Shapes 3

In[1]:=
```
i2 = {1, 0};
j2 = {0, 1};
i3 = {1, 0, 0};
j3 = {0, 1, 0};
k3 = {0, 0, 1};
```

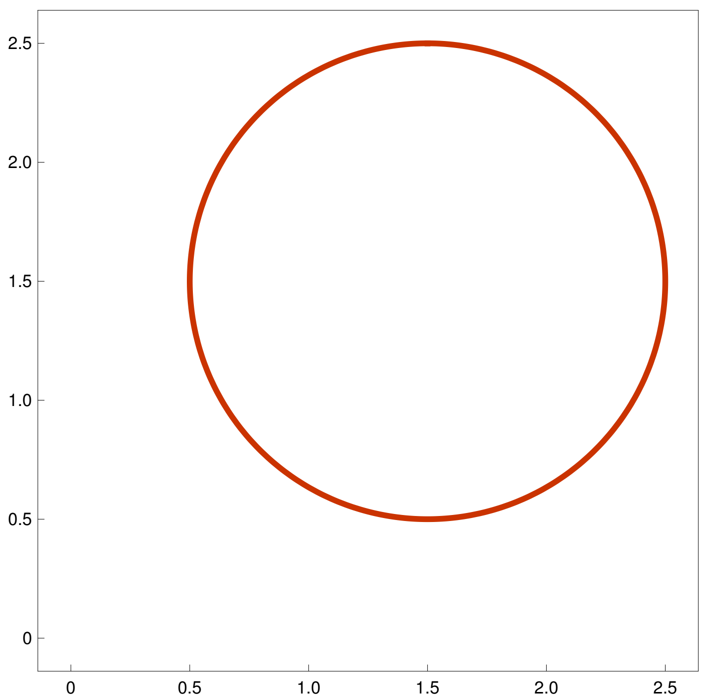
Question 1

In[6]:=
```
x1 = 1.5;
y1 = 1.5;
R = 1;
```

In[9]:=
```
x = R * Sin[t] + x1;
y = R * Cos[t] + y1;
```

In[11]:=
```
ParametricPlot[{x, y}, {t, 0, 2 Pi}, PlotTheme → "Web", AxesOrigin → {0, 0}]
```

Out[11]=



In[12]:=
```
r = x * i2 + y * j2
r' = D[r, t]
Integrate[Norm[r'], {t, 0, 2 Pi}]
```
Out[12]= {1.5 + Sin[t], 1.5 + Cos[t]}

Out[13]= {Cos[t], -Sin[t]}

Out[14]= 2π

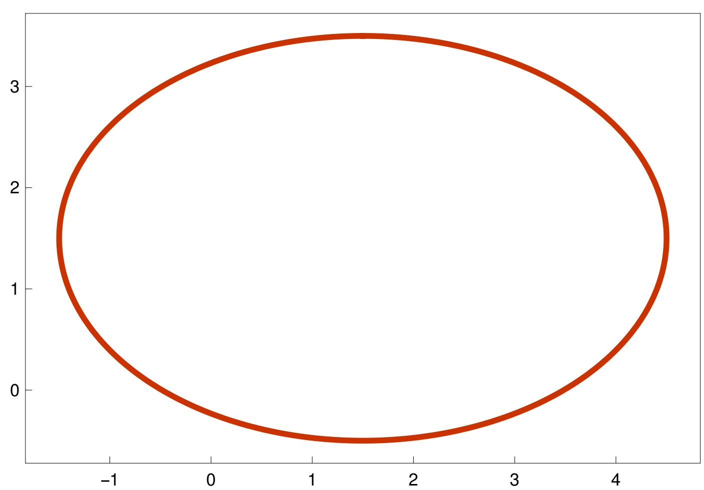
Question 2

In[15]:=
```
Clear["Global`*"]
i2 = {1, 0};
j2 = {0, 1};
i3 = {1, 0, 0};
j3 = {0, 1, 0};
k3 = {0, 0, 1};
```

In[21]:=
```
x1 = 1.5;
y1 = 1.5;
a = 3;
b = 2;
```

In[25]:=
```
x = a * Sin[t] + x1;
y = b * Cos[t] + y1;
```

In[27]:=
```
ParametricPlot[{x, y}, {t, 0, 2 Pi}, PlotTheme → "Web", AxesOrigin → {0, 0}]
```

