BB Review

1. Dot multiply matrix . vector

2. Rot= $\cancel{\cos\theta}$ $\begin{bmatrix} c\theta & -s\theta \\ s\theta & c\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$ $\theta = 30$


3. Reflection x = $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

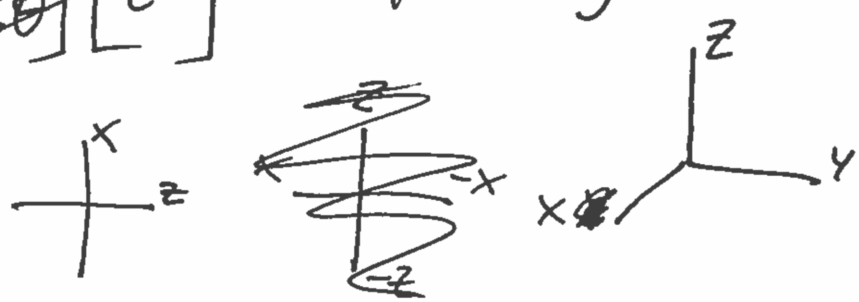
~~Ref.~~
rot . ref . V

4. Rot45 $\begin{bmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{bmatrix}$

5. Horiz shear
$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_1 & v_2 & v_3 & \dots \end{bmatrix}$

6. $\begin{bmatrix} -s\theta & 0 & c\theta \\ 0 & 1 & 0 \\ c\theta & 0 & s\theta \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

about
y axis, xz plane
10 deg

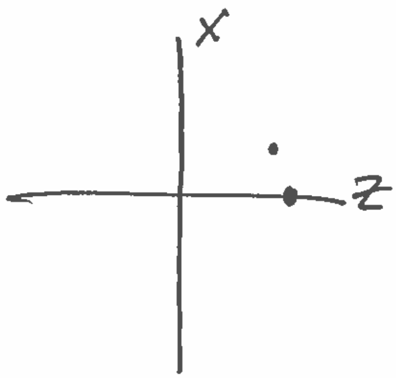


6

$$R_z = \begin{bmatrix} c\theta & -s\theta & 0 \\ s\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} cx - sy \\ sx + cy \\ z \end{bmatrix}$$

$$R_x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\theta & -s \\ 0 & s & c \end{bmatrix} = \begin{bmatrix} x \\ \\ \end{bmatrix}$$

$$R_y = \begin{bmatrix} c & 0 & s \\ 0 & 1 & 0 \\ -s & 0 & c \end{bmatrix} = \begin{bmatrix} cx + sz \\ y \\ -sx + cz \end{bmatrix}$$

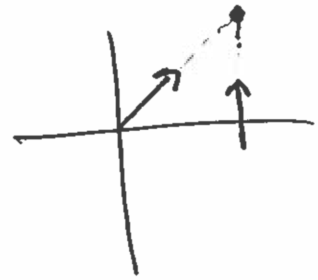



7

$$\text{trans} = \begin{bmatrix} 1 & 0 & d \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x + d \\ y \\ 0 \end{bmatrix}$$

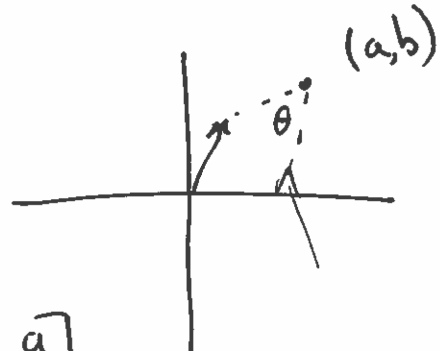
(8) $T \cdot R \cdot V$

$R^{-1} \cdot T^{-1} \cdot T \cdot R \cdot V = V$



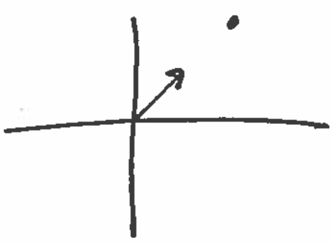
(9)

