Complex Variables Toolkit - Final Project - Nathan Yee

About

This notebook serves as a group of utilities (functions) that allow a user to interact with and understand complex functions. In particular it provides functions that give exponential and sphere (inverse stereograph) based spacings, point based circle creation, point based line and grid creation, complex image map utilities, additional color creation tools, stereographic projection utilities, 3D printing tools and examples, and finally Newton's method animation utilities. As an added benefit to the user, every function has a usage line (Mathematica equivalent of docstring), and at least one example of how it is used.

The line below tells Mathematica to save all initialization cells to a .m file. This is the file that will be imported in the future whenever one wants to use the toolkit.

In[59]:=
```
SetOptions[EvaluationNotebook[], AutoGeneratedPackage → Automatic]
```

Functions

Provided below are the functions, documentation, and examples.

Exponential, and Sphere based Spacings

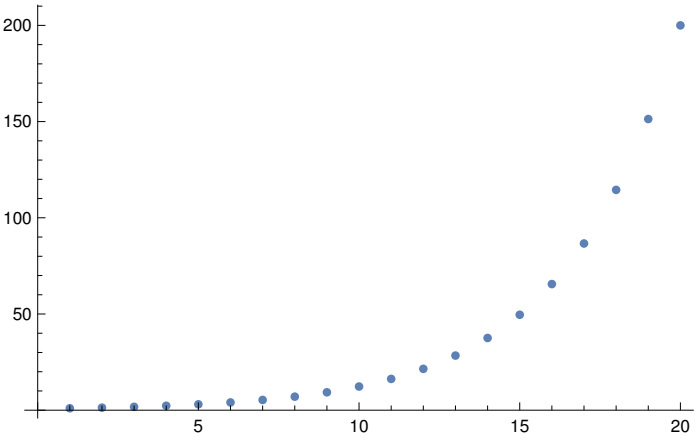
Various types of spacings serve as the foundation for circle, line, and grid point generator functions.

Inverse (default Log) spaced points

In[60]:=
```
fSpace[min_, max_, numPts_, f_: Log] := Module[{},
  N[InverseFunction[f] /@ Range[f@min, f@max, (f@max - f@min) / (numPts - 1)]]]
fSpace::usage = "fSpace[min, max, steps, Log] gives inverse f (default
    Log) spaced points from min to max over a given number of points";
```

In[62]:=
```
ListPlot[fSpace[1, 200, 20, Log]]
```

Out[62]=

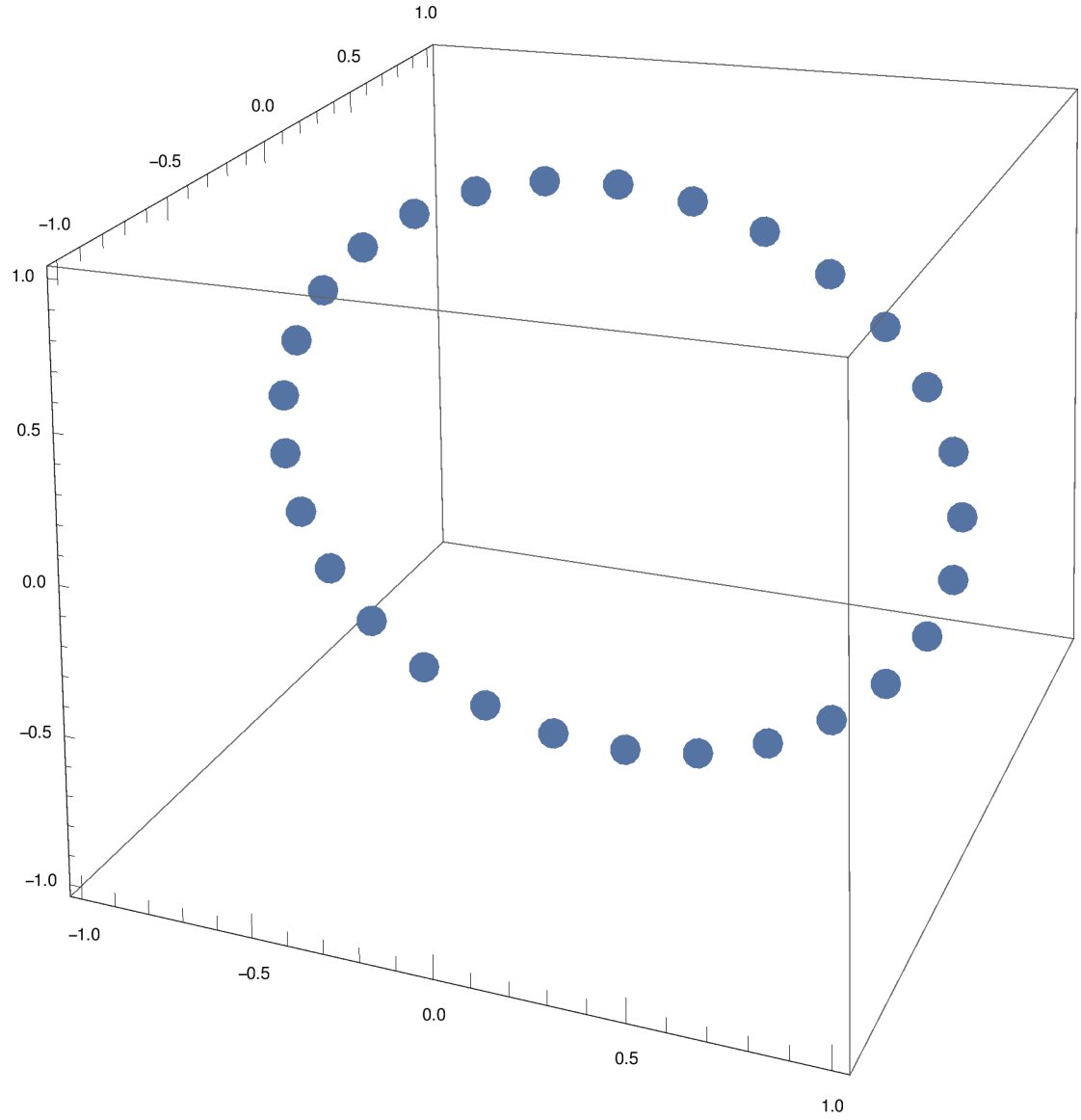


Sphere spaced points

```
makeRiemannRealAxis[numPts_] := Module[
  {angles, pts},
  angles = N@Range[0 Pi, 2 Pi, (2 Pi - 0 Pi)/(numPts - 1)];
  pts = Table[{Cos[ang], 0, Sin[ang]}, {ang, angles}];
  Return[pts]
]
makeRiemannRealAxis::usage =
  "makeRiemannRealAxis[numPts_] makeRiemannRealAxis returns equally
    spaced points around the positive real axis on the Riemann Sphere";
```

In[65]:=
```
realAxis3D = makeRiemannRealAxis[30];
ListPointPlot3D[realAxis3D, AspectRatio → 1,
 PlotStyle → PointSize[.03], ImageSize → Large]
```

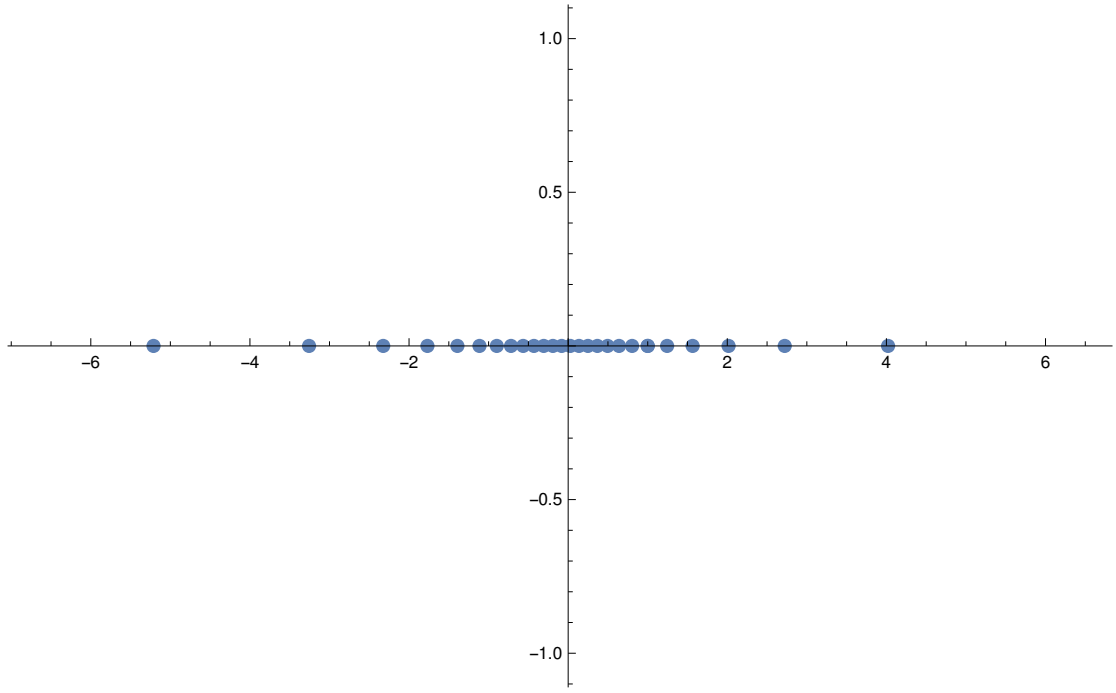
Out[66]=




```
riemannPointToComplexPlane[{X_,Y_,Z_}]:=Module[{sol,pair},
sol=Solve[{x+i y==(X+i Y)/(1-Z)},{x∈Reals,y∈Reals}];
pair=({x,y}/.sol)[[1]];
Return[pair]
]
riemannPointToComplexPlane::usage="riemannPointToComplexPlane[{X_,Y_,Z_}]
riemannPointToComplexPlane returns the inverse stereographic projection of a point dis
```

