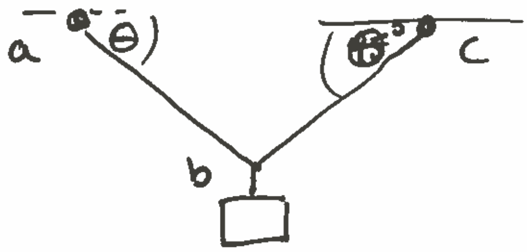
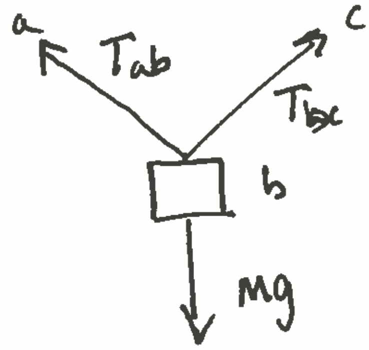
# Interactions 2.2

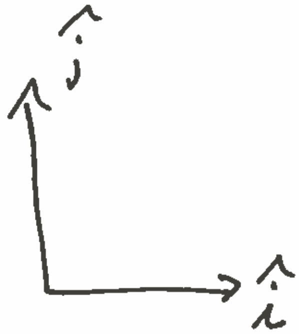
$\boxed{1}$

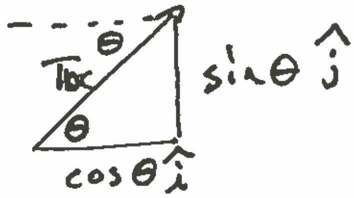



$\theta = 45^\circ$

ⓐ Tension in cables




$$\Sigma F = 0 : -mg\hat{j} + T_{ab}\left(-\cos\theta\,\hat{i} + \sin\theta\,\hat{j}\right) + T_{bc}\left(\cos\theta\,\hat{i} + \sin\theta\right.$$

$$i : \cancel{T_{ab}(-\cos\theta)\hat{i} = T_{bc}(\cos\theta)\hat{j}}$$

$$T_{ab}(-\cos\theta)\hat{i} + T_{bc}(\cos\theta)\hat{j} = 0$$

$$\boxed{T_{ab} = T_{bc}}$$

$$j : -mg\hat{j} + T_{ab}\sin\theta\,\hat{j} + T_{bc}\sin\theta\,\hat{j} = 0$$

$$2 \cdot T_{ab}\sin\theta = mg$$

$$\boxed{T_{ab} = \frac{mg}{2\sin\theta}}$$

1 b

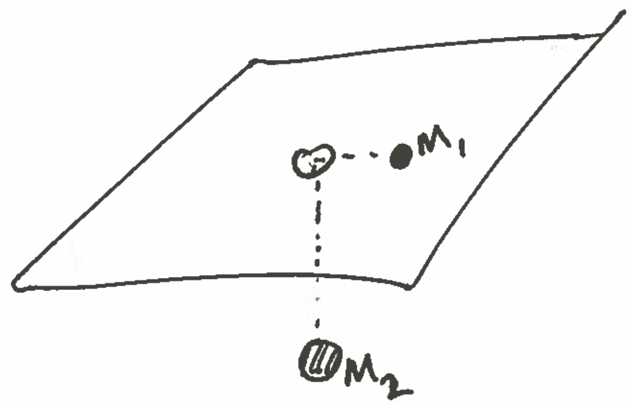
$$T_{ab} = \frac{mg}{2\sin\theta}$$

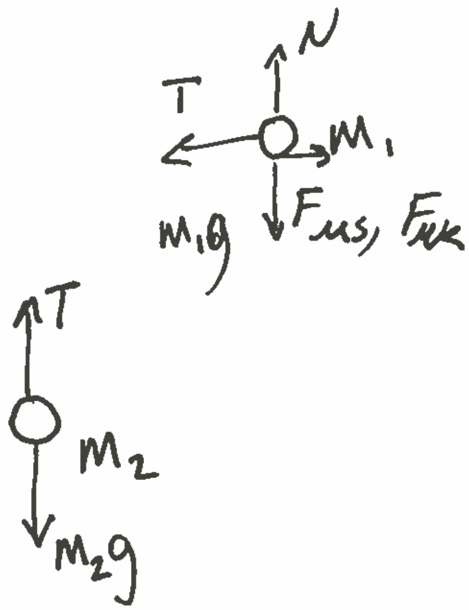
As $\theta \to 90$, $T_{ab} \to \frac{mg}{2}$

