Complex Variables - HW 7 - question 4

Define a function that allows us to plot the image plane for a set of set of points. I used a set of set to make plotting the graphic objects non connected independent objects.

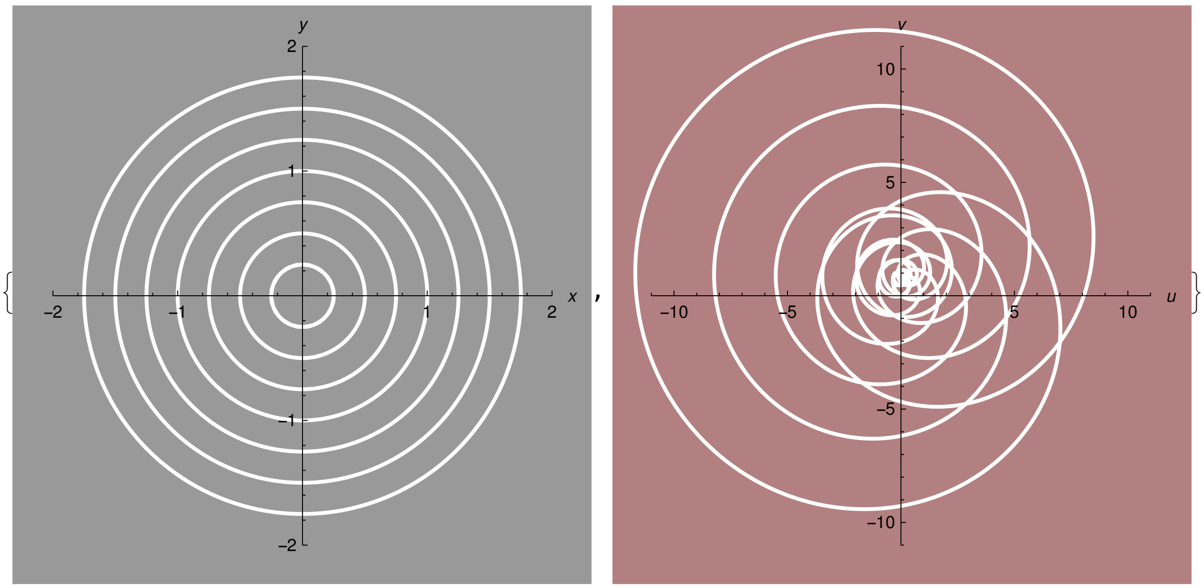
```
In[18]:= makeImage[pts_, expr_, pltRange1_, PltRange2_] := Module[{},
    {
     Graphics[{White, Thick, Line[# & /@ pts]},
      PlotRange → {{-pltRange1, pltRange1}, {-pltRange1, pltRange1}},
      Axes → True, Background → GrayLevel[.6], ImageSize → {300, 300},
      AxesLabel → {Style["x", Italic], Style["y", Italic]}, ImagePadding → 20],

     Graphics[{White, Thick, Line[
         {Re[expr /. z → #[[1]] + ⅈ #[[2]]], Im[expr /. z → #[[1]] + ⅈ #[[2]]]} & /@ # & /@
          pts]}, PlotRange → {{-PltRange2, PltRange2}, {-PltRange2, PltRange2}},
      Axes → True, Background → RGBColor[.7, .5, .5], ImageSize → {300, 300},
      AxesLabel → {Style["u", Italic], Style["v", Italic]}, ImagePadding → 20]
    }
   ]
```

First we will plot expanding circles from .25 to 1.75 as these circles will either touch or go between the roots.

```
In[19]:= expr = (z + 1) (z - .5) (z - 1.5 I)
Out[19]= (-0.5 + z) ((0. - 1.5 ⅈ) + z) (1 + z)
```

In[20]:=
```
ang = Range[0 Pi, 2 Pi, .001];
lists = Table[{r Cos[ang], r Sin[ang]}, {r, Range[.25, 1.75, .25]}];
pts = Transpose[#] & /@ lists;
n = 2;
m = 11;
makeImage[pts, expr, n, m]
```

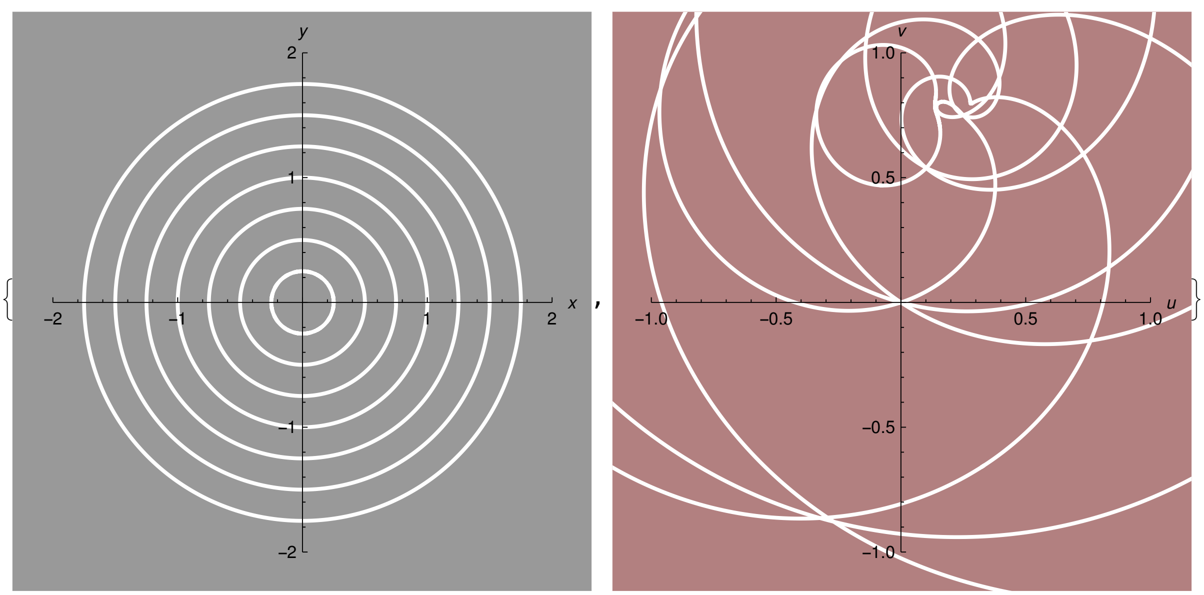
Out[25]=



Plot the same range but zoom in on the image

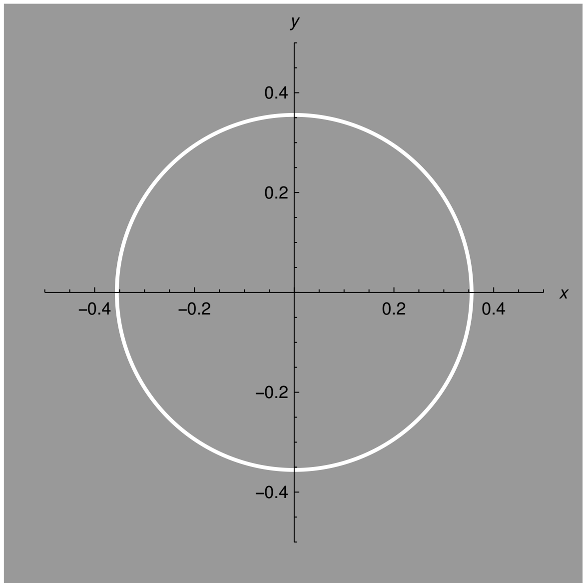
In[26]:=
```
n = 2;
m = 1;
makeImage[pts, expr, n, m]
```

Out[28]=

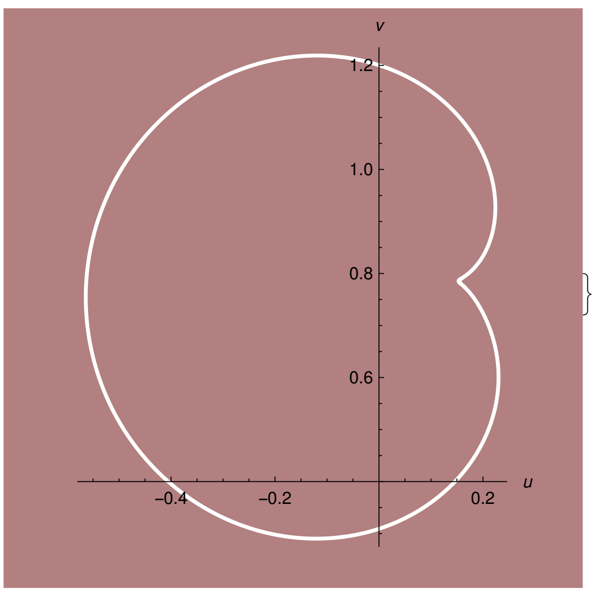


Next we will plot expanding circles from with radius's very close to the norm of the root of derivative of f. This is only done for one of the critical points to save ink!

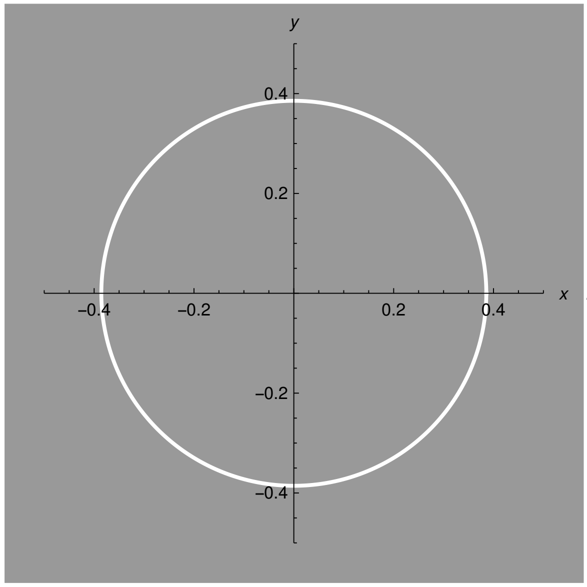
```
In[29]:= makeImage2[pts_, expr_, pltRange1_] := Module[{},
   {
    Graphics[{White, Thick, Line[# & /@ pts]},
     PlotRange → {{-pltRange1, pltRange1}, {-pltRange1, pltRange1}},
     Axes → True, Background → GrayLevel[.6], ImageSize → {300, 300},
     AxesLabel → {Style["x", Italic], Style["y", Italic]}, ImagePadding → 20],

    Graphics[{White, Thick, Line[
        {Re[expr /. z → #[[1]] + ⅈ #[[2]]], Im[expr /. z → #[[1]] + ⅈ #[[2]]]} & /@ # & /@
         pts]}, PlotRange → All, Axes → True,
     Background → RGBColor[.7, .5, .5], ImageSize → {300, 300},
     AxesLabel → {Style["u", Italic], Style["v", Italic]}, ImagePadding → 20]
   }
  ]

In[30]:= ∂z expr

Out[30]= (-0.5 + z) ((0. - 1.5 ⅈ) + z) + (-0.5 + z) (1 + z) + ((0. - 1.5 ⅈ) + z) (1 + z)

In[31]:= sol = Solve[(-0.5 + z) ((0. - 1.5 ⅈ) + z) + (-0.5 + z) (1 + z) + ((0. - 1.5 ⅈ) + z) (1 + z) == 0, z]

Out[31]= {{z → -0.315996 + 0.220975 ⅈ}, {z → -0.0173371 + 0.779025 ⅈ}}

In[32]:= z /. sol

Out[32]= {-0.315996 + 0.220975 ⅈ, -0.0173371 + 0.779025 ⅈ}

In[33]:= radius = Norm[-0.3159962460216397` + 0.22097512866074334` ⅈ]

Out[33]= 0.385595

In[34]:= Table[lists = Table[{r Cos[ang], r Sin[ang]}, {r, {x}}];
  pts = Transpose[#] & /@ lists;
  n = .5;
  ;
  Flatten[{x, makeImage2[pts, expr, n]}], {x, Range[radius - .03, radius + .03, .03]}]
```