Out[27]=


In[28]:=
```
r = x * i2 + y * j2;
r' = D[r, t];
N@Integrate[Norm[r'], {t, 0, 2 Pi}]
```
Out[30]= 15.8654

In[31]:=
```
"Unit Normal";
T = r' / Norm[r']
T' = D[T, t];
N1 = T' / Norm[T'];
Simplify[N1];
```

Out[32]= $\left\{\frac{3\,\mathrm{Cos}[t]}{\sqrt{9\,\mathrm{Abs}[\mathrm{Cos}[t]]^2 + 4\,\mathrm{Abs}[\mathrm{Sin}[t]]^2}}, -\frac{2\,\mathrm{Sin}[t]}{\sqrt{9\,\mathrm{Abs}[\mathrm{Cos}[t]]^2 + 4\,\mathrm{Abs}[\mathrm{Sin}[t]]^2}}\right\}$

Question 3

In[36]:=
```
Clear["Global`*"]
i2 = {1, 0};
j2 = {0, 1};
i3 = {1, 0, 0};
j3 = {0, 1, 0};
k3 = {0, 0, 1};
```

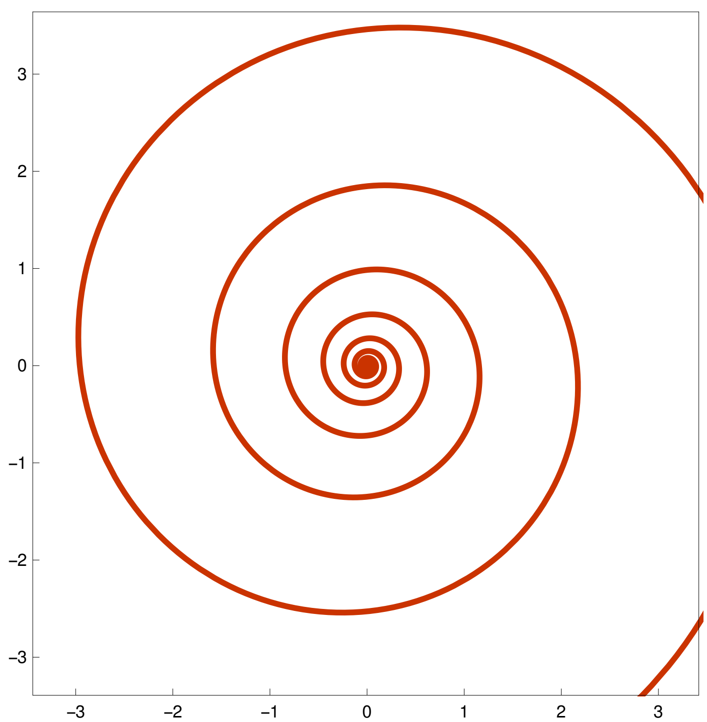
In[42]:=
```
r[u_] := a * Exp[b * u] * Cos[u] * i2 + a * Exp[b * u] * Sin[u] * j2
```

In[43]:=
```
a = 50
b = -.1
ParametricPlot[r[u], {u, 0, 100}, PlotTheme → "Web"]
```

Out[43]= 50

Out[44]= -0.1

Out[45]=



In[46]:=
```
T = r'[u] / Norm[r'[u]];
N1 = D[T, u] / Norm[D[T, u]];
```

- a makes the spiral larger
- b dictates how tight the spiral is
- u is the location of the sprial

In[48]:=
```
Integrate[Norm[r'[u]], {u, 0, Infinity}]
```

Out[48]=
```
502.494
```

Question 4

In[49]:=
```
Clear["Global`*"]
i2 = {1, 0};
j2 = {0, 1};
i3 = {1, 0, 0};
j3 = {0, 1, 0};
k3 = {0, 0, 1};
```

