Physical Interpretation of Swept Stroke Data

Supplementary information for the article:
A Physics-Based Approach to Aircraft Lightning Zoning: Zone 2

Nathanael Jenkins *Massachusetts Institute of Technology*

October 30, 2025

Swept stroke simulations compute the distributions of several metrics over an aircraft surface. This document explains how simulation metrics can be interpreted in a physical context, considering the effects of an unstructured grid representation of the aircraft surface. The physical interpretation of results is valuable when mapping simulation results to a lightning zoning diagram. Quantitative results are given for grids of triangular cells.

Nomenclature

P	Attachment probability.
P'	Attachment probability per unit length, $\mathrm{m^{-1}}$.
T	Dwell time, s.
T'	Dwell time per unit length, $\mathrm{s\,m^{-1}}$.
V	Arc sweeping speed $\parallel$ to streamline, $\mathrm{m\,s^{-1}}$.
U	Fluid velocity, $\parallel$ to streamline, $\mathrm{m\,s^{-1}}$.
n	Direction $\perp$ to streamlines.
s	Direction $\parallel$ to streamlines.
w	Projected cell width, $\perp$ to streamlines, m.
l	Traversal distance, $\parallel$ to streamline, m.
$\varnothing$	Arc diameter, m.
r	Arc radius, m, $r = \varnothing/2$.
C	Arc chord, m.
A	Cell area, $\mathrm{m^2}$.
δ	Shortest distance from arc center to cell edge, m.
L_{Z3}	Length of section ς categorized as zone 3, m.
M_ς	Number of arcs initialized upstream of section ς.
N	Total number of arcs.
$\mathbb{N}$	Total number of time steps.
ϕ_i	Quantity ϕ in cell i

Attachment Probability

For each individual arc simulation, the attachment probability in each surface cell is either 1 or 0, depending on whether the arc came into contact with the cell (so that $\delta < r$ during at least one time step). The sum of these values over every simulation, divided by the total number of arcs, N, represents the average probability, P, of an arc passing within radius, r, of the cell. This is illustrated in figure 1 and represented in equation 1, where $\mathbb{I}$ is the indicator function.

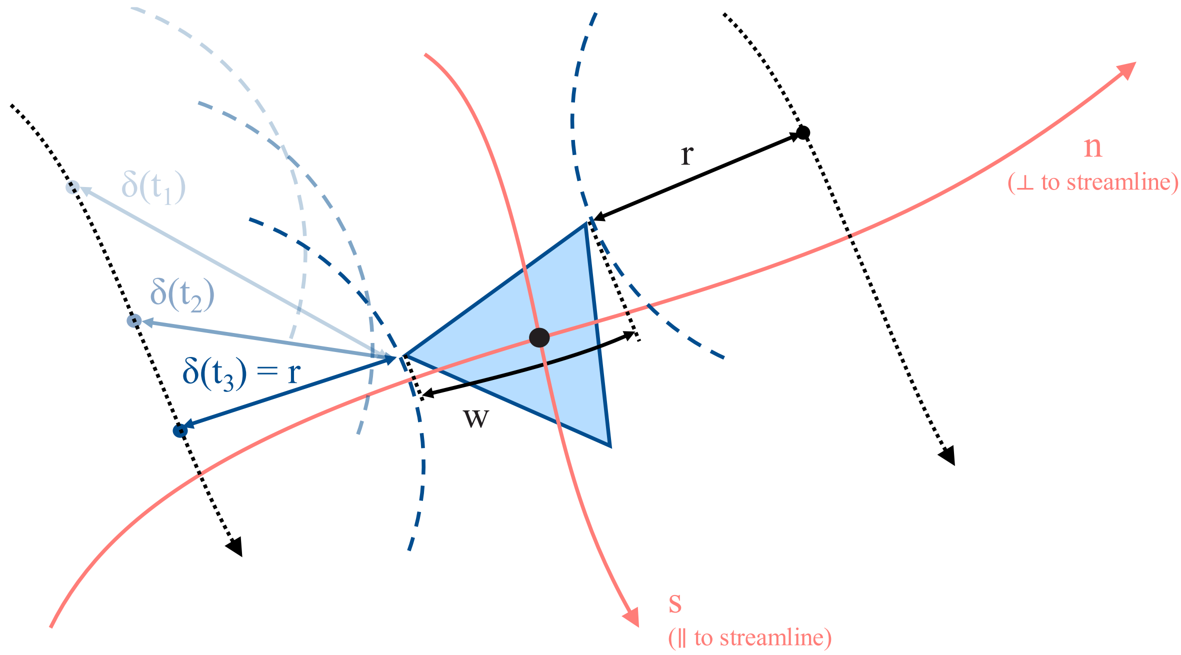

Figure 1: *Intersection of lightning arc with triangular cell in an unstructured grid.*

