

Controls Project

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1 Abstract

The project for Controls will involve controlling a doubly inverted pendulum with a linear “cart”. The goals of this project are to

1. Linearize the system about the stability point and design a controller for it.
2. Determine the limits of the controller with respect to each state variable(in isolation)

In order to simplify the model, the mass of the pendulums will be assumed to be at the end of the rod, rather than in the middle as it would be for a constant density rod.

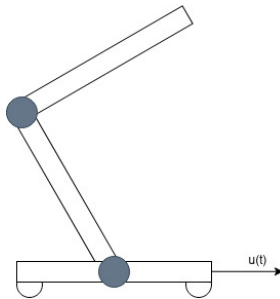


Figure 1: System to be modeled

2 Introduction

2.1 Defining the non-linear system

A doubly inverted pendulum is a pendulum on a pendulum. What makes this a classical control problem is the inverted nature of the system. That is instead of hanging the pendulum, the goal is to balance it upright.

The equations of motion were derived with assistance from **eq'of motion**. **TODO:**
re-add the filter

This will not be a derivation of the work, as the work follows closely from **eq'of motion** which I used as a source to help derive the non-linear equations of motion.

2.2 Defining the linear system

The Linearization was simply a jacobian taken at 0,0,0 which is an equilibrium point that the system will be controlled to.

3 The Controller

To begin with, we will develop a controller that will be used in order to apply feedback and make the system controllable.

the desired eigenvalues after feedback are:

$$\lambda = -3, -3, -4, -4, -5, -5 \quad (1)$$

Desired Characteristic equation

$$\Delta_d(\lambda) = \lambda^6 + 24\lambda^5 + 238\lambda^4 + 1248\lambda^3 + 3649\lambda^2 + 5640\lambda + 3600 \quad (2)$$

Desired Characteristic equation

$$\bar{\alpha} = [24 \quad 238 \quad 1248 \quad 3649 \quad 5640 \quad 3600] \quad (3)$$

Desired Characteristic equation

$$\Delta_d(\lambda) = \text{PurePoly} (1.0\lambda^6 + 0.2750244140625\lambda^5 - 14.6950073242188\lambda^4 - 1.71600341796875\lambda^3 - 48.1669921875\lambda^2 - 144.000000000000\lambda + 3600.00000000000) \quad (4)$$

Desired Characteristic equation

$$\alpha = [0.275 \quad -14.695 \quad -1.716 \quad -48.167 \quad 0 \quad 0] \quad (5)$$

\bar{k}

$$\bar{k} = \bar{\alpha} - \alpha = \quad (6)$$

$$\bar{k} = [24 \quad 238 \quad 1248 \quad 3649 \quad 5640 \quad 3600] - [0.275 \quad -14.695 \quad -1.716 \quad -48.167 \quad 0 \quad 0] = \quad (7)$$

$$\bar{k} = [23.725 \quad 252.695 \quad 1249.716 \quad 3697.167 \quad 5640.0 \quad 3600] \quad (8)$$

Calculate P

$$Q = P^{-1} = \begin{bmatrix} B & AB & A^2B \end{bmatrix} \begin{bmatrix} 1 & \alpha_1 & \alpha_2 & \alpha_3 & \alpha_4 & \alpha_5 \\ 0 & 1 & \alpha_1 & \alpha_2 & \alpha_3 & \alpha_4 \\ 0 & 0 & 1 & \alpha_1 & \alpha_2 & \alpha_3 \\ 0 & 0 & 0 & 1 & \alpha_1 & \alpha_2 \\ 0 & 0 & 0 & 0 & 1 & \alpha_1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (9)$$

$$Q = \begin{bmatrix} 0 & 0 & -0.025 & 14.722 & -4.171 & 169.324 \\ 0 & -0.5 & 0.088 & -12.277 & 3.804 & -205.383 \\ 0 & 1.0 & -0.125 & 4.922 & -1.964 & 120.82 \\ 1.0 & -0.125 & 4.922 & -1.964 & 120.82 & -59.665 \\ -0.5 & 0.088 & -12.277 & 3.804 & -205.383 & 95.517 \\ 0 & -0.025 & 14.722 & -4.171 & 169.324 & -83.786 \end{bmatrix} \begin{bmatrix} 1 & 0.275 & -14.695 & -1.716 & -48.167 \\ 0 & 1 & 0.275 & -14.695 & -1.716 \\ 0 & 0 & 1 & 0.275 & -14.695 \\ 0 & 0 & 0 & 1 & 0.275 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (10)$$

$$Q = \begin{bmatrix} 0 & 0 & -0.025 & 14.715 & 0.245 & -48.118 \\ 0 & -0.5 & -0.05 & -4.905 & 0 & 0 \\ 0 & 1.0 & 0.15 & -9.808 & -0.49 & 0 \\ 1.0 & 0.15 & -9.808 & -0.49 & 0 & 0 \\ -0.5 & -0.05 & -4.905 & 0 & 0 & 0 \\ 0 & -0.025 & 14.715 & 0.245 & -48.118 & 0 \end{bmatrix} \quad (11)$$

$$P = Q^{-1} = \begin{bmatrix} 0 & 0.1 & -0.075 & 0.5 & -1.0 & 0.001 \\ 0 & -1.0 & 0.5 & 0.003 & 0.005 & -0.005 \\ 0 & 0 & 0.003 & -0.051 & -0.102 & 0 \\ 0 & -0.102 & -0.051 & 0 & 0.001 & 0.001 \\ 0 & 0 & 0 & -0.016 & -0.031 & -0.021 \\ -0.021 & -0.031 & -0.016 & 0 & 0 & 0 \end{bmatrix} \quad (12)$$

$$k = \bar{k}P = \begin{bmatrix} -74.816 & -739.453 & -115.411 & -138.066 & -323.582 & -116.417 \end{bmatrix} \quad (13)$$

4 Help Recieved

This section is a thank you for people who caught issues or otherwised helped(collaboration is allowed)

1. People will be listed here