

FORECASTING THE CHEMICAL INFORMATION CONTENT OF STELLAR SPECTRA

NATHAN SANDFORD

UC BERKELEY

With:

Dan Weisz

Yuan-Sen Ting

Hans-Walter Rix

GOLDEN AGE OF STELLAR SPECTROSCOPY

Observations

- ▶ **Current Facilities**
 - ▶ Keck, VLT, Magellan
- ▶ **>10⁵ stars outside the MW**
- ▶ **Next Generation Facilities**
 - ▶ Space-based: JWST
 - ▶ Ground-based:
PFS, E-ELT, MSE

Techniques

- ▶ **Improved Spectral Models**
 - ▶ 3D, non-LTE
 - ▶ More complete linelists
- ▶ **Full Spectrum Fitting**
 - ▶ The Cannon (Ness+ 2015)
 - ▶ The Payne (Ting+ 2018)
 - ▶ StarNet (Fabbro+ 2017)
- ▶ **10+ Elements from
R~2000 spectra in MW**

BUT WAIT...

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What abundances do we need?
To what precision?

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To what precision?

Where is the information located?

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What abundances do we need?

To what precision?

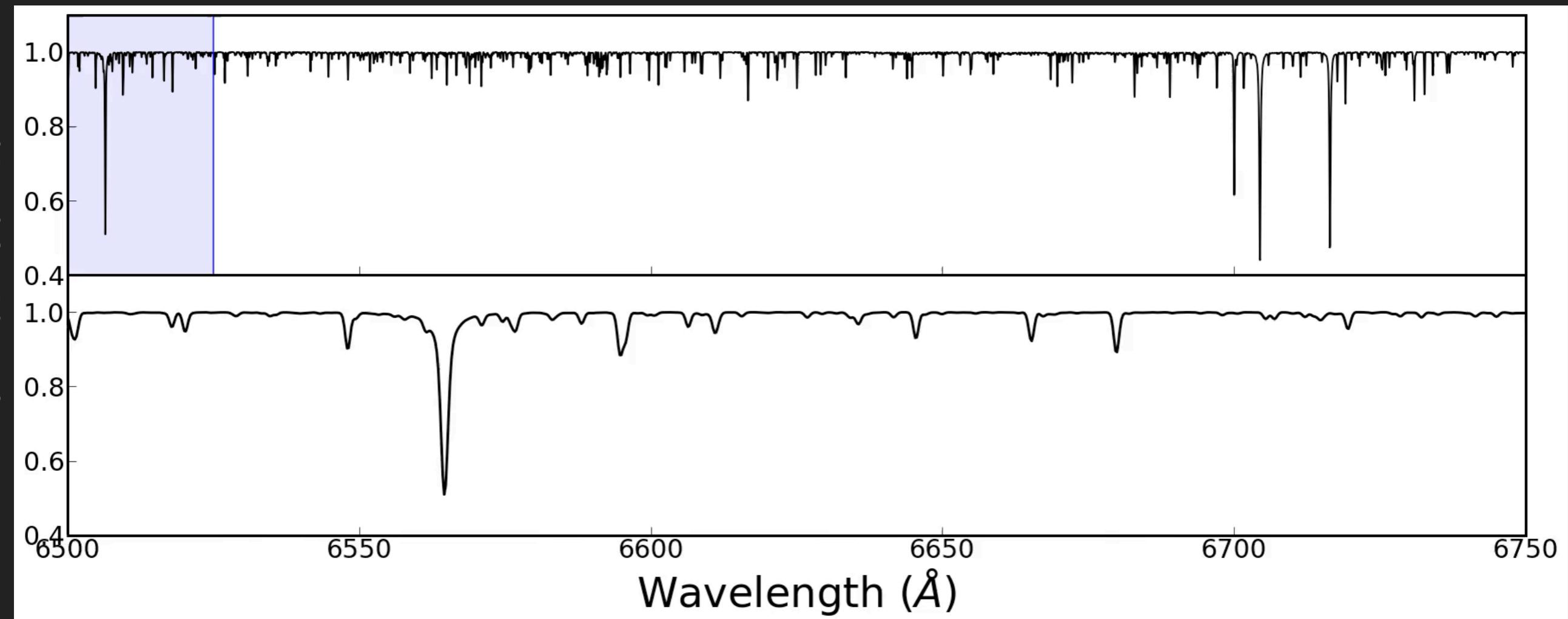
Where is the information located?

What observations best provide

that information?

SYNTHETIC DEIMOS RGB SPECTRUM

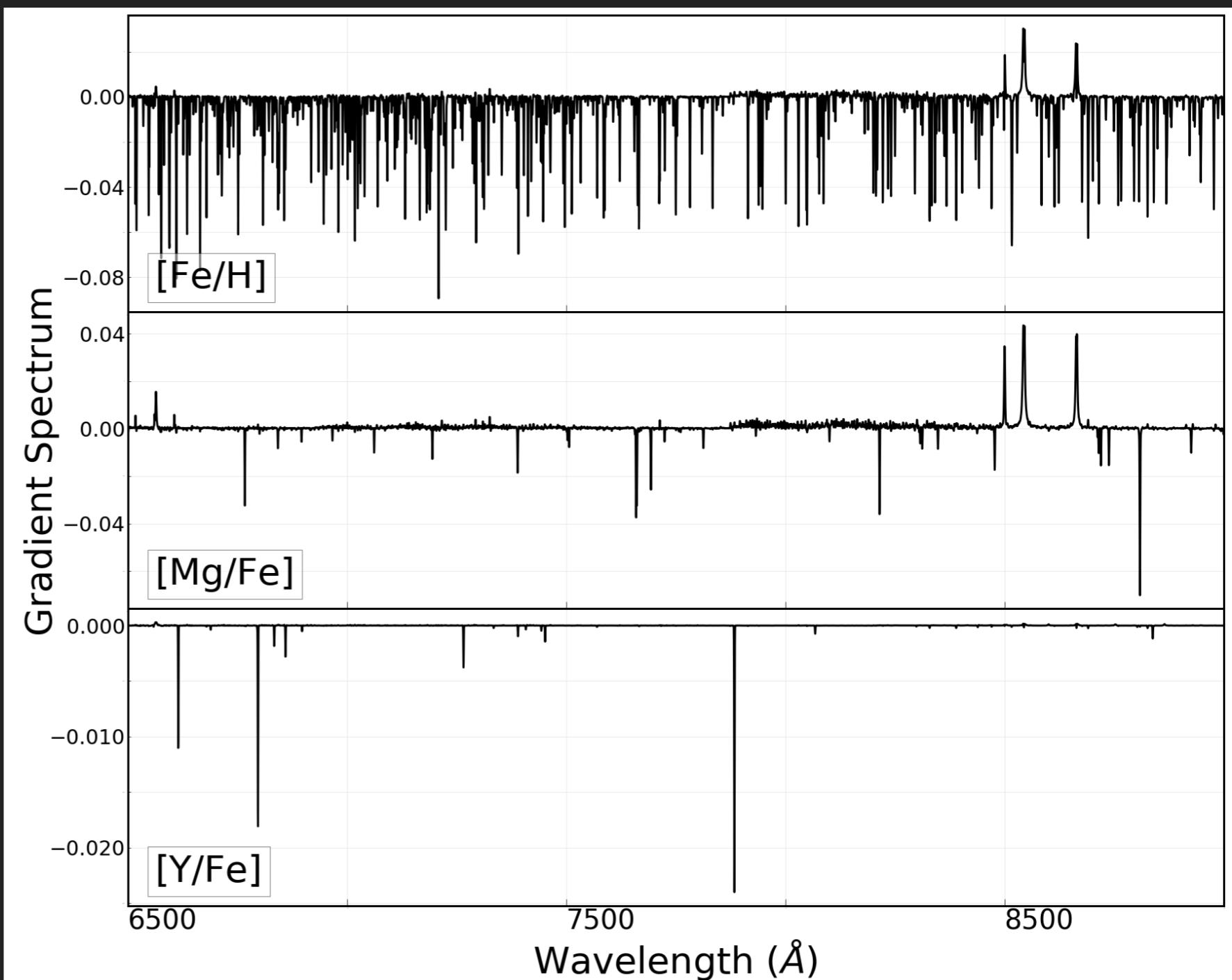
$6500\text{\AA} < \lambda < 9000\text{\AA}$
 $R = 6500$