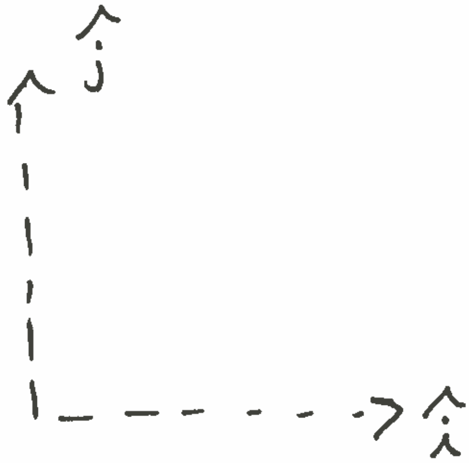
As $\theta \to 0$, $T_{ab} \to \infty$

c Load cell (strain gauge)

$\boxed{2}$




Since system is at rest we will use static friction ($F_{\mu s}$)

$m_1$ $\Sigma F = 0$

$i$: $F_{\mu s} - T = 0$ $\qquad$ $j$: $N - m_1 g = 0$

$m_1 g = N$

$m_2$ $i$: $\qquad$ $j$: $T - m_2 g = 0$

$T = m_2 g$

$F_{\mu s} = N \mu_s$

$F_{\mu s} = m_1 g \mu_s$

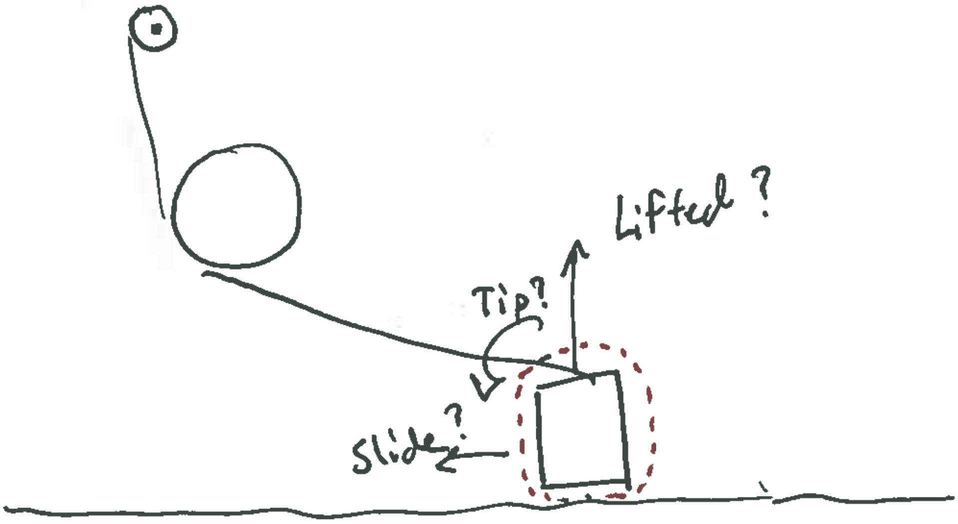
$m_1 g \mu_s - m_2 g = 0$

$m_1 \mu_s = m_2$

$\mu_s = \frac{m_2}{m_1}$

$\boxed{\mu_s > \frac{m_2}{m_1}}$

3





T assume a value

N constraining force "wherever it has to be"

$F_{\mu_s} < \mu_s N$

$\sum F_y \geq \emptyset$ Must be +

$$\sum \vec{F} = 0$$

$$\sum \vec{M} = 0$$

$$T\hat{r} - mg\hat{j} + N\hat{j} + f\hat{i} = 0$$

$\hat{j}$: $T\sin\theta + N - mg = 0$

$\hat{i}$: $-T\cos\theta + f = 0$

$$T\cos\theta = \mu_s N$$

$$T\sin\theta + \frac{1}{\mu_s} T\cos\theta = mg$$

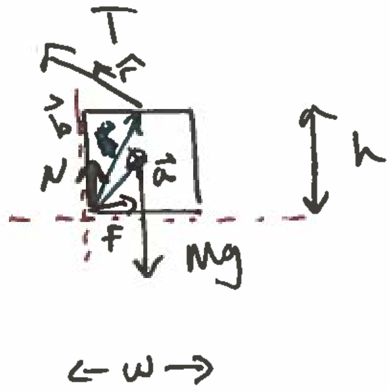
$$\boxed{T = \frac{mg}{\sin\theta + \frac{1}{\mu_s}\cos\theta}}$$

