HW2, Due: February 10, 2022 **Spring 2022**

Homework 2

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Instructions: *i) Paper size "ANSI A" (*8.5×11 *in) is preferred; ii) Write your answers in order; iii) Show all details for credit.*

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1. (20pts) A system is described by

$$\dot{\bar{x}}(t) = \begin{bmatrix} -2 & 1 \\ -1 & 0 \end{bmatrix} \bar{x}(t) + \begin{bmatrix} 3 \\ 1 \end{bmatrix} u(t)$$

Obtain the STM of the uncontrolled system using the following methods:

a) (7pts) via taking Laplace inverse of $(sI - A)^{-1}$;

b) (7pts) via modal decomposition of matrix A;

c) (6pts) via Cayley-Hamilton theorem.

2. (20pts) In the system in Problem 1,

a) (5pts) Obtain the zero-input solution $x_{ZI}(t)$ for initial condition $\bar{x}(0) = [10 \;\; 1]^T$.

b) (10pts) Obtain the zero-state solution $x_{ZS}(t)$ for input $u(t) = e^{2t}$ for $t > 0$.

c) (5pts) Obtain the total solution $\bar{x}(t)$ for the initial conditions and input in a) and b).

3. (10pts) Given

$$A = \begin{bmatrix} -5 & -6 & 0 \\ 2 & 2 & 0 \\ 0 & 0 & -3 \end{bmatrix}$$

a) (5pts) Find A^{-1} using the Cayley-Hamilton theorem.

b) (5pts) Obtain e^{At} using one of the three methods mentioned in parts a)-c) in Problem 1.

4. (15pts) Let the STM of the system $\dot{\bar{x}}(t) = A\bar{x}(t)$, where A is a constant matrix, be $\Phi(t, t_0)$. Also, let the STM of the system $\dot{\bar{z}}(t) = -A^T \bar{z}(t)$, where A^T is the transpose of A, be $\Theta(t, t_0)$. Use the properties of the STM on Slide #39 to show that $\Theta(t, t_0) = \Phi^T(t_0, t)$.

5. (15pt) Given a system in state space form

$$\begin{aligned} \dot{\bar{x}}(t) &= A\bar{x}(t) + B\bar{u}(t) \\ \bar{y}(t) &= C\bar{x}(t) + D\bar{u}(t) \end{aligned}$$

prove that the transfer function matrix is invariant to any similarity transformation of the state, i.e. $\bar{x} = T\bar{z}$, where T is a constant invertible matrix.

6. (20pt) Give the algebraic and geometric multiplicities of the repeated eigenvalue and find e^{Jt} for the matrices below:

a) $J = \begin{bmatrix} \lambda & 1 \\ 0 & \lambda \end{bmatrix}$, b) $J = \begin{bmatrix} \lambda & 1 & 0 \\ 0 & \lambda & 1 \\ 0 & 0 & \lambda \end{bmatrix}$, c) $J = \begin{bmatrix} \lambda & 1 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}$