Controls Project

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1 Abstract

The project for Controls will involve controlling a doubly inverted pendulum with a linear "cart". The goals of this project are to

1. Linearize the system about the stability point and design a controller for it.
2. Determine the limits of the controller with respect to each state variable(in isolation)

In order to simplify the model, the mass of the pendulums will be assumed to be at the end of the rod, rather than in the middle as it would be for a constant density rod.

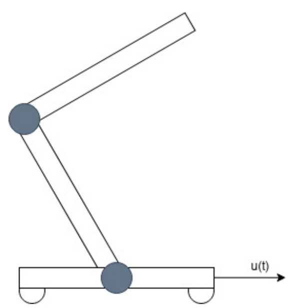

Figure 1: System to be modeled

2 Introduction

2.1 Defining the non-linear system

A doubly inverted pendulum is a pendulum on a pendulum. What makes this a classical control problem is the inverted nature of the system. That is instead of hanging the pendulum, the goal is to balence it upright.

The equations of motion were derived with assistance from **eq˙of˙motion**. TODO: re-add the filter

This will not be a derivation of the work, as the work follows closely from **eq˙of˙motion** which I used as a source to help derive the non-linear equations of motion.

2.2 Defining the linear system

The Linearization was simply a jacobian taken at 0,0,0 which is an equilibrium point that the system will be controlled to.

3 The Controller

To begin with, we will develop a controller that will be used in order to apply feedback and make the system controllable.

the desired eigenvalues after feedback are:

$$\lambda = (-3+3j), (-3-3j), (-4+4j), (-4-4j), (-5+5j), (-5-5j) \tag{1}$$

Desired Characteristic equation

$$\Delta_d(\lambda) = \lambda^6 + 24.0\lambda^5 + 288.0\lambda^4 + 2016.0\lambda^3 + 8836.0\lambda^2 + 22560.0\lambda + 28800.0 \tag{2}$$

Desired Characteristic equation

$$\bar{\alpha} = \begin{bmatrix} 24.0 & 288.0 & 2016.0 & 8836.0 & 22560.0 & 28800.0 \end{bmatrix} \tag{3}$$

Desired Characteristic equation

$$\Delta_d(\lambda) = \text{PurePoly}\left(1.0\lambda^6 + 0.27502441406250\lambda^5 - 14.6950073242188\lambda^4 - 1.71600341796875\lambda^3 - 48.1669921875\right. \tag{4}$$

Desired Characteristic equation

$$\alpha = \begin{bmatrix} 0.275 & -14.695 & -1.716 & -48.167 & 0 & 0 \end{bmatrix} \tag{5}$$

$\bar{k}$

$$\bar{k} = \bar{\alpha} - \alpha = \tag{6}$$

$$\bar{k} = \begin{bmatrix} 24.0 & 288.0 & 2016.0 & 8836.0 & 22560.0 & 28800.0 \end{bmatrix} - \begin{bmatrix} 0.275 & -14.695 & -1.716 & -48.167 & 0 & 0 \end{bmatrix} = \tag{7}$$

$$\bar{k} = \begin{bmatrix} 23.725 & 302.695 & 2017.716 & 8884.167 & 22560.0 & 28800.0 \end{bmatrix} \tag{8}$$

Calculate P

$$Q = P^{-1} = \begin{bmatrix} B & AB & A^2B \end{bmatrix} \begin{bmatrix} 1 & \alpha_1 & \alpha_2 & \alpha_3 & \alpha_4 & \alpha_5 \\ 0 & 1 & \alpha_1 & \alpha_2 & \alpha_3 & \alpha_4 \\ 0 & 0 & 1 & \alpha_1 & \alpha_2 & \alpha_3 \\ 0 & 0 & 0 & 1 & \alpha_1 & \alpha_2 \\ 0 & 0 & 0 & 0 & 1 & \alpha_1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{9}$$

$$Q = \begin{bmatrix} 0 & 0 & -0.025 & 14.722 & -4.171 & 169.324 \\ 0 & -0.5 & 0.088 & -12.277 & 3.804 & -205.383 \\ 0 & 1.0 & -0.125 & 4.922 & -1.964 & 120.82 \\ 1.0 & -0.125 & 4.922 & -1.964 & 120.82 & -59.665 \\ -0.5 & 0.088 & -12.277 & 3.804 & -205.383 & 95.517 \\ 0 & -0.025 & 14.722 & -4.171 & 169.324 & -83.786 \end{bmatrix} \begin{bmatrix} 1 & 0.275 & -14.695 & -1.716 & -48.167 \\ 0 & 1 & 0.275 & -14.695 & -1.716 \\ 0 & 0 & 1 & 0.275 & -14.695 \\ 0 & 0 & 0 & 1 & 0.275 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{10}$$

$$Q = \begin{bmatrix} 0 & 0 & -0.025 & 14.715 & 0.245 & -48.118 \\ 0 & -0.5 & -0.05 & -4.905 & 0 & 0 \\ 0 & 1.0 & 0.15 & -9.808 & -0.49 & 0 \\ 1.0 & 0.15 & -9.808 & -0.49 & 0 & 0 \\ -0.5 & -0.05 & -4.905 & 0 & 0 & 0 \\ 0 & -0.025 & 14.715 & 0.245 & -48.118 & 0 \end{bmatrix} \tag{11}$$

$$P = Q^{-1} = \begin{bmatrix} 0 & 0.1 & -0.075 & 0.5 & -1.0 & 0.001 \\ 0 & -1.0 & 0.5 & 0.003 & 0.005 & -0.005 \\ 0 & 0 & 0.003 & -0.051 & -0.102 & 0 \\ 0 & -0.102 & -0.051 & 0 & 0.001 & 0.001 \\ 0 & 0 & 0 & -0.016 & -0.031 & -0.021 \\ -0.021 & -0.031 & -0.016 & 0 & 0 & 0 \end{bmatrix} \tag{12}$$

$$k = \bar{k}P = \begin{bmatrix} -598.528 & -2103.927 & -741.308 & -438.832 & -925.114 & -464.341 \end{bmatrix} \tag{13}$$

First the goal is to see if the system behaves well in a non-linear system from various distances from the origin. In order to make sure none of the state space variable explode but also wanting to make sure that multiple starting states can be overlaid on the same graph the following C matrix was used in a linear model

