Complex Variables Toolkit - Final Project - Nathan Yee

About

This notebook serves as a group of utilities (functions) that allow a user to interact with and understand complex functions. In particular it provides functions that give exponential and sphere (inverse stereograph) based spacings, point based circle creation, point based line and grid creation, complex image map utilities, additional color creation tools, stereographic projection utilities, 3D printing tools and examples, and finally Newton's method animation utilities. As an added benefit to the user, every function has a usage line (Mathematica equivalent of docstring), and at least one example of how it is used.

The line below tells Mathematica to save all initialization cells to a .m file. This is the file that will be imported in the future whenever one wants to use the toolkit.

```
In[59]:= SetOptions[EvaluationNotebook[], AutoGeneratedPackage → Automatic]
```

Functions

Provided below are the functions, documentation, and examples.

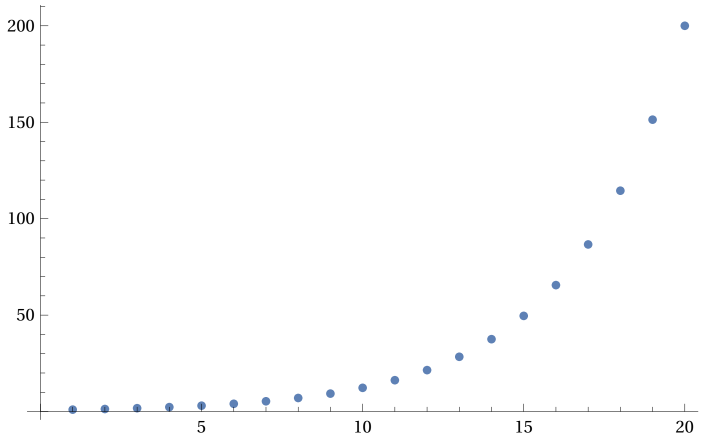
Exponential, and Sphere based Spacings

Various types of spacings serve as the foundation for circle, line, and grid point generator functions.

Inverse (default Log) spaced points

```
In[60]:= fSpace[min_, max_, numPts_, f_: Log] :=
          Module[{},
           N[InverseFunction[f] /@ Range[f@min, f@max, (f@max - f@min) / (numPts - 1)]]]
         fSpace::usage =
           "fSpace[min, max, steps, Log] gives inverse f (default Log) spaced
             points from min to max over a given number of points";
```

In[62]:=
```
ListPlot[fSpace[1, 200, 20, Log]]
```

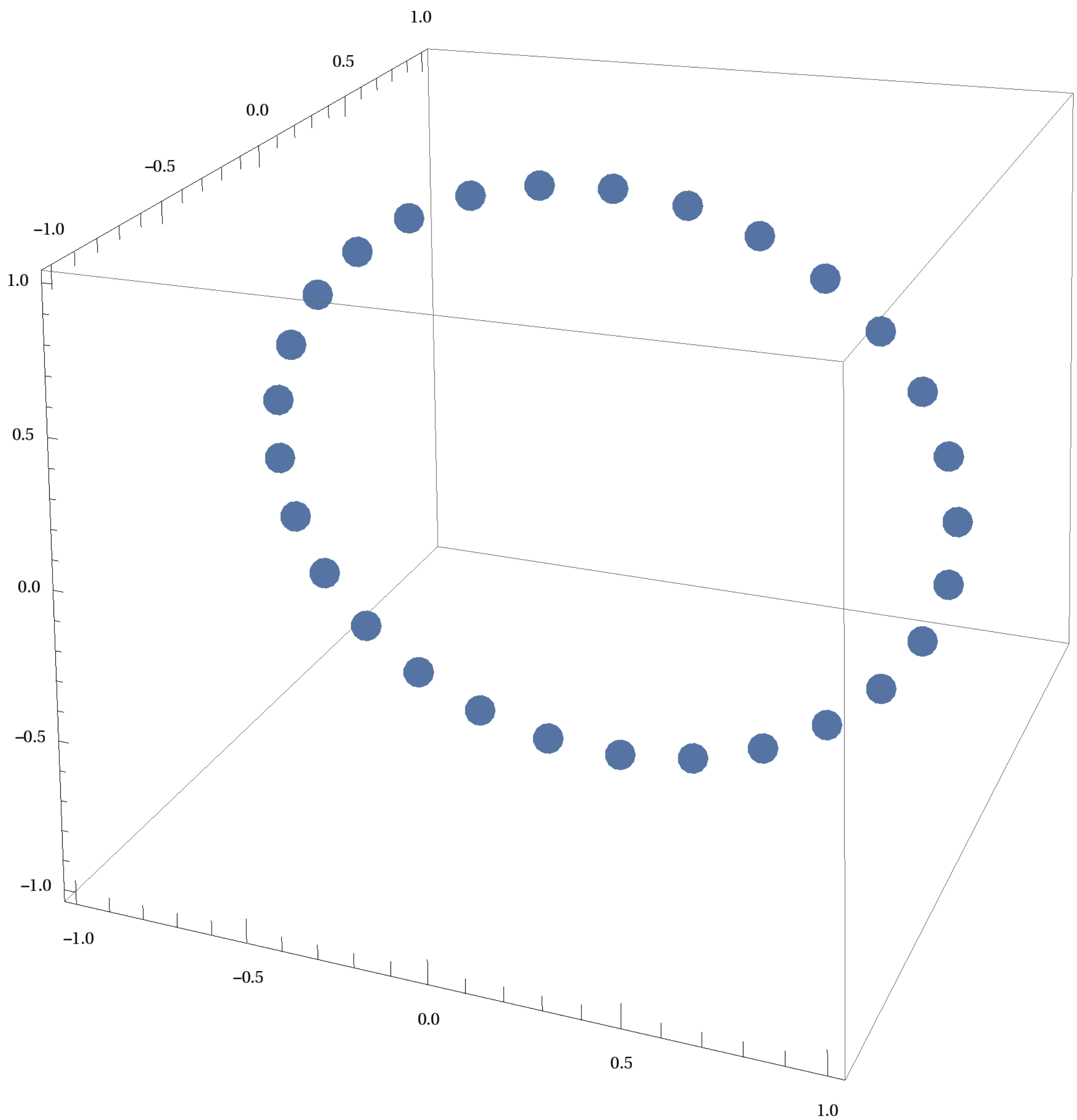
Out[62]=


Sphere spaced points

```
makeRiemannRealAxis[numPts_] := Module[
  {angles, pts},
  angles = N@Range[0 Pi, 2 Pi, (2 Pi - 0 Pi)/(numPts - 1)];
  pts = Table[{Cos[ang], 0, Sin[ang]}, {ang, angles}];
  Return[pts]
]
makeRiemannRealAxis::usage =
  "makeRiemannRealAxis[numPts_] makeRiemannRealAxis returns equally
    spaced points around the positive real axis on the Riemann Sphere";
```

In[65]:=
```
realAxis3D = makeRiemannRealAxis[30];
ListPointPlot3D[realAxis3D, AspectRatio → 1, PlotStyle → PointSize[.03],
 ImageSize → Large]
```

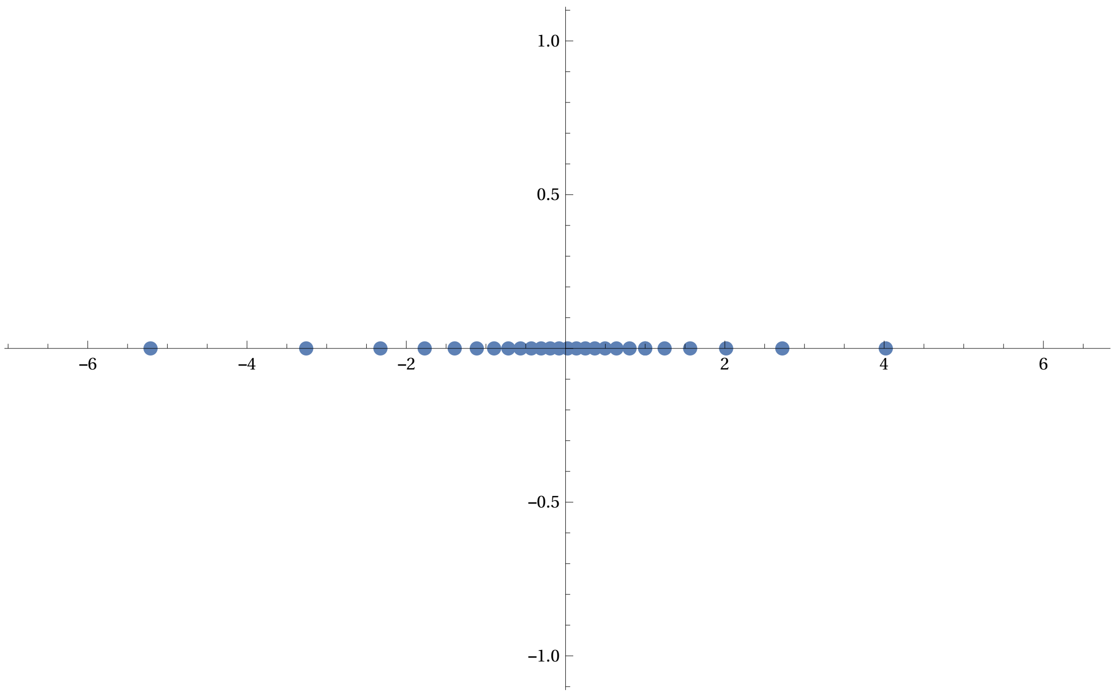
Out[66]=




```
riemannPointToComplexPlane[{X_,Y_,Z_}]:=Module[{sol,pair},
sol=Solve[{x+ⅈ y==(X+ⅈ Y)/(1-Z)},{x∈Reals,y∈Reals}];
pair=({x,y}/.sol)[[1]];
Return[pair]
]
riemannPointToComplexPlane::usage="riemannPointToComplexPlane[{X_,Y_,Z_}]
riemannPointToComplexPlane returns the inverse stereographic projection of a point (
```

