Complex Variables - HW 7 - question 3

Define a function that allows us to plot the image plane for a set of set of points. I used a set of set to make plotting the graphic objects non connected independent objects.

```
makeImage[pts_, expr_, pltRange1_, PltRange2_] := Module[{},
  {
   Rasterize@Graphics[{White, Thick, Line[# & /@ pts]},
     PlotRange → {{-pltRange1, pltRange1}, {-pltRange1, pltRange1}},
     Axes → True, Background → GrayLevel[.6], ImageSize → {500, 500},
     AxesLabel → {Style["x", Italic], Style["y", Italic]}, ImagePadding → 20],

   Rasterize@Graphics[{White, Thick, Line[{Re[expr /. z → #[[1]] + ⅈ #[[2]]],
          Im[expr /. z → #[[1]] + ⅈ #[[2]]]} & /@ # & /@ pts]},
     PlotRange → {{-PltRange2, PltRange2}, {-PltRange2, PltRange2}},
     Axes → True, Background → RGBColor[.7, .5, .5], ImageSize → {500, 500},
     AxesLabel → {Style["u", Italic], Style["v", Italic]}, ImagePadding → 20]
  }
 ]
```

i)

First find the preimage points

```
sol = Solve[E^(3 π z) == E^(3 π I) && Norm[z] ≤ 4/3, z]
```

$\left\{\left\{z \to -\frac{i}{3}\right\}, \left\{z \to \frac{i}{3}\right\}, \{z \to -i\}, \{z \to i\}\right\}$

```
preImages = z /. sol
```

$\left\{-\frac{i}{3}, \frac{i}{3}, -i, i\right\}$

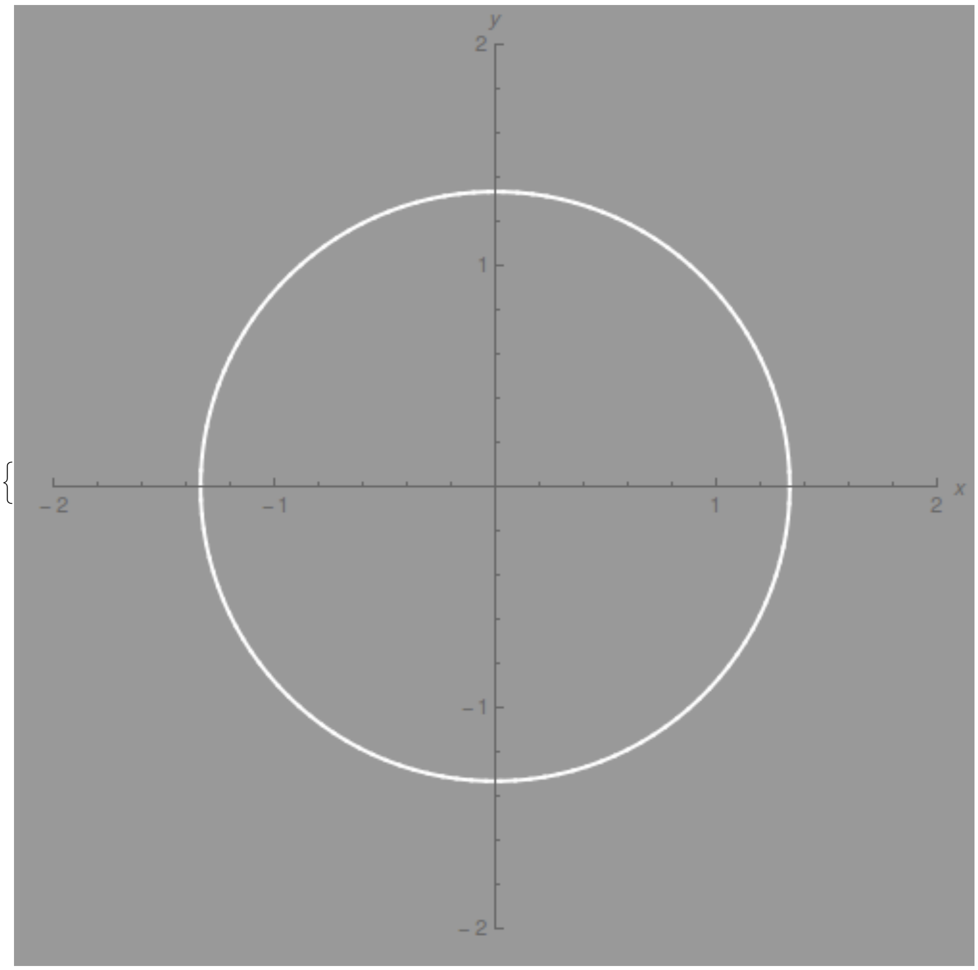
```
wrappingNumber = Length[preImages]
```

4

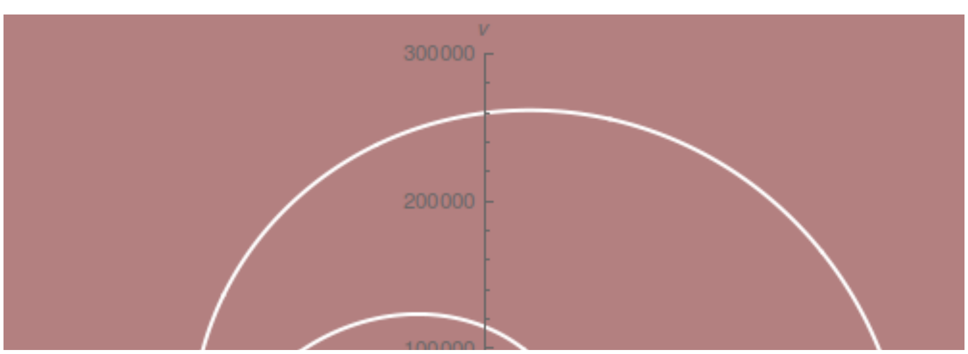
First plot I see a ton of windings, Image range = 300, 000

