EE 510 Take Home Exam

Please submit your answer as a **pdf file** through canvas by 5:00pm Monday 12/13/2021. **Word document will not be accepted. Please submit your Matlab codes (.m file) for each problem as attachment as well.**

1. (30 pts) A high-performance helicopter has a model shown in Figure 1.

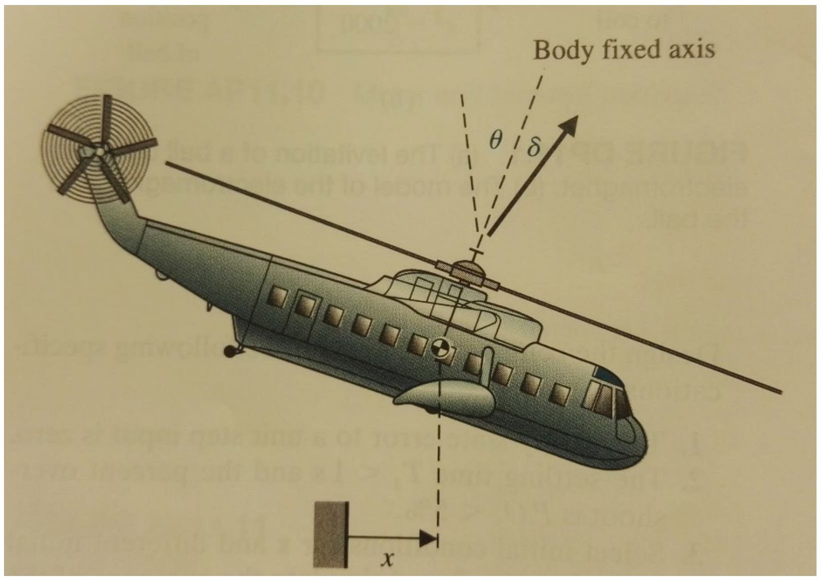


Figure 1.

The goal is to control the pitch angle θ of the helicopter by adjusting the rotor thrust angle δ. The equations of motion of the helicopter are

$$\frac{d^2\theta}{dt^2} = -\sigma_1 \frac{d\theta}{dt} - \alpha_1 \frac{dx}{dt} + n\delta$$

$$\frac{d^2x}{dt^2} = g\theta - \alpha_2 \frac{d\theta}{dt} - \sigma_2 \frac{dx}{dt} + g\delta$$

,

where *x* is the translation in the horizontal direction. For a military high-performance helicopter, we find that

$\sigma_1 = 0.415$

$\sigma_2 = 0.0198$

$\alpha_1 = 0.0111$

$\alpha_2 = 1.43$

$n = 6.27$

$g = 9.8$

all in appropriate SI units.

Find

(a) a state variable representation of this system.

(b) the transfer function representation for $\theta(s)/\delta(s)$.

(c) Use state variable feedback to achieve adequate performances for the controlled system.

Desired specifications include

(1) a steady-state for an input step command for $\theta_d(s)$, the desired pitch angle, less than 20% of the input step magnitude;

(2) an overshoot for a step input command less than 20%;

(3) a settling (with a 2% criterion) time for a step command of less than 1.5 seconds.

(d) If the state variable is not available, design the observer and control law to meet the design specifications in part (c).

2. (30 pts) The open-loop system

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & 4 & -3 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} u, \quad y = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} x, \quad x(0) = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}.$$

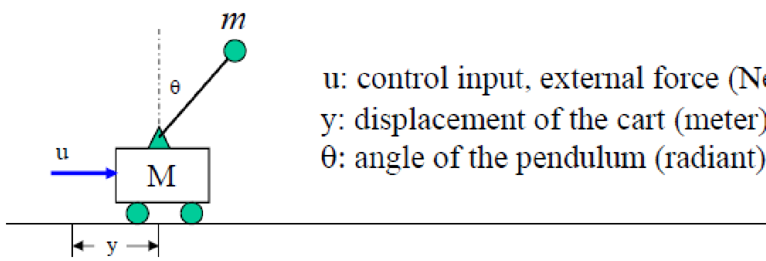
1) Assume that x is available for state feedback. Design an LQR control law by letting $R = 1$ and choosing Q so that all the elements of the feedback gain K have absolute value less than 50. Requirement: $|y_1(t)|, |y_2(t)| \le 0.05$ for all $t > 5$. Plot $y_1(t)$ and $y_2(t)$ in the same figure for $t \in [0, 20]$.

2) Assume that only the output y is available. Design an observer so that the poles of the observer are $-5 + j5,\ -5 - j5,\ -10$. Choose the observer gain so that all the elements have absolute value less than 80. Form a closed-loop system along with the LQR controller in part 1). Plot $y_1(t)$ and $y_2(t)$ in the same figure for $t \in [0, 20]$.

3. (40 pts)

A cart with an inverted pendulum (page33 , Chen's book)

State:



u: control input, external force (Newton)
y: displacement of the cart (meter)
θ: angle of the pendulum (radiant)

$$x = \begin{bmatrix} y \\ \dot{y} \\ \theta \\ \dot{\theta} \end{bmatrix}$$

The control problems are

1: Stabilization: Design a feedback law u=Fx such that $x(t) \to 0$ for x(0) close to the origin.

2: For $x(0)=(0,0,-\pi,0)$, apply an impulse force ($u(t)=u_{max}$ for $t\in[0,0.1]$) to bring θ to a certain range and then switch to the linear controller so that $x(t) \to 0$.

Assume that there is no friction or damping. The nonlinear model is as follows.

$$\begin{bmatrix} M+m & ml\cos\theta \\ \cos\theta & l \end{bmatrix}\begin{bmatrix} \ddot{y} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} u + ml\dot{\theta}^2\sin\theta \\ g\sin\theta \end{bmatrix}$$

$m = 1kg$: mass of the pendulum
$l = 0.2m$: length of the pendulum
$M = 5kg$: mass of the cart, $g = 9.8$

Linearize the system at x=0;

$$\begin{bmatrix} M+m & ml \\ 1 & l \end{bmatrix}\begin{bmatrix} \ddot{y} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} u \\ g\theta \end{bmatrix} \qquad x = \begin{bmatrix} y \\ \dot{y} \\ \theta \\ \dot{\theta} \end{bmatrix}$$

The state space description for the linearized system

$$\dot{x} = Ax + Bu$$

Problems:

1. Find matrices A, B for the state space equation.
2. Design a feedback law $u=F_1x$ so that $A+BF_1$ has eigenvalues at $-3\pm j3$;-6 and -8. Build a simulink model for the closed-loop linear system. Plot the response under initial condition x(0)=[-1.5,0,1,3].
3. Build a simulink model for the original nonlinear system, verify that stabilization is achieved by $u=F_1x$ when x(0) is close to the origin. Find the maximal θ_0 so that the nonlinear system can be stabilized from $x(0)=(0,0,\theta_0,0)$.
4. For $x(0)=(0,0,\pi/5,0)$, compare the response y(t) and θ(t) for the linearized system and the nonlinear system under the same feedback $u=F_1x$.