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Lab 3 : Bayesian Networks

Part II: Interference in an existing Bayesian network

5. a)
$P(meltdown) = 0.02578$
$P(meltdown \mid icyWeather) = 0.03472$

b)
$P(meltdown \mid pumpFailureWarning, waterLeakWarning) = 0.14535$
$P(meltdown \mid pumpFailure, waterLeak) = 0.2$

The difference comes from the observations, indeed it's not the same to have a warning as to have a real pump failure.

c) These conditional probabilities are difficult to obtain, because in some cases the repetition of experiment depends on random events. In our case it's a pump failure, which is unpredictable and rare, so it's hard to make a proper estimation. The same as the water leak, they don't occur often, so the statistic data used to simulations might not be accurate.

d) After making this alteration, the domain would change from $\{true, false\}$ to an interval domain (a discrete one), which consists of integer values, for instance $\{-20, -10, 0, 10, 20\}$.
The probability distribution of $P(WaterLeak \mid Temperature)$ will change according to the table of truth of water leak, which is dependent on the temperature.

6. a) A probability table represents a probability of a random variable X given its parent : $P(X \mid Parents(X))$.
For example, for the question 5, the table of truth of the random variable PumpFailureWarning is represented by :
$P(PumpFailureWarning \mid Parents(PumpFailureWarning)) =$
$P(PumpFailureWarning \mid PumpFailure)) =$
$P(pumpFailureWarning \mid pumpFailure))+$
$P(pumpFailureWarning \mid \neg pumpFailure))+$
$P(\neg pumpFailureWarning \mid pumpFailure))+$
$P(\neg pumpFailureWarning \mid \neg pumpFailure))$

b) Join probability distribution is a distribution made on multiple variables, where we present the probabilities of all combinations of the values of variables considered.

$P(\neg meltdown, \neg pumpFailureWarning, \neg pumpFailure, \neg waterLeakWarning, \neg waterLeak, \neg icyWeather)$

$= P(\neg meltdown \mid \neg pumpFailure, \neg waterLeak) \times P(\neg pumpFailureWarning \mid \neg pumpFailure) \times P(\neg pumpFailure) \times P(\neg waterLeakWarning \mid \neg waterLeak) \times P(\neg waterLeak \mid \neg icyWeather) \times P(\neg icyWeather)$

$= 0.999 \times 0.95 \times 0.9 \times 0.95 \times 0.9 \times 0.95$

$= 0.69378$

By common sense, this is basically a “normal” state for a nuclear plant, there is nothing anormal. Moreover, on basis of the calculations, we can see that this state is a common one for a nuclear plant to be in, because it’s almost 70% probable.

c)

$P(meltdown \mid waterLeak, pumpFailure) = 0.2$

The other variables will not change the result, because they are not dependent of Meltdown variable.

d)

$P(M \mid \neg pfw, \neg wl, \neg wlw, \neg iw)$

$= \alpha \times P(M, \neg pfw, \neg wl, \neg wlw, \neg iw)$

$= \alpha \times \sum_e P(M, \neg pfw, \neg wl, \neg wlw, \neg iw, PumpFailure = e)$

$= \alpha \times \sum_e P(M \mid PF = e, \neg wl) \times P(\neg pfw \mid PF = e) \times P(\neg wl \mid \neg iw) \times P(\neg wlw \mid \neg wl) \times P(\neg iw) \times P(PF = e)$

$= \alpha \times \Big[P(\neg wl \mid \neg iw) \times P(\neg wlw \mid \neg wl) \times P(\neg iw)\Big] \times \sum_e P(M \mid PF = e, \neg wl) \times P(\neg pfw \mid PF = e) \times P(PF = e)$

$= \alpha \times \Big[P(\neg wl \mid \neg iw) \times P(\neg wlw \mid \neg wl) \times P(\neg iw)\Big] \times \Big[\big(P(M \mid pf, \neg wl) \times P(\neg pfw \mid pf) \times P(pf)\big) + \big(P(M \mid \neg pf, \neg wl) \times P(\neg pfw \mid \neg pf) \times P(\neg pf)\big)\Big]$

