# Complex Variables – HW 7 – question 4

## Here we define Cauchy-Riemann equations

```
cauchyRiemannCartCart[u_, v_] := Module[{ux, uy, vx, vy},
  ux = D[u, x];
  uy = D[u, y];
  vx = D[v, x];
  vy = D[v, y];
  Return[ux == vy && vx == -uy]
 ]
```

## Modify first cauchyReimann into new Jacobian function

```
jacobian[u_, v_] := Module[{ux, uy, vx, vy},
  ux = D[u, x];
  uy = D[u, y];
  vx = D[v, x];
  vy = D[v, y];
  Return[(ux uy
          vx vy)]
 ]
```

## Find u and v

```
f = Abs[x + I y]^2 - I Conjugate[x + I y];
u = FullSimplify[Re[f], Element[{x, y}, Reals]]
v = FullSimplify[Im[f], Element[{x, y}, Reals]]
```

$x^2 + (-1 + y)\, y$

$-x$

## Get the Jacobian Matrix

```
jacMatrix = jacobian[u, v]
```

{{2 x, -1 + 2 y}, {-1, 0}}

```
jacMatrix // MatrixForm
```

$$\begin{pmatrix} 2x & -1+2y \\ -1 & 0 \end{pmatrix}$$

## Find the determinant

```
Det[jacMatrix]
```

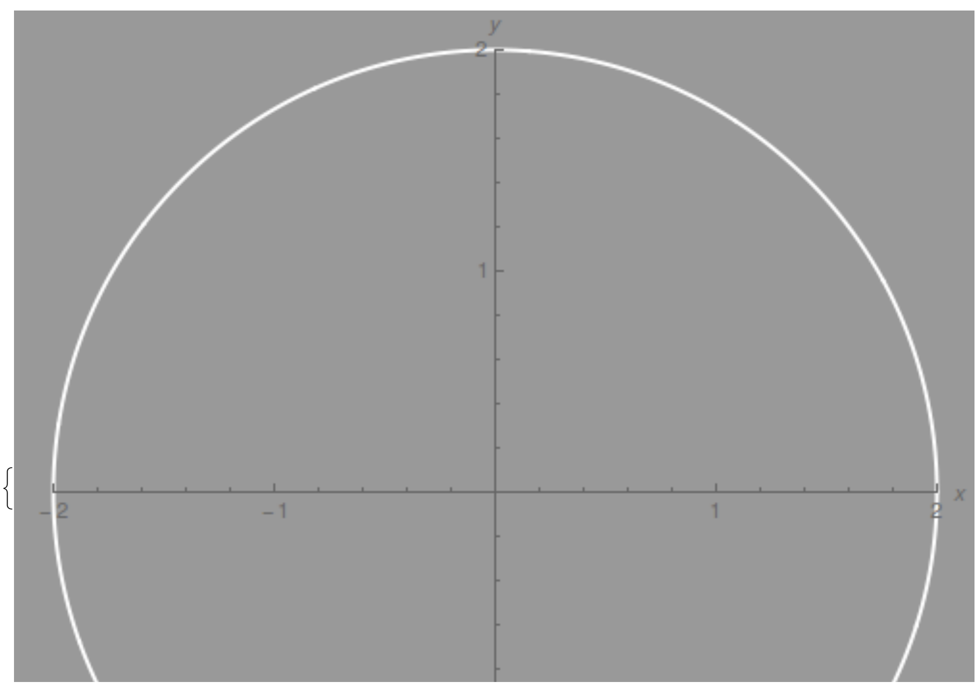
$-1 + 2y$

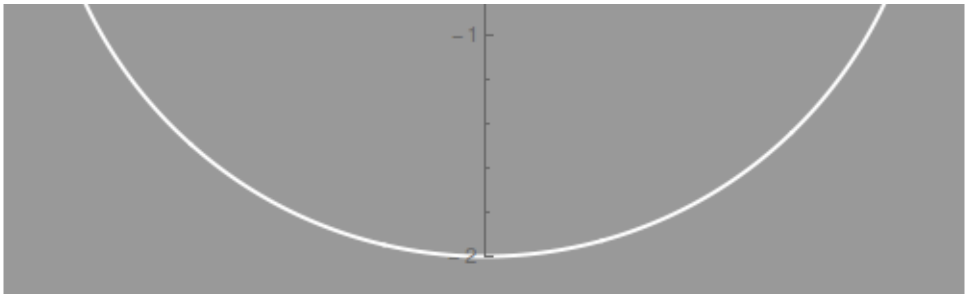
## Next make computer draw image plot. Make mapping function. I promise I will clean up how I import/make these functions in the future.

