HW1, Due: January 25, 2022 **Spring 2022**

Homework 1

Instructor: Morad Nazari

Instructions: *i) Paper size "ANSI A" (8.5×11 in) is preferred; ii) Write your answers in order; iii) Show all details for credit.*

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1. (25pts) Consider the following system consisting of a cart of mass m attached to a rigid wall by a spring and a damper. The spring stiffness $k(w)$ is a nonlinear function of displacement w such that the spring pull (force) applied to the mass is in the form of $f_s = k_1 w - k_3 w^3$, where k_1 and k_3 are constant scalars. Also, the damping coefficient c is a constant scalar. The cart is subject to a control force u. Assume that the cart rolls freely without friction. It can be shown that the equation of motion is $m\ddot{w} + c\dot{w} + k_1 w - k_3 w^3 = u$.

   a) The cart is instrumented with an accelerometer, which provides a measurement equation of the form $y = \ddot{w}$. Express the <u>nonlinear</u> equations of motion, including the output equation, in the state space form.

   b) Assume $m = 1$ (kg), $k_1 = 4$ $(\mathrm{N/m})$, $k_3 = 1$ $(\mathrm{N/m^3})$, and $c = 1$ $(\mathrm{N\,s/m})$. Determine all the equilibrium points of the system assuming $u = 0$.

   c) Using the numerical assumption above, linearize the equations of motion about the equilibrium point $\begin{bmatrix} 2 \\ 0 \end{bmatrix}$. That is, provide the linear state space model about that point in the form of

$$\begin{aligned} \delta\dot{\bar{x}}(t) &= A\,\delta\bar{x}(t) + B\,\delta\bar{u}(t) \\ \delta y(t) &= C\,\delta\bar{x}(t) + D\,\delta\bar{u}(t) \\ \delta\bar{x}(0) &= \delta\bar{x}_0 \end{aligned}$$

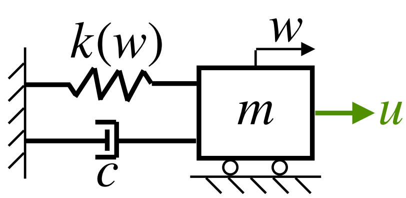

Figure 1: See Problem 1

2. (25pts) For the mass-spring-damper system given in Fig. 2, where k_1, k_2, c_1, and c_2 are constant scalars,

   a) obtain the differential equations of motion of the system;

   b) write the equations of motion in the state space form

$$\begin{aligned} \dot{\bar{x}} &= A\bar{x} + B\bar{u} \\ \bar{y} &= C\bar{x} + D\bar{u} \end{aligned}$$

such that the A matrix is in the form of

$$A = \begin{bmatrix} 0_{2\times 2} & I_2 \\ A_{21} & A_{22} \end{bmatrix}$$

where I_2 is the 2×2 identity matrix. In your solution, assume that the output is $y = \begin{bmatrix} w_2 \\ \dot{w}_1 \end{bmatrix}$. Your solution should clearly indicate the expressions for A_{21}, A_{22}, B, C, and D.

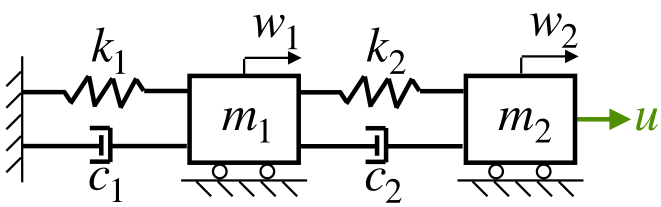

Figure 2: See Problem 2

3. (15pts) Find a state space representation for the system described by

$$\begin{aligned} \dot{w}_1 + 3(w_1 + w_2) &= u_1 \\ \ddot{w}_2 + 4\dot{w}_2 + 3w_2 &= u_2 \end{aligned}$$

Show the A and B matrices in your solution.

4. (15pts) Find the transfer function matrix for the system described as

$$\begin{aligned} \dot{\bar{x}} &= \begin{bmatrix} -7 & -2 & 6 \\ 2 & -3 & -2 \\ -2 & -2 & 1 \end{bmatrix} \bar{x} + \begin{bmatrix} 1 & 1 \\ 1 & -1 \\ 1 & 0 \end{bmatrix} \bar{u} \\ \bar{y} &= \begin{bmatrix} -1 & -1 & 2 \\ 1 & 1 & -1 \end{bmatrix} \bar{x} \end{aligned}$$

Ans. $H(s) = \begin{bmatrix} 0 & 0 \\ 1/(s+3) & 0 \end{bmatrix}$

5. (20pts) The model of a control system with input $u(t)$ and output $y(t)$ is given as

$$\ddot{y}(t) + 3\dot{y}(t) + 7y(t) = \dot{u}(t) + 2u(t)$$

All initial conditions are zero and the system is subjected to a unit step input.

a) Find the transfer function $\frac{Y(s)}{U(s)}$.
b) Find the values of all poles and zeros.
c) Find the steady state output response using the final value theorem.

[*Approximate Answer* for part c) $y_{ss} \approx 0.3$ (Obtain the exact value accurate to the fourth decimal.)]