Value of T for which we break frictional constraint

Minimiz T, $\theta \to 0°$ if $\mu_s < 1$

$T = \mu_s mg$ wheel as low as possible

3 | Condition 2!




$$\sum \vec{M} = 0$$

$$\underbrace{\vec{a} \times -mg\hat{j}} + \underbrace{\vec{b} \times T\hat{r}} = 0$$

$$\left(\frac{w}{2}\hat{i} + \frac{h}{2}\hat{j}\right) \times (-mg\hat{j})$$

$$\vec{a} = \frac{w}{2}\hat{i} + \frac{h}{2}\hat{j}$$

$$-\frac{mgw}{2}\hat{k}$$

$$\left(\frac{w}{2}\hat{i} + h\hat{j}\right) \times T(-\cos\theta\,\hat{i} + \sin\theta\,\hat{j}) \qquad \vec{b} = \frac{w}{2}\hat{i} + h\hat{j}$$

$$\left[\frac{wT}{2}\sin\theta + hT\cos\theta\right]\hat{k} = 0$$

$$\boxed{T - \frac{mg\,\frac{w}{2}}{\frac{w}{2}\sin\theta + h\cos\theta}}$$

Value of T for which we break 0 moment

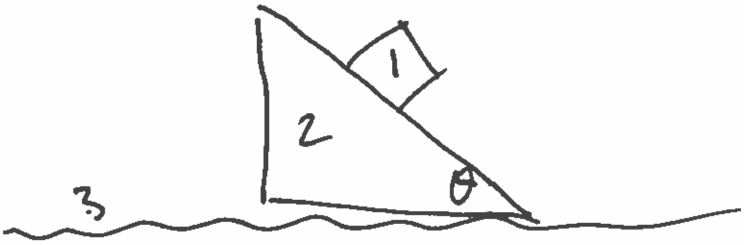
$$T = \frac{mg\,5}{5\sin\theta + 10\cos\theta}$$

We want to maximize denominator

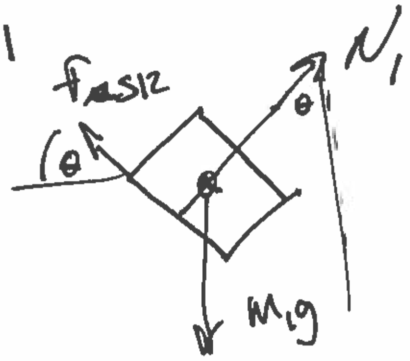
smallest T, $\Rightarrow$ $\theta \rightarrow 0^\circ$

