Hw 5 - chapter 4 - complex variables

2

$$12\, e^{i\pi}$$

$$-12$$

$$3\, r^2\, e^{i\, 2\, \theta}$$

$$3\, e^{2\, i\, \theta}\, r^2$$

$$\text{ExpToTrig}\left[3\, e^{2\, i\, \theta}\, r^2\right]$$

$$3\, r^2\, \text{Cos}[2\, \theta] + 3\, i\, r^2\, \text{Sin}[2\, \theta]$$

$$\text{Solve}\left[-12 == 3\, r^2\, \text{Cos}[2\, \theta]\ \&\&\ 0 == 3\, i\, r^2\, \text{Sin}[2\, \theta],\ \{\theta,\ r\}\right]$$

$$\left\{\left\{r \to -2,\ \theta \to \text{ConditionalExpression}\left[\frac{1}{2}\left(\pi + 2\,\pi\, C[1]\right),\ C[1] \in \text{Integers}\right]\right\},\right.$$
$$\{r \to -2\, i,\ \theta \to \text{ConditionalExpression}[\pi\, C[1],\ C[1] \in \text{Integers}]\},$$
$$\{r \to 2\, i,\ \theta \to \text{ConditionalExpression}[\pi\, C[1],\ C[1] \in \text{Integers}]\},$$
$$\left.\left\{r \to 2,\ \theta \to \text{ConditionalExpression}\left[\frac{1}{2}\left(\pi + 2\,\pi\, C[1]\right),\ C[1] \in \text{Integers}\right]\right\}\right\}$$

$$\text{First}[\%5]$$

$$\left\{r \to -2,\ \theta \to \text{ConditionalExpression}\left[\frac{1}{2}\left(\pi + 2\,\pi\, C[1]\right),\ C[1] \in \text{Integers}\right]\right\}$$

9

In[7]:=
```
cauchyRiemann[u_, v_] := Module[{ux, uy, vx, vy},
  ux = D[u, x];
  uy = D[u, y];
  vx = D[v, x];
  vy = D[v, y];
  Return[ux == vy && vx == -uy]
 ]
```

7i

```
In[8]:= With[{u = 3, v = 2},
  cauchyRiemann[u, v]
 ]
Out[8]= True
```

7ii

```
In[1]:= Solve[ux + I vx == 0 && uv - I uy == 0]
Out[1]= {{uy → -ⅈ uv, vx → ⅈ ux}}
```

7iii

8i

Cauchy Riemann equations are only satisfied if f(z) is constant or v = constant

```
In[10]:= cauchyRiemann[0, x I]
Out[10]= False

In[11]:= cauchyRiemann[0, y I]
Out[11]= False

In[12]:= cauchyRiemann[0, 3 I]
Out[12]= True
```

8ii

In the case of a circle,

```
In[15]:= cauchyRiemann[x^2, y^2]
Out[15]= 2 x == 2 y
```