$$P = \frac{\sum_N 1\mathbb{I}(\exists t : \delta < r)}{N} \tag{1}$$

Since the arcs move along surface streamlines, the attachment probability is only a function of the streamline-normal direction, n, so that $P = P(n)$, and $P' = P'(n) = \frac{dP}{dn}$. The average probability computed by the simulation, P, describes the average over a region $\Delta n = w + \varnothing = w + 2r$, as illustrated in figure 1. Equations 2 shows the trivial definition for attachment probability.

$$P(n) = \int P'(n) dn \tag{2}$$

Assuming $P(n)$ and $P'(n)$ are continuous, and knowing that P' is independent of n within the range $[n_1, n_2]$, this can be rearranged for equation 3. This is rearranged for probability per unit length, in each cell, in equation 4; this quantity is independent of the cell size and arc diameter, unlike P.

$$P = P' \int_{n_1}^{n_2} dn = P' \cdot \Delta n \tag{3}$$

$$P'_i = \frac{P_i}{\Delta n_i} = \frac{P_i}{w_{p_i} + \varnothing} \tag{4}$$

The result in equation 4 is based on a projected width, w_i, which is a function of the shape and orientation of the cell, θ_i, relative to the streamline. For a generic cell in a generic orientation, an *effective* projected width should be calculated. For triangular surface cells, the effective w_i can be approximated using equation 5 which is derived in appendix A.

$$w_i \approx \frac{2 \times 3^{\frac{3}{4}} \sqrt{A_i}}{\pi} \tag{5}$$

Zoning Mapping

Taking a 'slice' of the aircraft over an arbitrary n-normal plane ($s = const'$), we can identify a line which each arc root will cross exactly once (except, perhaps, in the case of an unusual jump reattachment). An example is illustrated in figure 2. Let us refer to this line as ς. Integrating the probability per unit length over this line yields the proportion of arcs initialized upstream of the section, equation 6.