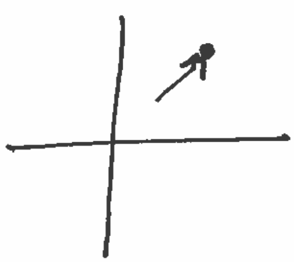
$$V = \begin{bmatrix} x_1 & x_2 \\ y_1 & y_2 \\ 1 & 1 \end{bmatrix}$$

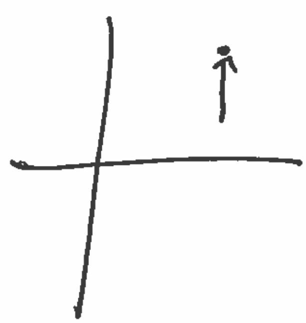


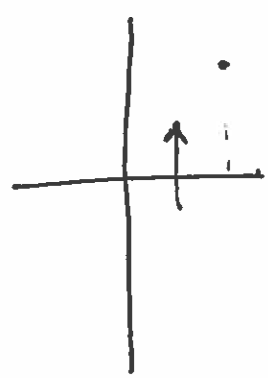

$$R = \begin{bmatrix} c\theta & -s\theta & 0 \\ s\theta & c\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad t = \begin{bmatrix} 1 & 0 & a \\ 0 & 1 & b \\ 0 & 0 & 1 \end{bmatrix}$$

$$(R\, t)\, t^{-1} \cdot R \cdot t \cdot V$$









10 Determinants

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$\mathrm{Det}(A) = ad - bc$

11

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{matrix} a & b & c \\ d & e & f \\ g & h & i \end{matrix}$$

$aei + bfg + cdh$
$- ceg - afh - bdi$

12 $\det \begin{bmatrix} a & c \\ b & d \end{bmatrix} = \text{Area}$

13 No area change, invertable

14 Area change, not invertable - get no area

15 $\det(AB) = \det(A)\det(B)$

Inverses

16 Inverse is the reverse transformation

get Identity matrix

$$AA^{-1} = I$$

$$A^{-1}A = I$$

17 $$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} d & b \\ c & a \end{bmatrix} / \det(A) = \begin{bmatrix} \frac{d}{ad-bc} & \frac{b}{ad-bc} \\ \frac{c}{ad-bc} & \frac{a}{ad-bc} \end{bmatrix}$$

$$\det(A) = ad - bc$$

18 $\det \neq 0$

19

$$C = AB$$

$$C^{-1} = B^{-1}A^{-1}$$

$$C \cdot C^{-1} = A(B\ B^{-1})A^{-1}$$

$$C \cdot C^{-1} = AA^{-1}$$

$$C \cdot C^{-1} = 1$$

20

(20) LSAE

$$Ax = b$$

$$x = A^{-1}b$$

$A^{-1}b$ is solution for unknowns

(21)

$$Ax = b$$

Unique A has inverse

No sol, Inf sol

$\text{Det}(A) = 0$

Not sure how to distinguish

Guess

$$\begin{bmatrix} A & b \end{bmatrix} = \begin{bmatrix} \sim & \sim & \sim \\ \sim & \sim & \sim \\ 0 & 0 & 0 \end{bmatrix} \text{ vs } \begin{bmatrix} 0 & 0 & - \\ \cdot & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(22) Add and subtract scaled rows to simplify matrix

(23)

$$Ax = b$$

$$A = LU$$

$$(LU)x = b$$

$$x = (LU)^{-1} b$$

$$x = U^{-1} L^{-1} b$$

(24) get from gauss elimination

Inverse

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \left[\begin{array}{cc|cc} a & b & 1 & 0 \\ c & d & 0 & 1 \end{array}\right]$$

