I have trouble understanding this series expansion $\frac{1}{\text{Sin}[z]}$

```
In[7]:= Series[1/Sin[z], {z, 0, 5}]
```

$$\text{Out[7]}= \frac{1}{z} + \frac{z}{6} + \frac{7\,z^3}{360} + \frac{31\,z^5}{15\,120} + 0[z]^6$$

I can find the residue by looking at the coefficient of the $\frac{1}{z}$ term

```
In[5]:= Series[Cos[z]/(z^4 Sin[z]), {z, 0, 4}]
```

$$\text{Out[5]}= \frac{1}{z^5} - \frac{1}{3\,z^3} - \frac{1}{45\,z} - \frac{2\,z}{945} - \frac{z^3}{4725} + 0[z]^5$$

Or I can calculate it directly with black magic

```
In[3]:= Residue[Cos[z]/(z^4 Sin[z]), {z, 0}]
```

$$\text{Out[3]}= -\frac{1}{45}$$

I also wanted to see what this function looked like with expanding circles. Here is one of the functions I've been working on for the final project. In the future I would like to be able to see what happens to horizontal and vertical lines rather than just circles.

```
In[8]:= makeImage[pts_, expr_, pltRange1_, PltRange2_] := Module[{},
  {
   Graphics[{White, Thick, Line[# & /@ pts]},
    PlotRange → {{-pltRange1, pltRange1}, {-pltRange1, pltRange1}},
    Axes → True, Background → GrayLevel[.6], ImageSize → {300, 300},
    AxesLabel → {Style["x", Italic], Style["y", Italic]}, ImagePadding → 20],

   Graphics[{White, Thick, Line[
      {Re[expr /. z → #[[1]] + i #[[2]]], Im[expr /. z → #[[1]] + i #[[2]]]} & /@ # & /@
       pts]}, PlotRange → {{-PltRange2, PltRange2}, {-PltRange2, PltRange2}},
    Axes → True, Background → RGBColor[.7, .5, .5], ImageSize → {300, 300},
    AxesLabel → {Style["u", Italic], Style["v", Italic]}, ImagePadding → 20]
  }
 ]
```

Below we see a plot zoomed to image axes of $5 * 10^{-7}$. This is probably one

of the strangest looking functions I've ever seen.

In[177]:=
```
expr = Cos[z] / (z^4 Sin[z])
ang = Range[0 Pi, 2 Pi, .0001];
lists = Table[{r Cos[ang], r Sin[ang]}, {r, Range[1, 101, 10]}];
pts = Transpose[#] & /@ lists;
n = 101;
m = 5 * 10^-8;
makeImage[pts, expr, n, m]
```

Out[177]=
$$\frac{\text{Cot}[z]}{z^4}$$

Out[183]=