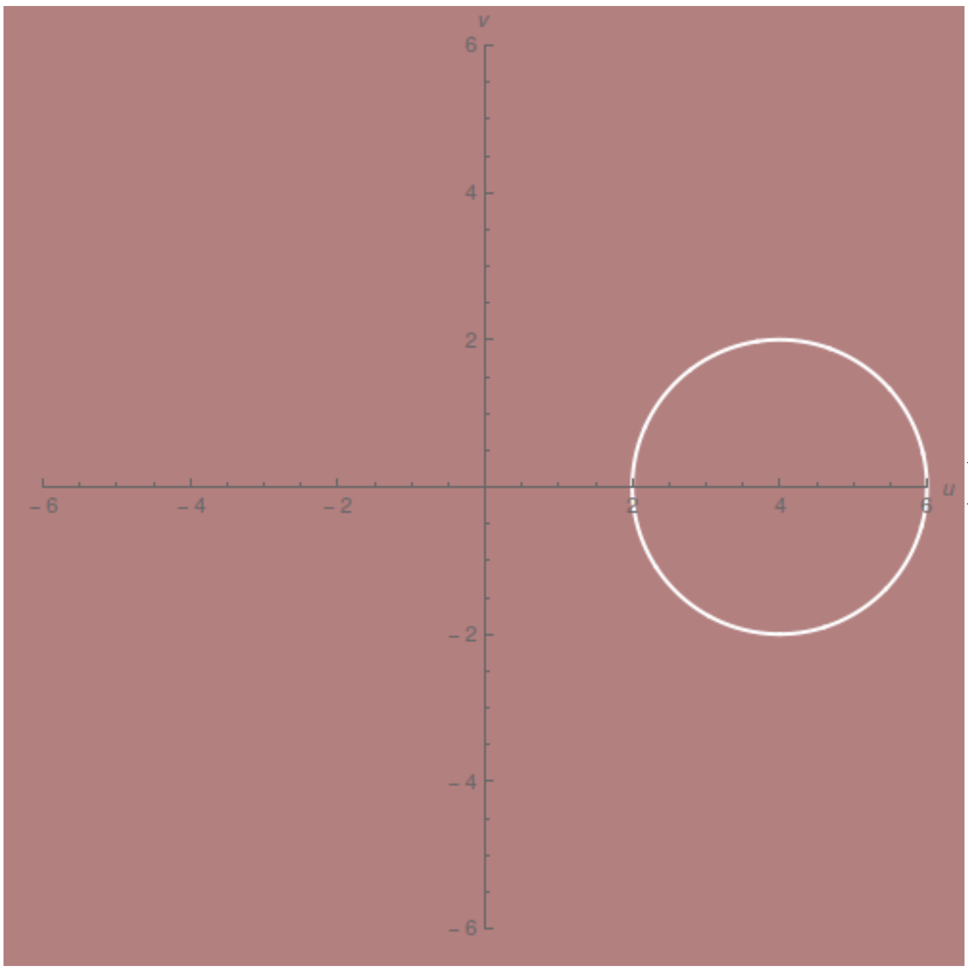
```
makeImage[pts_, expr_, pltRange1_, PltRange2_] := Module[{},
  {
   Rasterize@Graphics[{White, Thick, Line[# & /@ pts]},
     PlotRange → {{-pltRange1, pltRange1}, {-pltRange1, pltRange1}},
     Axes → True, Background → GrayLevel[.6], ImageSize → {500, 500},
     AxesLabel → {Style["x", Italic], Style["y", Italic]}, ImagePadding → 20],

   Rasterize@Graphics[{White, Thick, Line[{Re[expr /. z → #[[1]] + ⅈ #[[2]]],
          Im[expr /. z → #[[1]] + ⅈ #[[2]]]} & /@ # & /@ pts]},
     PlotRange → {{-PltRange2, PltRange2}, {-PltRange2, PltRange2}},
     Axes → True, Background → RGBColor[.7, .5, .5], ImageSize → {500, 500},
     AxesLabel → {Style["u", Italic], Style["v", Italic]}, ImagePadding → 20]
  }
 ]
```

## Now plot

```
expr = Abs[z]^2 - I Conjugate[z];
ang = Range[0 Pi, 2 Pi, .001];
lists = Table[{r Cos[ang], r Sin[ang]}, {r, {2}}];
pts = Transpose[#] & /@ lists;
n = 2;
m = 6;
makeImage[pts, expr, n, m]
```