$T = \frac{mg}{2}$, $\theta = 0^\circ$ wheel as ~~high as possible~~ low

4

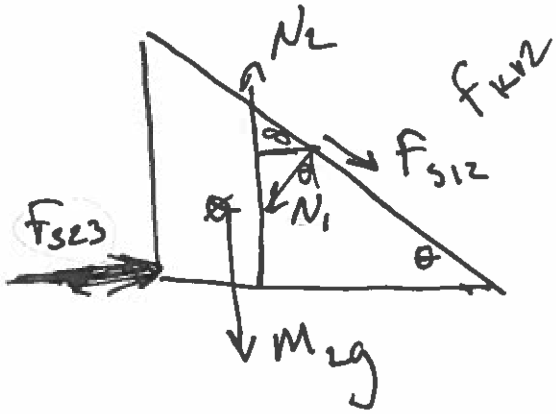



1




$$f_{s12} = N_1 \mu_{s12}$$

2




$$f_{s23} = N_2 \mu_{s23}$$

4

(a) $\sum F_1 \hat{j} = 0$ : $N_1 \cos\theta + f_{s12} \sin\theta - m_1 g = 0$

$\sum F_1 \hat{i} = 0$ : $N_1 \sin\theta - f_{s12} \cos\theta = 0$

$\sum F_2 \hat{j} = 0$ : $N_2 - m_2 g - N_1 \cos\theta - F_{s12} \sin\theta =$

$\sum F_2 \hat{i} = 0$ : $f_{s23} + f_{s12} \cos\theta - N_1 \sin\theta = 0$

$\mu_{s12} = \tan\theta$

$\mu_{s23} = 0$

(b) $N_2 = \{g, m_1, m_2, \theta\} \hat{j}$

$f_{s23} = 0 \hat{i}$

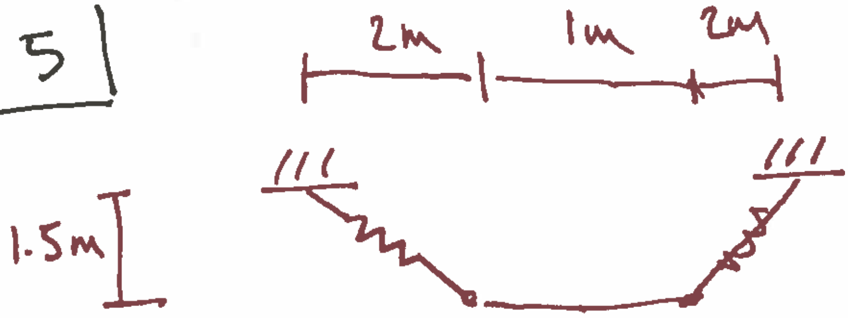
(c) $\sum F_2 \hat{i} = 0$ : $f_{s23} - N_1 \sin\theta + f_{k12} \cos\theta = 0$

$\sum F_2 \hat{j} = 0$ : $N_2 - mg - N_1 \cos\theta - f_{k12} \sin\theta = 0$

$N_2 = \{g, m_2, \theta, N_1, f_{k12}\}$

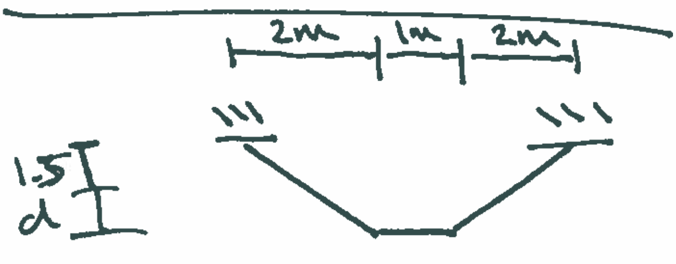
$\mu_{s23} = \{N_1, \mu_{s12}, \theta, g, m_2, f_{k12}\}$

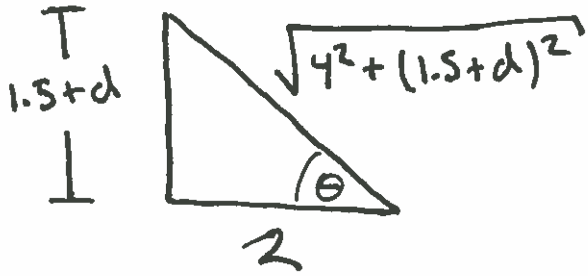
5

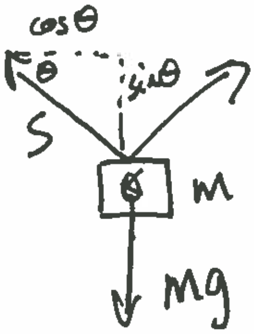




Rest length = 2.5 m




$$S = k\left(\sqrt{4^2+(1.5+d)^2} - 4\right)$$

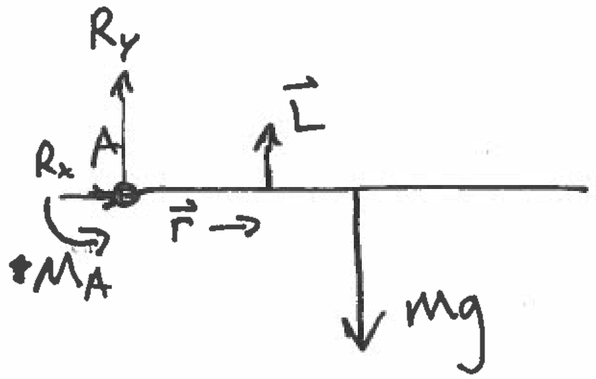
$$\sin\theta = \frac{1.5+d}{\sqrt{4^2+(1.5+d)^2}}$$