In[69]:=
```
realAxis2D = riemannPointToComplexPlane[#] & /@ realAxis3D;
ListPlot[realAxis2D, ImageSize → Large]
```

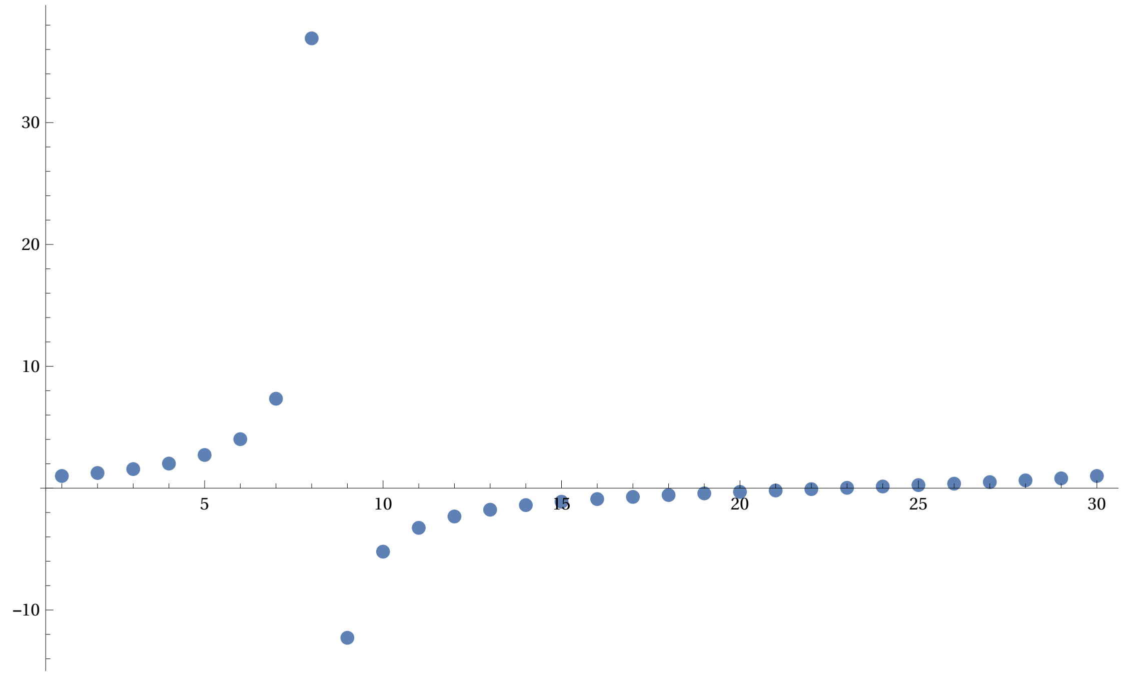
Out[70]=



In[71]:=
```
makeSphereSpacedPoints[numPts_] := Module[{pts, complexPts, realPts},
  pts = makeRiemannRealAxis[numPts];
  complexPts = riemannPointToComplexPlane[#] & /@ pts;
  realPts = complexPts[[All, 1]];
  Return[realPts]
 ]
makeSphereSpacedPoints::usage =
  "makeSphereSpacedPoints[numPts_] makeSphereSpacedPoints returns
    the inverse stereographic projection of points around the
    positive real axis on the Riemann Sphere. It gets the positive
    real axis of the Riemann Sphere by calling makeRiemannRealAxis.
    It finds the inverse stereographic projection of the points
    by mapping riemannPointToComplexPlane onto each point";
```

In[73]:=
```
ListPlot[makeSphereSpacedPoints[30], PlotRange → All, ImageSize → Large]
```

Out[73]=



Circle Creation

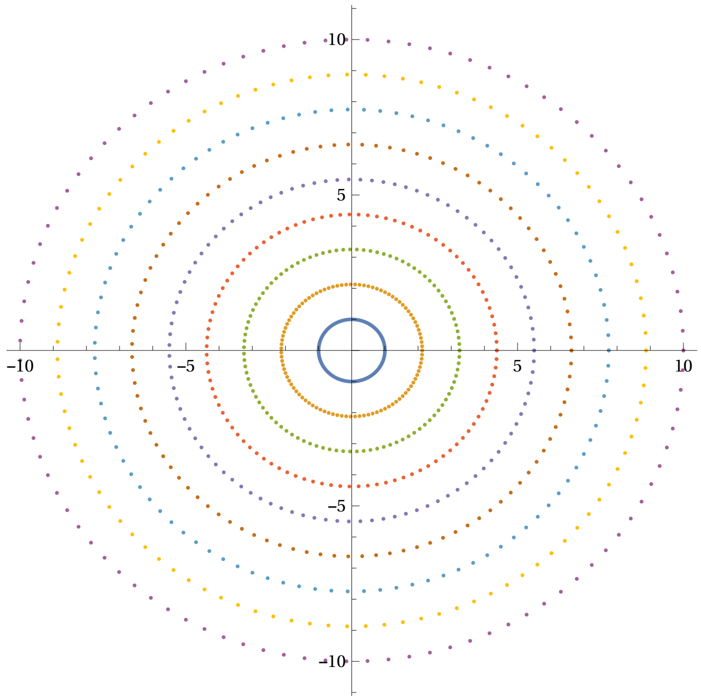
Point based circle generator functions are useful tool to visualize complex mappings.

Linear spaced circles

```
makeCirclePoints[smallRadius_,largeRadius_,numCircles_,ptsPerCircle_]:=Module[{ang,
ang=Range[0 Pi,2 Pi,(2π-0)/(ptsPerCircle-1)];
lists=Table[{r Cos[ang],r Sin[ang]},{r,Range[smallRadius,largeRadius,(largeRadius-sm)/(numCircl
pts=Transpose[#]&/@lists;
Return[pts]
]
makeCirclePoints::usage="makeCirclePoints[smallRadius_,largeRadius_,numCircles_,pts
makeCirclePoints returns expanding circles given smallest radius, largest radius, s
The constant step size dictates that the radius of the circles increases by a const
```