In[69]:=
```
realAxis2D = riemannPointToComplexPlane[#] & /@ realAxis3D;
ListPlot[realAxis2D, ImageSize → Large]
```

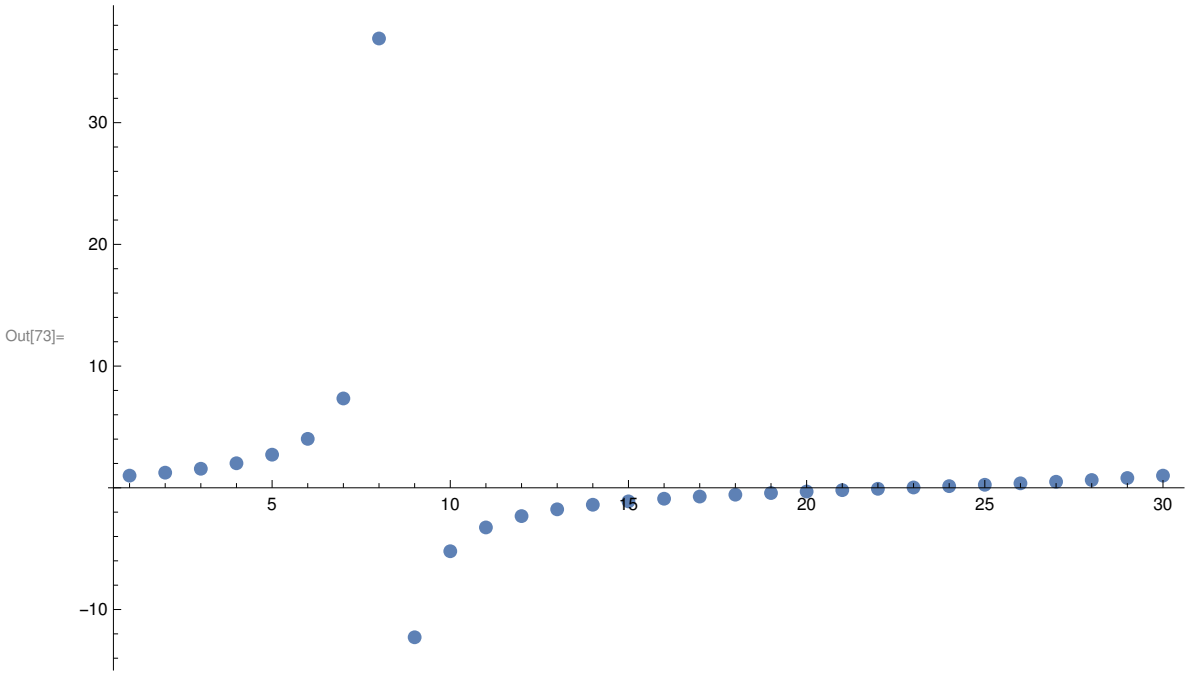
Out[70]=



In[71]:=
```
makeSphereSpacedPoints[numPts_] := Module[{pts, complexPts, realPts},
  pts = makeRiemannRealAxis[numPts];
  complexPts = riemannPointToComplexPlane[#] & /@ pts;
  realPts = complexPts[[All, 1]];
  Return[realPts]
 ]
makeSphereSpacedPoints::usage =
  "makeSphereSpacedPoints[numPts_] makeSphereSpacedPoints returns the inverse
    stereographic projection of points around the positive real axis on the
    Riemann Sphere. It gets the positive real axis of the Riemann Sphere by
    calling makeRiemannRealAxis. It finds the inverse stereographic projection
    of the points by mapping riemannPointToComplexPlane onto each point";
```

In[73]:=
```
ListPlot[makeSphereSpacedPoints[30], PlotRange → All, ImageSize → Large]
```

Out[73]=



Circle Creation

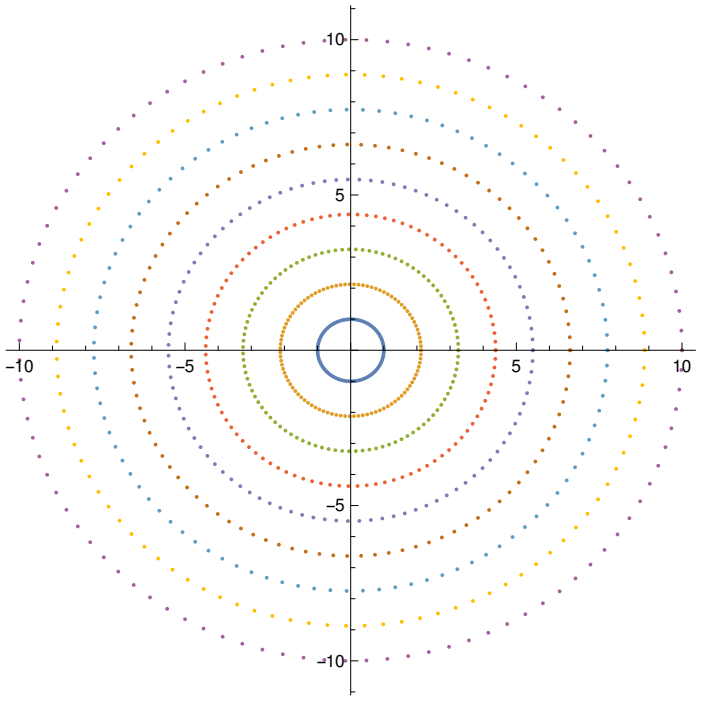
Point based circle generator functions are useful tool to visualize complex mappings.

Linear spaced circles

```
makeCirclePoints[smallRadius_,largeRadius_,numCircles_,ptsPerCircle_]:=Module[{ang, li
ang=Range[0 Pi,2 Pi, 2π-0/ptsPerCircle-1];
lists=Table[{r Cos[ang],r Sin[ang]},{r,Range[smallRadius,largeRadius, largeRadius-smal/numCircles-
pts=Transpose[#]&/@lists;
Return[pts]
]
makeCirclePoints::usage="makeCirclePoints[smallRadius_,largeRadius_,numCircles_,ptsPer
makeCirclePoints returns expanding circles given smallest radius, largest radius, step
The constant step size dictates that the radius of the circles increases by a constant
```