$$\hat{j}: \quad S\sin\theta + S\sin\theta - mg = 0$$

$$\hat{i}: \quad S\cos\theta - S\cos\theta = 0$$

$$\frac{mg}{2} = S\sin\theta = k\left(\sqrt{4^2+(1.5+d)^2} - 4\right)\left(\frac{1.5+d}{\sqrt{4^2+(1.5+d}}\right.$$

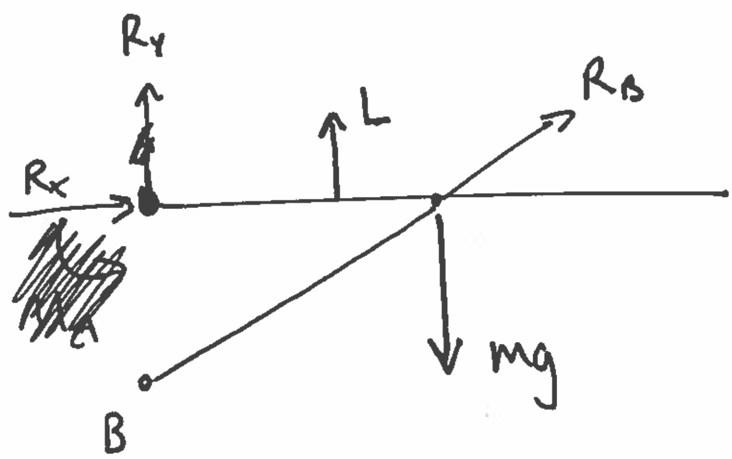
6

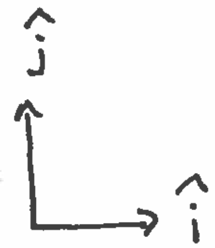





$$M_A = \vec{r} \times \vec{L} + \vec{r} \times mg(-\hat{j})$$

---

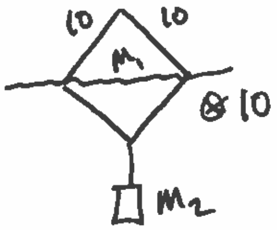



IF $R_B \hat{j} = mg\hat{j}$, then $R_x > 0$?

$\boxed{7}$

$\rho_l = 9.28 \frac{g}{cm^3}$




$d = 10$

$V_{\cancel{f}} = 10 \cdot 10 \cdot 10 = 1000\ cm^3$

$V_f = \frac{V}{2} = 500\ cm^3 = .0005\ m^3$

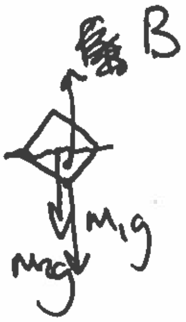
$\rho_F = 20 \frac{kg}{m^3}$

$B_l = \rho_{l} V_l\, g$

$B_l = \frac{1kg}{L} \left( \frac{1L}{9.28 kg} M_2 \right) g$

$B_F = \rho_w V_F\, g$

$B_F = \frac{1kg}{L} (.5L)\ g$




$m_1 = \rho_f V_F$

$m_1 = \frac{20kg}{m^3}\ .0005\ m^3$

$m_1 = .01\ kg$

$B_F + B_l - m_1 g - m_2 g = 0$

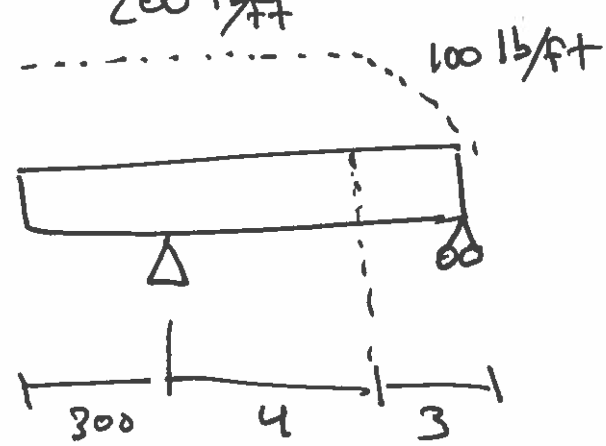
$.5g + \frac{1}{9.28} g - .01 g - m_2 g = 0$