$T_{\text{eff}} = 4750 \text{ K}$
 $\log(g) = 1.8$
 $v_{\text{turb}} = 1.9 \text{ km/s}$
 $[\text{Fe}/\text{H}] = -1.5$

CHEMICAL INFORMATION IN SPECTRAL GRADIENTS

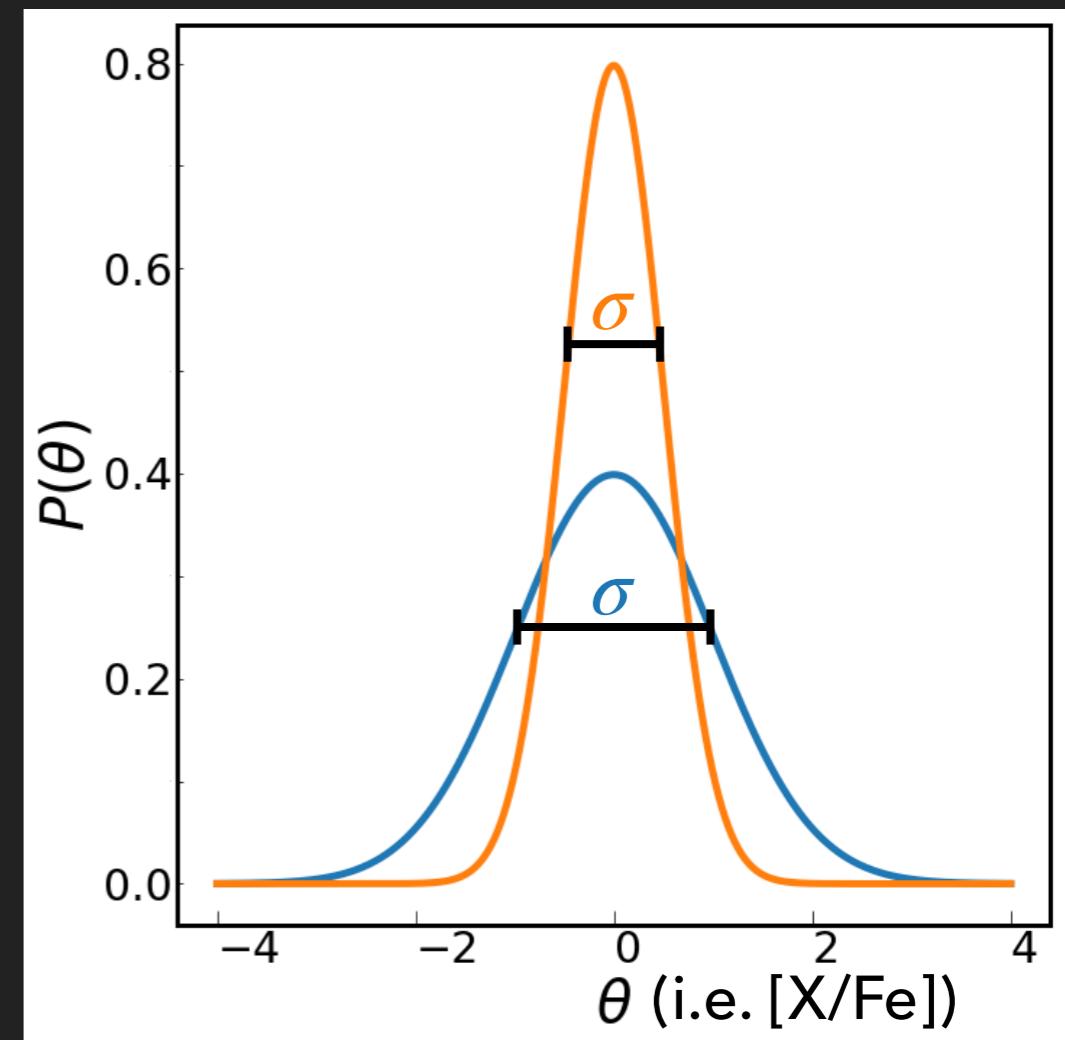
Strong Gradients →
Moderate Gradients →
Weak Gradients →



QUANTIFYING CHEMICAL INFORMATION CONTENT

MCMC Sampling of the Posterior

- ▶ Abundance fitting is many-dimensional
- ▶ Computationally expensive / intractable