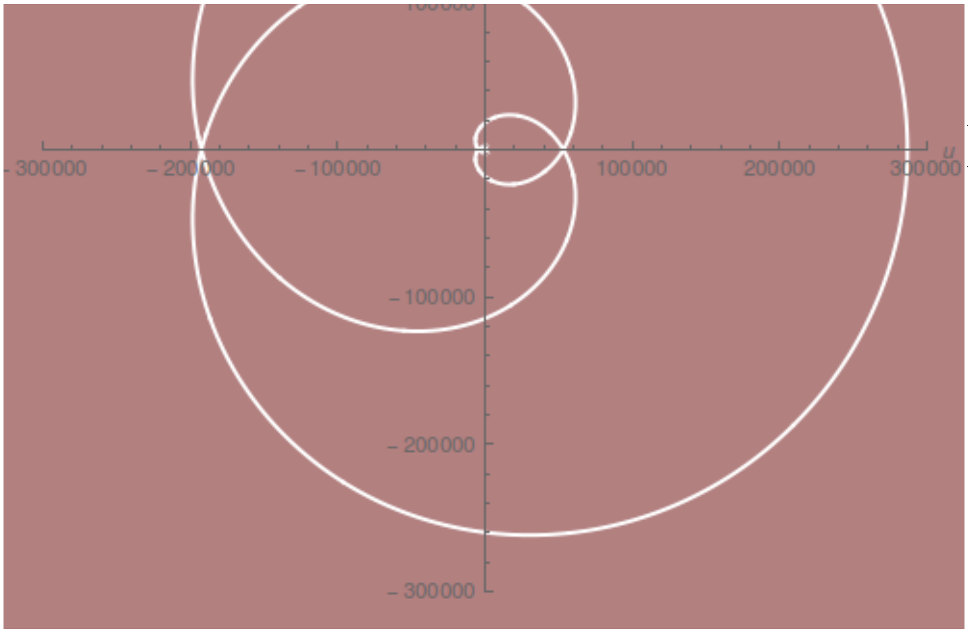
```
expr = E^(3 π z)
ang = Range[0 Pi, 2 Pi, .0001];
lists = Table[{r Cos[ang], r Sin[ang]}, {r, {4/3}}];
pts = Transpose[#] & /@ lists;
n = 2;
m = 300 000;
makeImage[pts, expr, n, m]
```

$e^{3\pi z}$

{ 

,





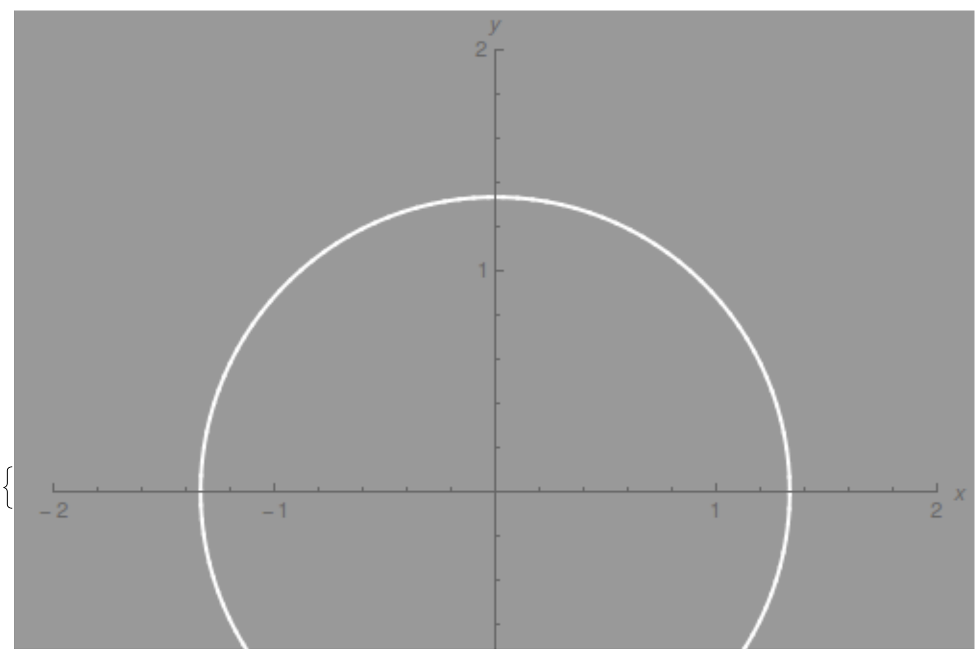
}

Second plot image range = .01

```
expr = E^(3 π z)
ang = Range[0 Pi, 2 Pi, .0001];
lists = Table[{r Cos[ang], r Sin[ang]}, {r, {4/3}}];
pts = Transpose[#] & /@ lists;
n = 2;
m = .01;
makeImage[pts, expr, n, m]
```

$e^{3 \pi z}$

{



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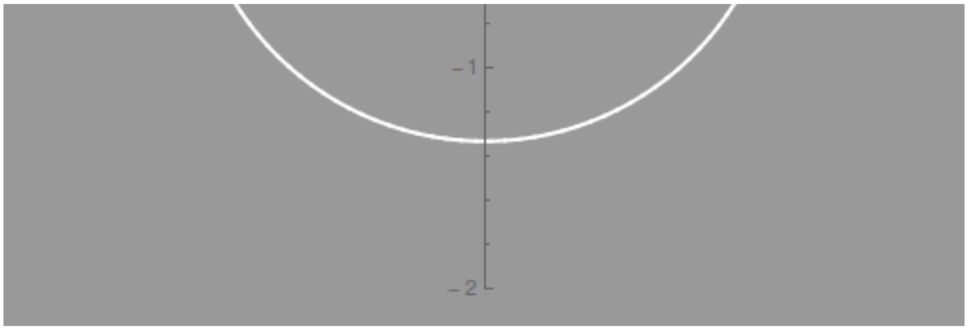
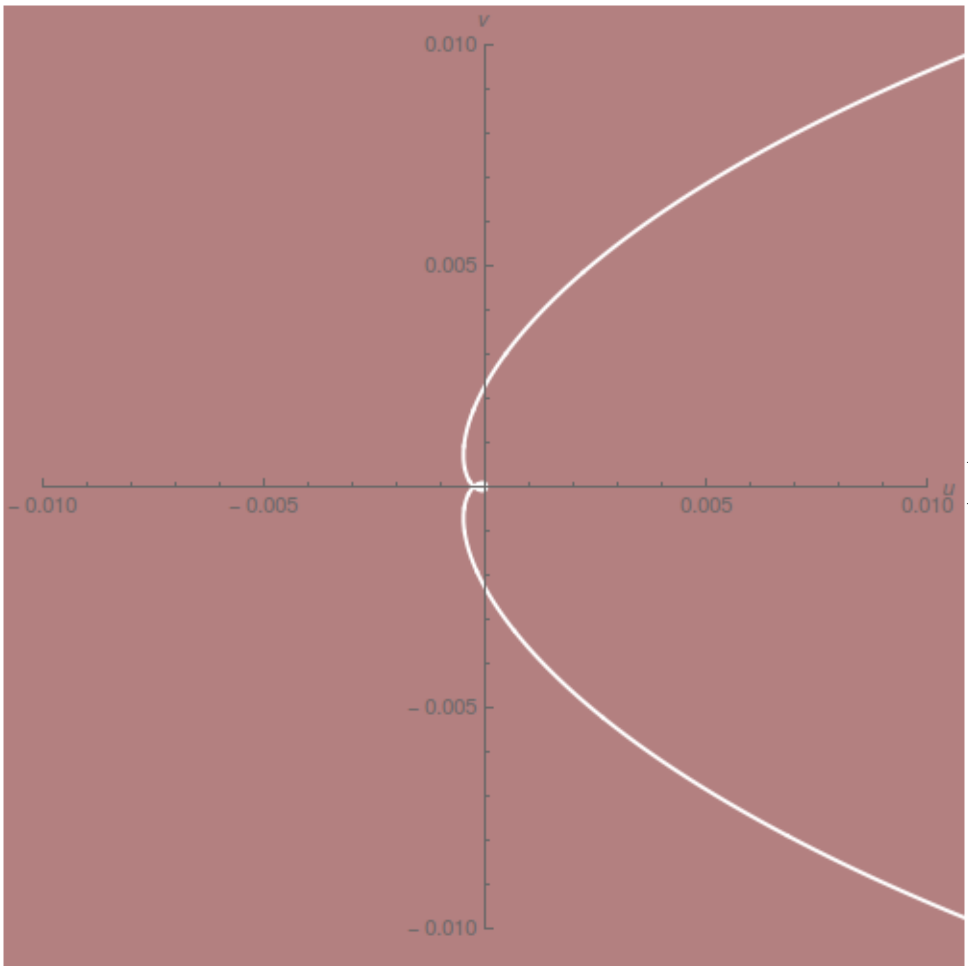
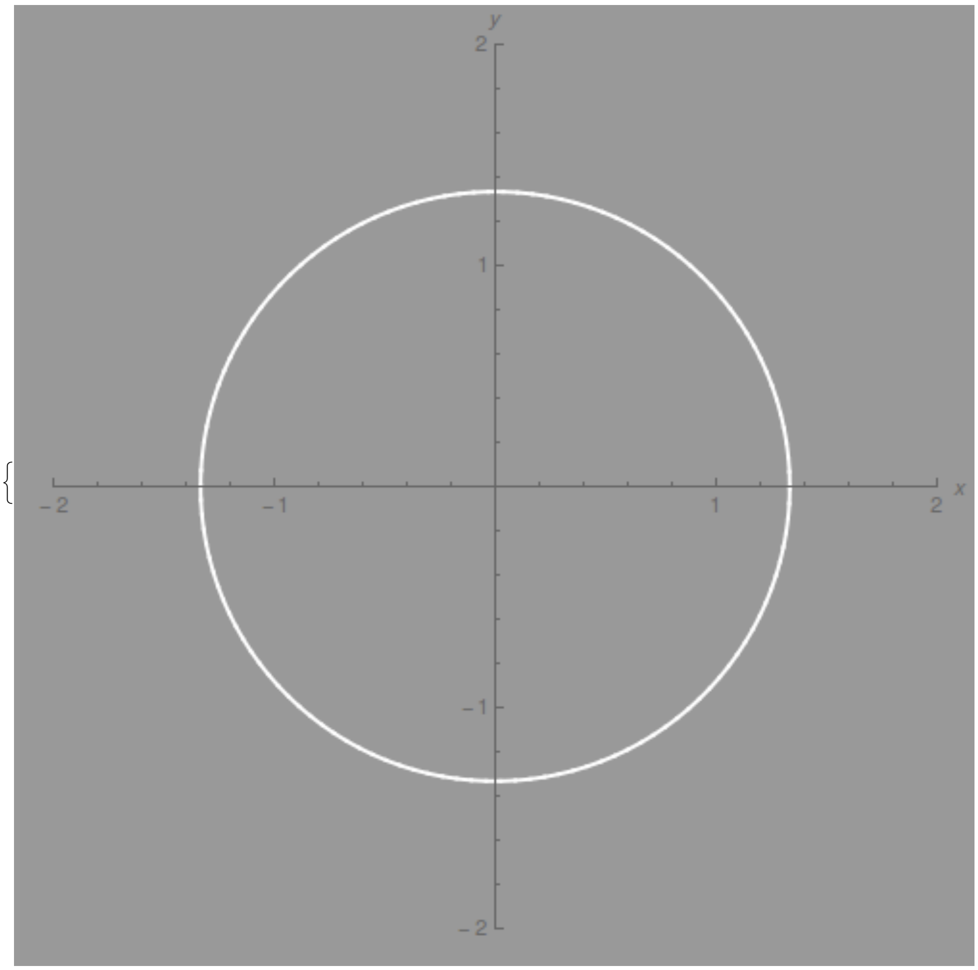