$.5 + \frac{1}{9.28} - .01 - m_2 = 0$

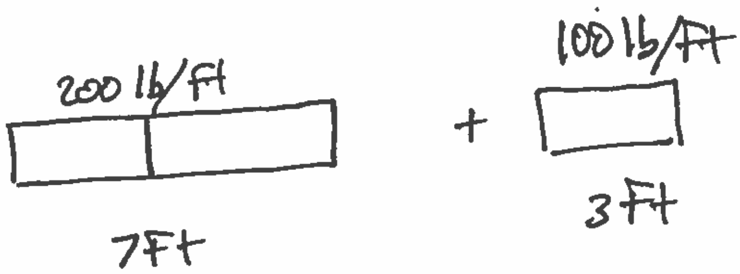
$m_2 = .5977 \approx .6\ kg$

$$\boxed{\begin{array}{l} d = 10 \\ m_2 = .6\ kg \end{array}}$$

8




Find com

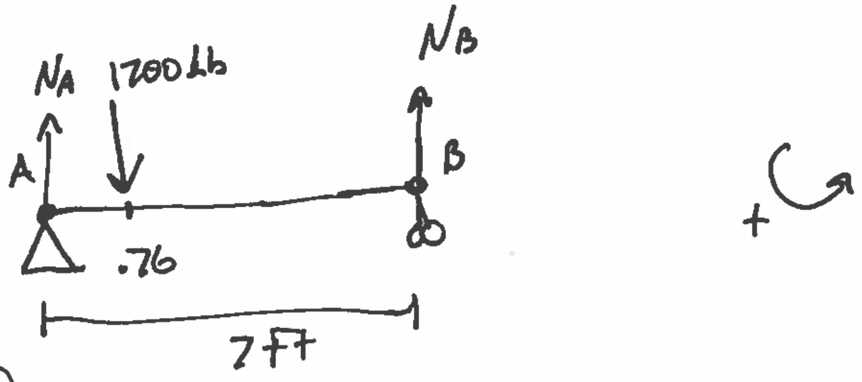



$\frac{1}{3}$ triangle

1400 lb at 3.5ft + 300 lb at 5ft

$$\frac{3.5 \cdot 1400 + 300 \cdot 5}{1200} = 3.76\,\text{ft} \qquad 1200\,\text{lb}$$

8 continued




$\Sigma F_{\hat{i}} = 0$

$\Sigma F_{\hat{j}} = 0$

$\Sigma M_A = 0$

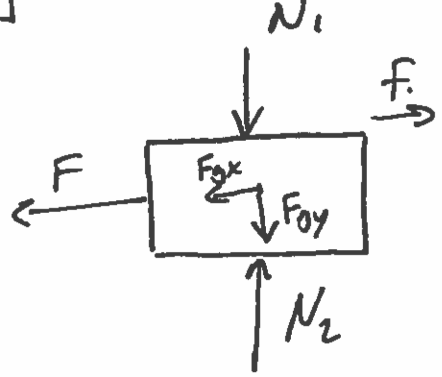
$F_{\hat{j}}: \; N_A + N_B - 1700 = 0$

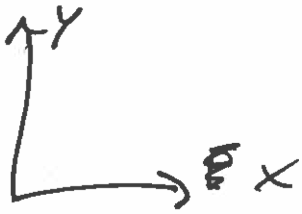
$M_A: \; -.76 \cdot 1700 + 7 \cdot N_B = 0$

