Let us consider polynomials of the fourth degree of the form $(x+a)^3(x-a)$ for $a \in \mathbb{R}$. First, we multiply to find

$$
\begin{aligned}
(x+a)^3(x-a) &= (x+a)^2(x^2-a^2) = (x^2+a^2+2ax)(x^2-a^2) && (1)\\
&= x^4 - a^2x^2 + a^2x^2 - a^4 + 2ax^3 - 2a^3x && (2)\\
&= x^4 + 2ax^3 - 2a^3x - a^4. && (3)
\end{aligned}
$$

Thus, with the standard notation

$$a_3 = 2a \quad a_2 = 0 \quad a_1 = -2a^3 \quad a_0 = -a^4. \tag{4}$$

We then compute the resolvent cubic,

$$
\begin{aligned}
0 &= y^3 - a_2y^2 + (a_1a_3 - 4a_0)y + (4a_2a_0 - a_1^2 - a_3^2a_0) && (5)\\
&= y^3 + (-2a^3 \times 2a - 4(-a^4))y + (-(-2a^3)^2 - (2a)^2(-a^4)) && (6)\\
&= y^3. && (7)
\end{aligned}
$$

Thus, the solutions are $y_1 = y_2 = y_3 = 0$.

Using one of these, the equation for x becomes

$$
\begin{aligned}
0 &= x^2 + \frac{1}{2}\left(a_3 \pm \sqrt{a_3^2 - 4a_2 + 4y_1}\right) + \frac{1}{2}\left(y_1 \mp \sqrt{y_1^2 - 4a_0}\right) && (8)\\
&= x^2 + \frac{1}{2}\left(2a \pm 2a \mp 2a^2\right). && (9)
\end{aligned}
$$

This produces two quadratic equations,

$$0 = x^2 + a^2 \quad \text{and} \quad 0 = x^2 + 2a - a^2, \tag{10}$$

neither of which produce the correct result.

A *very local* analysis suggests that the formula (**??**) could be replaced by

$$0 = x^2 - \frac{1}{2}\left(2a \pm 2n\right)x + \frac{1}{2}(\mp 2a^2), \tag{11}$$

i.e. both inserting an x (which could be a typo since the terms might otherwise have been collected), and changing the sign of the first degree term.

This new formula certainly produces the correct result.

It remains to be seen — by thoroughly following the deductions — where the discrepancy actually lies and how it should be corrected.