{
,

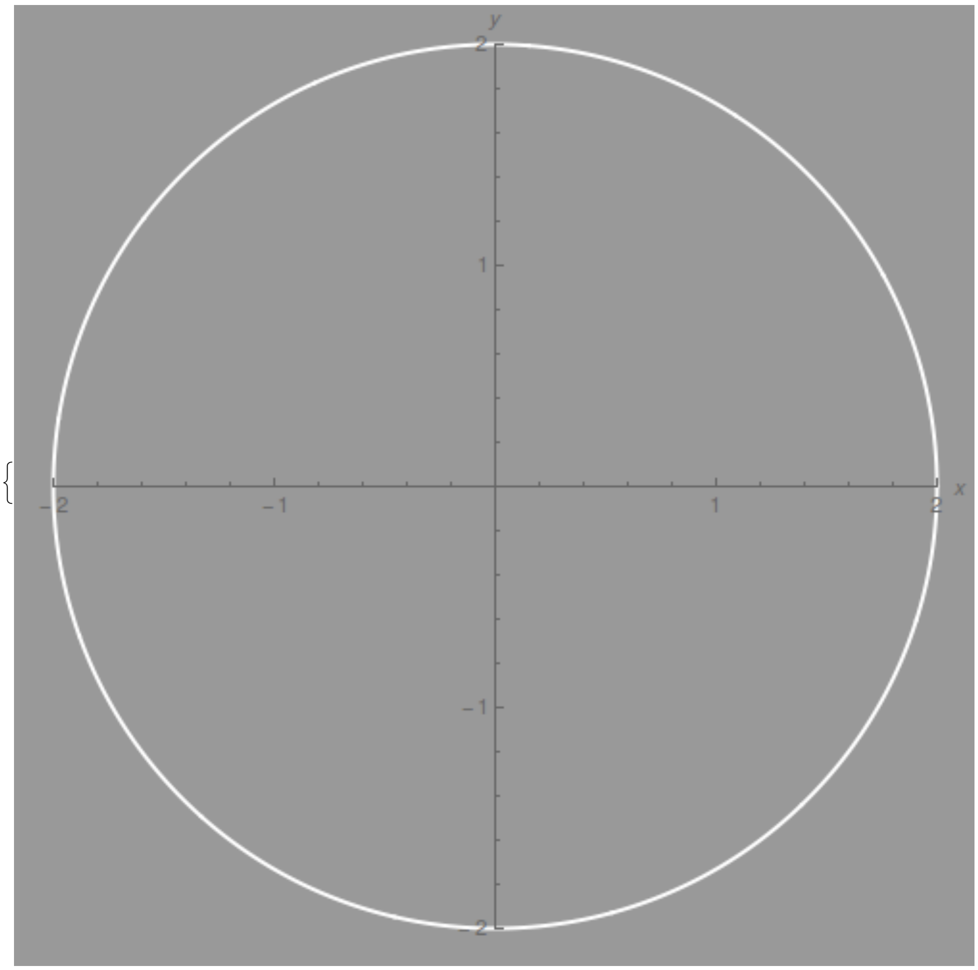


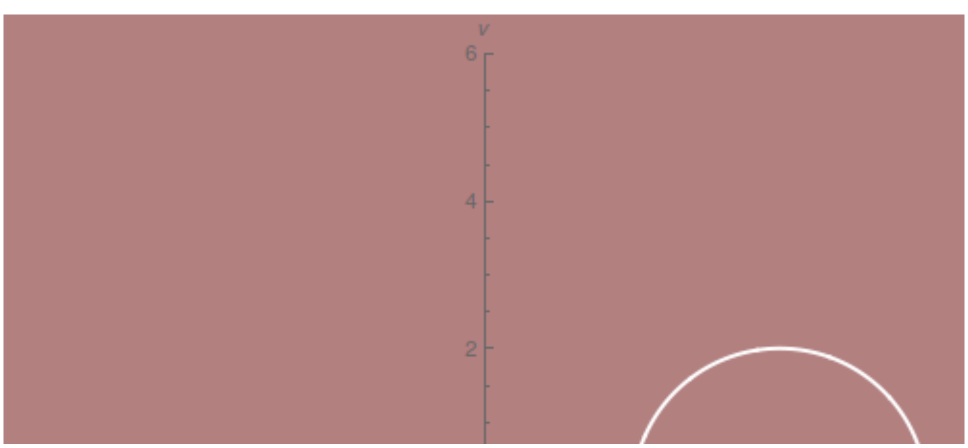

Yay, as we expect, the image doesn't encircle the origin because each point is going opposite direction!