Out[34]= {{0.355595, 

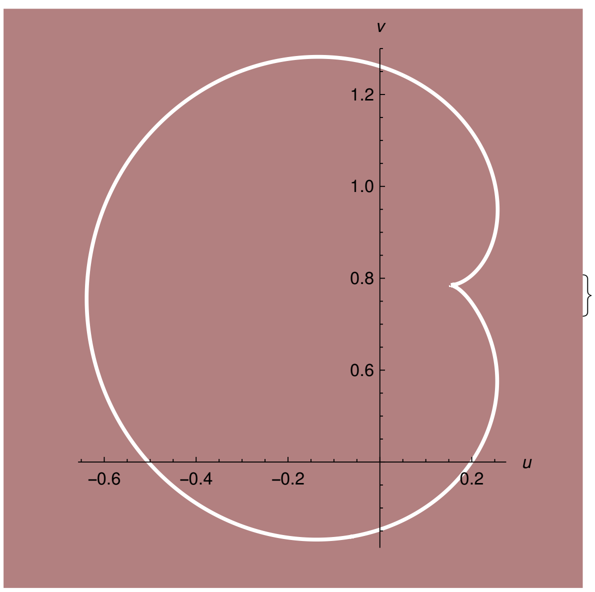
,



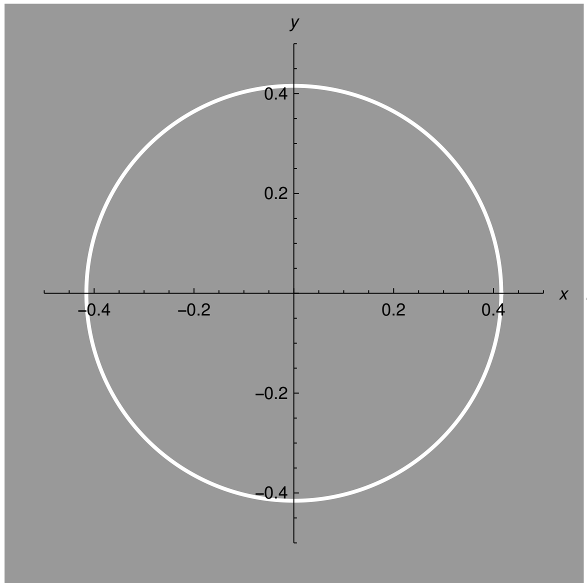
},

{0.385595, 

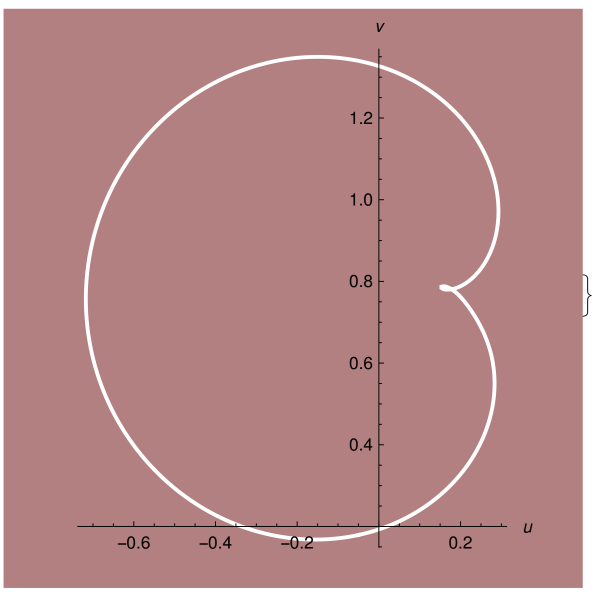
,



},

$\{0.415595,$ 

$,$



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