In[76]:=
```
ListPlot[makeCirclePoints[1, 10, 9, 100], AspectRatio → 1]
```

Out[76]=

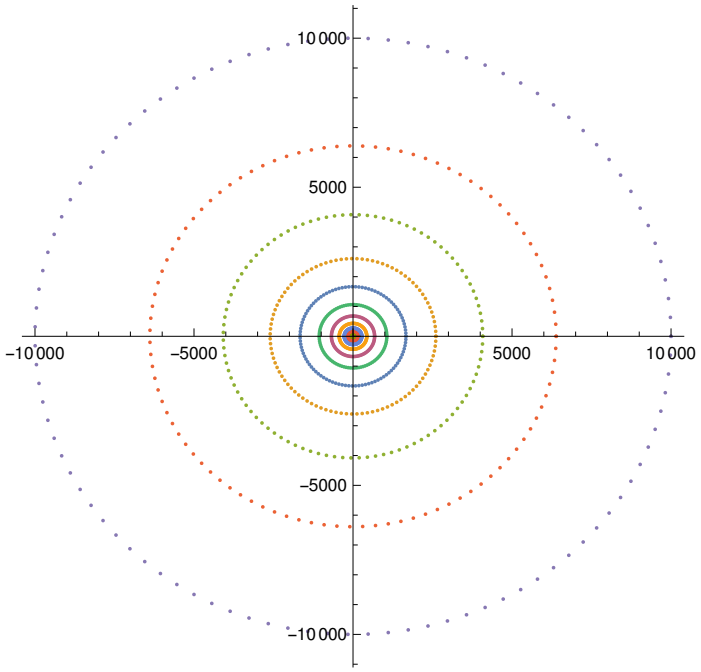


Exponentially spaced circles

In[77]:=
```
makeExponentialSpacedCirclePoints[smallRadius_, largeRadius_,
  numCircles_, ptsPerCircle_] := Module[{ang, lists, pts},
ang = Range[0 Pi, 2 Pi, (2 π - 0)/(ptsPerCircle - 1)];
  lists = Table[{r Cos[ang], r Sin[ang]},
    {r, fSpace[smallRadius, largeRadius, numCircles]}];
  pts = Transpose[#] & /@ lists;
Return[pts]
]
makeExponentialSpacedCirclePoints::usage =
  "makeExponentialSpacedCirclePoints[smallRadius_,largeRadius_,numCircles_,
    ptsPerCircle_] makeExponentialSpacedCirclePoints
    returns expanding circles given smallest radius, largest
    radius, number of circles, and number of points per circle.
    The number of circles dictates that the circles be spaced
    exponentially within the largestRadius - smallestRadius range";
```

In[79]:=
```
ListPlot[makeExponentialSpacedCirclePoints[2, 10 000, 20, 100],
 AspectRatio → 1, PlotRange → Full]
```

Out[79]=

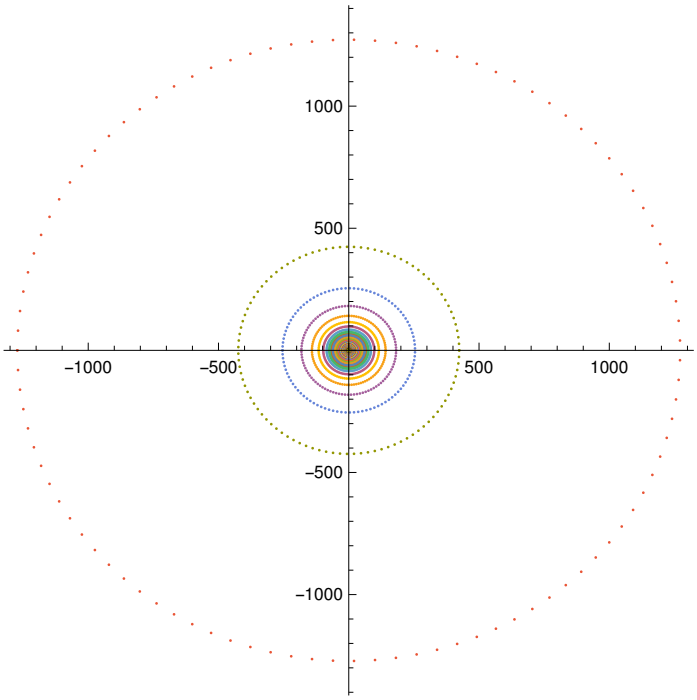


“Sphere” spaced circles

In[80]:=
```
makeSphereSpacedCirclePoints[numCircles_,ptsPerCircle_]:=Module[{ang, lists, pts},
  ang=Range[0 Pi,2 Pi, (2π-0)/(ptsPerCircle-1)];
  lists=Table[{r Cos[ang],r Sin[ang]},{r,makeSphereSpacedPoints[numCircles]}];
  pts=Transpose[#]&/@lists;
  Return[pts]
 ]
makeSphereSpacedCirclePoints::usage =
  "makeSphereSpacedCirclePoints[numCircles_,ptsPerCircle_]
    makeSphereSpacedCirclePoints returns circles based on inverse stereographic
    projection given a number of circles, and the number of points per circle";
```

In[82]:=
```
ListPlot[makeSphereSpacedCirclePoints[1000, 100], AspectRatio → 1, PlotRange → Full]
```

Out[82]=



Line and Grid creation

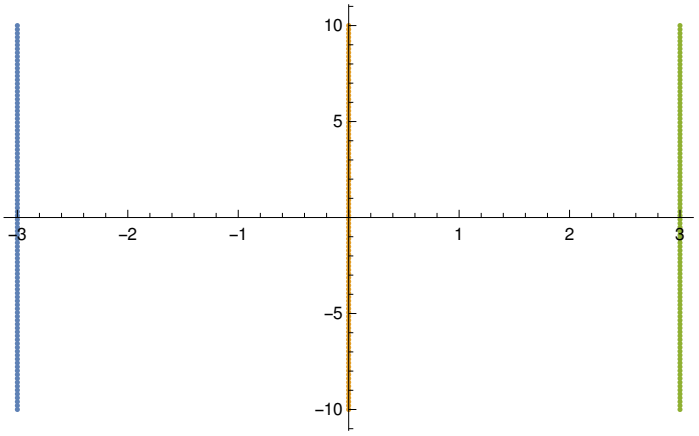
Point based line and grid generator functions are useful tool to visualize complex mappings.

Linearly spaced lines and grids

In[83]:=
```
makeVerticalPts[minX_,maxX_,minY_,maxY_,ptsPerLine_,numLines_]:=Module[{horiPts},
  horiPts=Table[
    Table[{x,y},{y,Range[minY,maxY,(maxY-minY)/(ptsPerLine-1)]},{x,minX,maxX,(maxX-minX)/(numLines-1)}];
  Return[horiPts]
 ]
makeVerticalPts::usage =
  "makeVerticalPts[minX_,maxX_,minY_,maxY_,ptsPerLine_,numLines_]
returns numLines number of vertical lines of equally
    spaced points starting from minX to maxX with length
    maxY-minY with ptsPerLine number of points per line";
```

In[85]:=
```
ListPlot[makeVerticalPts[-3, 3, -10, 10, 100, 3], PlotRange → Full]
```

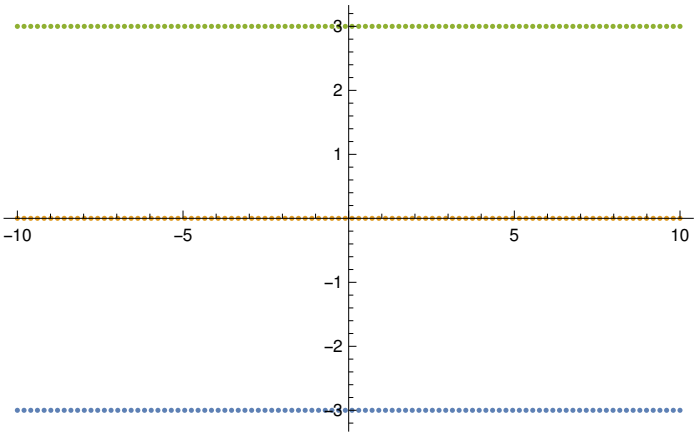
Out[85]=



In[86]:=
```
makeHorizontalPts[minX_, maxX_, minY_, maxY_, ptsPerLine_, numLines_] :=
 Module[{horiPts},
  horiPts = Table[Table[{x, y}, {x, Range[minX, maxX, (maxX - minX)/(ptsPerLine - 1)]}],
    {y, minY, maxY, (maxY - minY)/(numLines - 1)}];
  Return[horiPts]
 ]
makeHorizontalPts::usage =
  "makeHorizontalPts[minX_,maxX_,minY_,maxY_,ptsPerLine_,numLines_]
returns numLines number of horizontal lines of equally
    spaced points starting from minY to maxY with length
    maxX-minX with ptsPerLine number of points per line";
```

In[88]:=
```
ListPlot[makeHorizontalPts[-10, 10, -3, 3, 100, 3], PlotRange → Full]
```

