5 computational

Here we define Cauchy-Riemann equations

In[260]:=
```
cauchyRiemannCartCart[u_, v_] := Module[{ux, uy, vx, vy},
  ux = D[u, x];
  uy = D[u, y];
  vx = D[v, x];
  vy = D[v, y];
  Return[ux == vy && vx == -uy]
 ]
```

In[261]:=
```
cauchyRiemannPolarCart[u_, v_, r_] := Module[{uθ, ur, vθ, vr},
  uθ = D[u, θ];
  ur = D[u, r];
  vθ = D[v, θ];
  vr = D[v, r];
  Return[uθ == r ur && uθ == -r vr]
 ]
```

i) $f = e^{-y}(\cos(x) + i\sin(x))$

In[262]:=
```
Clear[f, u, v]
f = ⅇ^-y (Cos[x] + ⅈ Sin[x]);
u = FullSimplify[Re[f], Element[{x, y}, Reals]]
v = FullSimplify[Im[f], Element[{x, y}, Reals]]
```

Out[264]= e^{-y} Cos[x]

Out[265]= e^{-y} Sin[x]

In[266]:=
```
FullSimplify[cauchyRiemannCartCart[u, v], Element[{x, y}, Reals]]
```

Out[266]= True

$f = e^{-y}(\cos(x) + i\sin(x))$ is analytic

ii) $f = \cos(x) - i\sin(y)$

In[267]:=
```
Clear[f, u, v]
f = Cos[x] - ⅈ Sin[y];
u = FullSimplify[Re[f], Element[{x, y}, Reals]]
v = FullSimplify[Im[f], Element[{x, y}, Reals]]
```

Out[269]= Cos[x]

Out[270]= -Sin[y]

```
In[271]:= FullSimplify[cauchyRiemannCartCart[u, v], Element[{x, y}, Reals]]
Out[271]= Cos[y] == Sin[x]
```

$f = \cos(x) - i \sin(y)$ is not analytic

iii) $f = r^3 + i\, 3\, \theta$

```
In[272]:= Clear[f, u, v, r]
          f = r^3 + ⅈ 3 θ;
          u = FullSimplify[Re[f], Element[{r, θ}, Reals]]
          v = FullSimplify[Im[f], Element[{r, θ}, Reals]]
          r$length = Norm[f, Element[{r, θ}, Reals]]
Out[274]= r^3
Out[275]= 3 θ
Out[276]= Norm[r^3 + 3 ⅈ θ, (r | θ) ∈ Reals]
```

```
In[277]:= FullSimplify[cauchyRiemannPolarCart[u, v, r$length], Element[{u, v, r$length}, Reals]]
Out[277]= True
```

$f = r^3 + i\, 3\, \theta$ is analytic

iv) $f = r\, e^{r \cos(\theta) + i\, (\theta + r \cos(\theta))}$

Here I had to split up the function into two parts to make Mathematica evaluate it correctly

```
In[278]:= Clear[f, u, v, r]
          f = ⅇ^(ⅈ (θ+r Cos[θ]))
          u = r ⅇ^(r Cos[θ]) FullSimplify[Re[f], Element[{r, θ}, Reals]]
          v = r ⅇ^(r Cos[θ]) FullSimplify[Im[f], Element[{r, θ}, Reals]]
          r$length = Norm[r ⅇ^(r Cos[θ]) f, Element[{r, θ}, Reals]]
Out[279]= ⅇ^(ⅈ (θ+r Cos[θ]))
Out[280]= ⅇ^(r Cos[θ]) r Cos[θ + r Cos[θ]]
Out[281]= ⅇ^(r Cos[θ]) r Sin[θ + r Cos[θ]]
Out[282]= Norm[ⅇ^(r Cos[θ]+ⅈ (θ+r Cos[θ])) r, (r | θ) ∈ Reals]
```

```
In[283]:= FullSimplify[cauchyRiemannPolarCart[u, v, r$length], Element[{u, v, r$length}, Reals]]
Out[283]= r (r Cos[θ + r Cos[θ]] Sin[θ] + (1 - r Sin[θ]) Sin[θ + r Cos[θ]]) == 0
```

$f = r\, e^{r \cos(\theta) + i\, (\theta + r \cos(\theta))}$ is not analytic