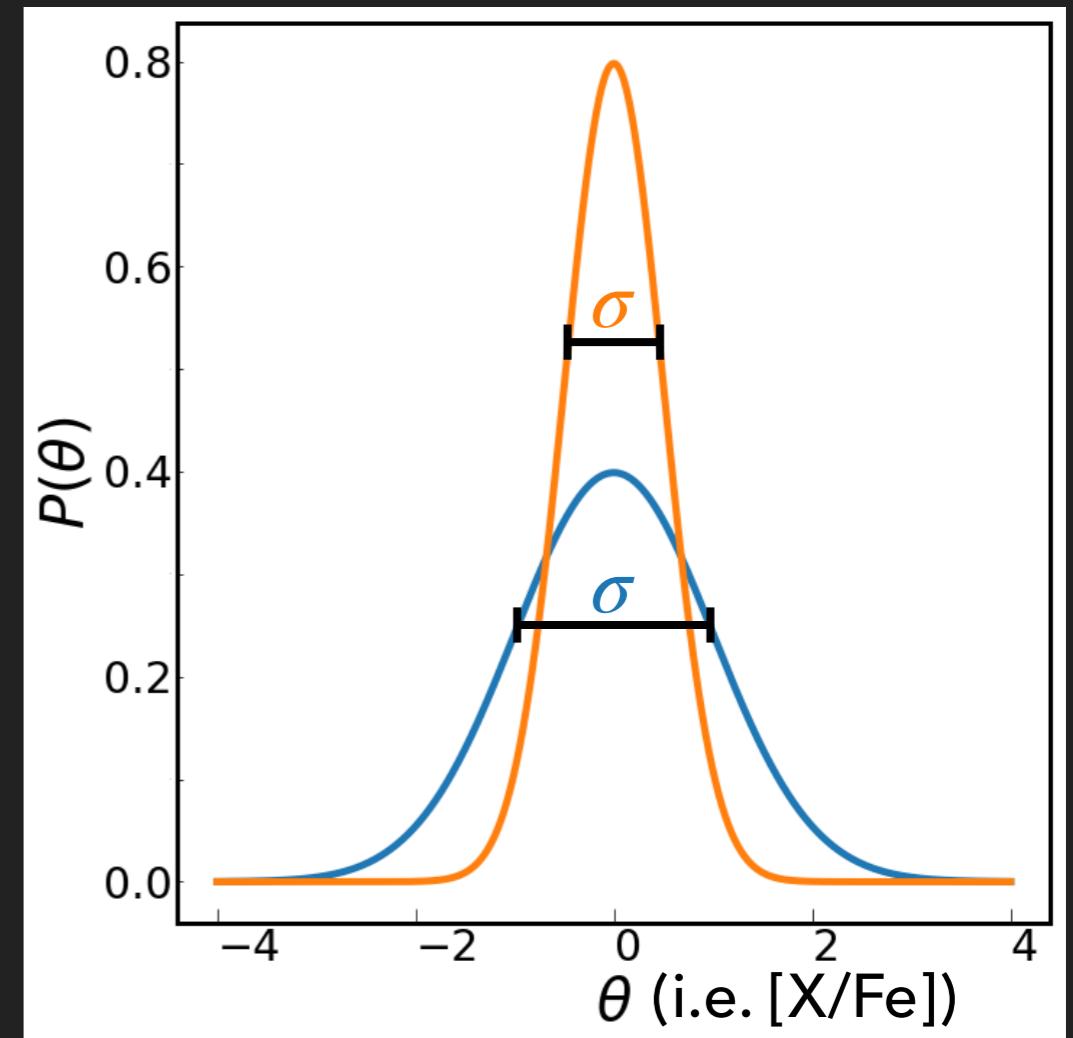


CRAMÉR–RAO LOWER BOUNDS

Fisher Information Matrix

- ▶ Negative curvature of the log-Posterior

$$F_{ij} = \left\langle \frac{\partial^2[-\ln P(\theta)]}{\partial\theta_i\partial\theta_j} \right\rangle$$



CRAMÉR-RAO LOWER BOUNDS

Fisher Information Matrix

- ▶ Negative curvature of the log-Posterior
- ▶ Can be written in terms spectral gradients (Ting+ 2017)

$$F_{ij} = \left[\frac{\partial f(\lambda, \theta)}{\partial \theta_i} \right]^T \Sigma^{-1} \left[\frac{\partial f(\lambda, \theta)}{\partial \theta_j} \right]$$



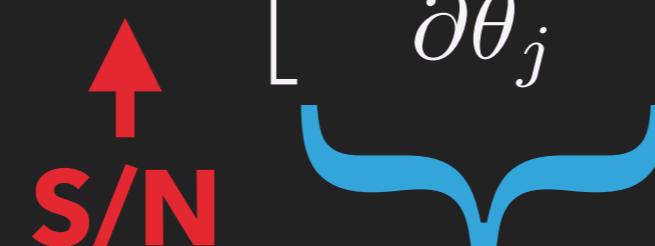
Spectral Gradient wrt θ

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Spectral Gradient wrt θ

$f(\lambda, \theta) = \begin{matrix} \text{Spectral} \\ \text{Model} \end{matrix}$

CRAMÉR-RAO LOWER BOUNDS

Fisher Information Matrix

- ▶ Negative curvature of the log-Posterior
- ▶ Can be written in terms spectral gradients (Ting+ 2017)

$$F_{ij} = \left[\frac{\partial f(\lambda, \theta)}{\partial \theta_i} \right]^T \Sigma^{-1} \left[\frac{\partial f(\lambda, \theta)}{\partial \theta_j} \right] \quad \left(f(\lambda, \theta) = \begin{array}{l} \text{Spectral} \\ \text{Model} \end{array} \right)$$