$$C = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \tag{14}$$

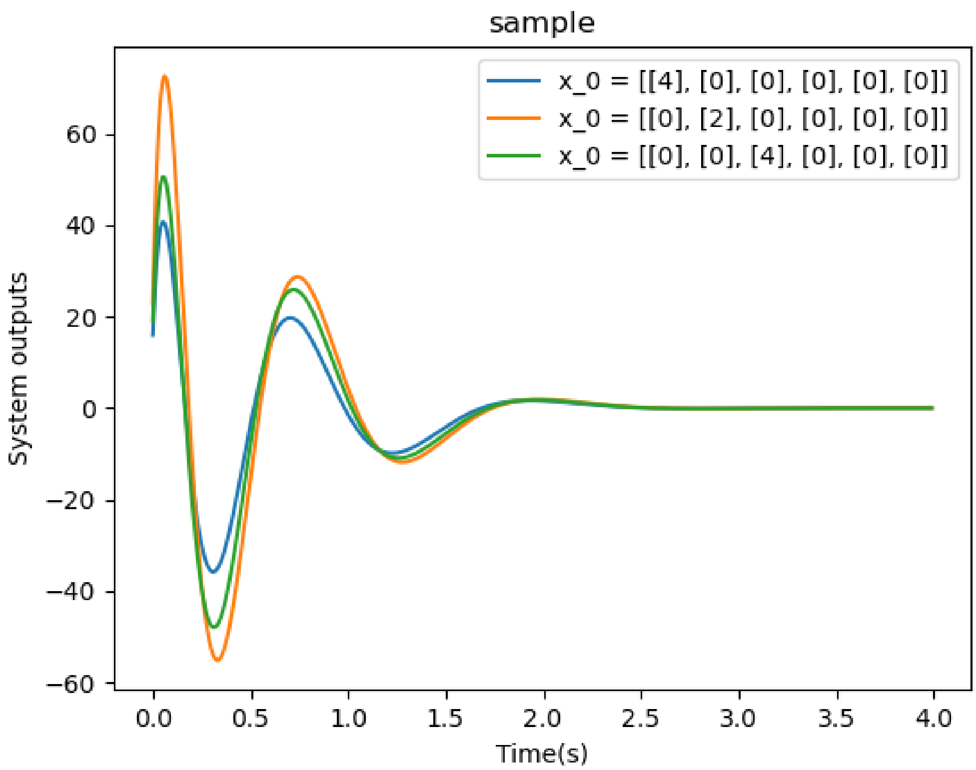

Figure 2: Linear analyis

from these graphs, it can be seen that even non-sensically high values can be controlled down to zero, thanks to the linear approximation and the controller being able to compensate for it. Howver this does show that the controller works in at least this limited case.

Now to see the usefulness of the controller, let us re-apply the system to the non-linear system. The inital states are adjusted by a factor of 100, which was necessary in order to make sure that all of the states converged. THe same C matrix was used for a linear output

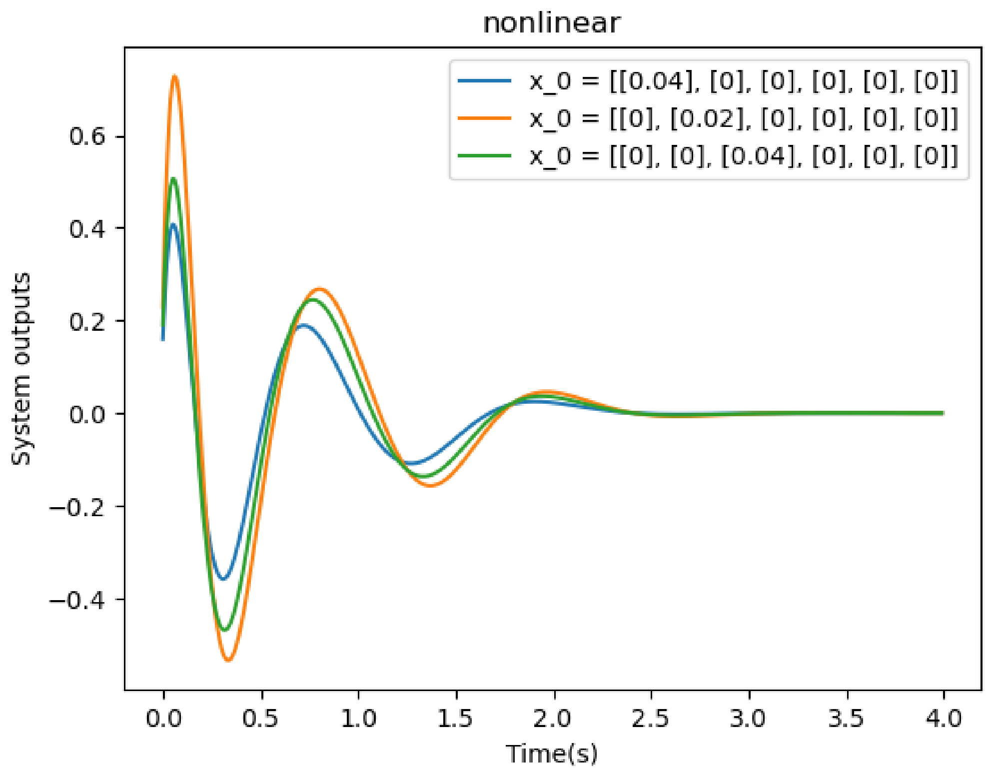

Figure 3: NonLinear analyis

4 The Observer

The previous section assumed that the state variables can be modeled, which is not necessaryily the case. To make it interesting I am going to assume that the system's velocities can be measured(ie only relative encoders are available for the system).

This is going to create a c matrix of:

$$\begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{15}$$

5 Help Recieved

This section is a thank you for people who caught issues or otherwised helped(collaboration is allowed)

1. People will be listed here