# Data + Matrices

2.5 Mean

$$\mu = \frac{1}{n} \sum_{i=1}^{n} a_i$$

Variance

$$\cancel{\frac{1}{n} \sum_{i=1}^{n} a_i^2}$$

$$\cancel{\sigma^2 = \sum X - \mu}$$

$$\sigma^2 = \frac{\sum_{i=1}^{N} (a_i - \mu)^2}{N}$$

Standard deviation

$$\sigma^2 = \sqrt{\frac{\sum_{i=1}^{N} (a_i - \mu)^2}{N}}$$

26

Correlation

$$\frac{\sum_{i=1}^{N} \left| (a_i - \mu_a) \cdot (b_i - \mu_b) \right|}{N \cdot \sigma_a \sigma_b}$$

$\sigma_a$ = standard dev a

Covariance

$$\frac{1}{N-1} b b^T, \quad b = \frac{\sum a_i - \mu}{\sigma_a}$$

27 Don't understand question.

I made generalized ~~fcn~~ function

# Eigenvalues & Eigenvectors

28

$Av = \lambda v$

29

$A - \lambda I = 0$

30 Solve $\lambda$ in $A - \lambda I = 0$

Plug $\lambda$'s ~~to~~ into A

31 $Av = \lambda v$

32 Values such that

$Av = \lambda v$

In a symmetric matrix

$v_1$ ~~to~~ is in direction of largest variance

$\lambda_1$ tells how much

$v$'s are orthogonal

# Basis Spanning and Decomposition

33

Orthagonal

$V_1 \cdot V_2 = 0$

Orthonormal

$V_1 \cdot V_2 = 0$

$||V_1|| = 1$

$||V_2|| = 1$

Linearly independent

No scalar multiples

No vector equals linear combination of other vectors in the set

Span

All linear combinations of vectors in a set

34

$$\begin{bmatrix} 3 \\ 2 \end{bmatrix} = x_1 \begin{bmatrix} a \\ b \end{bmatrix} + x_2 \begin{bmatrix} c \\ d \end{bmatrix}$$

$$\left[\begin{array}{cc|c} a & c & 3 \\ b & d & 2 \end{array}\right]$$

$\cancel{38}$

$$\left[\begin{array}{cc|c} 1 & 0 & 3 \\ 0 & 1 & 2 \end{array}\right]$$

$$\begin{bmatrix} 3 \\ 2 \end{bmatrix} = 3 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + 2 \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

35

$$A = Q \Lambda Q^{-1}$$

$$Q = \begin{bmatrix} v_1 & v_2 & v_3 \end{bmatrix}$$

$$\Lambda = \begin{bmatrix} \lambda_1 & & & \\ & \lambda_2 & & \\ & & \ddots & \\ & & & \lambda_n \end{bmatrix}$$

(36) Singular Value Decomposition

$$A_{n \times m} = U_{n \times r} \Sigma_{r \times r} V_{m \times r}^{T}$$

$$U_{n \times r} = \begin{bmatrix} v_1 & v_2 & v_3 & \cdots \end{bmatrix} \text{ of } AA^T$$

$$V_{m \times r} = \begin{bmatrix} v_1 & v_2 & v_3 & \cdots \end{bmatrix} \text{ of } A^TA$$

singular values $\Sigma_{r \times r} = \begin{bmatrix} \sqrt{\lambda_1} & & & \\ & \sqrt{\lambda_2} & & \\ & & \ddots & \\ & & & \sqrt{\lambda_r} \end{bmatrix} = \begin{bmatrix} \sigma_1 & & & \\ & \sigma_2 & & \\ & & \ddots & \\ & & & \sigma \end{bmatrix}$

$$\sigma_1 > \sigma_2 > \sigma_3$$

~~Nonzero~~
nonzero $\lambda$'s of $AA^T$ or $A^TA$

(37) $A = U \Sigma V^T$

(38) $v_1$'s direction of largest variance