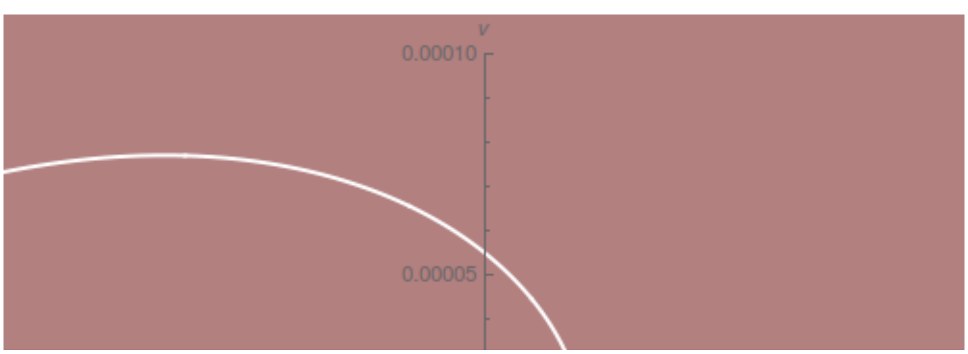
Third plot image range = .0001

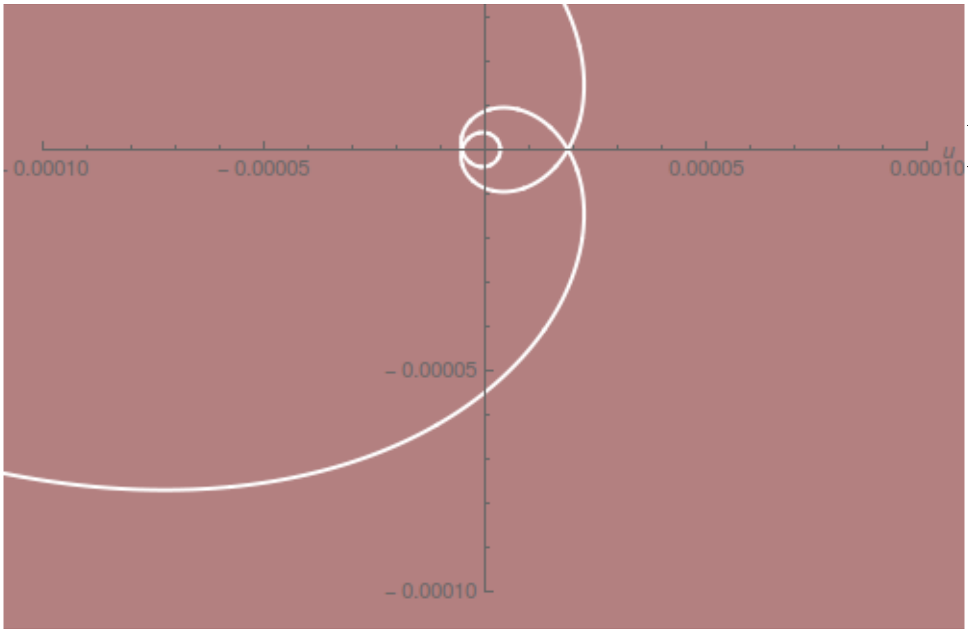
```
expr = E^(3 π z)
ang = Range[0 Pi, 2 Pi, .0001];
lists = Table[{r Cos[ang], r Sin[ang]}, {r, {4/3}}];
pts = Transpose[#] & /@ lists;
n = 2;
m = .0001;
makeImage[pts, expr, n, m]
```

$e^{3 \pi z}$

{

,





}

ii)

First find the preimage points and expected wrapping number

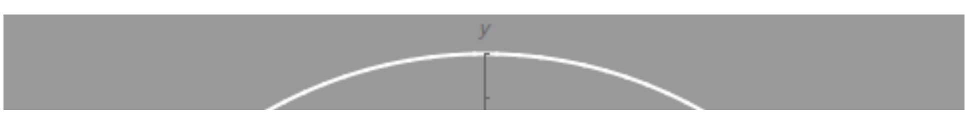
```
sol = Solve[Cos[z] == Cos[1] && Norm[z] ≤ 5, z]
{{z → -1}, {z → 1}}
```