|
Cramér-Rao Inequality

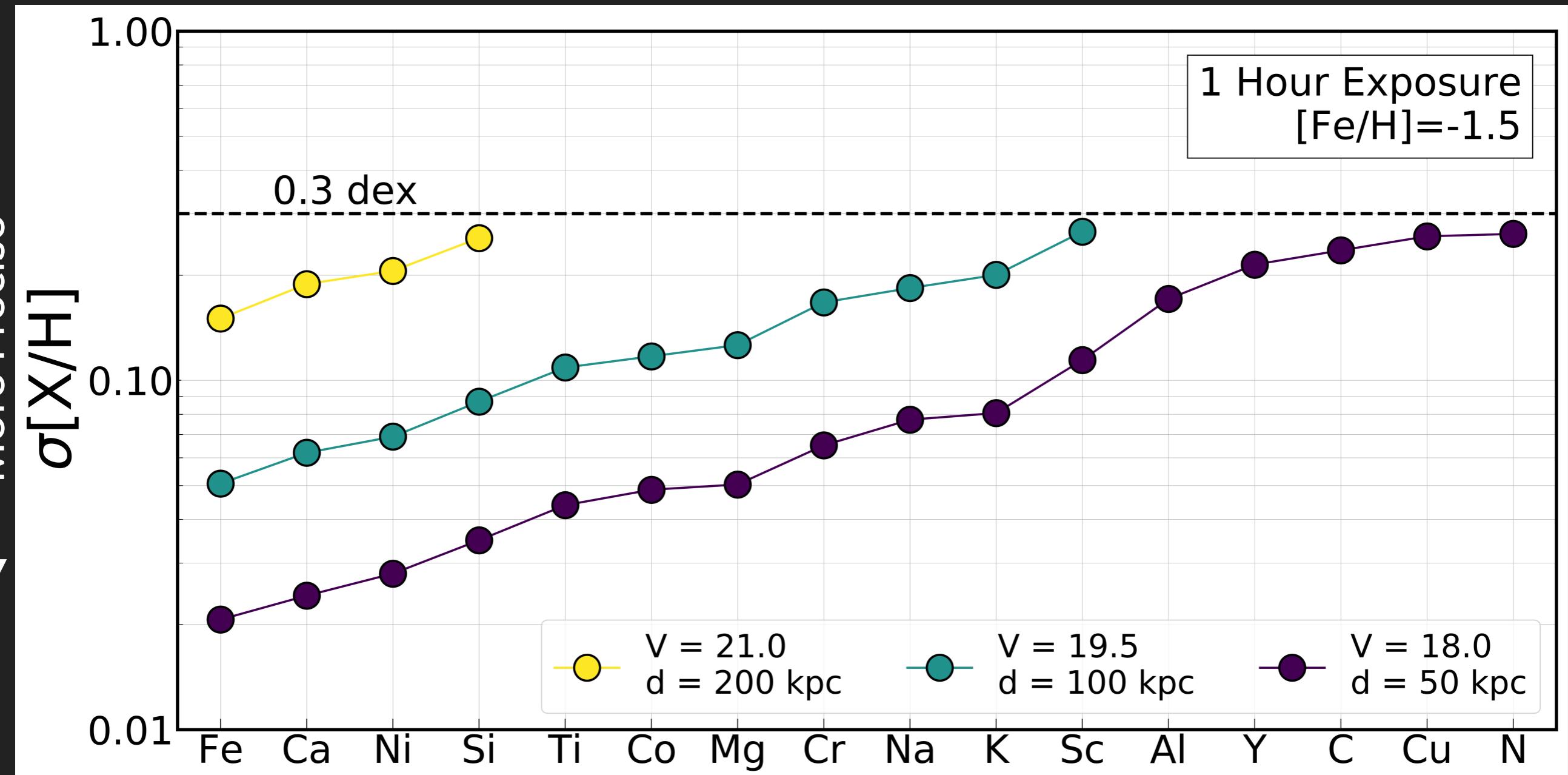


$$\sigma_i \leq \sqrt{(F^{-1})_{ii}}$$

CRLB: Highest precision achievable
from a set of observations

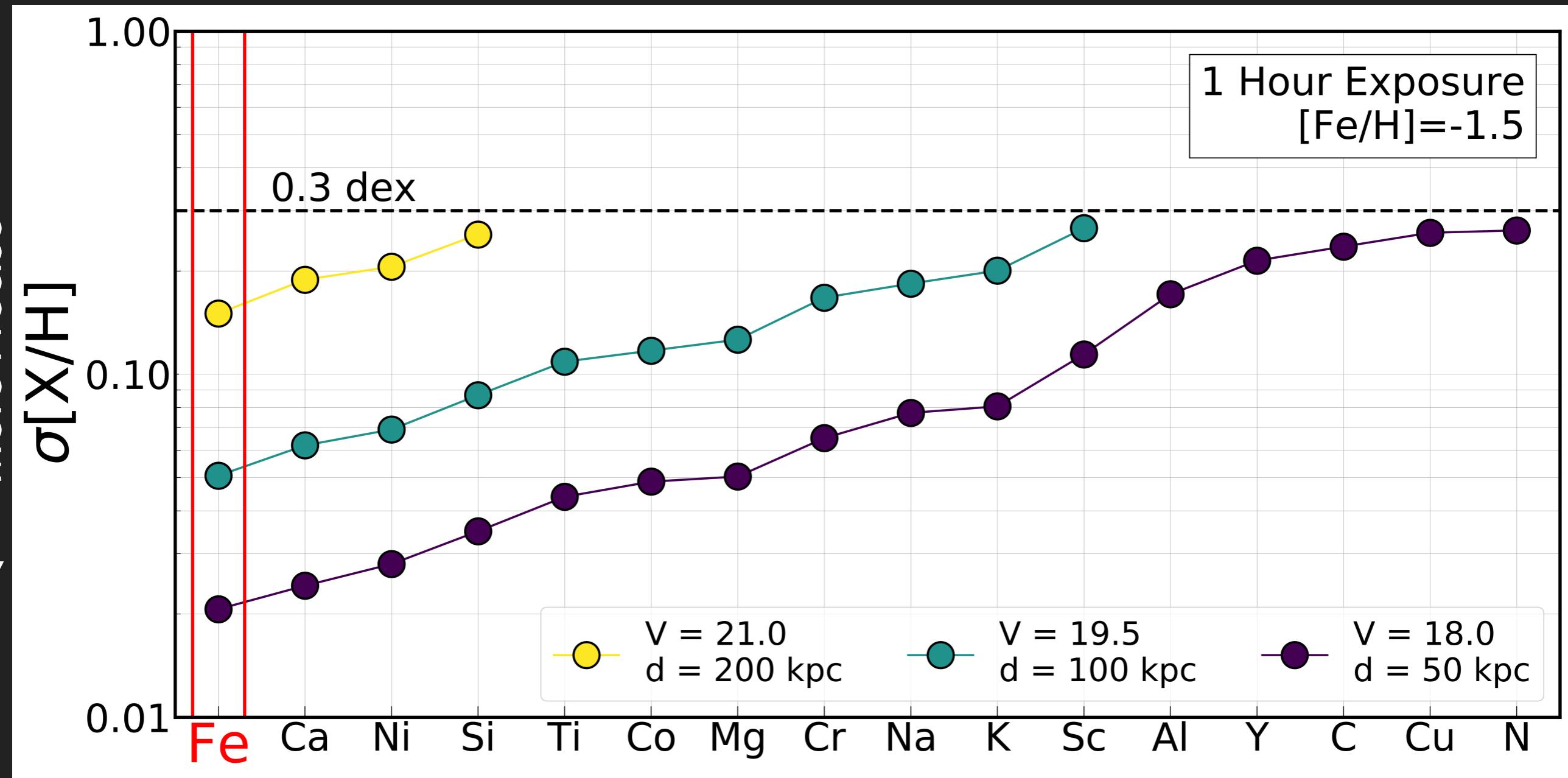
DEIMOS 1200G THEORETICAL PRECISION