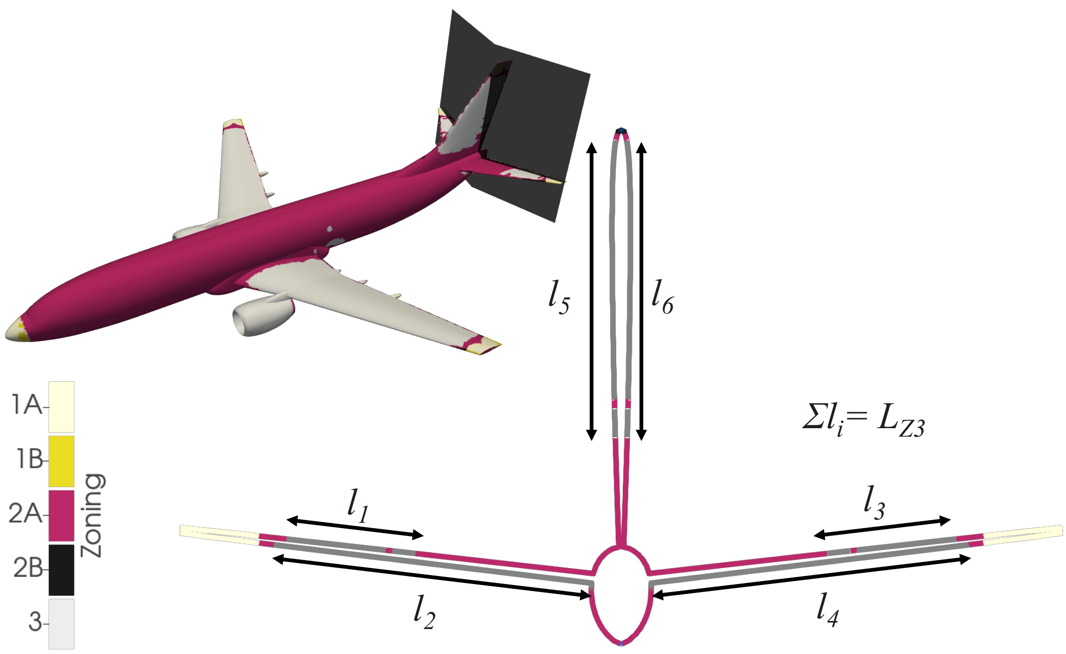

Figure 2: *Section taken approximately normal to streamlines for a conventional transport aircraft, using a simulated zoning result.*

$$P_\varsigma = \oint_\varsigma P' dn = \frac{M_\varsigma}{N} \tag{6}$$

Let $Z3$ be the section of the line ς which is classified as zone 3 (low attachment probability). We can define the net probability of a lightning arc passing through this region and constrain it based on the threshold probability (per unit length) $P' < P'_T$. This assumes that, **in the worse case**, $P' \approx P'_T$ for everywhere in zone 3 along line ς.

$$\frac{M_\varsigma}{N} P_{Z3} = \oint_\varsigma P' \mathbb{I}(P' < P'_T) dn \leq P'_T L_{Z3} \tag{7}$$

Rearranging equation 7, it is possible to identify a threshold probability per unit length, P'_T which ensures that attachment in zone 3 is no greater than the probability P_{Z3}. The result of this rearrangement is given in equation 8.

$$\begin{aligned} P'_T L_{Z3} &\geq \frac{M_\varsigma}{N} P_{Z3} \\ P'_T &\geq \frac{M_\varsigma}{N} \frac{P_{Z3}}{L_{Z3}} \end{aligned} \tag{8}$$

There are two important caveats to this result. First, the length of the zone 3 region, L_{Z3}, is coupled to the probability threshold, since increasing the threshold will increase the size of zone 3. Second, this calculation provides an upper limit for the attachment probability in zone 3; the actual attachment probability will be significantly lower than the computed quantity in every conceivable case. Therefore, this is a highly conservative approximation.

Example

Consider a conventional transport aircraft as shown in previous figures. ARP 5414B [1] lists in-service attachment probabilities for the tailplane surfaces of various conventional aircraft. Within zone 3, attachment probabilities up to approximately 3% are tolerated; these attachments occur on the horizontal stabilizer. Consider a representative aircraft where the zone 3 region of the horizontal tail is 14 m long when measured perpendicular to the streamlines. All lightning arcs attach upstream of the horizontal tail trailing edge. Therefore, equation 9 shows how a threshold probability of 0.0021 m^{-1} guarantees a zoning result within the bounds of the ARP at the horizontal tail. This means that $P' < 0.0021$ m^{-1} is a necessary, but not sufficient, condition for identifying this specific zone 3 region.

$$P'_T = \frac{M_\varsigma}{N} \frac{P_{Z3}}{L_{Z3}} = 100\% \times \frac{3\%}{14\ m} = 0.0021\ m^{-1} \tag{9}$$

Long Hang-On

Sweeping Speed

It is difficult to decouple the average dwell time from cell length while maintaining a physical understanding of the metric. A more intuitive metric is the average sweeping speed over the surface. The arc sweeping speed, V, may be different from the fluid velocity at the surface, U, even when using an inviscid fluid model because of reattachment and long hang-on. Identifying regions where $V << U$ is an easy way to identify long hang-on.

Figure 3 illustrates metrics required to define the sweeping speed, assuming that the arc is swept at the flow velocity, U, along the surface. Over the time the arc is in the cell, T_i, it moves a distance l_i in the s direction. Equation 10 shows a trivial relationship between T and V, where $T'(s,n) = \frac{dT}{ds}$.

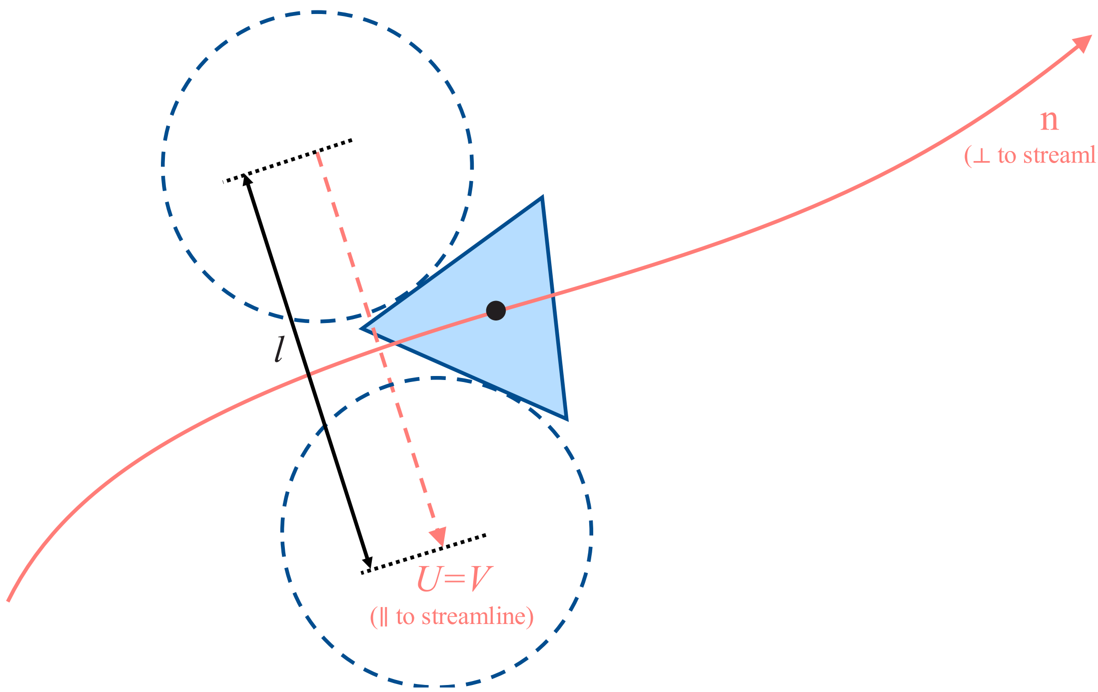

Figure 3: *Sweeping of a lightning arc through a triangular cell in an unstructured grid.*

$$T(s,n) = \int_{s_1}^{s_2} T'(s,n)ds = \int_{s_1}^{s_2} \frac{1}{V(s,n)}ds \tag{10}$$

Rearranging this result yields equation 11 which defines the average arc sweeping speed through a cell, as a function of the average dwell time computed by simulations. This mapping converts the simulated metrics, which are coupled to cell size, to a metric with greater relevance to lightning zoning. Again, this metric relies on the definition of an effective traversal length, l_i, which is defined in appendix B.

$$V_i = \frac{1}{T'_i} = \frac{l_i}{T_i} \tag{11}$$

References

[1] Aircraft Lightning Zone [Aerospace Recommended Practise]. SAE International; 2018. Available from: `https://www.sae.org/standards/content/arp5414b/`.

Appendix A: Derivation of w for a triangular cell

Metrics of cell size, w and l, can be approximated for a triangular grid, provided certain assumptions are made. (1) It is necessary to assume that the streamline-perpendicular coordinate, n, is a linear combination of body-fixed coordinates (i.e. it is not tightly curved in body-fixed coordinates) if the following derivations are to hold. This is a reliable assumption for most simple flowfields. (2) These derivations will assume that cells are regular (equilateral) triangles with a uniformly distributed orientation, so that no streamline-relative orientation is more or less likely than another.

Trivial results for the edge length and height of an equilateral triangle are given in equations 12 and 13, where A is the area of the triangle (cell).

$$L = \sqrt{\frac{4A}{\sqrt{3}}} \tag{12}$$

$$h = \frac{\sqrt{3}}{2} L \tag{13}$$

The projected width of an equilateral triangle is given in equation 14, for a rotation about an arbitrary axis by angle θ. The average of this width, taken over the range $\theta \in [-30°, +30°]$, is computed in equation 15.

$$w = L\cos(\theta) \forall \theta \in [-30°, +30°] \tag{14}$$

$$w = \frac{1}{\pi/3} \int_{-\pi/6}^{\pi/6} L\cos(\theta) d\theta = \frac{3L}{\pi} = \frac{2 \times 3^{\frac{3}{4}} \sqrt{A}}{\pi} \tag{15}$$

Appendix B: Derivation of l for a triangular cell

The traversal length is a more complex metric to compute, since it depends on the n position as well as the cell orientation, θ. Figures 4 and 5 show the two cases which must be considered, since traversal length depends on whether the arc core passes through the cell itself.

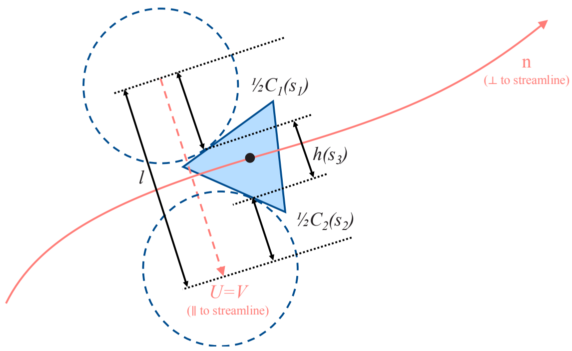

Figure 4: *Traversal length when arc core passes through cell*

The traversal length can be written using the general form of equation 16. Its average over n and θ is then approximated in equation 17. Since w has previously

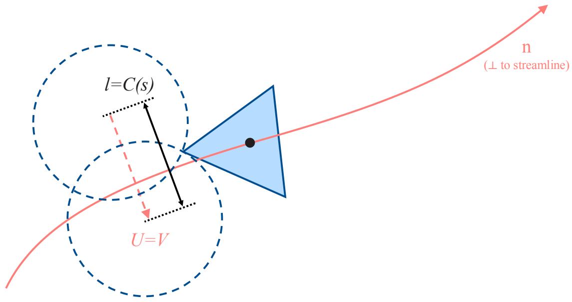

Figure 5: *Traversal length when arc core does not pass through cell*

been approximated for an equilateral triangle, the only remaining unknowns are h and C.

$$l(n) = \begin{cases} \frac{1}{2}(C_1(n) + C_2(n)), & \|\Delta n\| > w \\ \frac{1}{2}(C_1(n) + C_2(n)) + h(n), & \|\Delta n\| \leq w \end{cases} \qquad \forall\theta \in [0, \frac{\pi}{6}] \tag{16}$$

$$l = C + \frac{w}{w + \varnothing} h \tag{17}$$

C is approximated in equation 19 using the definition of a circle and its chord, where n is an arbitrary axis that passes through the center of the circle, and $r = \varnothing/2$ is the circle (arc) radius.

$$C(n) = 2r\sqrt{1 - \frac{y^2}{r^2}} \tag{18}$$

$$C = \frac{1}{2r} \int_{-r}^{r} 2r\sqrt{1 - \frac{y^2}{r^2}} dn = \frac{\pi r}{2} = \frac{\pi \varnothing}{4} \tag{19}$$

h is approximated below. Its derivation is not trivial, but simulation data has demonstrated that it cannot be neglected, particularly when the cell size becomes close to, or larger than, the arc size. This quantity represents the average height, h of the cell in the s direction, considering all n positions within the cell and all orientations θ. The solution in equation 22 was computed numerically.

$$\begin{aligned} s_1 &= y\tan(\frac{\pi}{3} - \theta) \\ s_2 &= -y\tan(\theta) \\ s_3 &= L\sin(\frac{\pi}{3} - \theta) - (n - L\cos(\frac{\pi}{3} - \theta)) \cdot \tan(\theta + \frac{\pi}{3}) \end{aligned} \tag{20}$$

$$h(n) = \begin{cases} s_1 - s_2, & s \in [0, L\cos(\frac{\pi}{3} - \theta)] \\ s_3 - s_2, & s \in [L\cos(\frac{\pi}{3} - \theta), L\cos(\theta)] \end{cases} \qquad \forall\theta \in [0, \frac{\pi}{6}] \tag{21}$$

$$h = \frac{6}{\pi} \int_0^{\frac{\pi}{6}} \frac{1}{L\cos(\theta)} \int_0^{L\cos(\theta)} h(n) dn d\theta \approx 0.430L = 0.653\sqrt{A} \tag{22}$$