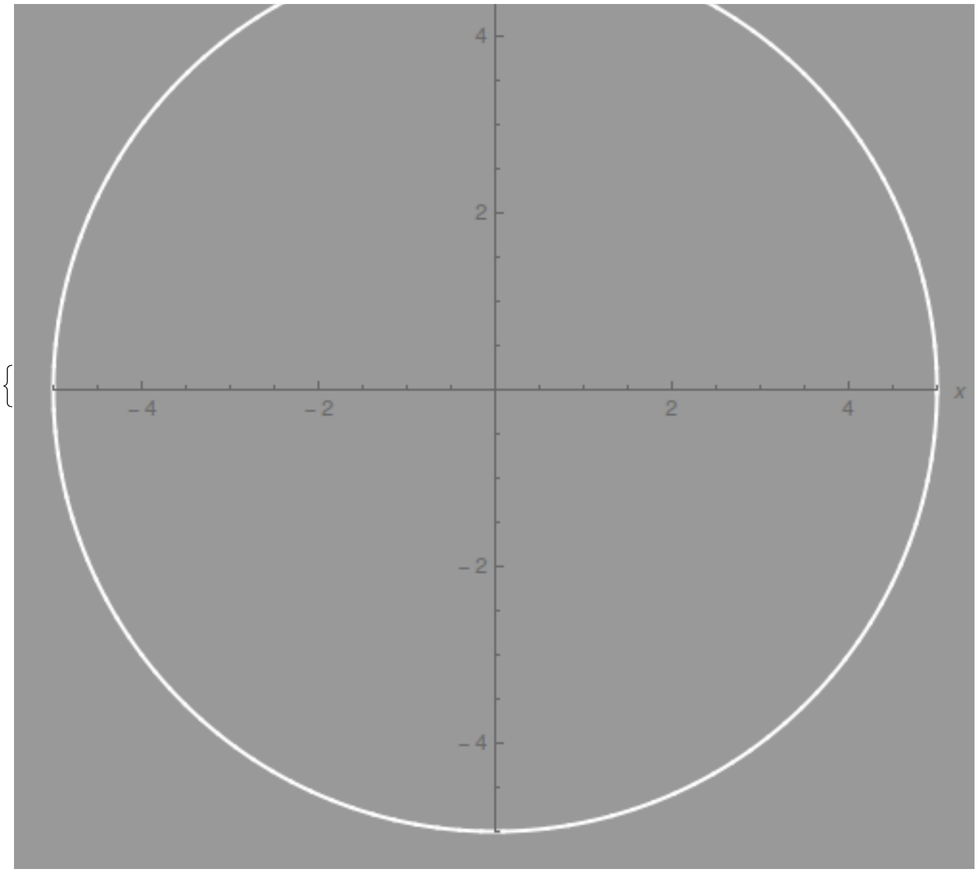
```
preImages = z /. sol
{-1, 1}
```

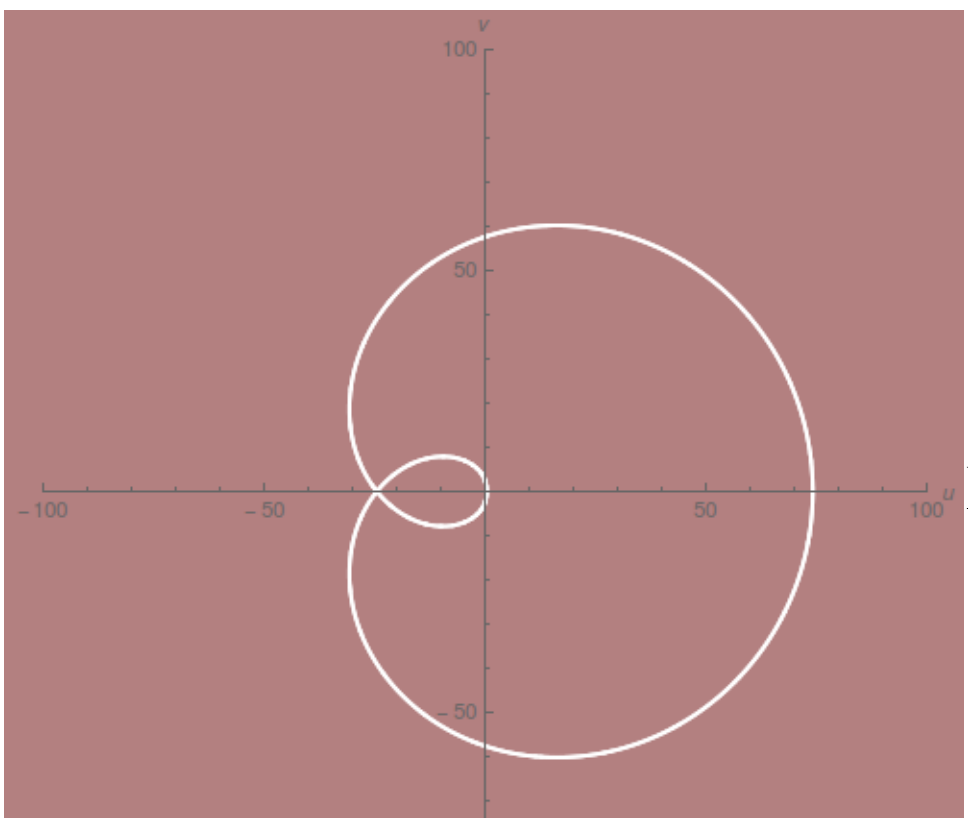
```
wrappingNumber = Length[preImages]
2
```

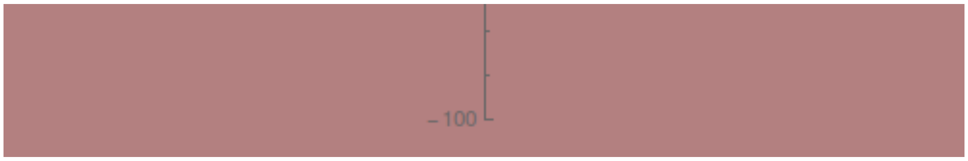
First plot I see two wrappings

```
expr = Cos[z]
ang = Range[0 Pi, 2 Pi, .0001];
lists = Table[{r Cos[ang], r Sin[ang]}, {r, {5}}];
pts = Transpose[#] & /@ lists;
n = 5;
m = 100;
makeImage[pts, expr, n, m]
Cos[z]
```



