

Calculations Pertaining to Analytically Determining Arc Lengths of Gaussian Porcesses

Nathan Wycoff

December 31, 2017

1 Lines as parametric equations

In higher dimensional spaces, lines need to be expressed as parametric equations.

In particular, the line segment connecting a and b may be expressed as such:

$$f(t) = a + t(b - a), t \in [0, 1]$$

It may also be desireable to scale t such that a one unit change in t corresponds to a movement of one unit in euclidean terms in the high D space:

$$f(t) = a + \frac{b - a}{\|b - a\|_2} t, t \in [0, \|b - a\|_2]$$

2 Arc Length of a GP Mean Surface for General Kernel

I will seek to derive the equation for the arc length along a line segment from a to b for a GP with arbitrary kernel function.

In general, to derive the arc length of a function $f : \mathbb{R}^n \rightarrow \mathbb{R}$ over the curve S , this is the quantity of interest:

$$\int_{s \in S} \sqrt{\|s\|_2 + f(s)^2} ds$$

In our case, the mean surface of a GP is defined as such:

$$\mu(x) = k(x)' K^{-1} y$$

Where $k(x)$ is the kernel function evaluated with respect to each training point, stored in a vector, K is the kernel matrix evaluated for training points, and y is the observed training responses.

Also, the curve is simply a line from a to b :

$$S = \{s : \exists t \in [0, 1] \text{ s.t. } s = a + t(b - a)\}$$

So we can rewrite the integral above as:

$$\int_{t=0}^1 \sqrt{\|s(t)\|_2 + (k(x)'K^{-1}y)^2} = \int_{t=0}^1 \sqrt{\|a + t(b-a)\|_2 + (k(x)'K^{-1}y)^2}$$

We will denote $K^{-1}y$ as α , an n-vector.