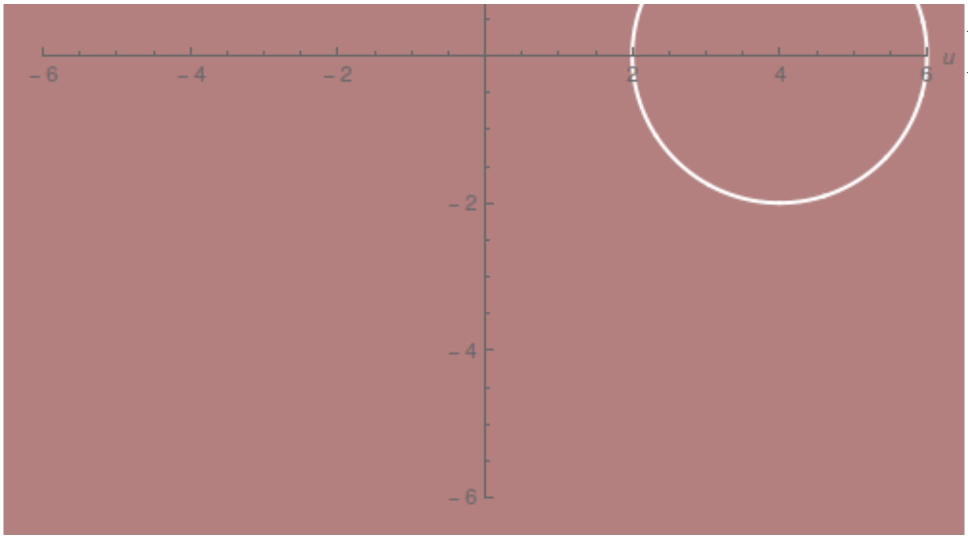
Now, let's make a quick plot of part v. We see that it is the exact same as the previous plot.

```
expr = Conjugate[z] (z - I);
ang = Range[0 Pi, 2 Pi, .001];
lists = Table[{r Cos[ang], r Sin[ang]}, {r, {2}}];
pts = Transpose[#] & /@ lists;
n = 2;
m = 6;
makeImage[pts, expr, n, m]
```