{
4
2
-4
-2
2
4
x
-2
-4
,

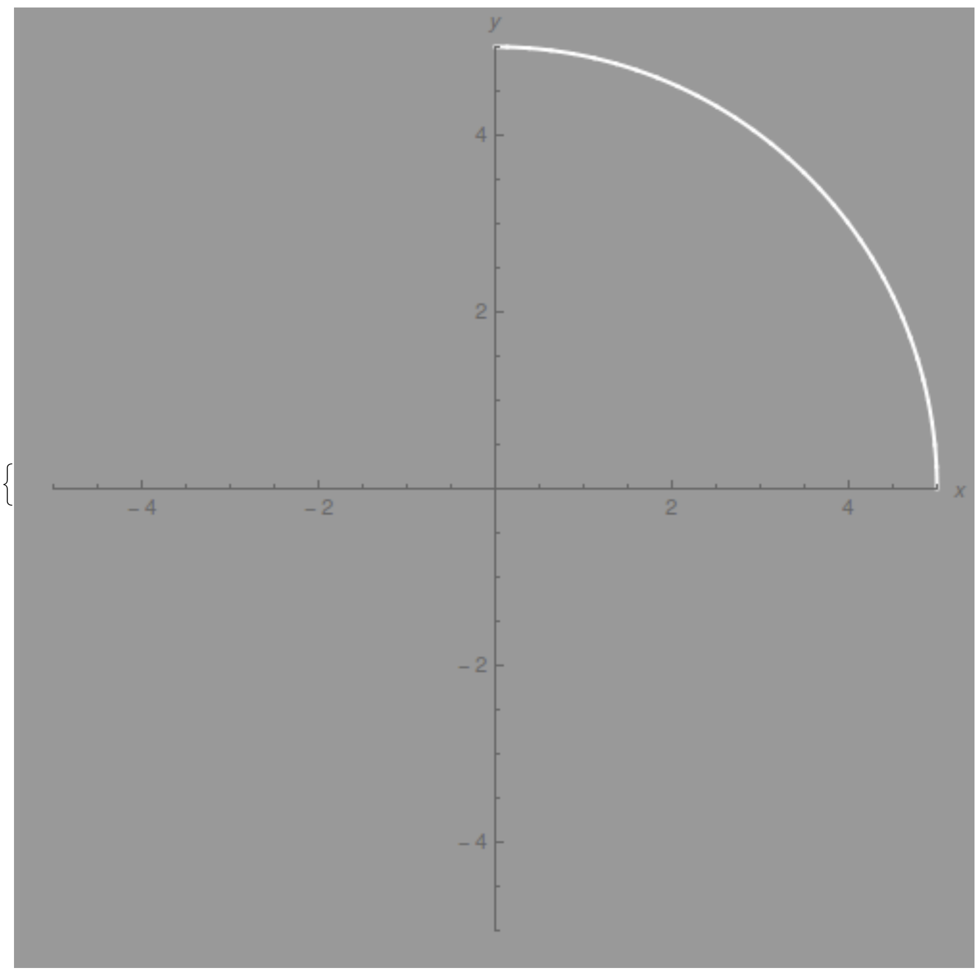

v
100
50
-100
-50
50
100
u
-50
}



Interesting, I see that I am only getting two wrappings. Let's try not plotting the entire circle.

```
expr = Cos[z]
ang = Range[0 Pi, .5 Pi, .0001];
lists = Table[{r Cos[ang], r Sin[ang]}, {r, {5}}];
pts = Transpose[#] & /@ lists;
n = 5;
m = 100;
makeImage[pts, expr, n, m]
```

Cos[z]

{ 

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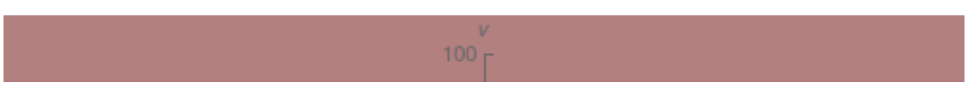
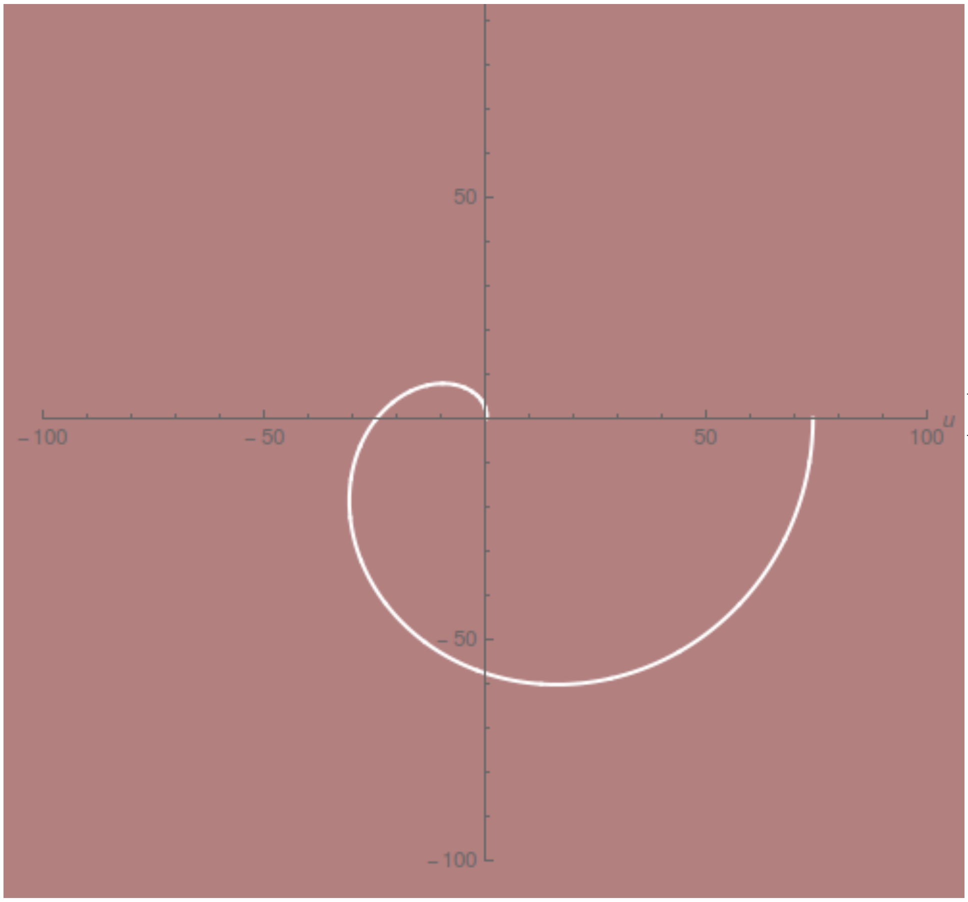




