HW4, Due: March 24, 2022 **Spring 2022**

Homework 4

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Instructions: *i) Paper size "ANSI A" (8.5×11 in) is preferred; ii) Write your answers in order; iii) Show all details for credit.*

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1. (40pts) For each of the systems (a) and (b) below

   a) $A = \begin{bmatrix} -6 & 1 & 0 \\ -11 & 0 & 1 \\ -6 & 0 & 0 \end{bmatrix}, \bar{b} = \begin{bmatrix} 1 \\ 6 \\ 5 \end{bmatrix}, \underline{c} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$

   b) $A = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -2 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -3 & 0 & 0 \\ 0 & 0 & 0 & 0 & -3 & 1 \\ 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 1 \\ 0 & 0 \\ 1 & 0 \end{bmatrix}, C = \begin{bmatrix} 1 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{bmatrix}$

   do the following:

   i) Obtain eigenvalues, eigenvectors, and the modal matrix V.
   ii) Use the similarity transformation $\bar{x}(t) = V\bar{z}(t)$ to express the system in the modal form.
   iii) Use the modal form to study controllability and observability of the system (see Slides #77 & 85).
   iv) Indicate controllability and observability of each mode.
   v) Study stabilizability and detectability of the system.
   vi) Plot the block diagram of the modal form.

2. (30pts) For each of the systems (a) and (b) below

   a) $A = \begin{bmatrix} -6 & 1 & 0 & 0 \\ 0 & -6 & 0 & 0 \\ 0 & 0 & -6 & 0 \\ 0 & 0 & 0 & 6 \end{bmatrix}, B = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 1 & 1 \\ 2 & 0 & 2 \\ 0 & 1 & 0 \end{bmatrix}, C = \begin{bmatrix} 3 & 1 & 4 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}, D = 0_{2\times 3}$

   b) Same A and B as in (a), $C = \begin{bmatrix} 3 & 1 & 4 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}, D = 0_{2\times 3}$

   do the following:

   i) Obtain the transfer function matrix.
   ii) Verify if the system is minimal (irreducible).
   If the system is not minimal, then do parts (iii) and (iv) below.
   iii) Use Approach (a) discussed in Slide #92 to obtain the minimal realization of the system.
   iv) Show that the transfer function matrix of the minimal realization is the same as that in part (i).

3. (30pts) For the system with transfer function matrix $H(s) = \begin{bmatrix} \frac{-s}{(s+1)^2} & \frac{1}{s+1} \\ \frac{2s+1}{s(s+1)} & \frac{1}{s+1} \end{bmatrix}$

   a) Find a minimal realization using Approach (b), Jordan canonical form method, discussed in Slides #94-96.
   b) Show that the transfer function matrix of the minimal realization is the same as $H(s)$ given above.