$6500\text{\AA} < \lambda < 9000\text{\AA}$
 $R = 6500$



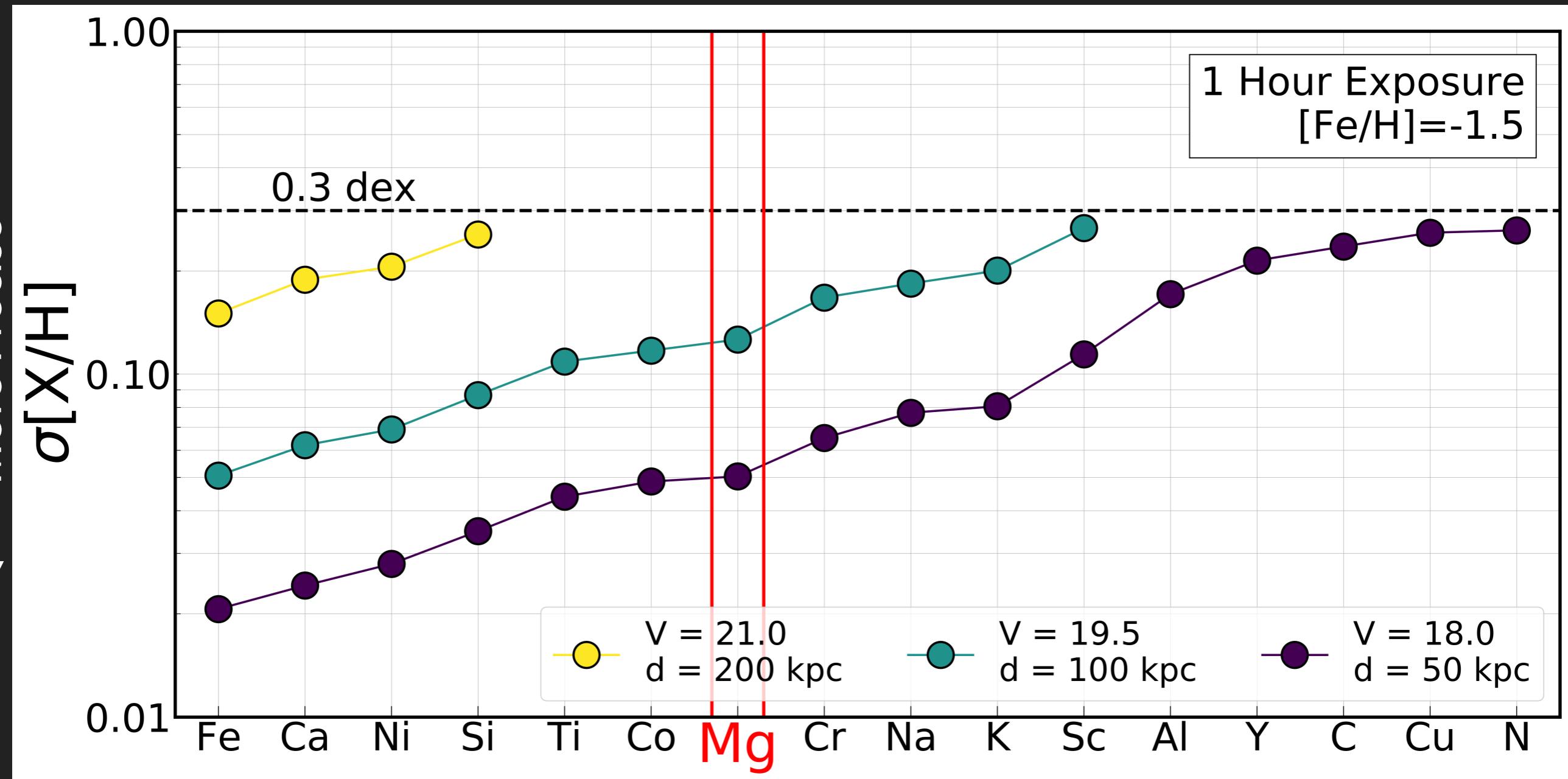
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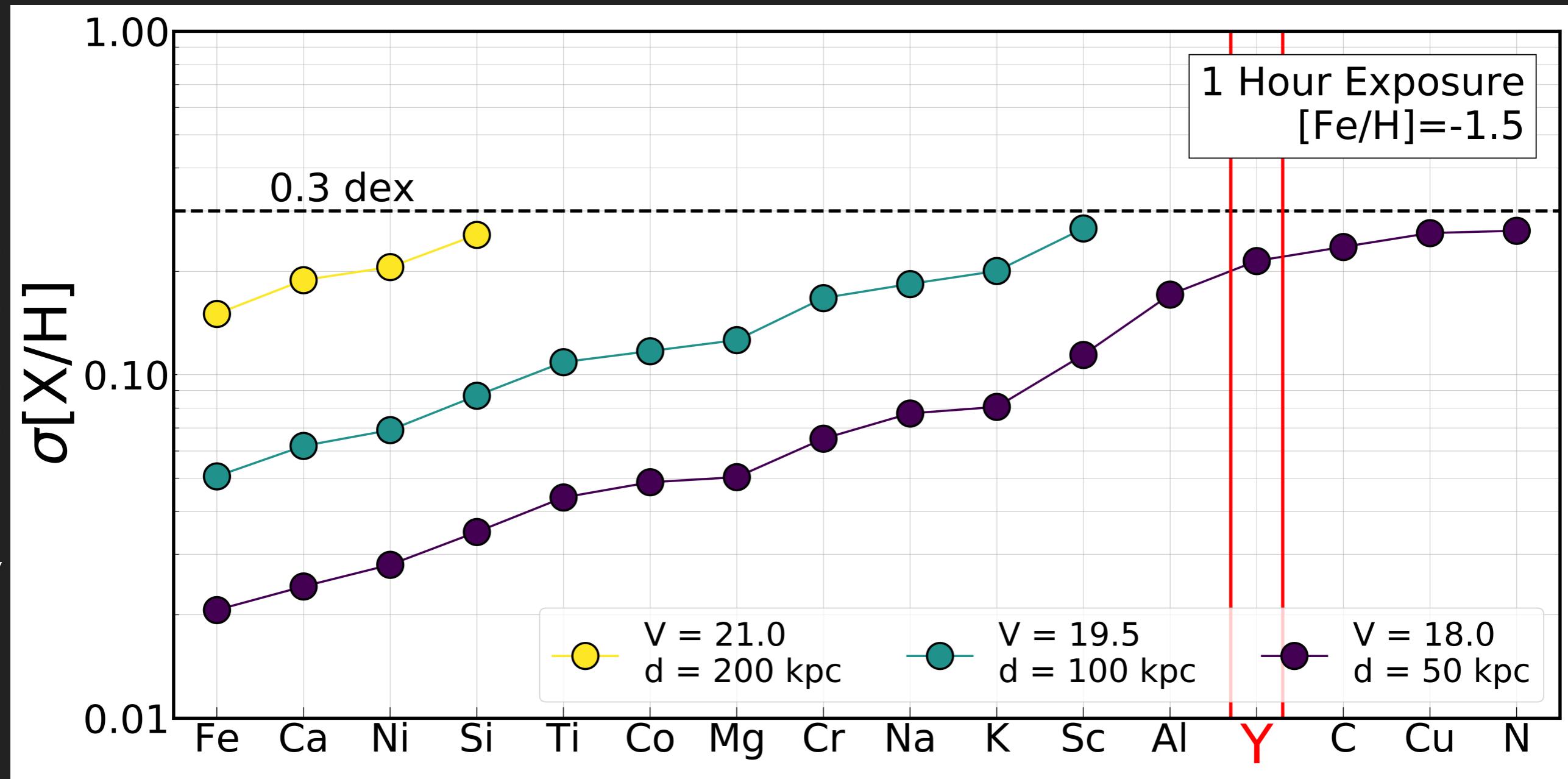
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BLUE SPECTRA ARE INFORMATION RICH