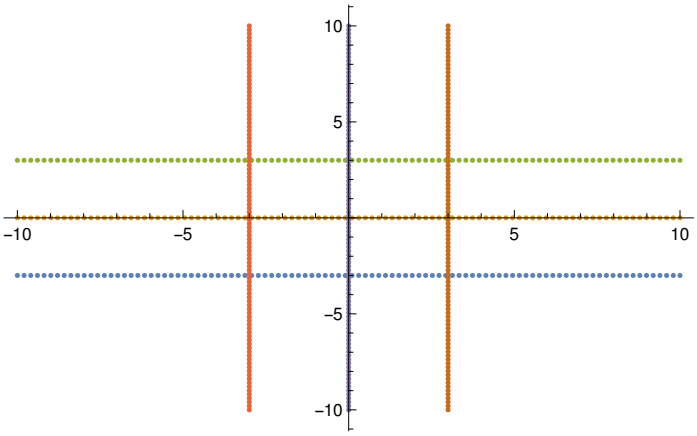
Out[88]=



```
makeGridPts[minX_,maxX_,minY_,maxY_,ptsPerLine_,numLines_]:=Module[{vertPts,horiPts,pt
vertPts=makeVerticalPts[minY,maxY,minX,maxX,ptsPerLine,numLines];
horiPts=makeHorizontalPts[minX,maxX,minY,maxY,ptsPerLine,numLines];
pts=Join[horiPts,vertPts];
Return[pts]
]
makeGridPts::usage="makeGridPts[minX_,maxX_,minY_,maxY_,ptsPerLine_,numLines_]
returns numLines number of vertical of equally spaced points lines starting from minX
length maxY-minY with ptsPerLine number of points per line and
returns numLines number of horizontal of equally spaced points lines starting from mi
length maxX-minX with ptsPerLine number of points per line.
NOTE: enter args as if doing horizontal line";
```

In[91]:=
```
ListPlot[makeGridPts[-10, 10, -3, 3, 100, 3], PlotRange → Full]
```

Out[91]=

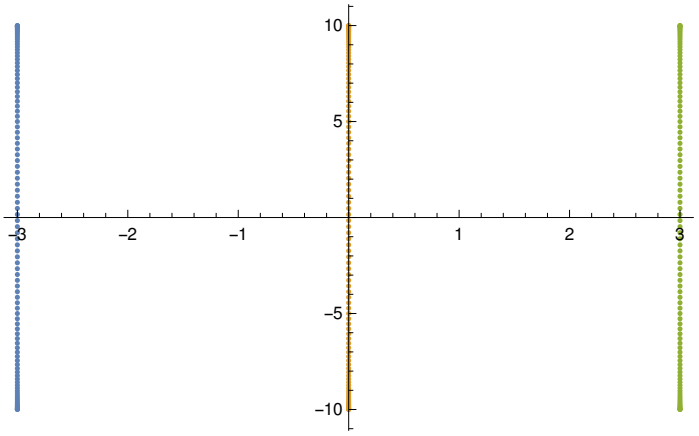


Exponentially spaced lines and grids

In[92]:=
```
makeLogVerticalPts2[minX_, maxX_, minY_, maxY_, ptsPerLine_, numLines_] :=
 Module[{vertPts},
  vertPts = Table[Table[{x, y}, {y, fSpace[minY, maxY, ptsPerLine]}],
    {x, minX, maxX, (maxX - minX)/(numLines - 1)}];
  Return[Re[vertPts]]
 ]
makeLogVerticalPts2::usage =
  "makeLogVerticalPts2[minX_,maxX_,minY_,maxY_,ptsPerLine_,numLines_]
returns numLines number of vertical lines of exponentially
    spaced points starting from minX to maxX with length
    maxY-minY with ptsPerLine number of points per line.
NOTE: makeLogVerticalPts2 is depreciated due to not handling
    negative minY to positive maxY ranges";
```

In[94]:=
```
ListPlot[makeLogVerticalPts2[-3, 3, -10, 10, 100, 3], PlotRange → Full]
```

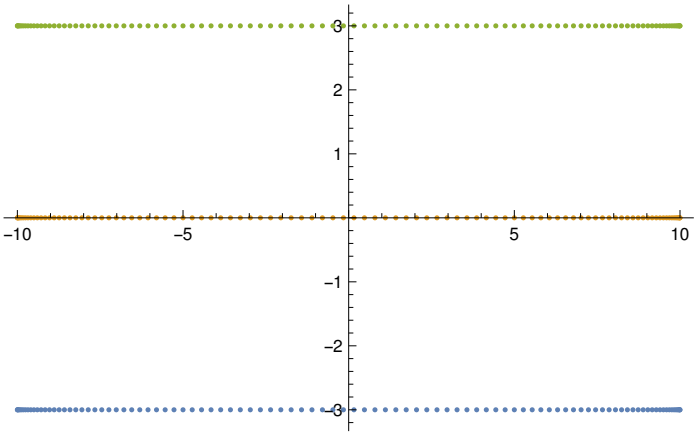
Out[94]=




```
makeLogHorizontalPts2[minX_,maxX_,minY_,maxY_,ptsPerLine_,numLines_]:=Module[{horiPts}
horiPts=Table[Table[{x,y},{x,fSpace[minX,maxX,ptsPerLine]}],{y,minY,maxY,(maxY-minY)/(numLines-1)}]
Return[Re[horiPts]]
]
makeLogHorizontalPts2::usage="makeLogHorizontalPts2[minX_,maxX_,minY_,maxY_,ptsPerLine
returns numLines number of horizontal lines of exponentially spaced points starting fr
length maxX-minX with ptsPerLine number of points per line
NOTE: makeLogHorizontalPts2 is depreciated due to not handling negative minX to positi
```

In[97]:=
```
ListPlot[makeLogHorizontalPts2[-10, 10, -3, 3, 100, 3], PlotRange → Full]
```

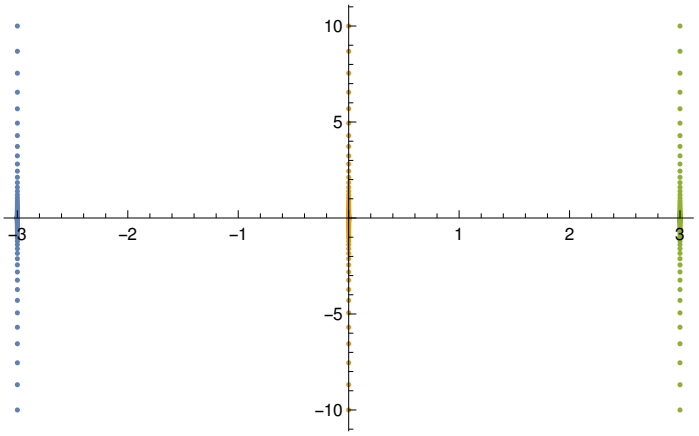
Out[97]=



In[98]:=
```
makeLogVerticalPts[minX_,maxX_,minY_,maxY_,ptsPerLine_,numLines_]:=Module[
{posVertPts,negVertPts,pts},
  posVertPts=Table[Table[{x,y},{y,fSpace[.01,maxY,Floor[ptsPerLine/2]]}],
    {x,minX,maxX,(maxX-minX)/(numLines-1)}];
  negVertPts=Table[Table[{x,y},{y,fSpace[minY,-.01,Floor[ptsPerLine/2]]}],
    {x,minX,maxX,(maxX-minX)/(numLines-1)}];
  pts=MapThread[Join,{negVertPts,posVertPts}];
  Return[Re[pts]]
 ]
makeLogVerticalPts::usage =
  "makeLogVerticalPts[minX_,maxX_,minY_,maxY_,ptsPerLine_,numLines_]
returns numLines number of vertical lines of exponentially
    spaced points starting from minX to maxX with length
    maxY-minY with ptsPerLine number of points per line.";
```

In[100]:=
```
ListPlot[makeLogVerticalPts[-3, 3, -10, 10, 100, 3], PlotRange → Full]
```

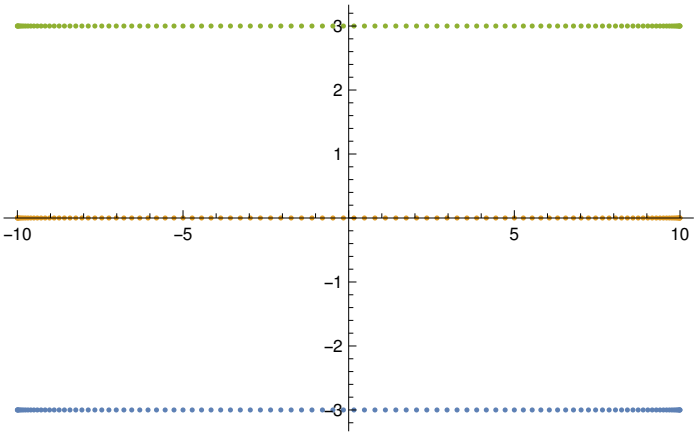
Out[100]=



In[101]:=
```
makeLogHorizontalPts[minX_,maxX_,minY_,maxY_,ptsPerLine_,numLines_]:=Module[
{posHoriPts,negHoriPts,pts},
  posHoriPts=Table[Table[{x,y},{x,fSpace[.01,maxX,Floor[ptsPerLine/2]]}],
    {y,minY,maxY,(maxY-minY)/(numLines-1)}];
  negHoriPts=Table[Table[{x,y},{x,fSpace[minX,-.01,Floor[ptsPerLine/2]]}],
    {y,minY,maxY,(maxY-minY)/(numLines-1)}];
  pts=MapThread[Join,{negHoriPts,posHoriPts}];
  Return[Re[pts]]
 ]
makeLogHorizontalPts::usage =
  "makeLogHorizontalPts[minX_,maxX_,minY_,maxY_,ptsPerLine_,numLines_]
returns numLines number of horizontal lines of exponentially
    spaced points starting from minY to maxY with length
    maxX-minX with ptsPerLine number of points per line";
```