{,





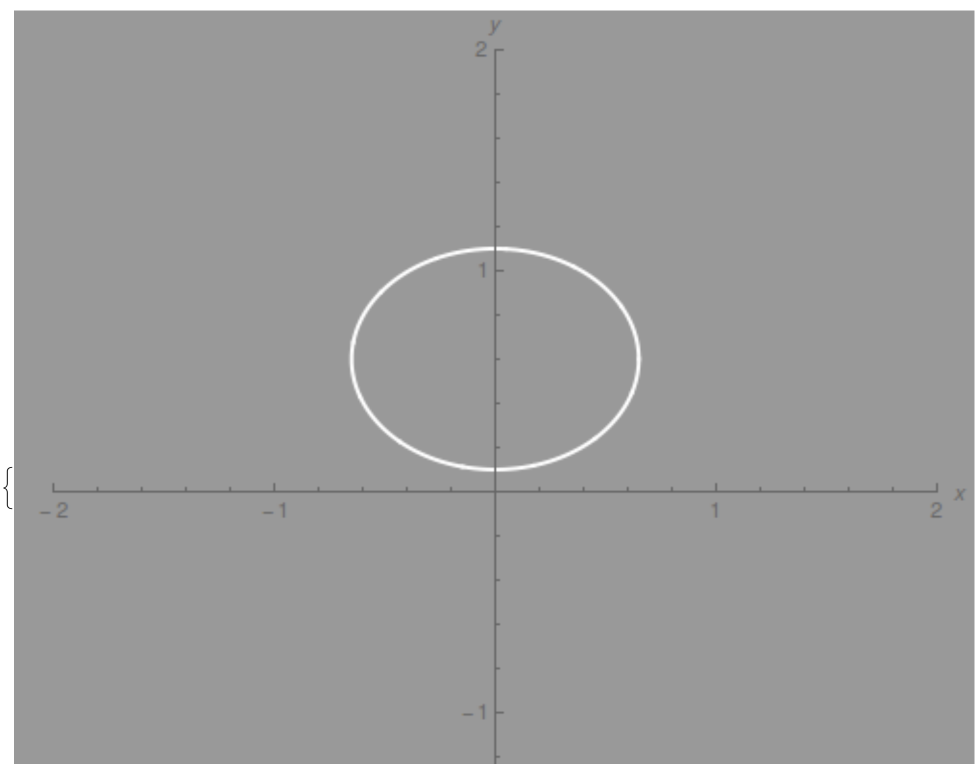


}

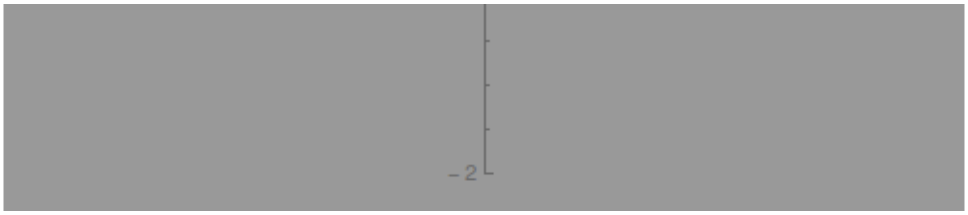
## Quick plot of an offset ellipse to see how many times we wind around $\frac{i}{2}$

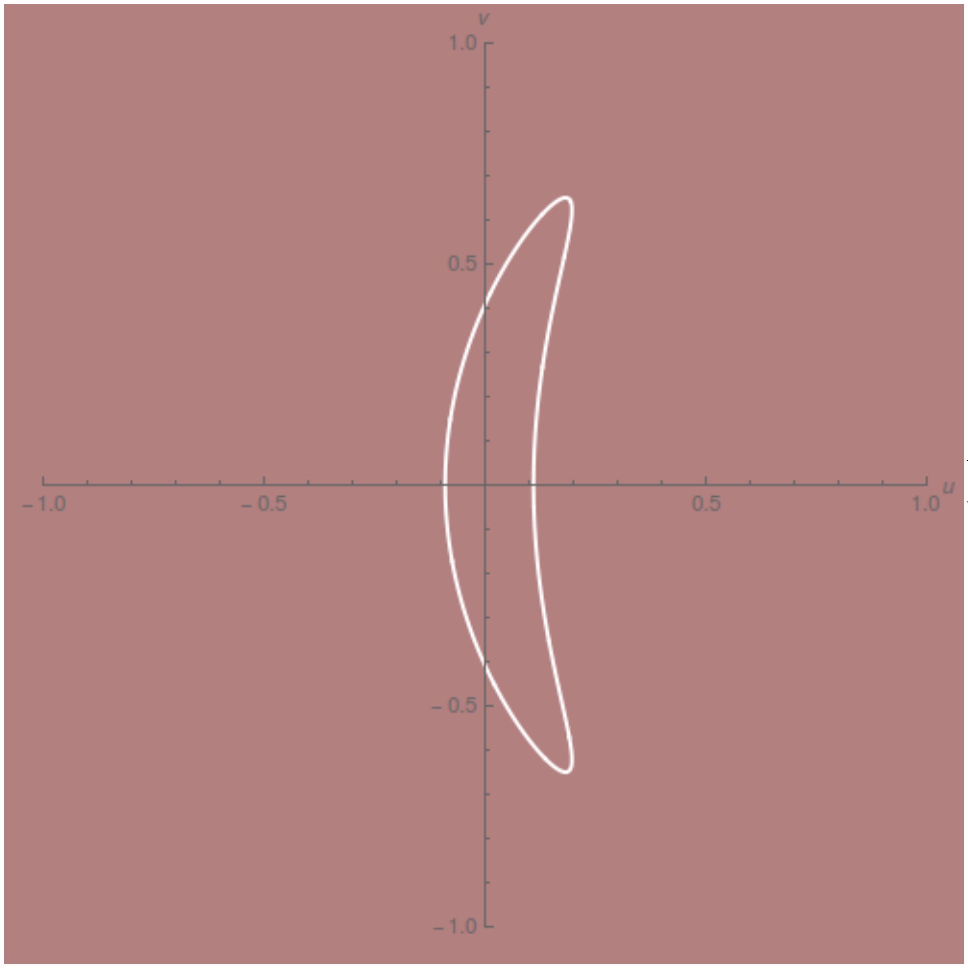
```
expr = Conjugate[z] (z - I);
ang = Range[.0 Pi, 2 Pi, .001];
lists = Table[{1.3 * r Cos[ang], 1.2 * 1/2 + r Sin[ang]}, {r, {1/2}}];
pts = Transpose[#] & /@ lists;
n = 2;
m = 1;
makeImage[pts, expr, n, m]
```

{



,




Looks like we wind around $\frac{i}{2}$ a single time