In[76]:=
```
ListPlot[makeCirclePoints[1, 10, 9, 100], AspectRatio → 1]
```

Out[76]=

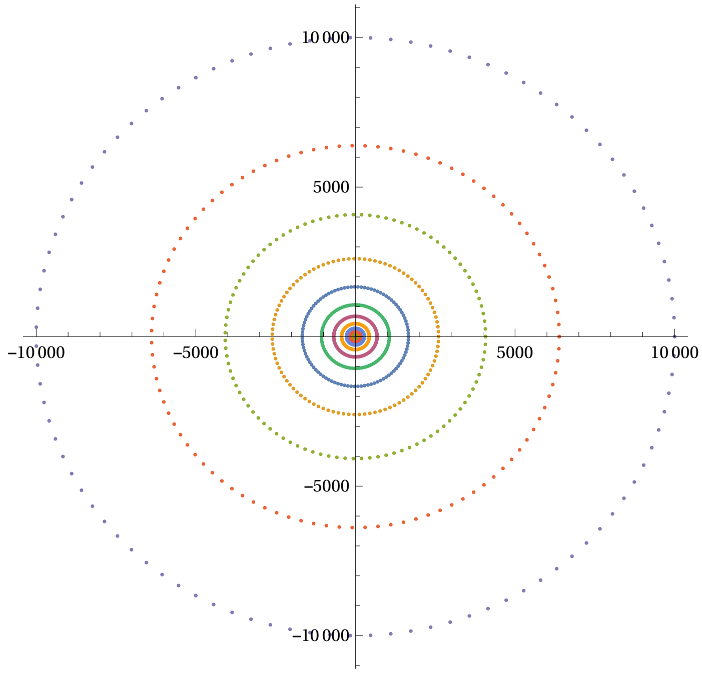


Exponentially spaced circles

In[77]:=
```
makeExponentialSpacedCirclePoints[smallRadius_, largeRadius_, numCircles_,
  ptsPerCircle_] := Module[{ang, lists, pts},
ang = Range[0 Pi, 2 Pi, (2 π - 0)/(ptsPerCircle - 1)];
  lists = Table[{r Cos[ang], r Sin[ang]},
    {r, fSpace[smallRadius, largeRadius, numCircles]}];
  pts = Transpose[#] & /@ lists;
Return[pts]
]
makeExponentialSpacedCirclePoints::usage =
  "makeExponentialSpacedCirclePoints[smallRadius_,largeRadius_,numCircles_,
    ptsPerCircle_] makeExponentialSpacedCirclePoints returns
    expanding circles given smallest radius, largest radius, number
    of circles, and number of points per circle. The number of
    circles dictates that the circles be spaced exponentially
    within the largestRadius - smallestRadius range";
```

In[79]:=
```
ListPlot[makeExponentialSpacedCirclePoints[2, 10 000, 20, 100], AspectRatio → 1,
 PlotRange → Full]
```

Out[79]=

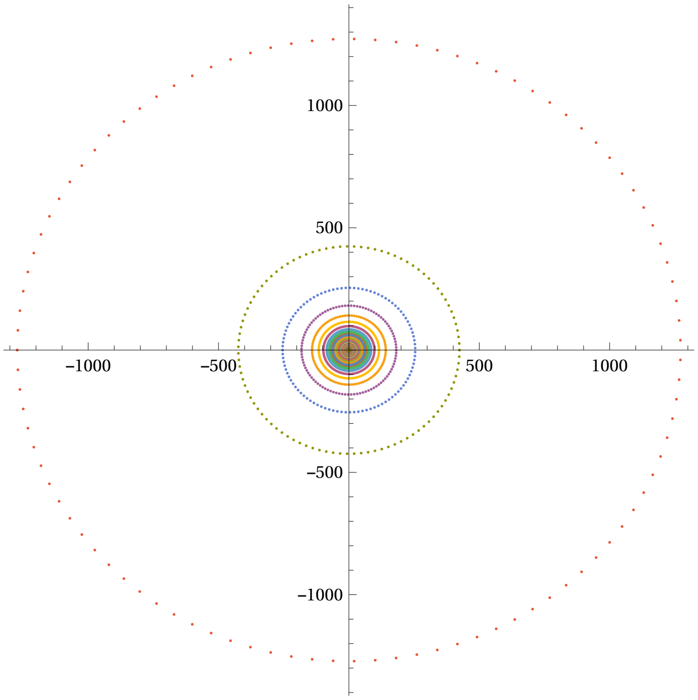


“Sphere” spaced circles

In[80]:=
```
makeSphereSpacedCirclePoints[numCircles_,ptsPerCircle_]:=
 Module[{ang, lists, pts},
  ang=Range[0 Pi,2 Pi,(2π-0)/(ptsPerCircle-1)];
  lists=Table[{r Cos[ang],r Sin[ang]},{r,makeSphereSpacedPoints[numCircles]}];
  pts=Transpose[#]&/@lists;
  Return[pts]
 ]
makeSphereSpacedCirclePoints::usage =
  "makeSphereSpacedCirclePoints[numCircles_,ptsPerCircle_]
    makeSphereSpacedCirclePoints returns circles based on inverse
    stereographic projection given a number of circles, and the
    number of points per circle";
```

In[82]:=
```
ListPlot[makeSphereSpacedCirclePoints[1000, 100], AspectRatio → 1,
  PlotRange → Full]
```

Out[82]=



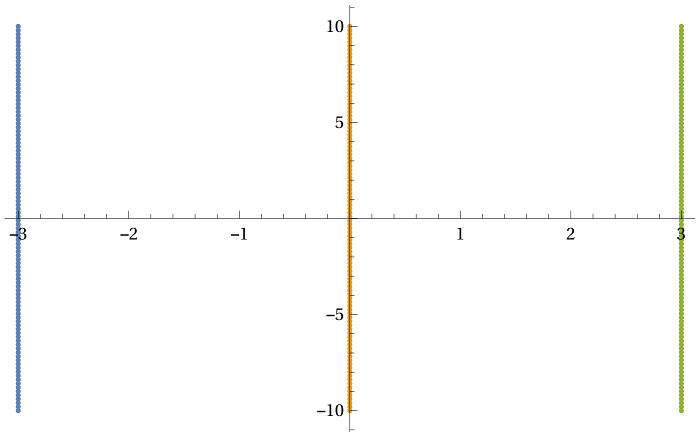
Line and Grid creation

Point based line and grid generator functions are useful tool to visualize complex mappings.

Linearly spaced lines and grids

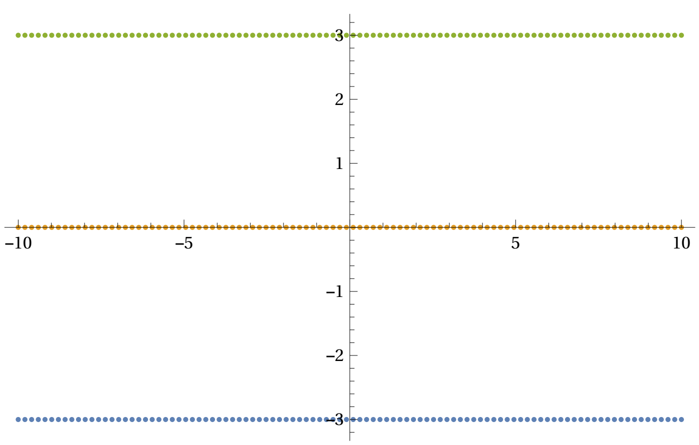
In[83]:=
```
makeVerticalPts[minX_,maxX_,minY_,maxY_,ptsPerLine_,numLines_]:=
 Module[{horiPts},
  horiPts=Table[Table[{x,y},{y,Range[minY,maxY,(maxY-minY)/(ptsPerLine-1)]}],
    {x,minX,maxX,(maxX-minX)/(numLines-1)}];
  Return[horiPts]
 ]
makeVerticalPts::usage =
  "makeVerticalPts[minX_,maxX_,minY_,maxY_,ptsPerLine_,numLines_]
returns numLines number of vertical lines of equally spaced points
    starting from minX to maxX with length maxY-minY with ptsPerLine
    number of points per line";
```

In[85]:=
```
ListPlot[makeVerticalPts[-3, 3, -10, 10, 100, 3], PlotRange → Full]
```

Out[85]=


In[86]:=
```
makeHorizontalPts[minX_, maxX_, minY_, maxY_, ptsPerLine_, numLines_] :=
 Module[{horiPts},
  horiPts = Table[Table[{x, y}, {x, Range[minX, maxX, (maxX - minX)/(ptsPerLine - 1)]}],
    {y, minY, maxY, (maxY - minY)/(numLines - 1)}];
  Return[horiPts]
 ]
makeHorizontalPts::usage =
  "makeHorizontalPts[minX_,maxX_,minY_,maxY_,ptsPerLine_,numLines_]
returns numLines number of horizontal lines of equally spaced points
    starting from minY to maxY with length maxX-minX with ptsPerLine
    number of points per line";
```

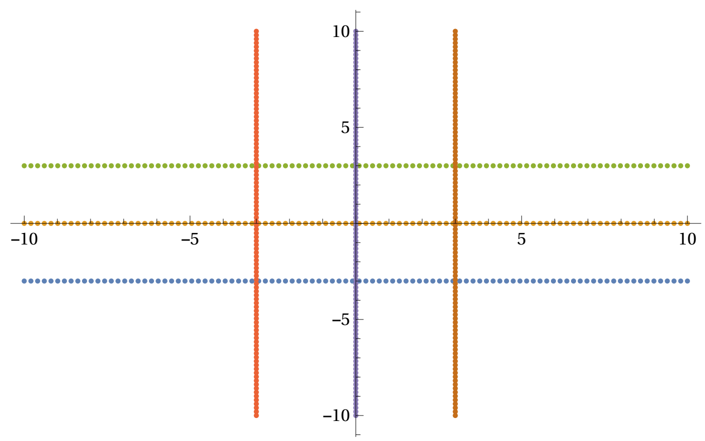
In[88]:=
```
ListPlot[makeHorizontalPts[-10, 10, -3, 3, 100, 3], PlotRange → Full]
```

Out[88]=


```
makeGridPts[minX_,maxX_,minY_,maxY_,ptsPerLine_,numLines_]:=Module[{vertPts,horiPts
vertPts=makeVerticalPts[minY,maxY,minX,maxX,ptsPerLine,numLines];
horiPts=makeHorizontalPts[minX,maxX,minY,maxY,ptsPerLine,numLines];
pts=Join[horiPts,vertPts];
Return[pts]
]
makeGridPts::usage="makeGridPts[minX_,maxX_,minY_,maxY_,ptsPerLine_,numLines_]
returns numLines number of vertical of equally spaced points lines starting from mi
length maxY-minY with ptsPerLine number of points per line and
returns numLines number of horizontal of equally spaced points lines starting from
length maxX-minX with ptsPerLine number of points per line.
NOTE: enter args as if doing horizontal line";
```

In[91]:=
```
ListPlot[makeGridPts[-10, 10, -3, 3, 100, 3], PlotRange → Full]
```

Out[91]=

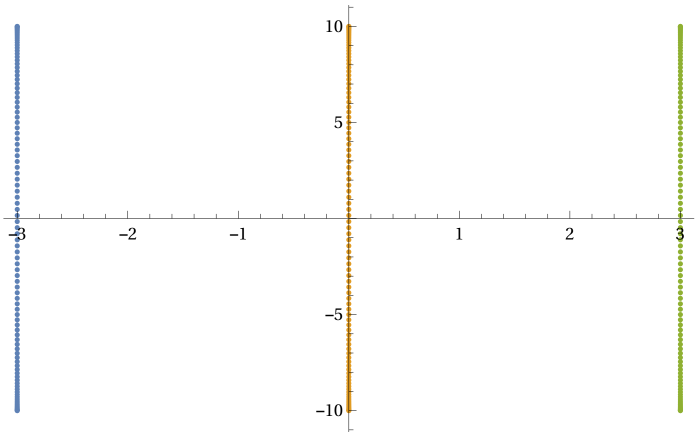


Exponentially spaced lines and grids

In[92]:=
```
makeLogVerticalPts2[minX_, maxX_, minY_, maxY_, ptsPerLine_, numLines_] :=
 Module[{vertPts},
  vertPts = Table[Table[{x, y}, {y, fSpace[minY, maxY, ptsPerLine]}],
    {x, minX, maxX, (maxX - minX)/(numLines - 1)}];
  Return[Re[vertPts]]
 ]
makeLogVerticalPts2::usage =
  "makeLogVerticalPts2[minX_,maxX_,minY_,maxY_,ptsPerLine_,numLines_]
returns numLines number of vertical lines of exponentially spaced
    points starting from minX to maxX with length maxY-minY with
    ptsPerLine number of points per line.
NOTE: makeLogVerticalPts2 is depreciated due to not handling negative
    minY to positive maxY ranges";
```

In[94]:= ListPlot[makeLogVerticalPts2[-3, 3, -10, 10, 100, 3], PlotRange → Full]

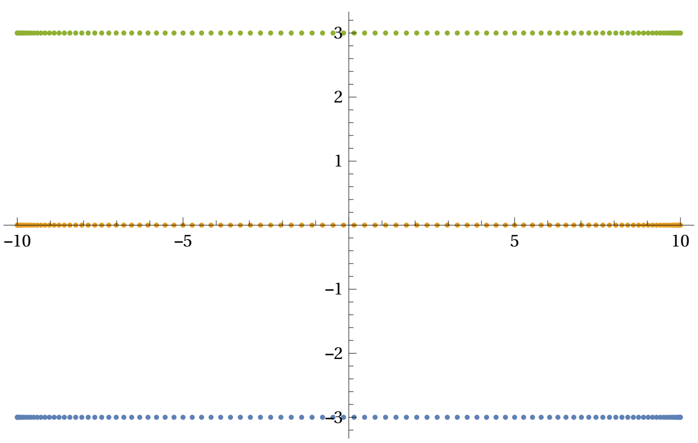
Out[94]=




```
makeLogHorizontalPts2[minX_,maxX_,minY_,maxY_,ptsPerLine_,numLines_]:=Module[{horiP
horiPts=Table[Table[{x,y},{x,fSpace[minX,maxX,ptsPerLine]}],{y,minY,maxY,(maxY-minY)/(numLines-1
Return[Re[horiPts]]
]
makeLogHorizontalPts2::usage="makeLogHorizontalPts2[minX_,maxX_,minY_,maxY_,ptsPerL
returns numLines number of horizontal lines of exponentially spaced points starting
length maxX-minX with ptsPerLine number of points per line
NOTE: makeLogHorizontalPts2 is depreciated due to not handling negative minX to pos
```

In[97]:= ListPlot[makeLogHorizontalPts2[-10, 10, -3, 3, 100, 3], PlotRange → Full]

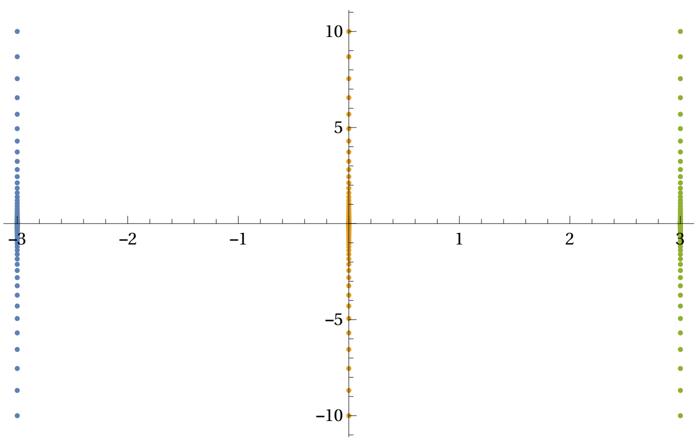
Out[97]=



In[98]:=
```
makeLogVerticalPts[minX_,maxX_,minY_,maxY_,ptsPerLine_,numLines_]:=Module[
{posVertPts,negVertPts,pts},
  posVertPts=Table[Table[{x,y},{y,fSpace[.01,maxY,Floor[ptsPerLine/2]]}],
    {x,minX,maxX,(maxX-minX)/(numLines-1)}];
  negVertPts=Table[Table[{x,y},{y,fSpace[minY,-.01,Floor[ptsPerLine/2]]}],
    {x,minX,maxX,(maxX-minX)/(numLines-1)}];
  pts=MapThread[Join,{negVertPts,posVertPts}];
  Return[Re[pts]]
 ]
makeLogVerticalPts::usage =
  "makeLogVerticalPts[minX_,maxX_,minY_,maxY_,ptsPerLine_,numLines_]
returns numLines number of vertical lines of exponentially spaced
    points starting from minX to maxX with length maxY-minY with
    ptsPerLine number of points per line.";
```

In[100]:=
```
ListPlot[makeLogVerticalPts[-3, 3, -10, 10, 100, 3], PlotRange → Full]
```

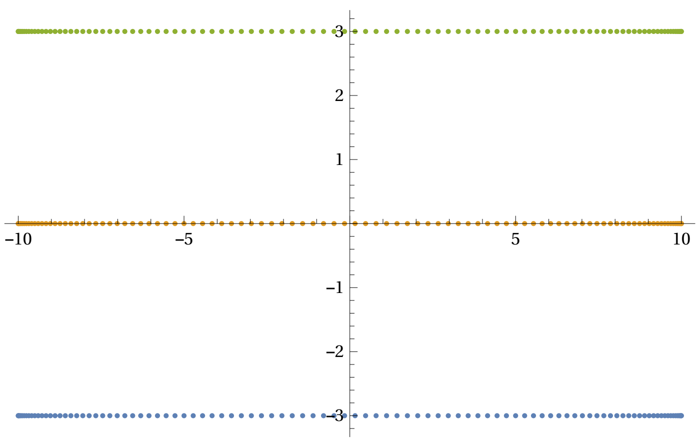
Out[100]=



In[101]:=
```
makeLogHorizontalPts[minX_,maxX_,minY_,maxY_,ptsPerLine_,numLines_]:=Module[
{posHoriPts,negHoriPts,pts},
  posHoriPts=Table[Table[{x,y},{x,fSpace[.01,maxX,Floor[ptsPerLine/2]]}],
    {y,minY,maxY,(maxY-minY)/(numLines-1)}];
  negHoriPts=Table[Table[{x,y},{x,fSpace[minX,-.01,Floor[ptsPerLine/2]]}],
    {y,minY,maxY,(maxY-minY)/(numLines-1)}];
  pts=MapThread[Join,{negHoriPts,posHoriPts}];
  Return[Re[pts]]
 ]
makeLogHorizontalPts::usage =
  "makeLogHorizontalPts[minX_,maxX_,minY_,maxY_,ptsPerLine_,numLines_]
returns numLines number of horizontal lines of exponentially spaced
    points starting from minY to maxY with length maxX-minX with
    ptsPerLine number of points per line";
```

In[103]:=
```
ListPlot[makeLogHorizontalPts2[-10, 10, -3, 3, 100, 3], PlotRange → Full]
```

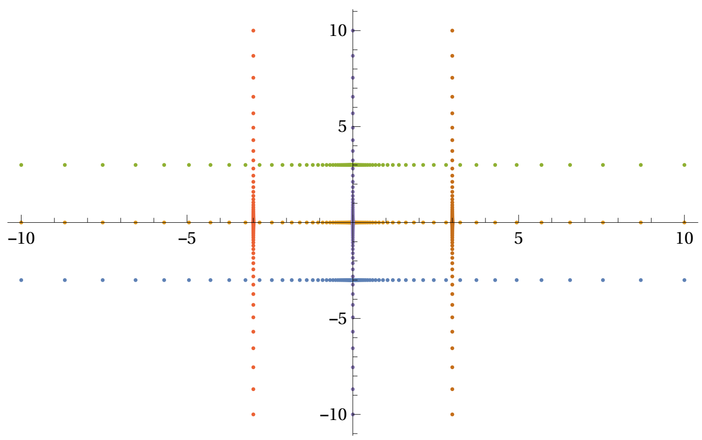
Out[103]=



In[104]:=
```
makeLogGridPts[minX_,maxX_,minY_,maxY_,ptsPerLine_,numLines_]:=Module[
{vertLogPts,horiLogPts,pts},
vertLogPts=makeLogVerticalPts[minY,maxY,minX,maxX,ptsPerLine,numLines];
horiLogPts=makeLogHorizontalPts[minX,maxX,minY,maxY,ptsPerLine,numLines];
pts=Join[horiLogPts,vertLogPts];
Return[pts]
]
makeLogGridPts::usage="makeLogGridPts[minX_,maxX_,minY_,maxY_,ptsPerLine_,numLines_
returns numLines number of vertical of exponentially spaced points lines starting f
with length maxY-minY with ptsPerLine number of points per line and
returns numLines number of horizontal of exponentially spaced points lines starting
with length maxX-minX with ptsPerLine number of points per line.
NOTE: enter args as if doing horizontal line";
```

In[106]:=
```
ListPlot[makeLogGridPts[-10, 10, -3, 3, 100, 3], PlotRange → Full]
```

Out[106]=

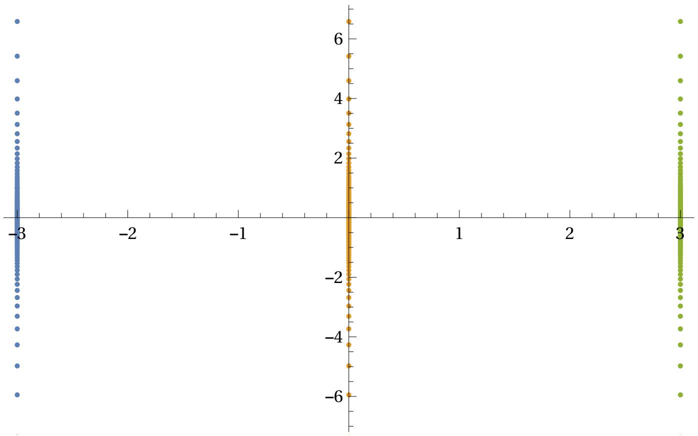


Sphere spaced lines and grids

In[107]:=
```
makeSphereVerticalLines[min_,max_,ptsPerLine_,numLines_]:=Module[{horiPts,spherePts
spherePts=makeSphereSpacedPoints[ptsPerLine];
horiPts=Table[Table[{x,y},{y,spherePts}],{x,min,max,(max-min)/(numLines-1)}];
Return[horiPts]
]
makeSphereVerticalLines::usage=
    "makeSphereVerticalLines[min_,max_,ptsPerLine_,numLines_] makes vertical lines
    sphere spaced points given line bounds of min, max, ptsPerLine, and numLines";
```

In[109]:=
```
ListPlot[makeSphereVerticalLines[-3, 3, 100, 3]]
```

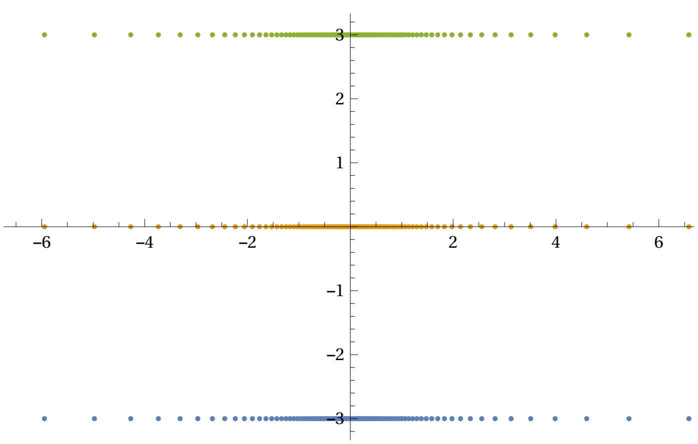
Out[109]=



In[110]:=
```
makeSphereHorizontalLines[min_, max_, ptsPerLine_, numLines_] :=
 Module[{horiPts, spherePts},
  spherePts = makeSphereSpacedPoints[ptsPerLine];
  horiPts = Table[Table[{x, y}, {x, spherePts}], {y, min, max, (max - min)/(numLines - 1)}];
  Return[horiPts]
 ]
makeSphereHorizontalLines::usage =
  "makeSphereHorizontalLines[min_,max_,ptsPerLine_,numLines_] makes
    horizontal lines of sphere spaced points given line bounds of
    min, max, ptsPerLine, and numLines";
```

In[112]:=
```
ListPlot[makeSphereHorizontalLines[-3, 3, 100, 3]]
```

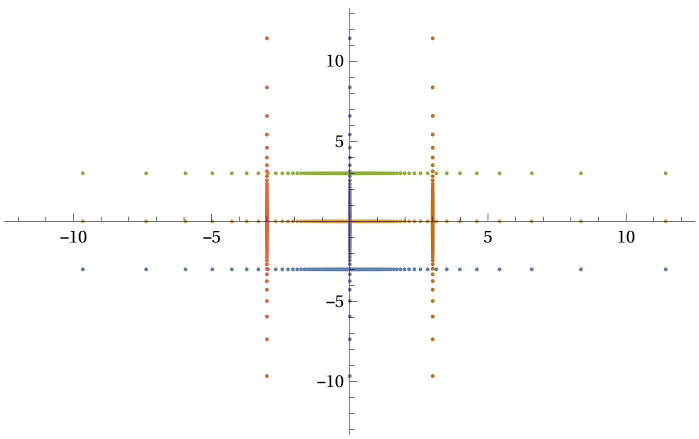
Out[112]=



In[113]:=
```
makeSphereGrid[min_, max_, ptsPerLine_, numLines_] :=
 Module[{vertPts, horiPts, pts},
  vertPts = makeSphereVerticalLines[min, max, ptsPerLine, numLines];
  horiPts = makeSphereHorizontalLines[min, max, ptsPerLine, numLines];
  pts = Join[horiPts, vertPts];
  Return[pts]
 ]
makeSphereGrid::usage =
  "makeSphereGrid[min,max,ptsPerLine,numLines] makes a grid of sphere
    spaced points given line bounds of min, max, ptsPerLine, and numLines";
```

In[115]:=
```
ListPlot[makeSphereGrid[-3, 3, 100, 3]]
```

Out[115]=



Complex Plane to Image Plane

Here we make functions that let us deal with lists of points on the complex plane, a mapping expression, and the resulting image plane.

Get image of points

In[116]:=
```
getImagePts[expr_, pts_] := Module[{imgPts},
  imgPts =
   {Re[expr /. z → #[[1]] + ⅈ #[[2]]], Im[expr /. z → #[[1]] + ⅈ #[[2]]]} & /@
     # & /@ pts;
  Return[imgPts]]
getImagePts::usage =
  "getImagePts[expr[z],pts] returns the image points given takes in
    an expression of z and a list of lists of lists of points";
```

In[118]:=
```
getImagePts[z + 1 - ⅈ, {{{1, 1}, {2, 2}}}]
```

Out[118]=
```
{{{2, 0}, {3, 1}}}
```

Plot image of points

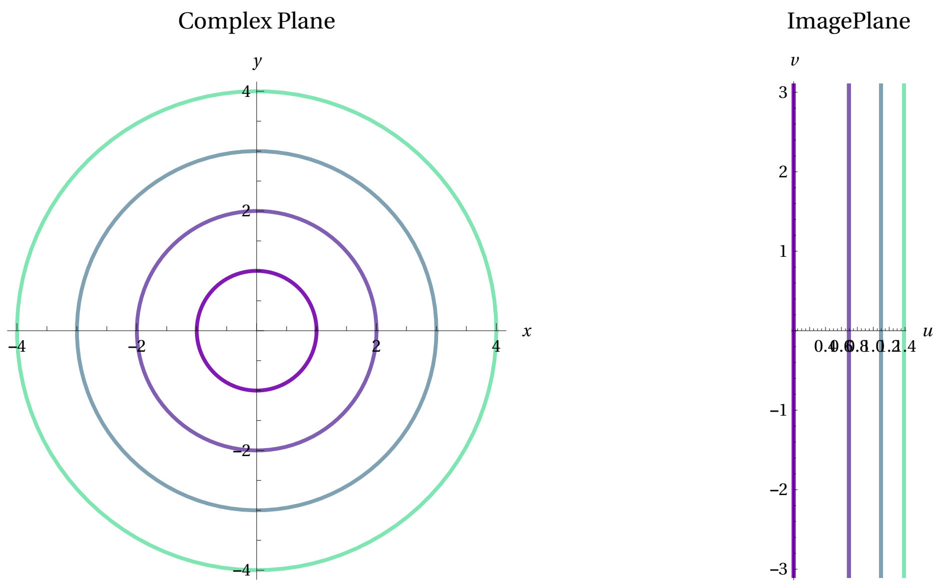
In[119]:=

```
plotImage[pts_, expr_, pltRange1_: Automatic, PltRange2_: Automatic, colors_: 1] :=
 Module[{},
  If[colors == 1, colors = RGBColor[1, 1, 1], colors = colors];
  {
   Graphics[MapThread[{#1, Thick, Line[#2]} &, {colors, pts}],
    PlotRange → pltRange1, Axes → True, Background → White, ImageSize → {300, 300},
    AxesLabel → {Style["x", Italic], Style["y", Italic]}, ImagePadding → 20],
   Graphics[
    MapThread[
     {#1, Thick,
        Line[{Re[expr /. z → #[[1]] + I #[[2]]], Im[expr /. z → #[[1]] + I #[[2]]]} & /@
          #2]} &, {colors, pts}], PlotRange → PltRange2, Axes → True,
    Background → White, ImageSize → {300, 300},
    AxesLabel → {Style["u", Italic], Style["v", Italic]}, ImagePadding → 20]
  }
 ]
plotImage::usage =
  "plotImage[pts,expr,pltRange1:Automatic,PltRange2:Automatic,colors:1]
    returns a list containing the complex and image plots given a
    list of lists of lists of points, an expression of z, two plot
    ranges (can be Automatic), and a list of colors. Note that
    plotImage doesn't call getImagePts so we can MapThread with
    color gradients";
```

In[121]:=
```
circles = Quiet[makeCirclePoints[1, 4, 4, 100]];
circleColors = makeColorGradient[circles]
Quiet@
  Grid[{{"Complex Plane", "ImagePlane"},
    plotImage[circles, Log[z], Automatic, Automatic, circleColors]}] //
 TraditionalForm
```

Out[122]= {■, ■, ■, ■}

Out[123]//TraditionalForm=



Colors

Color functions further help the user understand what is going on in complex plane to image plane or Riemann Sphere mappings.

In[124]:=
```
makeRandomColors[pts_]:=Module[{colors},
 colors=RandomColor[Length[pts]];
 Return[colors]]
makeRandomColors::usage="makeRandomColors[pts_] returns length of pts number of ran
```

In[126]:= `makeRandomColors[{{0, 0}, {1, 1}, {2, 2}, {3, 3}, {4, 4}}]`

Out[126]= {■, ■, ■, ■, ■}

In[127]:=
```
makeColorGradient[pts_] := Module[{colors},
  colors = Table[RGBColor[.5, x, .7], {x, .1, .9, (.9 - .1)/(Length[pts] - 1)}];
  Return[colors]]
makeColorGradient::usage =
  "makeColorGradient[pts_] returns length of pts number of colors in
    a purple -> teal gradient";
```

In[129]:=
```
makeColorGradient[{{0, 0}, {1, 1}, {2, 2}, {3, 3}, {4, 4}}]
```

Out[129]= {■, ■, ■, ■, ■}

Riemann Sphere Stereographic Projection

Here we make functions that allow the the projection of points on the complex plane to the Riemann Sphere through the use of stereographic projection

Single point

In[130]:=
```
complexTo3D[point_]:=Module[{x,y,z,mix,X,Y,Z},
x=point[[1]];
y=point[[2]];
mix=(2x+ⅈ 2y)/(1+x^2+y^2);
X=Re[mix];
Y=Im[mix];
z=x+ⅈ y;
Z=(Abs[z]^2-1)/(Abs[z]^2+1);
Return[{X,Y,Z}]
]
complexTo3D::usage="complexTo3D[point_] returns the stereographic projection of a p
```

In[132]:=
```
complexTo3D[{1, 1}]
```

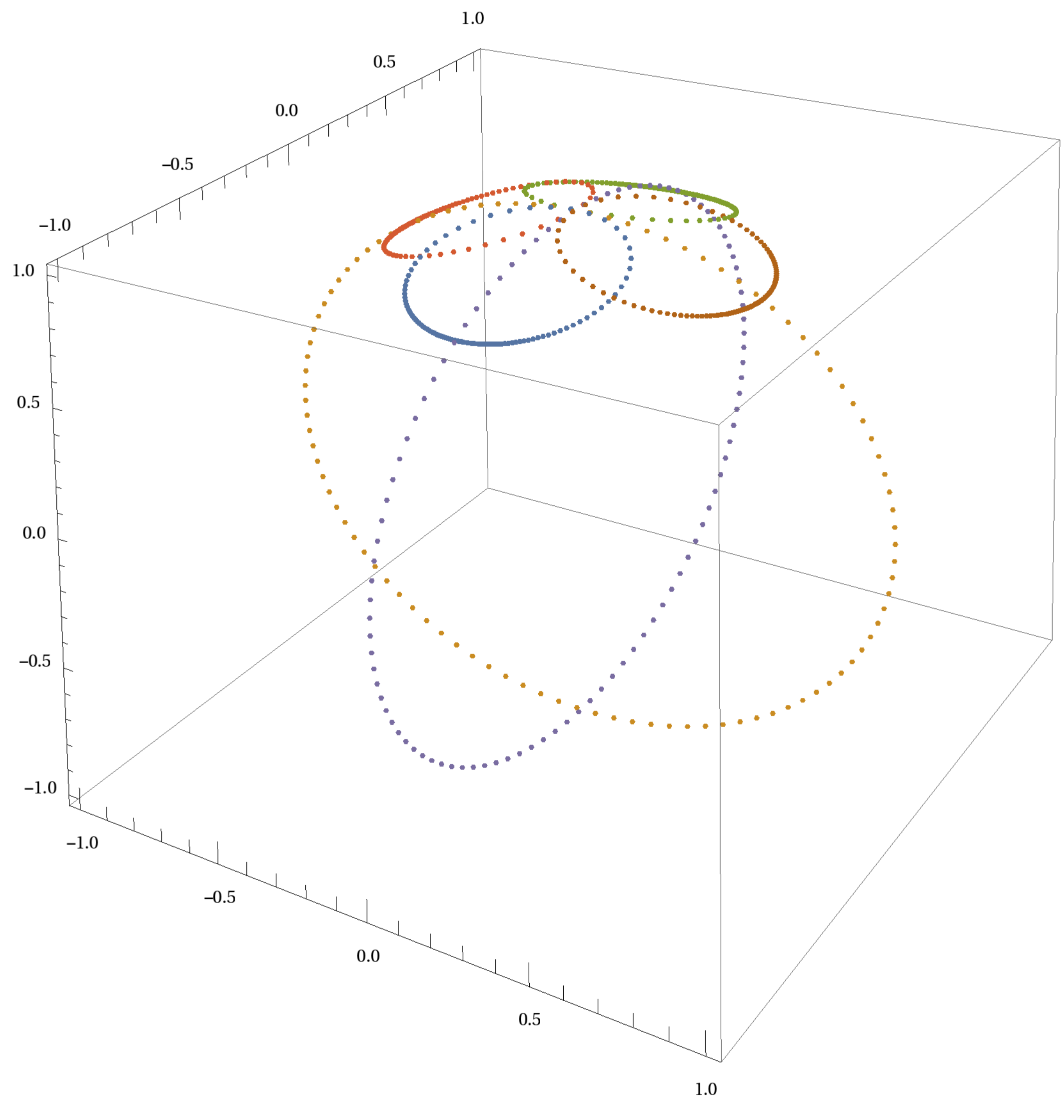
Out[132]= $\left\{\frac{2}{3}, \frac{2}{3}, \frac{1}{3}\right\}$

Multiple points

In[133]:=
```
complexPtsTo3D[points_]:=Module[{spherePts3D},
spherePts3D=complexTo3D[#]&/@#&/@points;
Return[spherePts3D]
]
complexPtsTo3D::usage="complexPtsTo3D[points_] returns the stereographic projection
of points on the complex plane";
```

In[135]:=
```
gridPts2D = makeSphereGrid[-3, 3, 100, 3];
gridPts3D = complexPtsTo3D[gridPts2D];
ListPointPlot3D[gridPts3D, PlotRange → All, AspectRatio → 1, ImageSize → Large]
```

Out[137]=



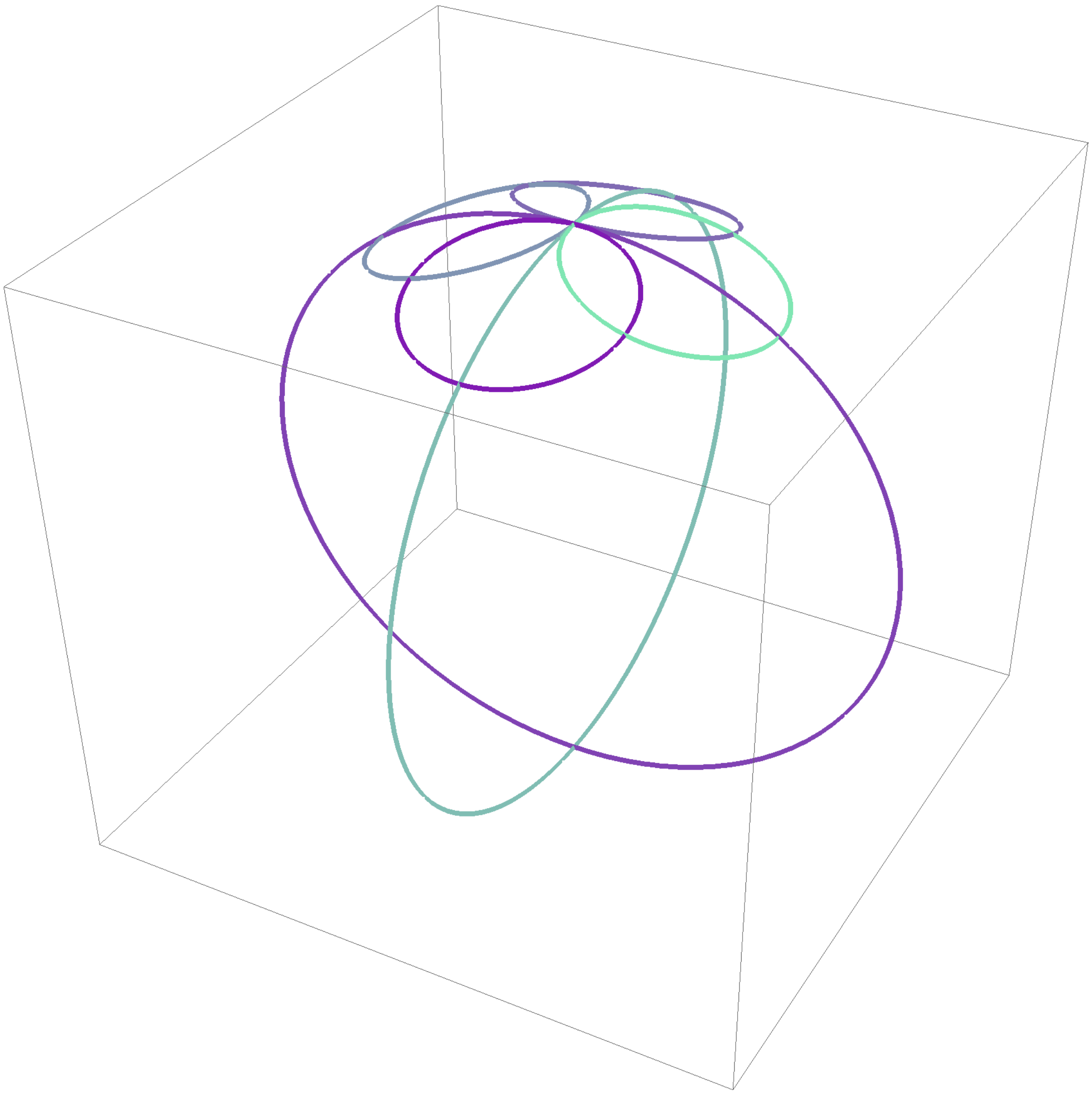
In[138]:=

Color Graphics3D test

In[139]:=
```
colors3D = makeColorGradient[gridPts3D]
Graphics3D[MapThread[{#1, Thick, Line[#2]} &, {colors3D, gridPts3D}],
 AspectRatio → 1, ImageSize → Large]
```

Out[139]= {■, ■, ■, ■, ■, ■}

Out[140]=



3D printing

3D printing allows us to take what we see in the computer and hold it in real life.

Connecting cylinders

In[141]:=
```
takingPts[takes_,pts_]:=Module[{newPts},
 newPts=Take[pts,#]&/@takes;
 Return[newPts]]
takingPts::usage="takingPts[takes_,pts_] returns a list of taken points
given a list of takes and a list of points";
```

In[143]:=
```
takingPts[{{1, 2}, {2, 3}, {3, 4}}, {2, 7, 4, 0}]
```

Out[143]=
```
{{2, 7}, {7, 4}, {4, 0}}
```

In[144]:=
```
cylinderPts[pts_]:=Module[{takes},
 takes=Table[{x,x+1},{x,1,Dimensions[pts][[2]]-1}];
 Return[takingPts[takes,#]&/@pts]
 ]
cylinderPts::usage="cylinderPts[pts_] returns a list of taken points ready to be co
```

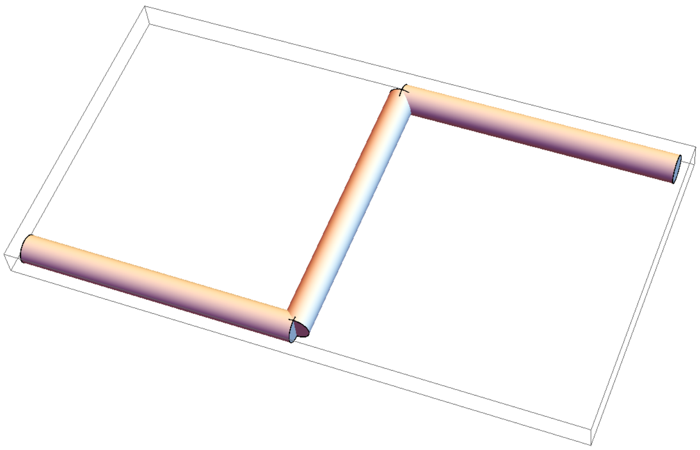
In[146]:=
```
cylinderPts[{{{1, 2, 3}, {4, 5, 6}, {7, 8, 9}, {10, 11, 12}}}]
```

Out[146]=
```
{{{{1, 2, 3}, {4, 5, 6}}, {{4, 5, 6}, {7, 8, 9}}, {{7, 8, 9}, {10, 11, 12}}}}
```

In[147]:=
```
cylinders[pts_, radius_] := Module[{},
  Return[Cylinder[#, radius] & /@ # & /@ cylinderPts[pts]]]
cylinders::usage =
  "cylinders[pts_,radius_] returns cylinders linearly along the path
    of the list of points with specified radius";
```

In[149]:=
```
Graphics3D[cylinders[{{{1, 2, 3}, {2, 2, 3}, {2, 3, 3}, {3, 3, 3}}}, .05]]
```

Out[149]=

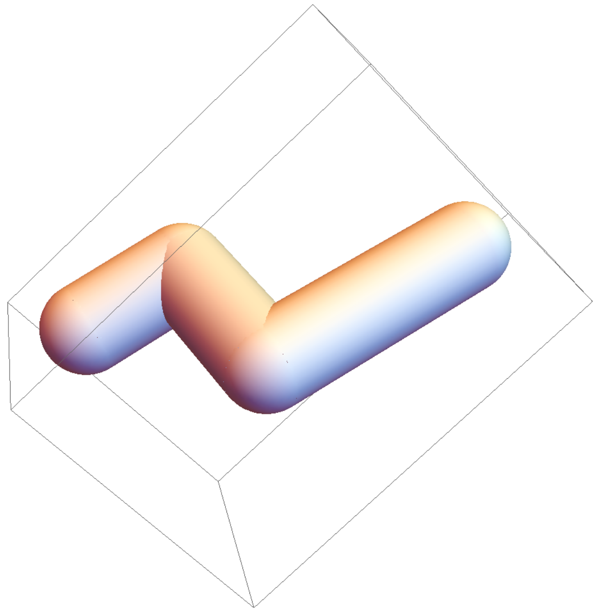


Connecting tubes

In[150]:=
```
tubes[pts_,radius_]:=Module[{},Tube[#,radius]&/@complexPtsTo3D[pts]]
tubes::usage = "tubes[pts_,radius_] returns tubes linearly
 along the path of the list of points with specified radius. This is
sometimes used as it gives a better result than cylinders but is less
reliable";
```

In[152]:=
```
Graphics3D[tubes[{{{1, 2, 3}, {2, 2, 3}, {2, 3, 3}, {3, 3, 3}}}, .05]]
```

Out[152]=



Exporting stls

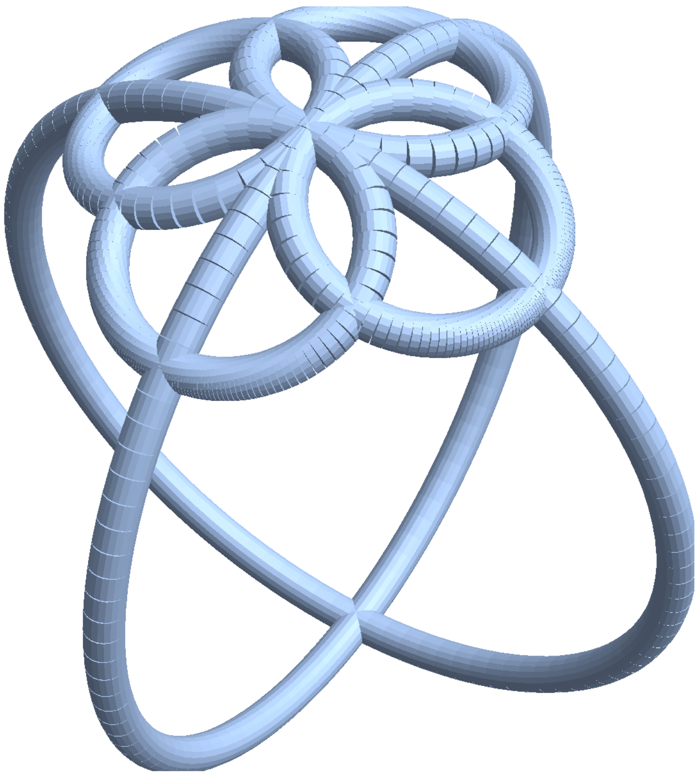
In[153]:=
```
gridPts2D = makeSphereGrid[-3, 3, 100, 3];
gridPts3D = complexPtsTo3D[gridPts2D];
```

In[155]:=
```
Export["grid2.stl", Graphics3D[cylinders[gridPts3D, .05]],
  {"STL,", "BinaryFormat" → True}]
```

Out[155]=
```
grid2.stl
```

In[156]:=
```
Import["grid2.stl"]
```

Out[156]=

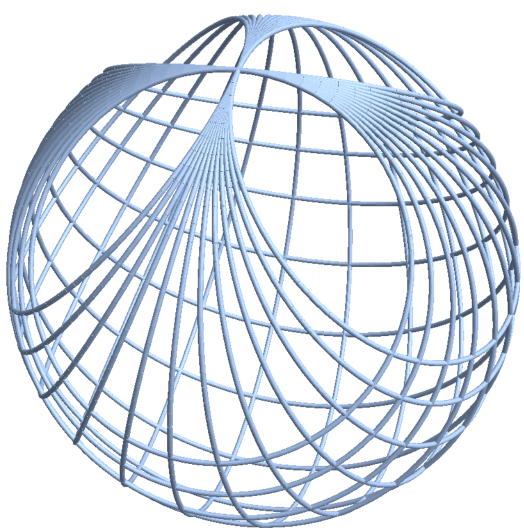


In[157]:=
```
Export["grid_10_7.stl",
 Graphics3D[cylinders[complexPtsTo3D[makeSphereGrid[-1, 1, 100, 12]], .01]],
 {"STL,", "BinaryFormat" → True}]
```

Out[157]= grid_10_7.stl

In[158]:=
```
grid107 = Import["grid_10_7.stl"]
```

Out[158]=



Newtons Method

Newton’s Method illustrates how visualizations of complex mappings helps us understand numerical methods.

In[159]:=
```
makeNewtonMethodAnimation[plotRange_, map_, depth_, points_] :=
 Module[{localPts, plots},
localPts = points;
plots = Flatten[{
{ListPlot[localPts,
       PlotRange → {{-plotRange, plotRange}, {-plotRange, plotRange}},
       PlotStyle → PointSize[.02], AspectRatio → 1, ImageSize → Large]},
Table[
(*Apply map to grid pts*) localPts = getImagePts[map, localPts];
ListPlot[localPts, PlotRange → {{-plotRange, plotRange},
          {-plotRange, plotRange}}, PlotStyle → PointSize[.02],
       AspectRatio → 1, ImageSize → Large]
      , {n, 1, depth}]
}];
Return[Manipulate[plots[[n]], {n, 1, Length[plots], 1}]]]
makeNewtonMethodAnimation::usage =
  "makeNewtonMethodAnimation[plotRange,map,depth,points] makes
    animation of successive mappings";
```

In[161]:=
```
f[z_]:=z^3-1
```

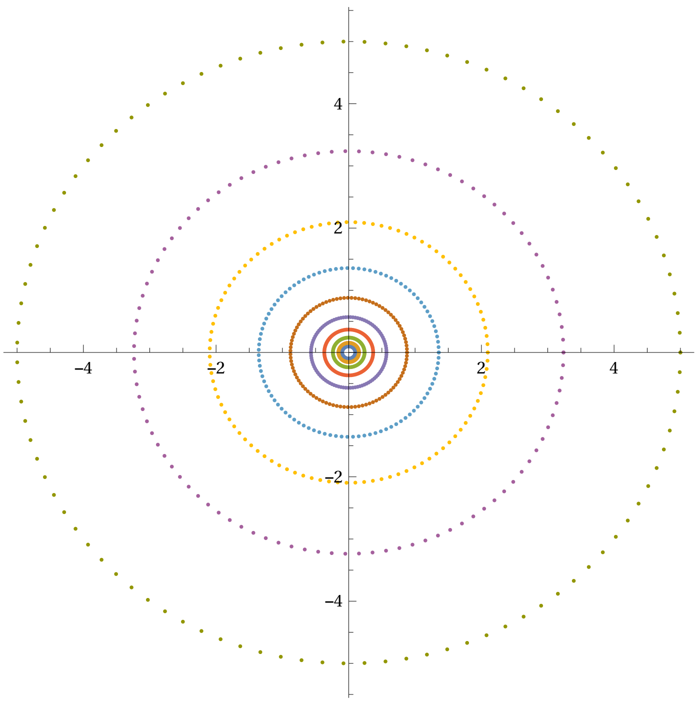
Use definition of Newton’s Method to derive mapping function

In[162]:= $\texttt{map}=\left(z-\frac{f[z]}{f'[z]}\right);$

Make a circular grid:

In[163]:=
```
smallestRadius=.1;
largestRadius=5;
numberOfCircles=10;
pointsPerCircle=100;
circleGrid = makeExponentialSpacedCirclePoints[smallestRadius,largestRadius,numberOf(
circleGridCopy1=circleGrid;
ListPlot[circleGridCopy1,AspectRatio→1,PlotRange→Full]
```

Out[169]=



In[170]:= `makeNewtonMethodAnimation[5,map,35,circleGrid]`

Out[170]=