Ah Ha! Now we see that a quarter of a circle makes half a loop around our image plane. But let's see what happens very close to the origin.

---

iii)

First find the preimage points.

```
sol = Solve[Sin[z^4] == Sin[0^4] && Norm[z] ≤ 2, z];
```

```
preImages = z /. sol
```

$$\left\{0,0,0,0,-\pi^{1/4},(-1-i)\,\pi^{1/4},(-1+i)\,\pi^{1/4},-i\,\pi^{1/4},i\,\pi^{1/4},\pi^{1/4},(1-i)\,\pi^{1/4},(1+i)\,\pi^{1/4},-\frac{(1+i)\,\pi^{1/4}}{\sqrt{2}},-\frac{(1-i)\,\pi^{1/4}}{\sqrt{2}},\frac{(1-i)\,\pi^{1/4}}{\sqrt{2}},\frac{(1+i)\,\pi^{1/4}}{\sqrt{2}},-\sqrt{2}\,\pi^{1/4},-i\sqrt{2}\,\pi^{1/4},i\sqrt{2}\,\pi^{1/4},\sqrt{2}\,\pi^{1/4},-(2\pi)^{1/4},-i\,(2\pi)^{1/4},i\,(2\pi)^{1/4},(2\pi)^{1/4},-(3\pi)^{1/4},-i\,(3\pi)^{1/4},i\,(3\pi)^{1/4},(3\pi)^{1/4},-\frac{(1+i)\,(3\pi)^{1/4}}{\sqrt{2}},-\frac{(1-i)\,(3\pi)^{1/4}}{\sqrt{2}},\frac{(1-i)\,(3\pi)^{1/4}}{\sqrt{2}},\frac{(1+i)\,(3\pi)^{1/4}}{\sqrt{2}},-(5\pi)^{1/4},-i\,(5\pi)^{1/4},i\,(5\pi)^{1/4},(5\pi)^{1/4},-\frac{(1+i)\,(5\pi)^{1/4}}{\sqrt{2}},-\frac{(1-i)\,(5\pi)^{1/4}}{\sqrt{2}},\frac{(1-i)\,(5\pi)^{1/4}}{\sqrt{2}},\frac{(1+i)\,(5\pi)^{1/4}}{\sqrt{2}},-\left(\frac{\pi}{2}\right)^{1/4}-i\sqrt{3\sqrt{\frac{\pi}{2}}-\sqrt{2\pi}},\left(\frac{\pi}{2}\right)^{1/4}-i\sqrt{3\sqrt{\frac{\pi}{2}}-\sqrt{2\pi}},-\left(\frac{\pi}{2}\right)^{1/4}+i\sqrt{3\sqrt{\frac{\pi}{2}}-\sqrt{2\pi}},\left(\frac{\pi}{2}\right)^{1/4}+i\sqrt{3\sqrt{\frac{\pi}{2}}-\sqrt{2\pi}}\right\}$$

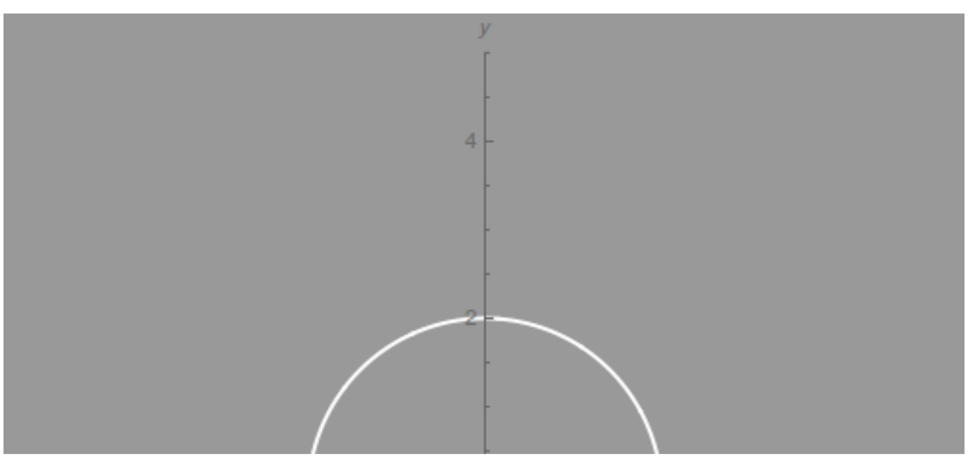
```
wrappingNumber = Length[preImages]
```