In[55]:=
```
r[u_] = a * Cos[u] * i3 + a * Sin[u] * j3 + b * u * k3
```

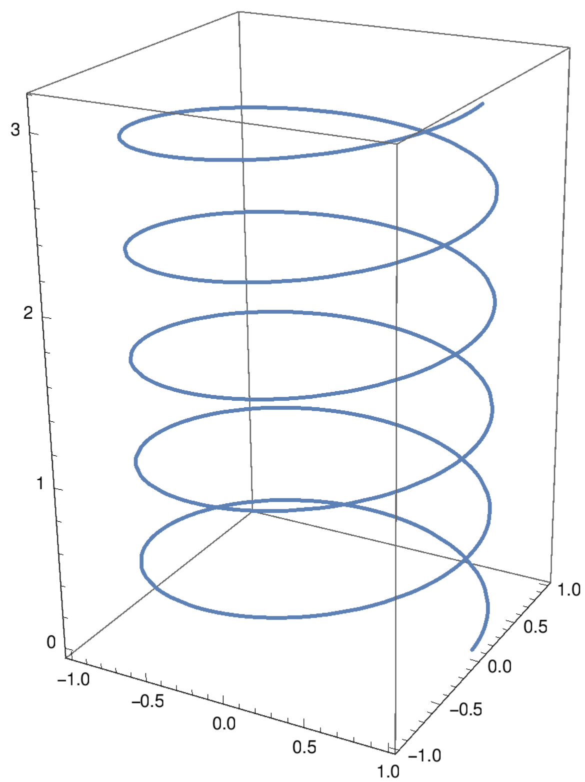
Out[55]=
```
{a Cos[u], a Sin[u], b u}
```

In[56]:=
```
a = 1
b = .1
ParametricPlot3D[r[u], {u, 0, 10 Pi}]
```

Out[56]= 1

Out[57]= 0.1

Out[58]=



In[59]:=
```
T = r'[u] / Norm[r'[u]]
```

Out[59]=
$$\left\{-\frac{\text{Sin}[u]}{\sqrt{0.01+\text{Abs}[\text{Cos}[u]]^2+\text{Abs}[\text{Sin}[u]]^2}},\ \frac{\text{Cos}[u]}{\sqrt{0.01+\text{Abs}[\text{Cos}[u]]^2+\text{Abs}[\text{Sin}[u]]^2}},\ \frac{0.1}{\sqrt{0.01+\text{Abs}[\text{Cos}[u]]^2+\text{Abs}[\text{Sin}[u]]^2}}\right\}$$

- a is the diameter of the spiral
- b makes the spiral heigher or lower
- u determines the number of spirals

In[60]:=
```
Integrate[Norm[r'[u]], {u, 0, 10 Pi}]
```

Out[60]= 31.5726

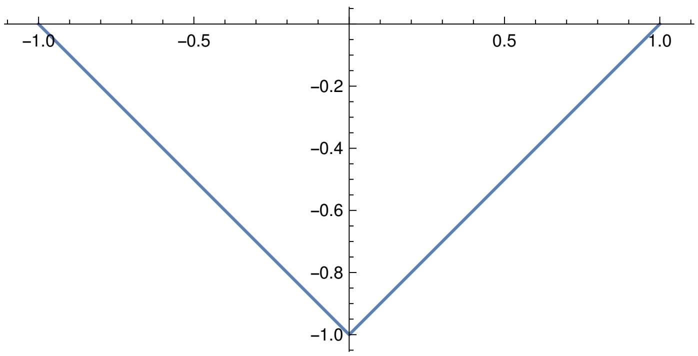
Question 5

In[61]:=
```
Clear["Global`*"]
i2 = {1, 0};
j2 = {0, 1};
i3 = {1, 0, 0};
j3 = {0, 1, 0};
k3 = {0, 0, 1};
```

In[67]:=
```
r[u_] := u * i2 + (Abs[u]^n - 1) * j2
```

In[68]:=
```
n = 1
ParametricPlot[r[u], {u, -1, 1}]
```

Out[68]= 1

Out[69]=


In[70]:=
```
Integrate[Norm[r'[u]], {u, -1, 1}]
```

Out[70]= $2\sqrt{2}$

In[71]:=
```
Integrate[1, {x, -1, 1}, {y, Abs[x]^n - 1, d}]
```

Out[71]= 1 + 2 d

In[72]:=
```
Solve[Sqrt[2] == 1 + 2 d, d]
```

Out[72]= $\left\{\left\{d \to \frac{1}{2}\left(-1+\sqrt{2}\right)\right\}\right\}$