DEIMOS 1200G

$6500\text{\AA} < \lambda < 9000\text{\AA}$
 $R = 6500$

DEIMOS 1200B

$4000\text{\AA} < \lambda < 6400\text{\AA}$
 $R = 4000$

LRIS 600/4000

$3900\text{\AA} < \lambda < 5500\text{\AA}$
 $R = 1800$

BLUE SPECTRA ARE INFORMATION RICH

DEIMOS 1200G

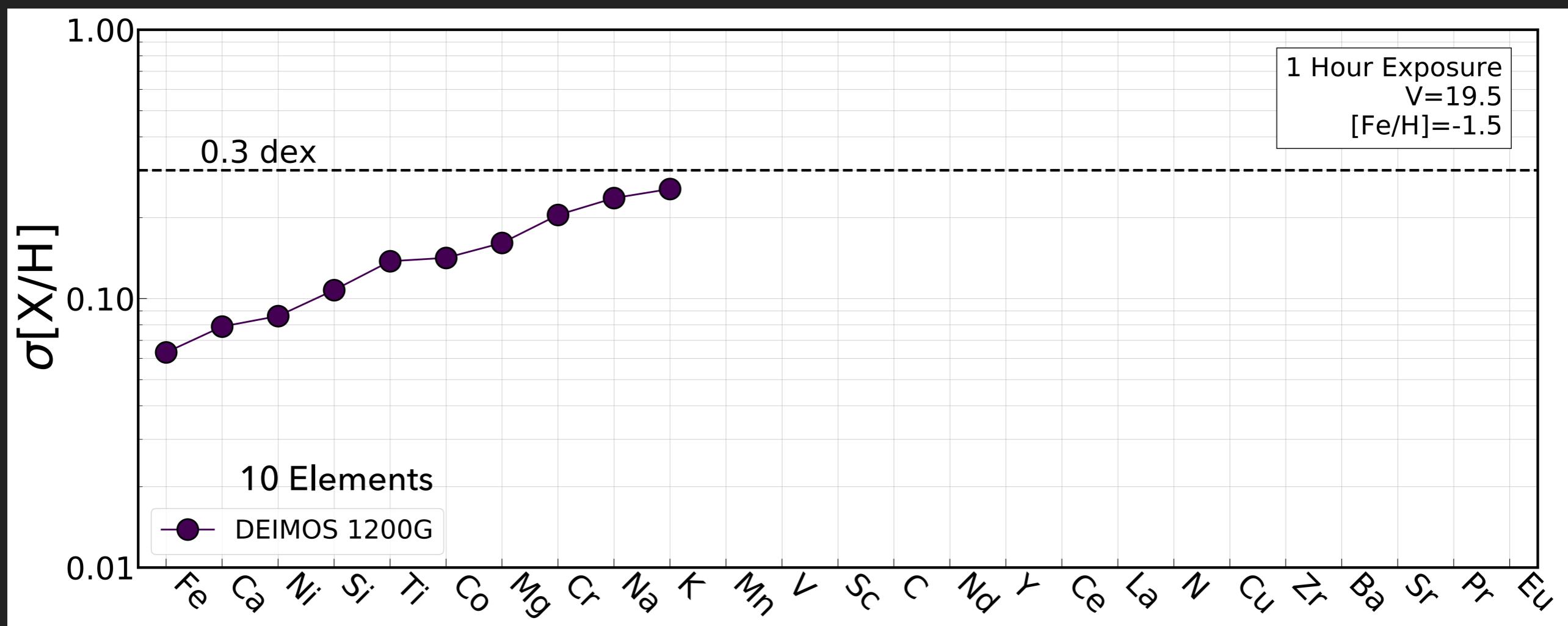
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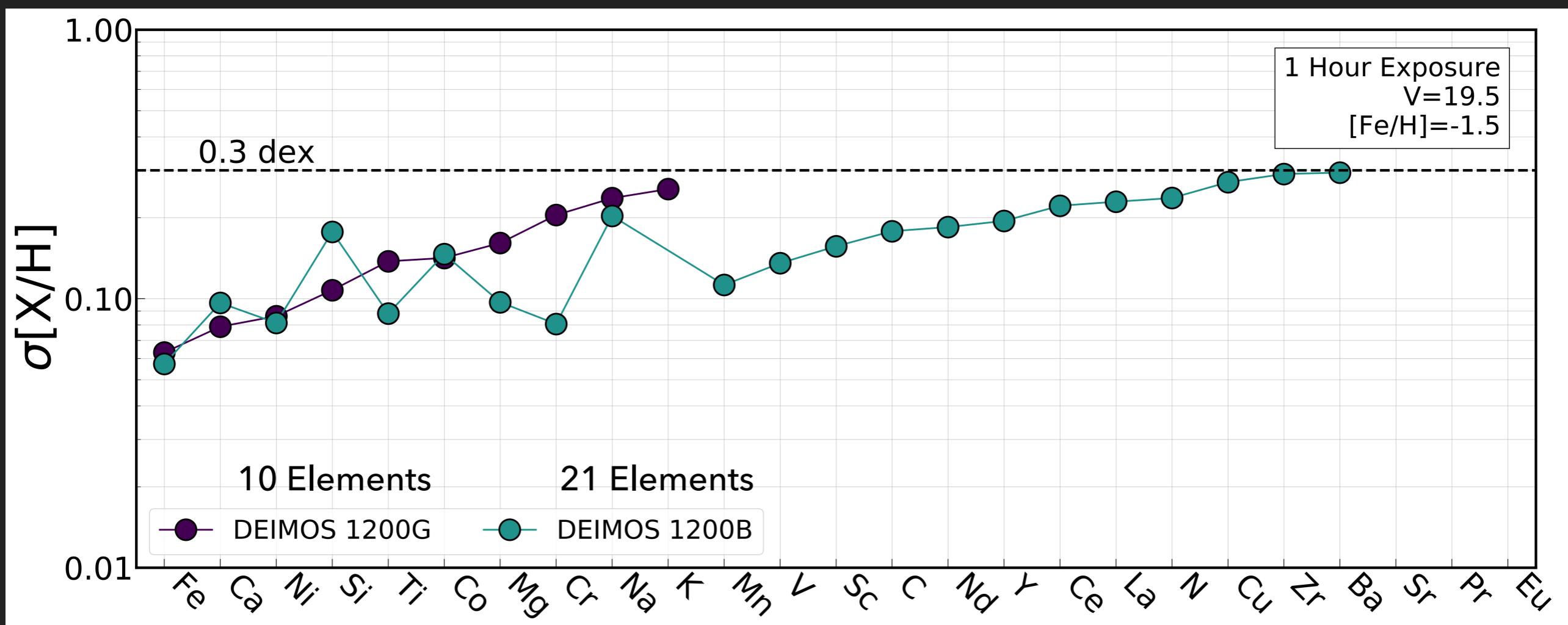
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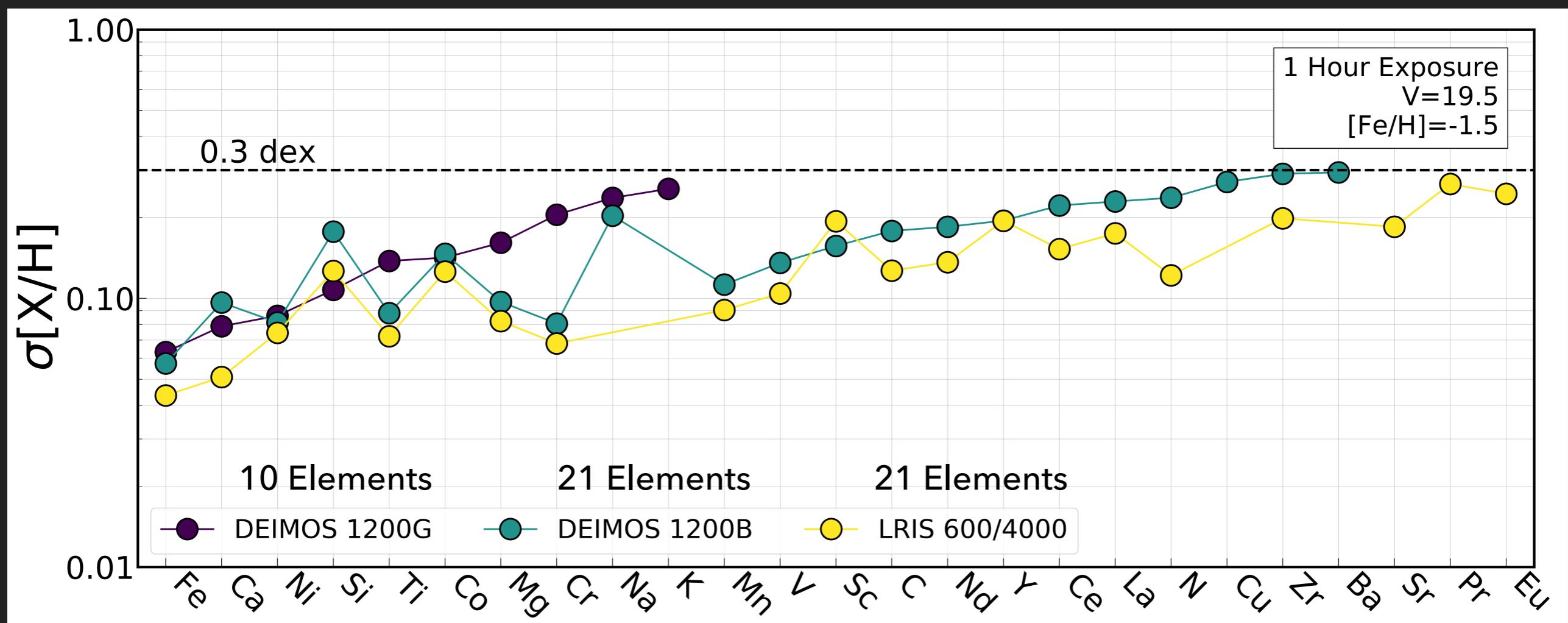
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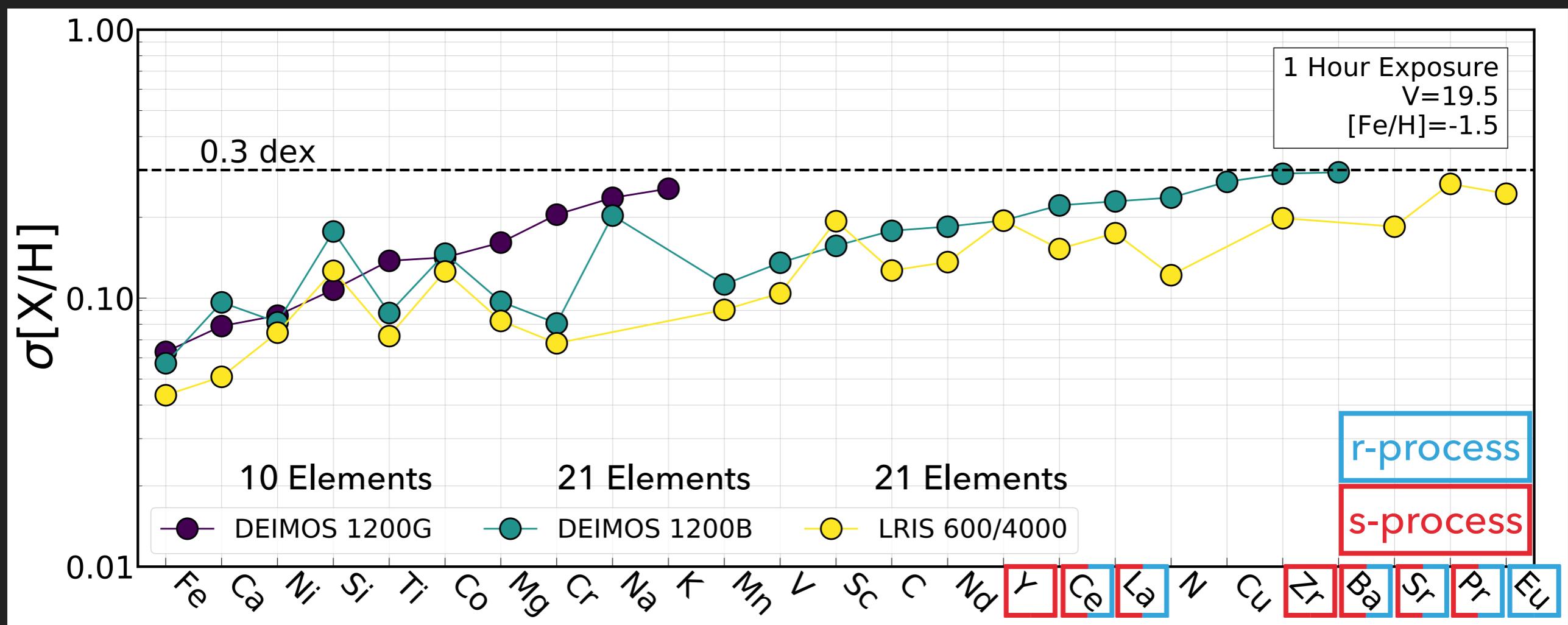
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DEIMOS 1200B