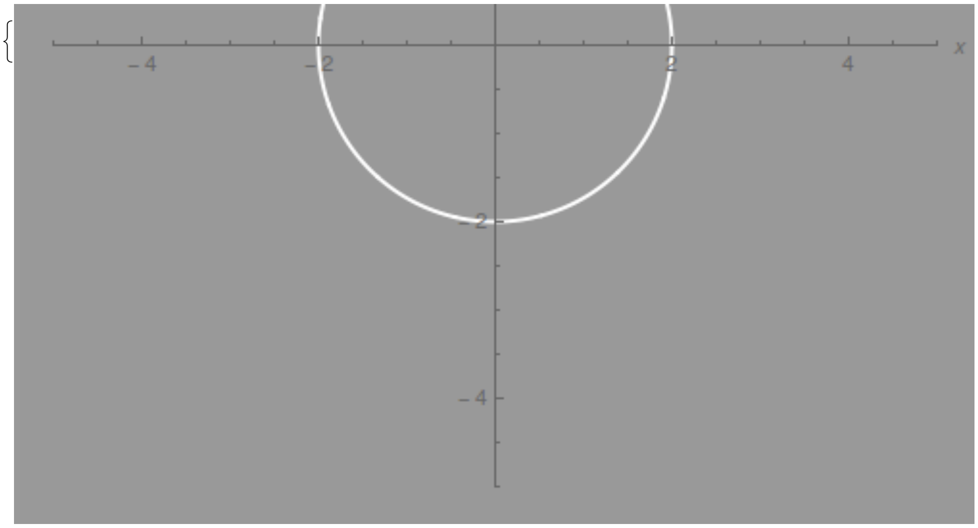
44

Wow that's a lot of wraps, let's see what our pictures show. Let's start by looking at a extremely zoomed out image

```
expr = Sin[z^4]
ang = Range[0 Pi, 2 Pi, .001];
lists = Table[{r Cos[ang], r Sin[ang]}, {r, {2}}];
pts = Transpose[#] & /@ lists;
n = 5;
m = 5 * 10^6;
makeImage[pts, expr, n, m]
```

$\text{Sin}[z^4]$



{

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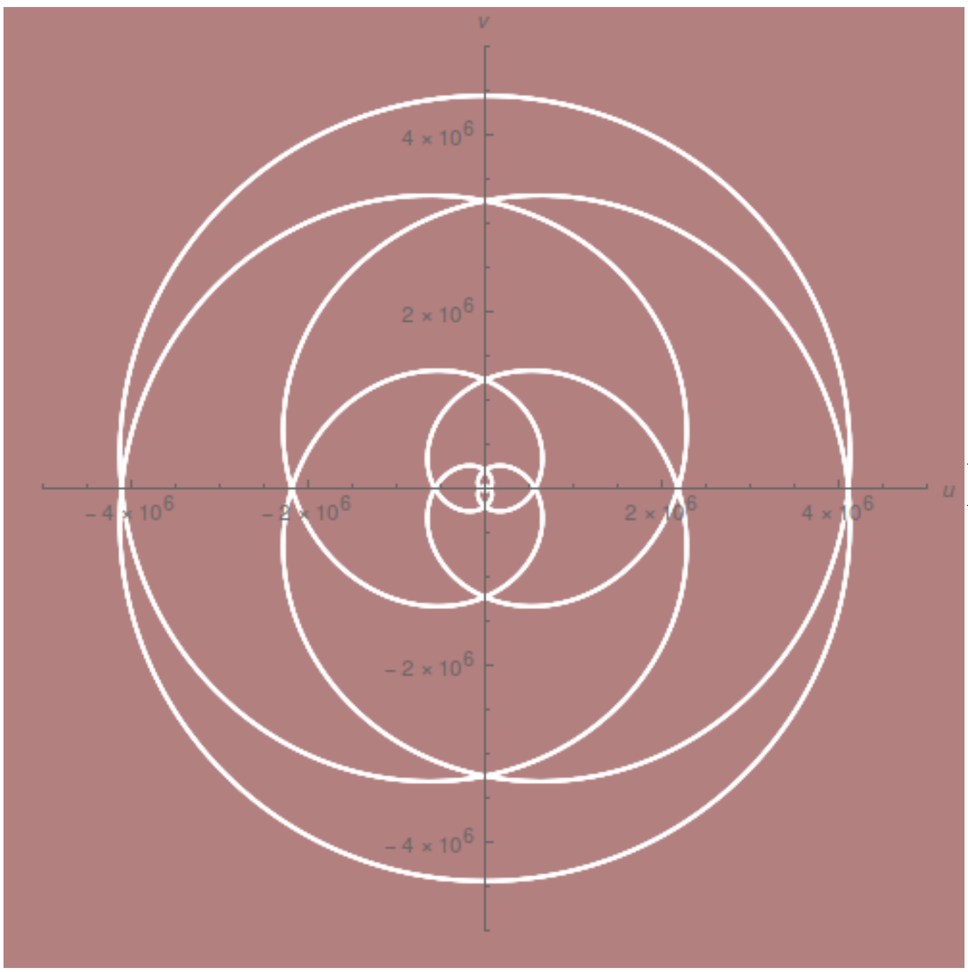

}

Based on the above plot, I think it's reasonable to say that each point has multiplicity +1 and 44 loops here is not unreasonable.