(f) Confirm eigenvalues of $A - bk$

$$A - bk = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 27 & 27 & 9 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ -27 & -27 & -7 \end{bmatrix} \tag{11}$$

eigenvalues are: $-1, -1, -1$

1 Help Recieved

This section is a thank you for people who caught issues or otherwised helped(collaboration is allowed)

1. List people here