HW6, Due: April 26, 2022 **Spring 2022**

Homework 6

Instructor: Morad Nazari

Instructions: *i) Paper size "ANSI A" (8.5×11 in) is preferred; ii) Write your answers in order; iii) Show all details for credit.*
iv) This assignment is out of ***60 points****.*

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1. (45pts) Consider the LTI MIMO system in *Problem 2 of Homework 5*, i.e.

$$A = \begin{bmatrix} -2 & -2 & 0 \\ 0 & 0 & 1 \\ 0 & -3 & -4 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}, \quad D = 0_{2\times 2}$$

Recall that, in that homework, you designed a state feedback control law $\bar{u}(t) = -K\bar{x}(t)$ such that the eigenvalues of the closed-loop system became $\lambda_{d_1} = \lambda_{d_2} = -3$ and $\lambda_{d_3} = -4$, i.e. you have already obtained the control gain matrix K.

   a) (15pts) Design a full-order observer with observer poles $\mu_{d_1} = -6$, $\mu_{d_2} = -7$, and $\mu_{d_3} = -8$ using either Method 1 in Slide #121 or Method 2 in Slide #122 and verify the eigenvalues of $(A - LC)$.

   b) (15pts) We know that using separation principle, we can design an observer-based feedback control. In Eqs. (69) and (70) of the slides, let the control input be $\bar{u}(t) = -K\hat{x}(t)$ with the control gain matrix K obtained in *either part (a) or part (b) of Problem 2 of Homework 5*, and let the observer gain matrix L be that obtained in part (a) above. Simulate and plot the state response of the closed-loop system assuming that the initial conditions of the states and estimated states are $\bar{x}_0 = \begin{bmatrix} 1 \\ -1 \\ 2.3 \end{bmatrix}$ and $\hat{x}_0 = \begin{bmatrix} 1.7 \\ -0.3 \\ 2.5 \end{bmatrix}$, respectively. In your plots, ensure that the transient response and system convergence are depicted clearly.

   c) (15pts) Design a reduced-order observer with observer pole $\mu_{d_1} = -8$. (No need for any computer simulation for this part!)

2. (15pts) Use Eqs. (77)-(81) to verify Eq. (82) of the slides.