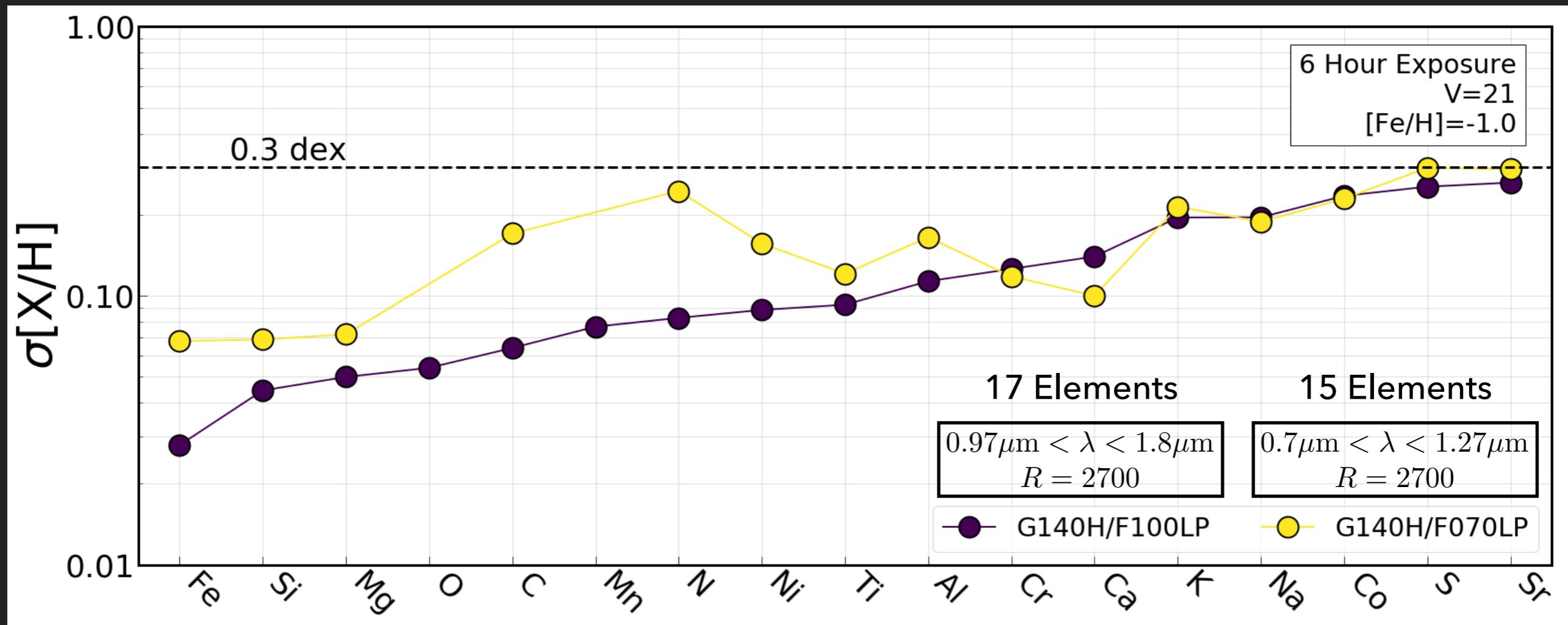
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 $R = 4000$

LRIS 600/4000

$3900\text{\AA} < \lambda < 5500\text{\AA}$
 $R = 1800$



JWST SPECTRA: FAINT STARS, CROWDED FIELDS



ARE THE CRLBs REALISTIC?

In Practice:

- ▶ Information lost
 - ▶ Detector gaps, Sky lines, Masked non-LTE lines
- ▶ Imperfect Calibrations
 - ▶ Continuum normalization, Sky subtraction, Wavelength solution
- ▶ Imperfect Models
 - ▶ Non-LTE lines, Incomplete linelists
- ▶ Model Interpolation
 - ▶ Many-dimensional fitting

These are all being actively improved upon!

EXPLORE ON YOUR OWN!

Chem-I-Calc

▼ 1. Select Reference Star

Reference Star(s):

RGB_m1.5

x ▾

▼ 2. Set Instrument Specifications

Instrument Combination: Combo-0

DESI (b) x ▾

Name: DESI (b)

Wavelength (Å): 3600 - 5550
Min: 3000 Å Max: 18000 Å

Res. Power ($\Delta\lambda/\lambda$): 2000

Pixels / FWHM:
3

▼ SNR Configuration

Constant x ▾

SNR / pixel: 30

REMOVE

DESI (r) x ▾

Name: DESI (r)

Wavelength (Å): 5550 - 6560
Min: 3000 Å Max: 18000 Å

Res. Power ($\Delta\lambda/\lambda$): 3200

Pixels / FWHM:
3

▼ SNR Configuration

Constant x ▾

SNR / pixel: 30

REMOVE

DESI (i) x ▾

Name: DESI (i)