In[103]:=
```
ListPlot[makeLogHorizontalPts2[-10, 10, -3, 3, 100, 3], PlotRange → Full]
```

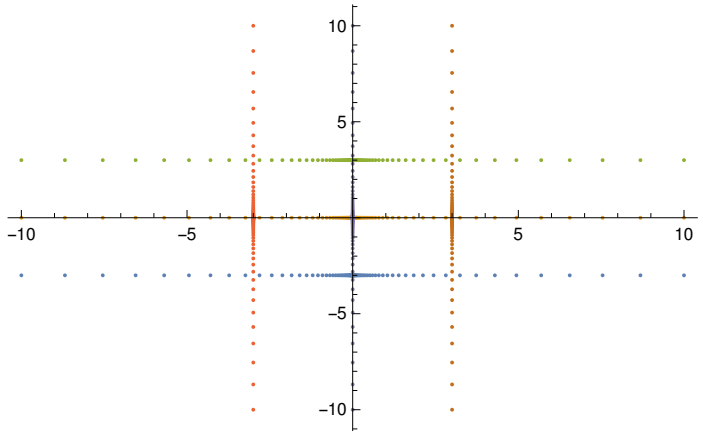
Out[103]=



In[104]:=
```
makeLogGridPts[minX_,maxX_,minY_,maxY_,ptsPerLine_,numLines_]:=Module[
{vertLogPts,horiLogPts,pts},
vertLogPts=makeLogVerticalPts[minY,maxY,minX,maxX,ptsPerLine,numLines];
horiLogPts=makeLogHorizontalPts[minX,maxX,minY,maxY,ptsPerLine,numLines];
pts=Join[horiLogPts,vertLogPts];
Return[pts]
]
makeLogGridPts::usage="makeLogGridPts[minX_,maxX_,minY_,maxY_,ptsPerLine_,numLines_]
returns numLines number of vertical of exponentially spaced points lines starting fro
with length maxY-minY with ptsPerLine number of points per line and
returns numLines number of horizontal of exponentially spaced points lines starting f
with length maxX-minX with ptsPerLine number of points per line.
NOTE: enter args as if doing horizontal line";
```

In[106]:=
```
ListPlot[makeLogGridPts[-10, 10, -3, 3, 100, 3], PlotRange → Full]
```

Out[106]=

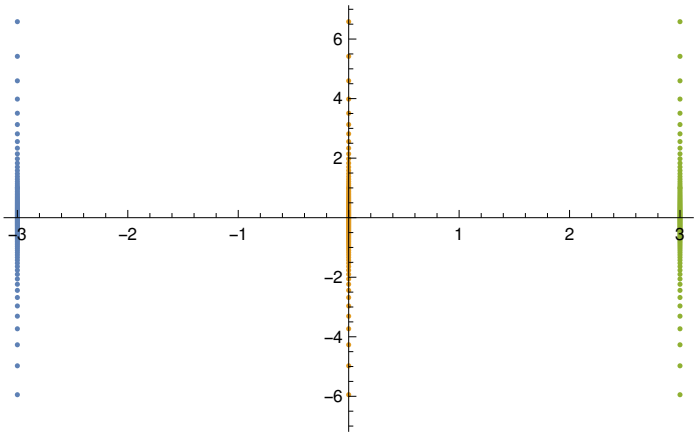


Sphere spaced lines and grids

In[107]:=
```
makeSphereVerticalLines[min_,max_,ptsPerLine_,numLines_]:=Module[{horiPts,spherePts},
spherePts=makeSphereSpacedPoints[ptsPerLine];
horiPts=Table[Table[{x,y},{y,spherePts}],{x,min,max,(max-min)/(numLines-1)}];
Return[horiPts]
]
makeSphereVerticalLines::usage=
    "makeSphereVerticalLines[min_,max_,ptsPerLine_,numLines_] makes vertical lines of
    sphere spaced points given line bounds of min, max, ptsPerLine, and numLines";
```

In[109]:=
```
ListPlot[makeSphereVerticalLines[-3, 3, 100, 3]]
```

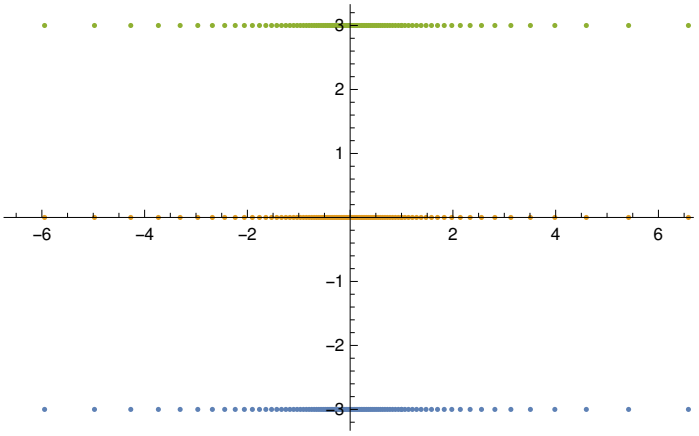
Out[109]=



In[110]:=
```
makeSphereHorizontalLines[min_, max_, ptsPerLine_, numLines_] :=
 Module[{horiPts, spherePts},
  spherePts = makeSphereSpacedPoints[ptsPerLine];
  horiPts = Table[Table[{x, y}, {x, spherePts}], {y, min, max, (max - min)/(numLines - 1)}];
  Return[horiPts]
 ]
makeSphereHorizontalLines::usage =
  "makeSphereHorizontalLines[min_,max_,ptsPerLine_,numLines_]
    makes horizontal lines of sphere spaced points given
    line bounds of min, max, ptsPerLine, and numLines";
```

In[112]:=
```
ListPlot[makeSphereHorizontalLines[-3, 3, 100, 3]]
```

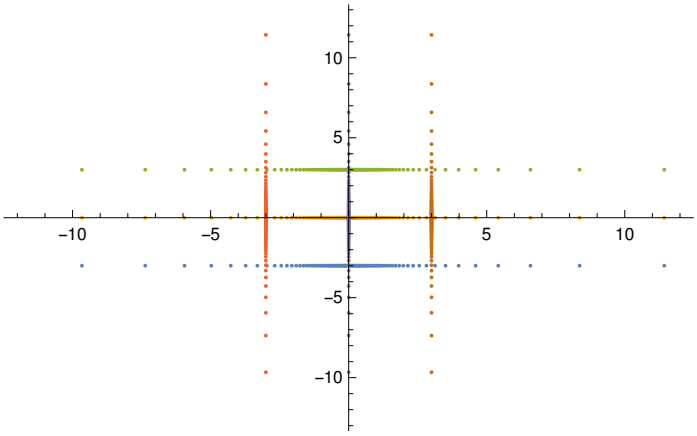
Out[112]=



In[113]:=
```
makeSphereGrid[min_, max_, ptsPerLine_, numLines_] := Module[{vertPts, horiPts, pts},
  vertPts = makeSphereVerticalLines[min, max, ptsPerLine, numLines];
  horiPts = makeSphereHorizontalLines[min, max, ptsPerLine, numLines];
  pts = Join[horiPts, vertPts];
  Return[pts]
 ]
makeSphereGrid::usage =
  "makeSphereGrid[min,max,ptsPerLine,numLines] makes a grid of sphere spaced
    points given line bounds of min, max, ptsPerLine, and numLines";
```

In[115]:=
```
ListPlot[makeSphereGrid[-3, 3, 100, 3]]
```

Out[115]=



Complex Plane to Image Plane

Here we make functions that let us deal with lists of points on the complex plane, a mapping expression, and the resulting image plane.

