

# Homework Eleven

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1 - A High performance helicopter has a model shown in Figure 1. The goal is to control the pitch angle  $\theta$  of the helicopter by adjusting the rotor thrust  $\delta$ . The equations of motion of the helicopter are

$$\frac{d^2\theta}{dt^2} = -\sigma_1 \frac{d\theta}{dt} - \alpha \frac{dx}{dt} + n\delta \quad (1)$$

$$\frac{d^2x}{dt^2} = g\theta - \alpha_2 \frac{d\theta}{dx} - \sigma_2 \frac{dx}{dt} + g\delta \quad (2)$$

Where  $x$  is the translation in the horizontal direction. For a military high-performance helicopter we find:  $\sigma_1 = 0.415$ ,  $\sigma_2 = 0.0198$ ,  $\alpha_1 = 0.0111$ ,  $\alpha_2 = 1.43$ ,  $n = 6.27$ ,  $g = 9.8$  all in appropriate SI units. Find:

- (a) A state variable representation of this system
- (b) The transfer function representation for  $\frac{\theta(s)}{\delta(s)}$
- (c) Use state variable feedback to achieve adequate performances for the controlled system. Desired specifications include:
  - (1) A steady-state for an input step command for  $\theta_d(s)$ , the desired pitch angle, less than 20% of the input step magnitude
  - (2) An overshoot for a step input command is less than 20%
  - (3) a settling (with a 2% criterion) time for a step command of less than 1.5 seconds
- (d) If the state variable is not available, design the observer and control law to meet the design specifications included in part (c)

Final Answer: