Math 154

Instructor : Professor Verstraete

Homework 1

20 points = 4 + 6 + 4 + 3 + 3

Instructions

1. Use this pdf file to submit your homework to gradescope (**5 questions**, **5 pages**).
2. Present your solutions **in the order the problems are listed**.
3. Show all your work on all problems and make sure your solutions are legible.
4. On proof-oriented problems, you must write a complete, coherent proof, as in the course notes.
5. You may study with others currently enrolled in the class, but work turned in should be your own; do not copy someone else's answer, or copy from an answer manual, answer key, or the internet.

4 POINTS

Homework 1 Question 1

Student Name
Student ID
SectionNumber

Question 1.3° For $n \geq 2$, let G_n be the ***grid graph***, whose vertex set is

$$V = \{(x, y) \in \mathbb{Z} \times \mathbb{Z} : 0 \leq x < n, 0 \leq y < n\}$$

and whose edge set is

$$E = \{\{(x, y), (x', y')\} : (x - x')^2 + (y - y')^2 = 1\}.$$

Determine the number of vertices and number of edges in G_n for each $n \geq 2$.

2+2+2=6 POINTS

Homework 1 Question 2

Student Name
Student ID
SectionNumber
TA Name

Question 1.9. Let $K_{n:r}$ denote the ***Kneser graph***, whose vertex set is the set of r-element subsets of an n-element sets, and where two vertices form an edge if the corresponding sets are disjoint.

(a) Describe $K_{n:1}$ for $n \geq 1$.
(b) Draw $K_{4:2}$ and $K_{5:2}$.
(c) Determine $|E(K_{n:r})|$ for $n \geq 2r \geq 1$.

2+2=4 POINTS

Student Name
Student ID
SectionNumber

Homework 1 Question 3

Question 2.7°

(a) Draw the de Bruijn graph $\vec{G}(3, 2)$.

(b) Find a de Bruijn sequence for words of length two over the alphabet $\{0, 1, 2\}$.

3 POINTS

Student Name
Student ID
SectionNumber

Homework 1 Question 4

Question 2.11° A *tournament* is an orientation of a complete graph. Prove that every tournament contains a directed path containing all of its vertices.

3 POINTS

Homework 1 Question 5

Student Name
Student ID
SectionNumber
TA Name

Question 2.16. Prove that a graph of minimum degree at least $k \geq 2$ containing no triangles contains a cycle of length at least $2k$.