# Homework 1

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1. Consider the following system consisting of a cart of mass m attached to a rigid wall by a spring and a damper. The spring stiffness $k(w)$ is a nonlinear function of displacement $w$ such that the spring pull(force) applied to the mass is in the form of $f_s = k_1 w - k_3 w^3$, where $k_1$ and $k_3$ are constant scalars. Assume that the cart rolls freely without friction. It can be shown that the motion is $m\ddot{w} + c\dot{w} + k_1 w - k_3 w^3 = u$

   (a) The cart is instrumented with an accelerometer, which provides a measurement equation of the form $y = \ddot{w}$. Express the nonlinear equations of motion, including the output equation in state space form
   This system has two state variables: position and velocity. In order to keep it consistent with the notation in question 1c. the following will be used.

$$x = \begin{bmatrix} position \\ velocity \end{bmatrix} = \begin{bmatrix} w \\ \dot{w} \end{bmatrix} \tag{1}$$

$$k_1 w = u + k_3 w^3 - c\dot{w} - m\ddot{w} \tag{2}$$

$$w = \frac{u + k_3 w^3 - c\dot{w} - m\ddot{w}}{k_1} \tag{3}$$

$$c\dot{w} = u + k_3 w^3 - k_1 w - m\ddot{w} \tag{4}$$

$$\dot{w} = \frac{u + k_3 w^3 - k_1 w - m\ddot{w}}{c} \tag{5}$$

   (b) Assume $m = 1$, $k_1 = 4$, $k_3 = 1$, $c = 1$. Determine all the equilibrium points of the system assuming $u = 0$
   TODO: Figure out how to find equilibrium points

(c) Using the numerical assumptions above, linearize the equations of motion about the equilibrium point

$$\begin{bmatrix} 2 \\ 0 \end{bmatrix} \tag{6}$$

That is provide the lienar state space model about the point in the form of:

$$\begin{aligned} \delta\dot{\bar{x}} &= A\delta\bar{x}(t) + B\delta\bar{u}(t) \\ \delta y &= C\delta\bar{x}(t) + D\delta\bar{u}(t) \\ \delta\bar{x} &= C\delta\bar{x}(t) \end{aligned}$$

For the system that is described by:

$$w = \frac{0 + 1w^3 - 1\dot{w} - 1\ddot{w}}{4} \tag{7}$$

$$\dot{w} = \frac{0 + 1w^3 - 4w - 1\ddot{w}}{1} \tag{8}$$

It can be linearized into:

$$w = 1w + 2\dot{w} + 5u \tag{9}$$

$$w = 3w + 4\dot{w} + 6u \tag{10}$$

which in state space can be represented as:

$$\dot{x} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} x + \begin{bmatrix} 5 \\ 6 \end{bmatrix} u \tag{11}$$

$$y = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \end{bmatrix} u \tag{12}$$