In[73]:=
```
Integrate[Norm[r'[u]], {u, -1, 1}]
```

Out[73]= $2\sqrt{2}$

Question 6

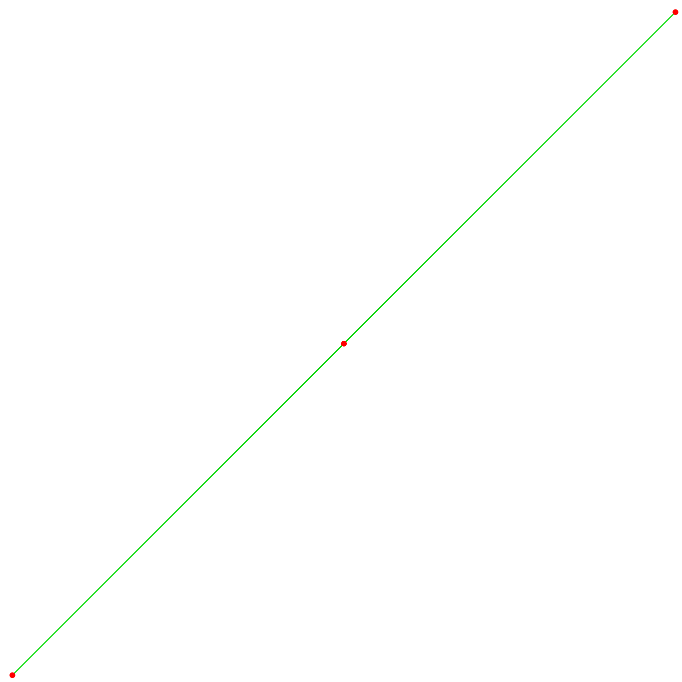
In[74]:=
```
Clear["Global`*"]
i2 = {1, 0};
j2 = {0, 1};
i3 = {1, 0, 0};
j3 = {0, 1, 0};
k3 = {0, 0, 1};
```

In[80]:=
```
a1 = {0, 0};
a2 = {1, 1};
```

In[82]:=
```
r[u_] = (1 - u) a1 + u * a2
```
Out[82]= {u, u}

In[83]:=
```
pts = {r[0], r[1 / 2], r[1]}
```
Out[83]= $\left\{\{0, 0\}, \left\{\frac{1}{2}, \frac{1}{2}\right\}, \{1, 1\}\right\}$

In[84]:=
```
Graphics[{BezierCurve[pts], Green, Line[pts], Red, Point[pts]}]
```
Out[84]=



Question 7

In[85]:=
```
Clear["Global`*"]
i2 = {1, 0};
j2 = {0, 1};
i3 = {1, 0, 0};
j3 = {0, 1, 0};
k3 = {0, 0, 1};
```

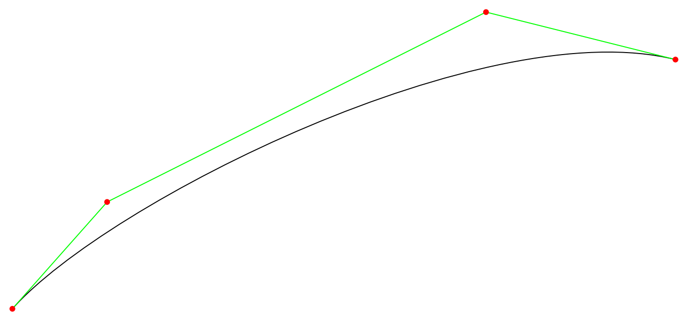
In[91]:=
```
a1 = {0, 0};
a2 = {2, 3};
a3 = {4, 5};
```

In[94]:= $r[u\_] := (1 - u)^2\, a1 + 2 * u * (1 - u)\, a2 + u^2 * a3$

In[95]:=
```
pts = {r[1 / 2], r[1], r[3], r[4]}
```
Out[95]= $\left\{\left\{2, \frac{11}{4}\right\}, \{4, 5\}, \{12, 9\}, \{16, 8\}\right\}$

In[96]:=
```
Graphics[{BezierCurve[pts], Green, Line[pts], Red, Point[pts]}]
```

Out[96]=



Question 8

In[97]:=
```
Clear["Global`*"]
i2 = {1, 0};
j2 = {0, 1};
i3 = {1, 0, 0};
j3 = {0, 1, 0};
k3 = {0, 0, 1};
```