$N_A = 1514 \text{ lb}$

$N_B = 186 \text{ lb}$

9



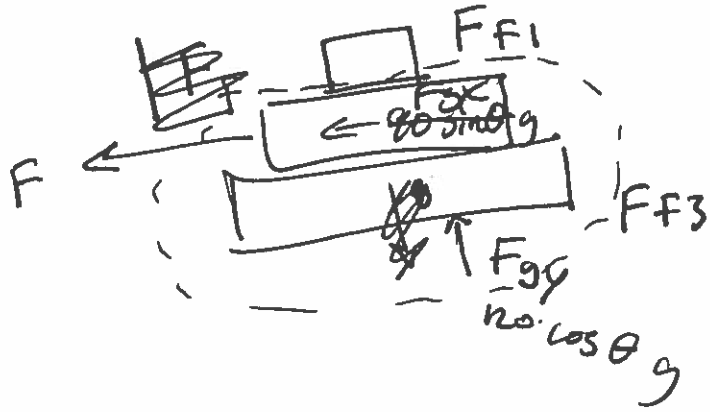

$$|F_{f_1}| \le N_1 \mu_1$$

$$|F_{f2}| \le N_2 \mu_2$$

$$F_{gx} + F = F_{f1} + F_{f2}$$

$$F = N_1\mu_1 + N_2\mu_2 - 50g\sin$$

$$F = 30(.3)\cos\theta + 80(.4)\cos\theta - 50g\cdot$$




$$F + F_x = F_{f1} + F_{f3}$$

$$F = N_1\mu_1 + N_3\mu_3 - F_{gx}$$

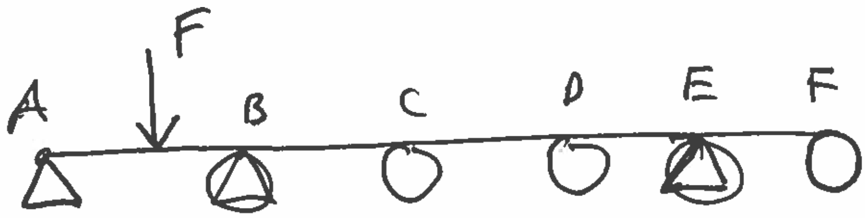
$$\cancel{F_2 = \ldots}$$

$$F = 30(.3)\cancel{\sin\theta}\cos\theta + 120(.45)\cancel{\sin\theta}\cos\theta - 90\,kg\,g\sin\theta$$

10

Statically indeterminate

More variables than equations




Max 3 equations

$\sum F\hat{i} = 0$ $\quad\quad \cancel{\sum F_j}$

$\sum F\hat{j} = 0$

$\sum M_A = 0$

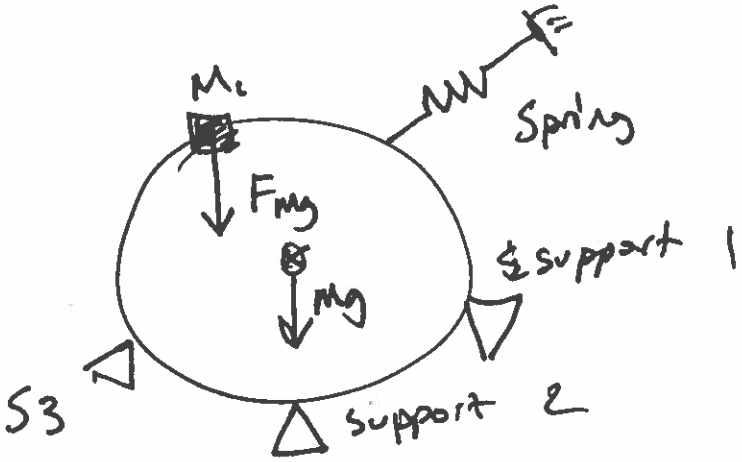
$\sum F\hat{j} \Rightarrow \hat{j}(F_A + F_B + F_C + F_D + F_E + F_F) = 0$

$\sum M_A \Rightarrow \vec{r} \times F_A + \vec{r}_2 \times F_B + \vec{r}_3 \times F_C \ldots$

Many more variables than equations

10

# Circular beam




$$\sum F\hat{i} = 0 \Rightarrow -S_3 + S_2 F_{spring}\hat{i} - S_1 = 0$$

$$\sum F\hat{j} = 0 \Rightarrow \cancel{\text{Support 1} + S2}\ S_3 + S1\hat{j} + S2\hat{j} - M_1 - mg - \text{Spring}\,\hat{j} =$$

$$\sum M_2 = 0 \Rightarrow \vec{r}_3 \times S_3 + \vec{r}_2 \times S_2 + \vec{r}_1 \times S_1 + \vec{r}_4 \times M_1 + \vec{r}_5 \times mg = 0$$