Get image of points

In[116]:=
```
getImagePts[expr_, pts_] := Module[{imgPts},
  imgPts =
   {Re[expr /. z → #[[1]] + i #[[2]]], Im[expr /. z → #[[1]] + i #[[2]]]} & /@ # & /@
    pts;
  Return[imgPts]]
getImagePts::usage =
  "getImagePts[expr[z],pts] returns the image points given takes
    in an expression of z and a list of lists of lists of points";
```

In[118]:=
```
getImagePts[z + 1 - i, {{{1, 1}, {2, 2}}}]
```

Out[118]=
```
{{{2, 0}, {3, 1}}}
```

Plot image of points

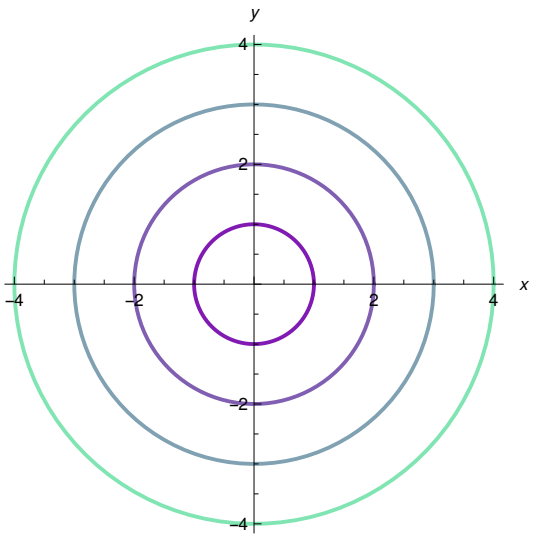
In[119]:=
```
plotImage[pts_, expr_, pltRange1_: Automatic,
  PltRange2_: Automatic, colors_: 1] := Module[{},
  If[colors == 1, colors = RGBColor[1, 1, 1], colors = colors];
  {
   Graphics[MapThread[{#1, Thick, Line[#2]} &, {colors, pts}],
    PlotRange → pltRange1, Axes → True, Background → White, ImageSize → {300, 300},
    AxesLabel → {Style["x", Italic], Style["y", Italic]}, ImagePadding → 20],
   Graphics[MapThread[{#1, Thick, Line[{Re[expr /. z → #[[1]] + I #[[2]]],
           Im[expr /. z → #[[1]] + I #[[2]]]} & /@ #2]} &, {colors, pts}],
    PlotRange → PltRange2, Axes → True, Background → White, ImageSize → {300, 300},
    AxesLabel → {Style["u", Italic], Style["v", Italic]}, ImagePadding → 20]
  }
 ]
plotImage::usage =
  "plotImage[pts,expr,pltRange1:Automatic,PltRange2:Automatic,colors:1]
    returns a list containing the complex and image plots given a list
    of lists of lists of points, an expression of z, two plot ranges
    (can be Automatic), and a list of colors. Note that plotImage
    doesn't call getImagePts so we can MapThread with color gradients";
```

In[121]:=
```
circles = Quiet[makeCirclePoints[1, 4, 4, 100]];
circleColors = makeColorGradient[circles]
Quiet@Grid[{{"Complex Plane", "ImagePlane"},
    plotImage[circles, Log[z], Automatic, Automatic, circleColors]}] // TraditionalForm
```

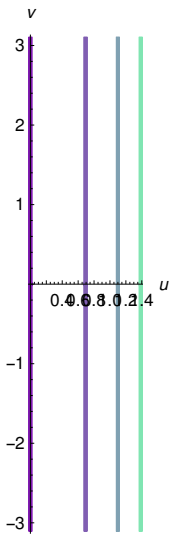
Out[122]= {■, ■, ■, ■}

Out[123]//TraditionalForm=

Complex Plane



ImagePlane



Colors

Color functions further help the user understand what is going on in complex plane to image plane or Riemann Sphere mappings.

In[124]:=
```
makeRandomColors[pts_]:=Module[{colors},
colors=RandomColor[Length[pts]];
Return[colors]]
makeRandomColors::usage="makeRandomColors[pts_] returns length of pts number of randon
```

In[126]:=
```
makeRandomColors[{{0, 0}, {1, 1}, {2, 2}, {3, 3}, {4, 4}}]
```

Out[126]=


In[127]:=
```
makeColorGradient[pts_] := Module[{colors},
  colors = Table[RGBColor[.5, x, .7], {x, .1, .9, (.9 - .1)/(Length[pts] - 1)}];
  Return[colors]]
makeColorGradient::usage =
  "makeColorGradient[pts_] returns length of pts number
    of colors in a purple -> teal gradient";
```

In[129]:=
```
makeColorGradient[{{0, 0}, {1, 1}, {2, 2}, {3, 3}, {4, 4}}]
```

Out[129]=


Riemann Sphere Stereographic Projection

Here we make functions that allow the the projection of points on the complex plane to the Riemann Sphere through the use of stereographic projection

Single point

In[130]:=
```
complexTo3D[point_]:=Module[{x,y,z,mix,X,Y,Z},
x=point[[1]];
y=point[[2]];
mix=(2x+i 2y)/(1+x^2+y^2);
X=Re[mix];
Y=Im[mix];
z=x+i y;
Z=(Abs[z]^2-1)/(Abs[z]^2+1);
Return[{X,Y,Z}]
]
complexTo3D::usage="complexTo3D[point_] returns the stereographic projection of a poi
```

In[132]:=
```
complexTo3D[{1, 1}]
```

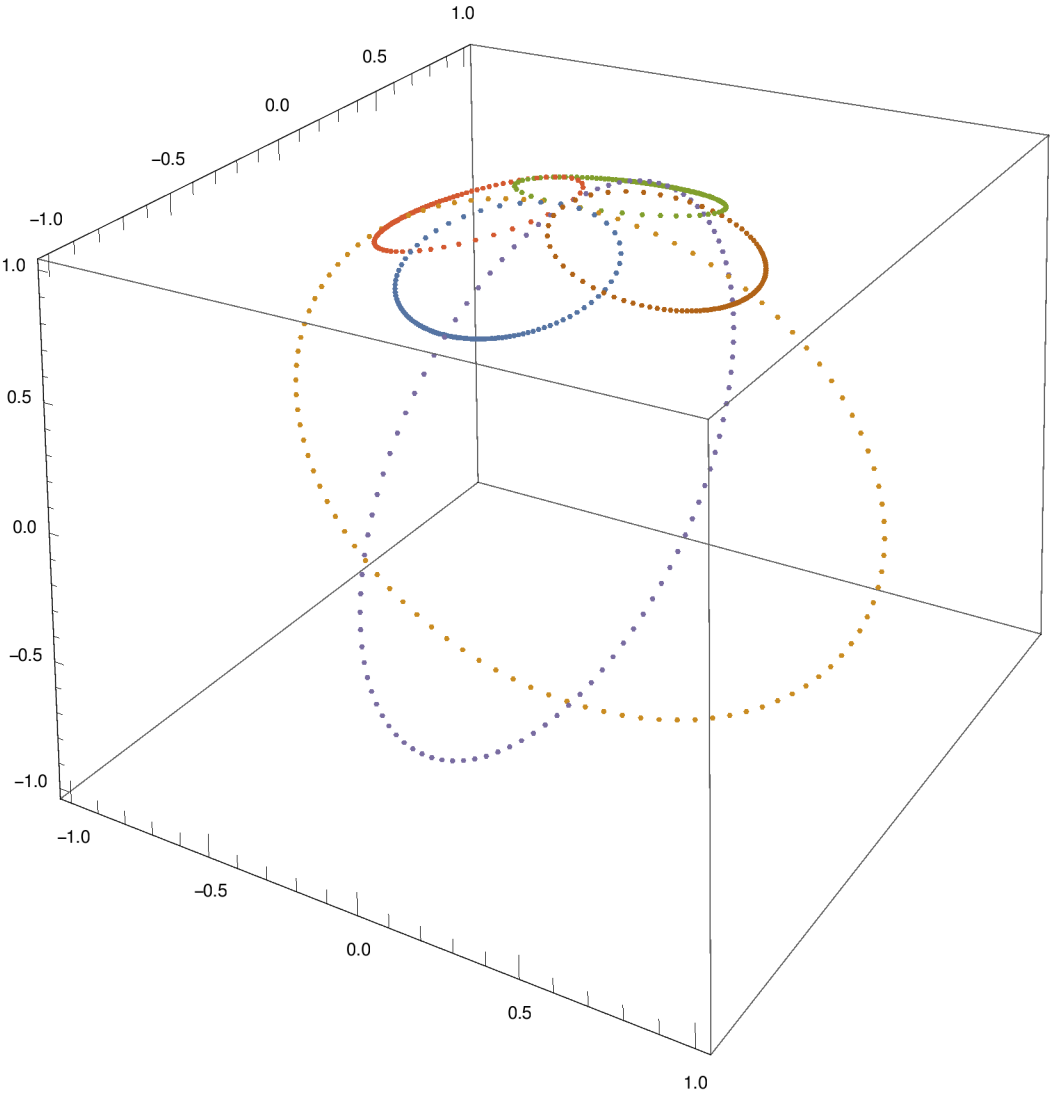
Out[132]=
$$\left\{\frac{2}{3}, \frac{2}{3}, \frac{1}{3}\right\}$$