$= \alpha \times \big[0.9 \times 0.95 \times 0.95\big] \times \Big[\big(\langle 0.15, 0.85\rangle \times 0.1 \times 0.1\big) + \big(\langle 0.001, 0.999\rangle \times 0.95 \times 0.9\big)\Big]$

$M = True$

$(0.15 \times 0.1 \times 0.1) + (0.001 \times 0.95 \times 0.9) = 0.002355$

$M = False$

$(0.85 \times 0.1 \times 0.1) + (0.999 \times 0.95 \times 0.9) = 0.862645$

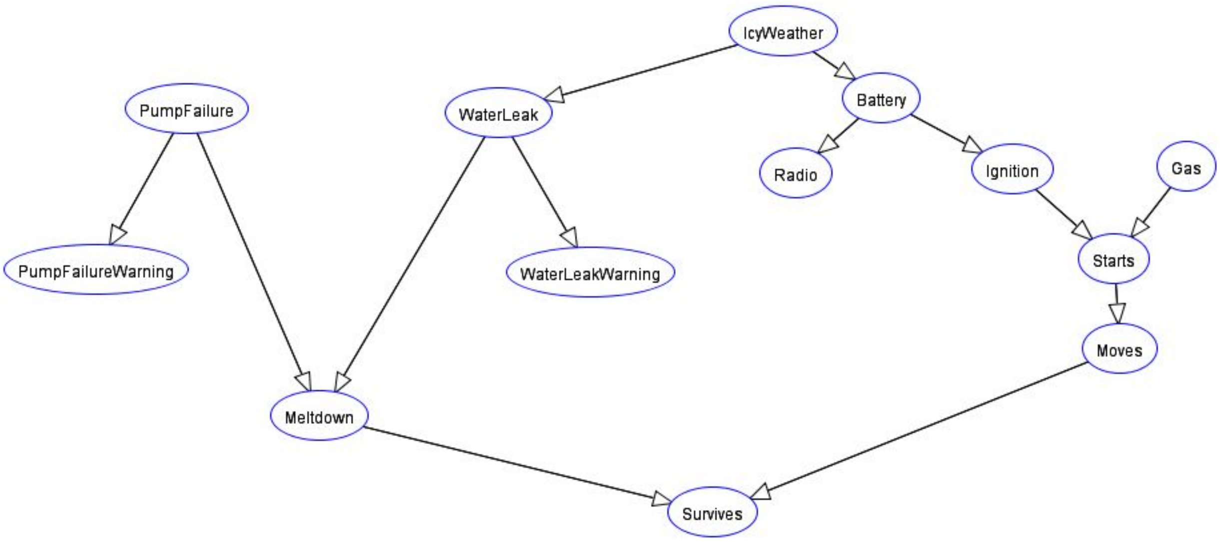
$P(M \mid \neg pfw, \neg wl, \neg wlw, \neg iw) = \alpha \times 0.9 \times 0.95 \times 0.95 \times \langle 0.002355, 0.862645\rangle$

$= \alpha \times \langle 0.00191, 0.70068\rangle$

$\implies \alpha = \frac{1}{0.00191 + 0.70068} = 1.4233$

$P(M \mid \neg pfw, \neg wl, \neg wlw, \neg iw) = \langle 0.00272, 0.99728\rangle$

Part III: Extending a network



Answers:

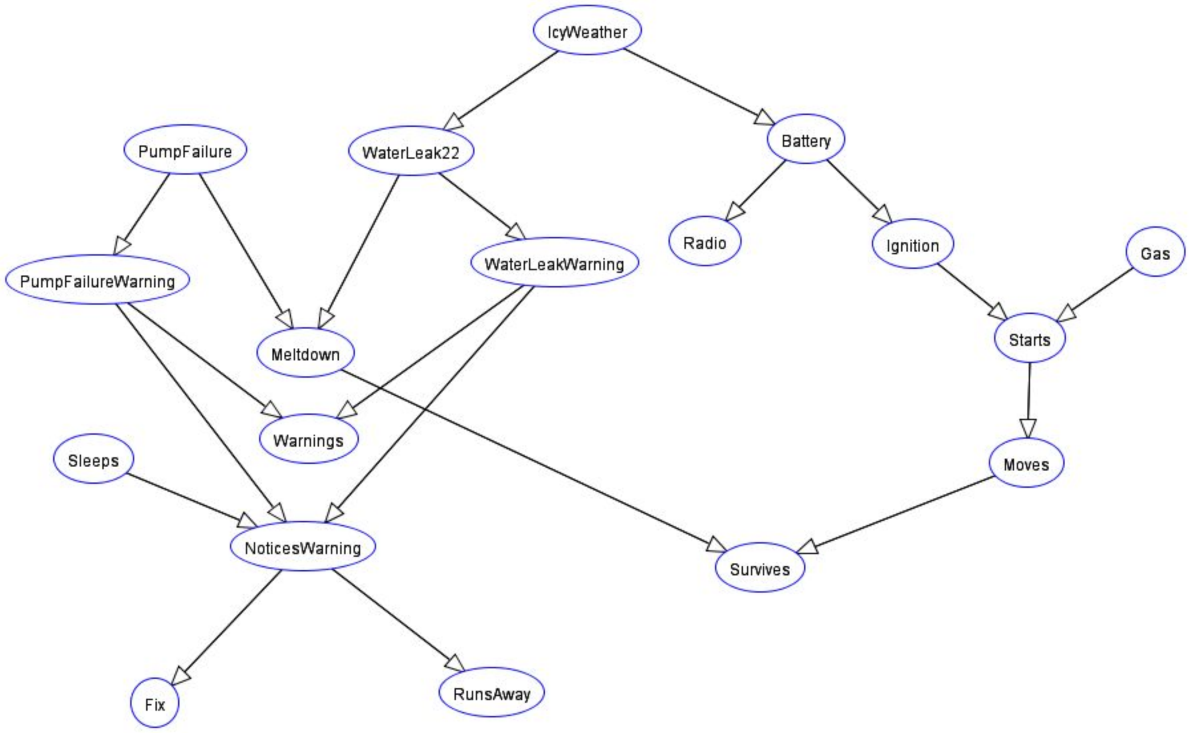
1. $P(survives) = 0.98253$
   $P(survives \mid \neg radio) = 0.97758$
   After this observation the chances of the owner's survival dropped by approximately 0.01.

2. Adding the bicycle, it will increase the probabilities.
   $P(survives) = 0.99178$

3. The complexity of exact inference in Bayesian networks is strongly dependent on the structure of the network. The simpler the structure, the smaller the complexity of exact inference. In means of complexity, we distinguish two types of Bayesian networks, which are singly connected networks (polytrees) and multiply connected networks. The complexity of exact inference in polytrees is linear in the size of the network. On the other hand, in case of multiply connected networks variable elimination can have exponential time and space complexity in the worst scenario, which leaves us with conclude that inference in Bayesian networks is NP-hard. The alternative to variable elimination algorithm is using one of clustering algorithms instead. The basic idea of all join tree algorithms is to join individual nodes of the network and form them into clusters, in such a way to get a new network as a result, which will be a polytree. If we look at the time costs we can easily state that this method is much more sufficient in comparison to variable elimination algorithms.

Part IV: More extensions

Below the diagram for the part IV is presented.



Answers:

1. Yes, we could consider decreasing the probability of pump failure occurrence as getting a better pump. If we decreased the probability respectively to our specialist chances of repairing the fault, then this approach should give better results, as we avoid adding a human model to our network, which is hard to model properly.

2. It's difficult to compute logical proposition into Bayesian networks, because the observations made are on separate variables, and here we want a disjunction on two random variables so the solution is to manually add a disjunction node and make an observations on it.

3. In case of creating a Bayesian network model of a person we assume that a person can act only in limited, certain ways described by the network. We assume that all the people are the same in terms of behaviour, which is not true in reality. We cannot measure human behaviour properly. Also we cannot assure that the person will choose only one action from the available ones in the diagram. For example in the diagram presented above, in the first place a specialist would try to fix the problem and then after realising that nothing more can be done he or she would decide to run away.

4. An example of a dynamic world can be presented with a sequence of three nodes, which state whether on a certain day there was an icy weather or not. We can observe that the probability of a water leak is higher if the icy weather occured in previous days. The example of the described world is presented on the diagram below.