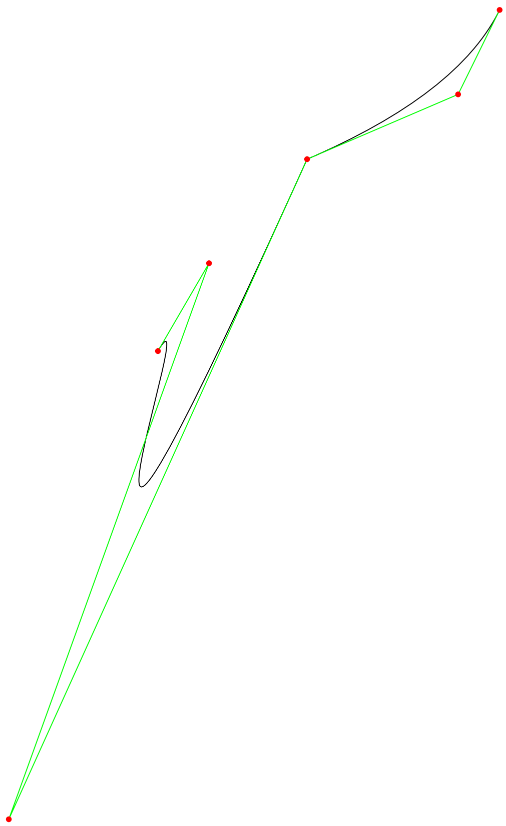
In[103]:=
```
a1 = {0, 0};
a2 = {1, 2};
a3 = {2, 1};
a4 = {2, 2};
```

In[107]:=
```
r[u_] := (1 - u)^3 * a1 + 3 * u * (1 - u)^2 * a2 + 3 * u^2 * (1 - u) * a3 + u^3 * a4
```

In[108]:=
```
pts = {r[0], r[.1], r[-.3], r[.3], r[.7], r[1]}
Graphics[{BezierCurve[pts], Green, Line[pts], Red, Point[pts]}]
```

Out[108]= {{0, 0}, {0.299, 0.515}, {-0.873, -2.745}, {0.873, 1.125}, {1.757, 1.505}, {2, 2}}

Out[109]=



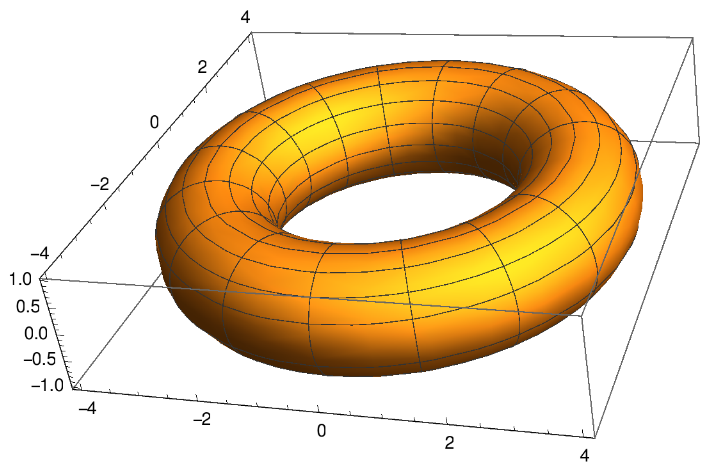
Question 9

In[110]:=
```
Clear["Global`*"]
i2 = {1, 0};
j2 = {0, 1};
i3 = {1, 0, 0};
j3 = {0, 1, 0};
k3 = {0, 0, 1};
```

In[116]:=
```
a = 3;
R = 1;
r[u_, v_] := (a + R * Cos[u]) * Cos[v] * i3 + (a + R * Cos[u]) * Sin[v] * j3 + R * Sin[u] * k3
```

In[119]:=
```
ParametricPlot3D[r[u, v], {u, 0, 2 π}, {v, 0, 2 π}]
```

Out[119]=



In[120]:=
```
du = D[r[u, v], u];
dv = D[r[u, v], v];
cross = Cross[du, dv];
```

In[123]:=
```
n = cross / Norm[cross];
```

In[124]:=
```
Integrate[Norm[cross], {u, 0, 2 π}, {v, 0, 2 π}]
```

Out[124]= $12\,\pi^2$

Question 9

In[125]:=
```
Clear["Global`*"]
i2 = {1, 0};
j2 = {0, 1};
i3 = {1, 0, 0};
j3 = {0, 1, 0};
k3 = {0, 0, 1};
```

In[131]:=
```
x[x_] := x
z[x_] := -H + H * (16 / L^4) * x^4
y[x_] := (W / 2) (4 / L^2) x^2 - (W / 2) Sqrt[(z[u] + H) / H]
r[u_, v_] := x[v] * i3 + y[v] * j3 + z[u] * k3
```

In[135]:=
```
W = 10
L = 30
H = 10
r[u]
ParametricPlot3D[r[u, v], {u, -100, 100}, {v, -100, 100}]
```

Out[135]= 10

Out[136]= 30

Out[137]= 10

Out[138]= r[u]

Out[139]=