Multiple points

In[133]:=
```
complexPtsTo3D[points_]:=Module[{spherePts3D},
spherePts3D=complexTo3D[#]&/@#&/@points;
Return[spherePts3D]
]
complexPtsTo3D::usage="complexPtsTo3D[points_] returns the stereographic projection of
of points on the complex plane";
```

In[135]:=
```
gridPts2D = makeSphereGrid[-3, 3, 100, 3];
gridPts3D = complexPtsTo3D[gridPts2D];
ListPointPlot3D[gridPts3D, PlotRange → All, AspectRatio → 1, ImageSize → Large]
```

Out[137]=



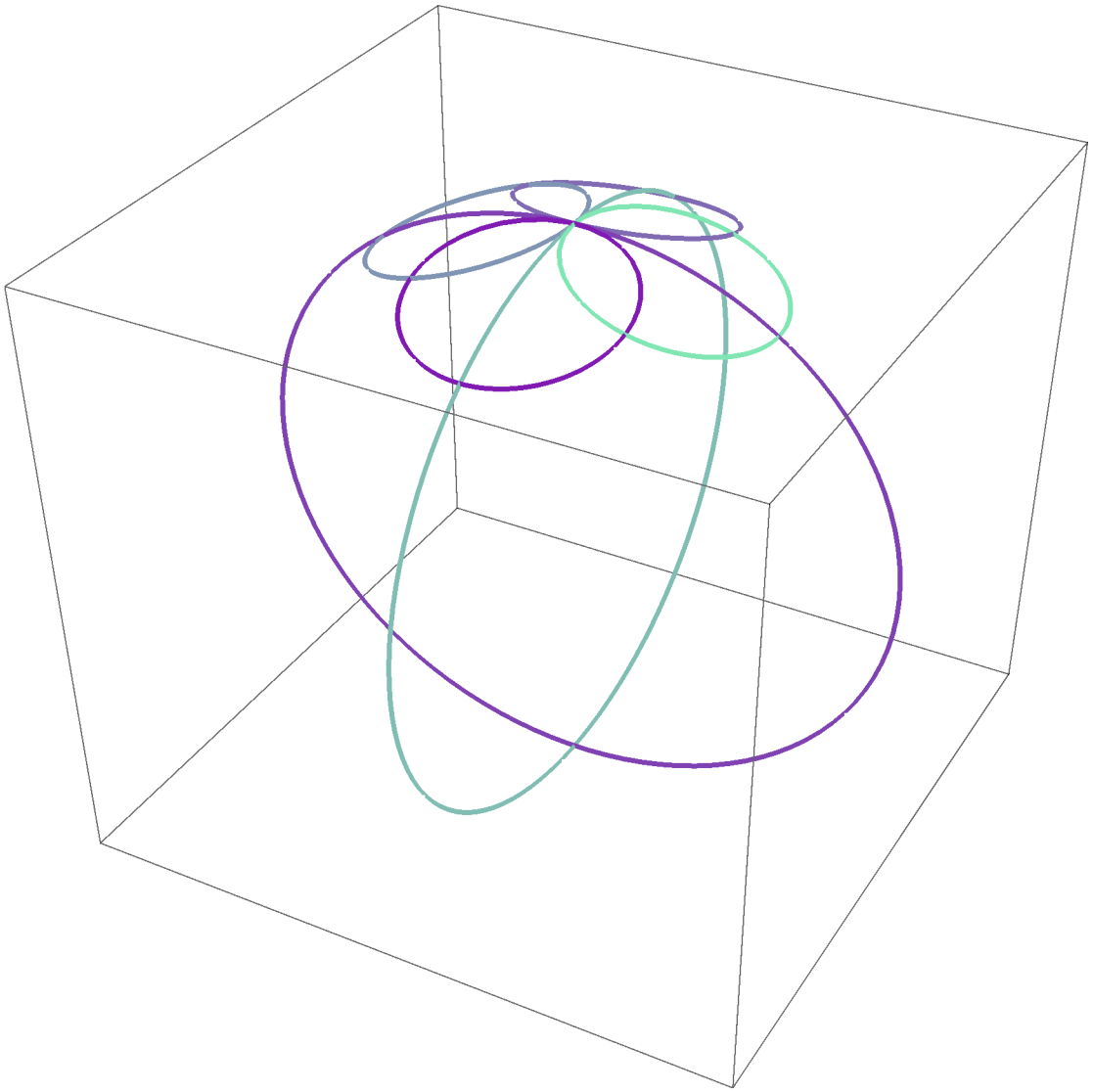
In[138]:=

Color Graphics3D test

In[139]:=
```
colors3D = makeColorGradient[gridPts3D]
Graphics3D[MapThread[{#1, Thick, Line[#2]} &, {colors3D, gridPts3D}],
 AspectRatio → 1, ImageSize → Large]
```

Out[139]= {■, ■, ■, ■, ■, ■}

Out[140]=



3D printing

3D printing allows us to take what we see in the computer and hold it in real life.

Connecting cylinders

In[141]:=
```
takingPts[takes_,pts_]:=Module[{newPts},
newPts=Take[pts,#]&/@takes;
Return[newPts]]
takingPts::usage="takingPts[takes_,pts_] returns a list of taken points
given a list of takes and a list of points";
```

In[143]:=
```
takingPts[{{1, 2}, {2, 3}, {3, 4}}, {2, 7, 4, 0}]
```

Out[143]=
```
{{2, 7}, {7, 4}, {4, 0}}
```

In[144]:=
```
cylinderPts[pts_]:=Module[{takes},
takes=Table[{x,x+1},{x,1,Dimensions[pts][[2]]-1}];
Return[takingPts[takes,#]&/@pts]
]
cylinderPts::usage="cylinderPts[pts_] returns a list of taken points ready to be conve
```

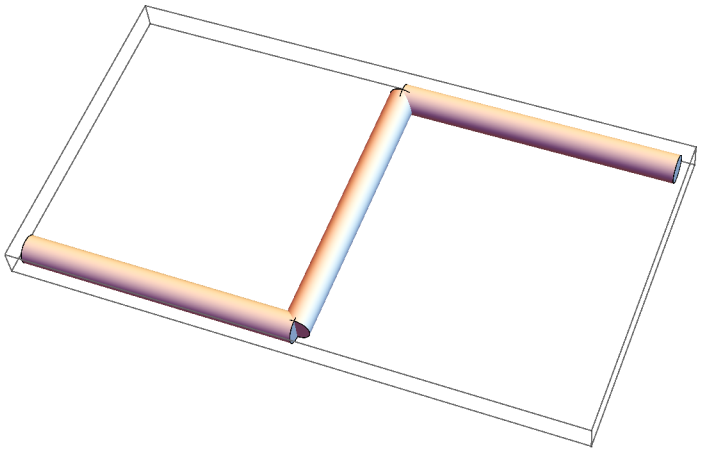
In[146]:=
```
cylinderPts[{{{1, 2, 3}, {4, 5, 6}, {7, 8, 9}, {10, 11, 12}}}]
```

Out[146]=
```
{{{{1, 2, 3}, {4, 5, 6}}, {{4, 5, 6}, {7, 8, 9}}, {{7, 8, 9}, {10, 11, 12}}}}
```

In[147]:=
```
cylinders[pts_, radius_] := Module[{},
  Return[Cylinder[#, radius] & /@ # & /@ cylinderPts[pts]]]
cylinders::usage =
  "cylinders[pts_,radius_] returns cylinders linearly along the
    path of the list of points with specified radius";
```

In[149]:=
```
Graphics3D[cylinders[{{{1, 2, 3}, {2, 2, 3}, {2, 3, 3}, {3, 3, 3}}}, .05]]
```