Wavelength (Å): 6560 - 9800
Min: 3000 Å Max: 18000 Å

Res. Power ($\Delta\lambda/\lambda$): 4100

Pixels / FWHM:
3

▼ SNR Configuration

Constant x ▾

SNR / pixel: 30

REMOVE

CONCLUSION

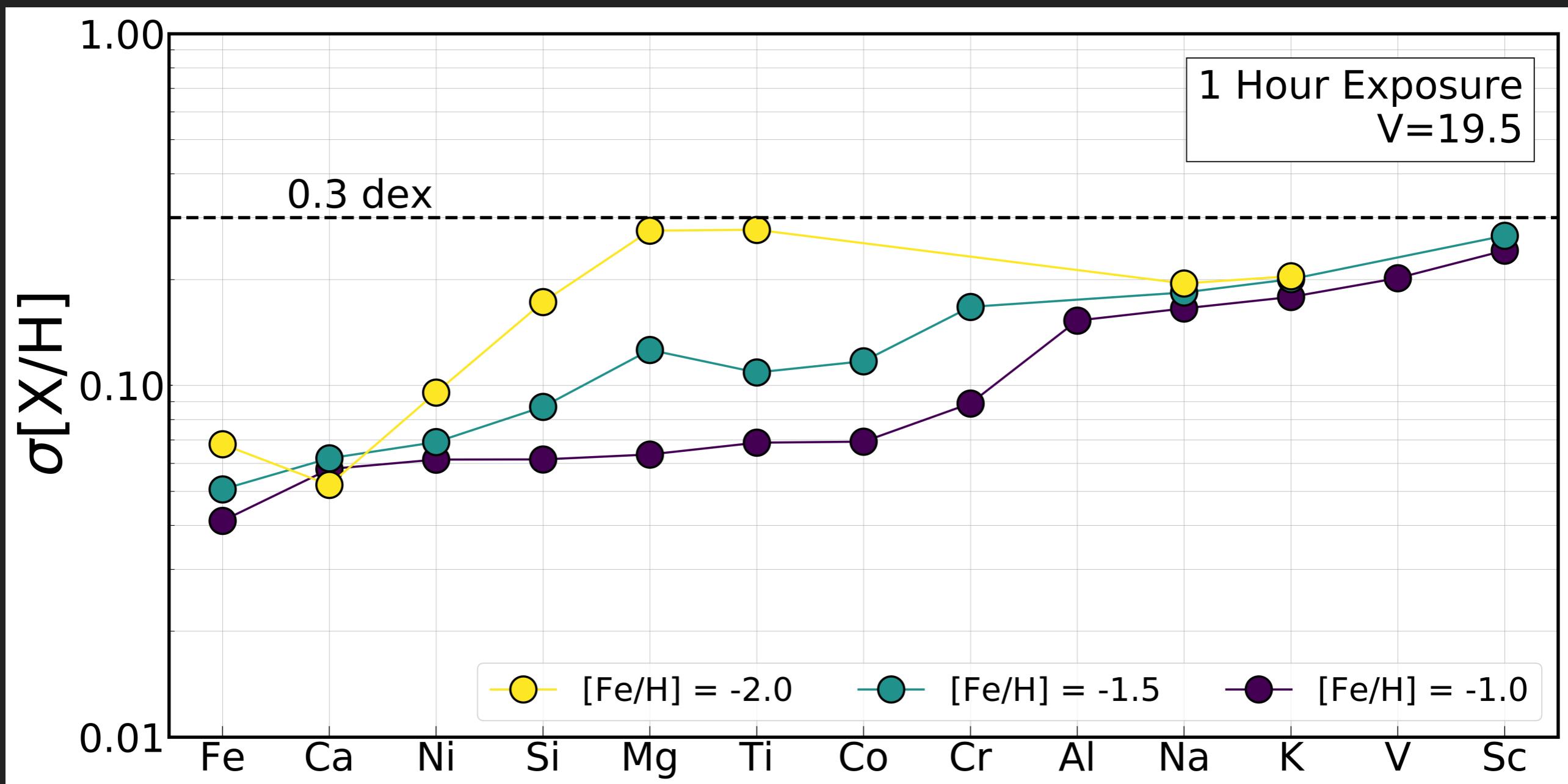
Take-Aways:

- ▶ CRLBs provide a quantitative measure of the information content of spectra
- ▶ Benchmark for achievable precision
- ▶ Useful tool for designing observations / instruments
- ▶ CRLBs are applicable to all spectral analysis (e.g. galaxies, HII regions)

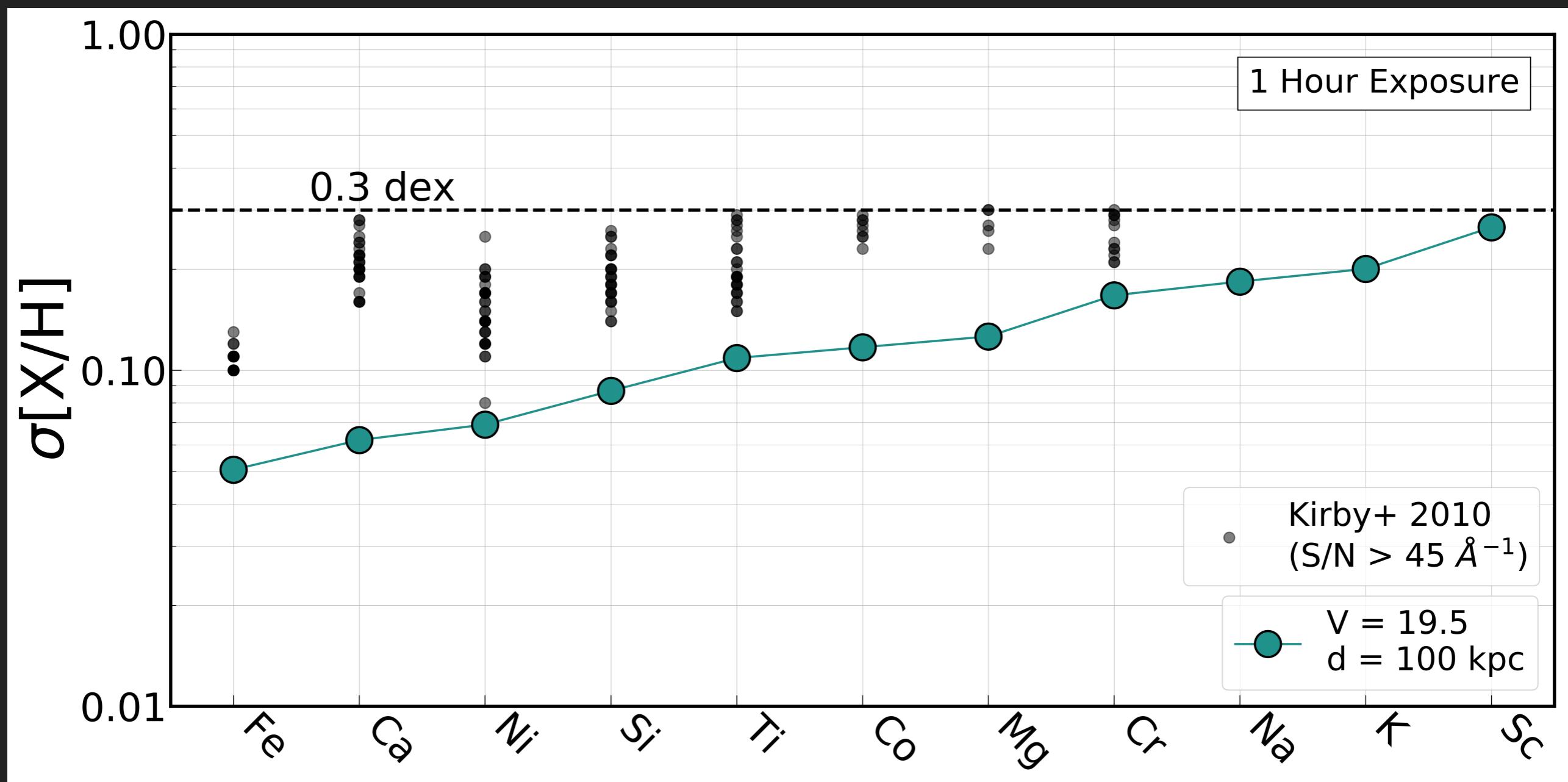
Explore on your own!

- ▶ Github: <https://github.com/NathanSandford/Chem-I-Calc>
- ▶ PyPi: pip install Chem-I-Calc
- ▶ Web Applet: Coming soon...

DEIMOS 1200G CRLB METALLICITY DEPENDENCE



DEIMOS 1200G CRLBs v. OBSERVATIONS



BAYESIAN CRLBs

$$F_{ij} = \left[\frac{\partial f(\lambda, \theta)}{\partial \theta_i} \right]^T \Sigma^{-1} \left[\frac{\partial f(\lambda, \theta)}{\partial \theta_j} \right] + \boxed{F_{ij,prior}}$$

For Gaussian Priors:

$$F_{ii,prior} = \left(\frac{1}{\sigma_{i,prior}} \right)^2$$

$$\sigma_i \leq \sqrt{(F + \sigma_{prior}^{-2})_{ii}^{-1}}$$