Homework UPDATE ME

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March 22, 2022

1. For each of the systems (a) and (b) do the following:
   (a) Obtain the eigenvalues

      i. A

$$0 = \Delta(\lambda) = |\lambda I - A| = \begin{vmatrix} \lambda + 6 & 1 & 0 \\ 11 & \lambda & 1 \\ 6 & 0 & \lambda \end{vmatrix} \tag{1}$$

$$0 = (\lambda + 6)(\lambda^2) + 11\lambda + 6 = \lambda^3 + 6\lambda^2 + 11\lambda + 6 = \tag{2}$$

$$0 = \lambda^3 + 6\lambda^2 + 11\lambda + 6 = (x+1)(x^2 + 5x + 6) = (x+1)(x+2)(x+3) \tag{3}$$

$$\lambda = -1, -2, -3 \tag{4}$$

$$0 = |\lambda I - A| = \begin{bmatrix} \lambda + 6 & 1 & 0 \\ 11 & \lambda & 1 \\ 6 & 0 & \lambda \end{bmatrix} v \tag{5}$$

$$0 = \begin{bmatrix} 5 & 1 & 0 \\ 11 & -1 & 1 \\ 6 & 0 & -1 \end{bmatrix} v_1 \tag{6}$$

$$v_1 = \begin{bmatrix} \frac{1}{2} \\ \frac{3}{2} \\ 1 \end{bmatrix} \tag{7}$$

$$0 = \begin{bmatrix} 4 & 1 & 0 \\ 11 & -2 & 1 \\ 6 & 0 & -2 \end{bmatrix} v_1 \tag{8}$$

$$v_2 = \begin{bmatrix} \frac{1}{3} \\ \frac{4}{3} \\ 1 \end{bmatrix} \tag{9}$$

$$0 = \begin{bmatrix} 3 & 1 & 0 \\ 11 & -3 & 1 \\ 6 & 0 & -3 \end{bmatrix} v_1 \tag{10}$$

$$v_3 = \begin{bmatrix} \frac{1}{6} \\ \frac{5}{6} \\ 1 \end{bmatrix} \tag{11}$$

$$V = \begin{bmatrix} \frac{1}{2} & \frac{1}{3} & \frac{1}{6} \frac{3}{2} & \frac{4}{3} & \frac{5}{6} \\ 1 & 1 & 1 & & \end{bmatrix} \tag{12}$$

ii. B

eigenvalues are: $\lambda = -1, -1, -1, -2, -3, -3$ from sympy.jordan_from, eigenvectors are:

$$V = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{13}$$

(b) Use the similarity transformation $x(t) = Vz(t)$ to express the system in the modal form

i. A

$$\bar{A} = J = \begin{bmatrix} -3 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -1 \end{bmatrix} \tag{14}$$

$$\bar{B} = V^{-1}B = \begin{bmatrix} -4 \\ 9 \\ 0 \end{bmatrix} \tag{15}$$

$$\bar{C} = CV = \begin{bmatrix} \frac{1}{2} & \frac{1}{3} & \frac{1}{6} \end{bmatrix} \tag{16}$$

ii. B

jordan form:

$$\bar{A} = J = \begin{bmatrix} -3 & 0 & 0 & 0 & 0 & 0 \\ 0 & -3 & 0 & 0 & 0 & 0 \\ 0 & 0 & -2 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix} \tag{17}$$

$$\bar{B} = V^{-1}B = \begin{bmatrix} 0 & 1 \\ -\frac{1}{2} & 0 \\ 0 & -1 \\ 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix} \tag{18}$$

$$\bar{C} = CV = \begin{bmatrix} 0 & 0 & 2 & 1 & 2 & 0 \\ 1 & 1 & 0 & 0 & 0 & \frac{1}{2} \end{bmatrix} \tag{19}$$

(c) Use the modal form to study the controllability and observability of the system(slides 77, 85)

i. A

modal state variables 1 and 2 are controllable do to non-zero values but 3 is not. all state variables are observable due to non-zero values.

ii. B

TODO: this

(d) idicate teh controllability and onbservability of each mode

i. A

System is observable not controllable

ii. B

TODO: this

(e) Study stabalizability and detectability of the system

i. A

system is stabalizable(and obviously) detectable

ii. B
TODO: this

(f) Plot the block diagram of the modal form

i. A
TODO: this

ii. B
TODO: this

1 Help Recieved

This section is a thank you for people who caught issues or otherwised helped(collaboration is allowed)

1. List people here