Out[149]=

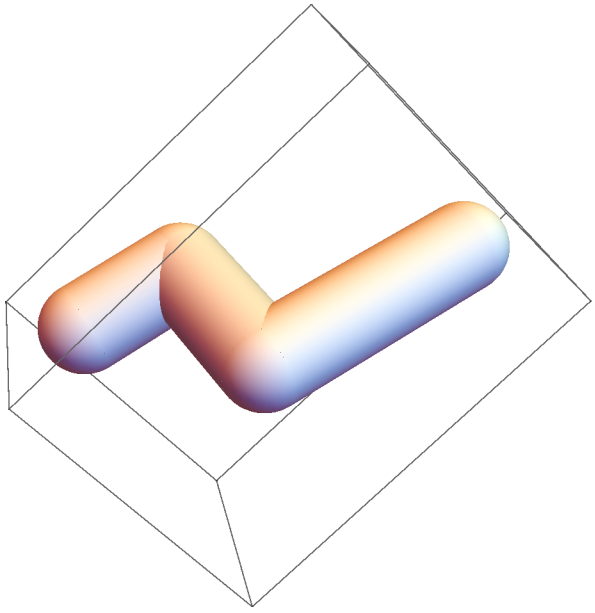


Connecting tubes

In[150]:=
```
tubes[pts_,radius_]:=Module[{},Tube[#,radius]&/@complexPtsTo3D[pts]]
tubes::usage = "tubes[pts_,radius_] returns tubes linearly
 along the path of the list of points with specified radius. This is
sometimes used as it gives a better result than cylinders but is less
reliable";
```

In[152]:=
```
Graphics3D[tubes[{{{1, 2, 3}, {2, 2, 3}, {2, 3, 3}, {3, 3, 3}}}, .05]]
```

Out[152]=



Exporting stls

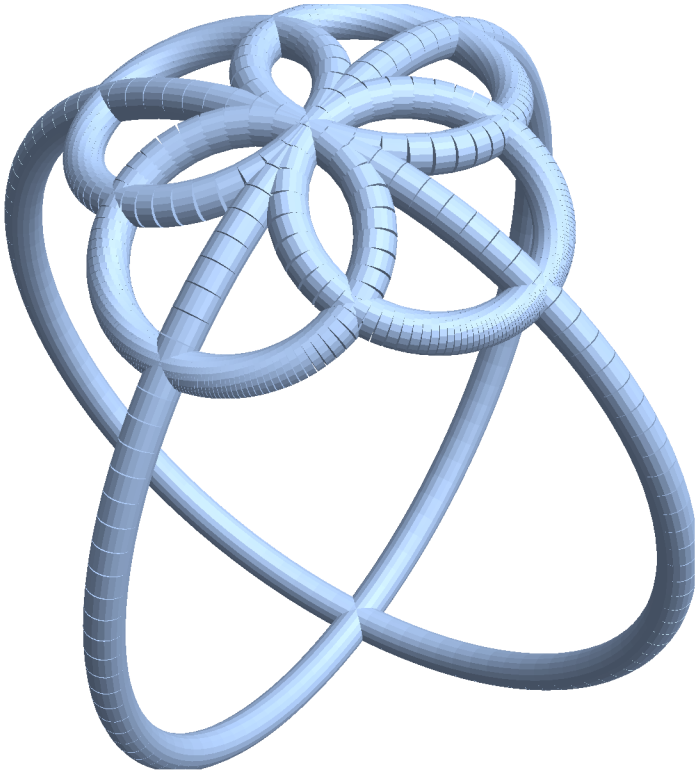
In[153]:=
```
gridPts2D = makeSphereGrid[-3, 3, 100, 3];
gridPts3D = complexPtsTo3D[gridPts2D];
```

In[155]:=
```
Export["grid2.stl", Graphics3D[cylinders[gridPts3D, .05]],
 {"STL,", "BinaryFormat" → True}]
```

Out[155]=
```
grid2.stl
```

In[156]:=
```
Import["grid2.stl"]
```

Out[156]=

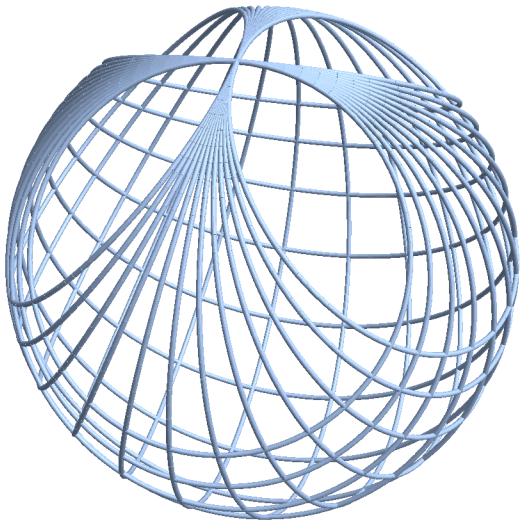


In[157]:=
```
Export["grid_10_7.stl",
 Graphics3D[cylinders[complexPtsTo3D[makeSphereGrid[-1, 1, 100, 12]], .01]],
 {"STL,", "BinaryFormat" → True}]
```

Out[157]=
```
grid_10_7.stl
```

In[158]:=
```
grid107 = Import["grid_10_7.stl"]
```

Out[158]=



Newtons Method

Newton's Method illustrates how visualizations of complex mappings helps us understand numerical methods.

In[159]:=
```
makeNewtonMethodAnimation[plotRange_, map_, depth_, points_] :=
 Module[{localPts, plots},
localPts = points;
plots = Flatten[{
{ListPlot[localPts,
        PlotRange → {{-plotRange, plotRange}, {-plotRange, plotRange}},
        PlotStyle → PointSize[.02], AspectRatio → 1, ImageSize → Large]},
Table[
(*Apply map to grid pts*) localPts = getImagePts[map, localPts];
ListPlot[localPts, PlotRange → {{-plotRange, plotRange}, {-plotRange, plotRange}},
        PlotStyle → PointSize[.02], AspectRatio → 1, ImageSize → Large]
      , {n, 1, depth}]
}];
Return[Manipulate[plots[[n]], {n, 1, Length[plots], 1}]]]
makeNewtonMethodAnimation::usage =
  "makeNewtonMethodAnimation[plotRange,map,depth,points]
    makes animation of successive mappings";
```

In[161]:=
```
f[z_]:=z^3-1
```

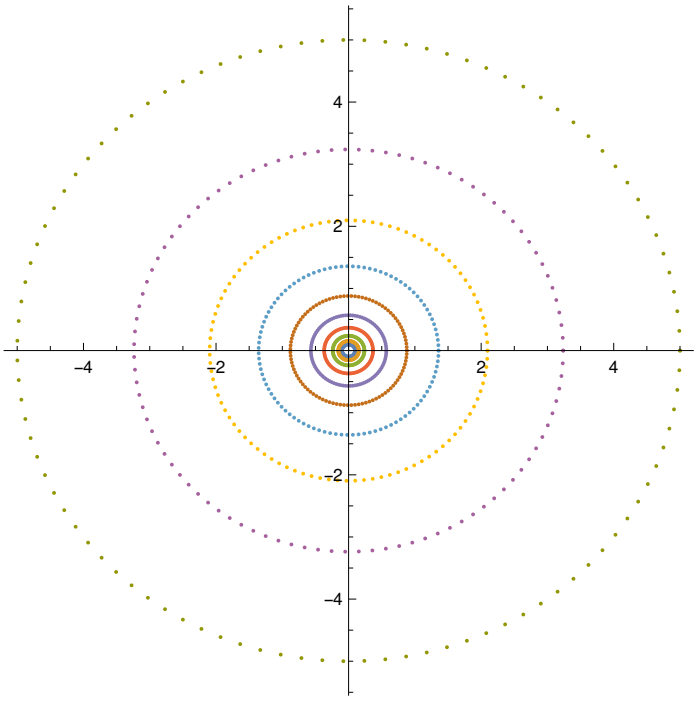
Use definition of Newton's Method to derive mapping function

In[162]:=
```
map=\left(z-\frac{f[z]}{f'[z]}\right);
```

Make a circular grid:

In[163]:=
```
smallestRadius=.1;
largestRadius=5;
numberOfCircles=10;
pointsPerCircle=100;
circleGrid = makeExponentialSpacedCirclePoints[smallestRadius,largestRadius,numberOfCi
circleGridCopy1=circleGrid;
ListPlot[circleGridCopy1,AspectRatio→1,PlotRange→Full]
```

Out[169]=



In[170]:=
```
makeNewtonMethodAnimation[5,map,35,circleGrid]
```

Out[170]=