Extreme Value Theory – Project

Study of extreme returns on LVMH stock

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I. Introduction to the study

1. Context of the study

There are a large number of protective tools available for measuring market risks, and more specifically potential losses in asset markets (such as equities). Indeed, the subprime crisis (2009) has tightened regulators control on the risks taken by financial institutions and asset management companies.

One of the main objectives is to predict certain events or behavior based on the extreme values (returns) of an asset, and to be able to measure the maximum losses incurred in rare cases.

The Central Limit Theorem (*CLT*) describes the asymptotic behavior of the mean of a large number of independent variables, but it proves ineffective when it comes to rare or extreme events. Between 1920 and 1940, Extreme Value Theory (*EVT*) was developed to evaluate rare events and the losses associated with their occurrence. In other words, when a major loss occurs, this theory is used to assess its magnitude. This theory relies on distribution tails and extreme data to estimate the parameters of *EVT* models, ensuring a better fit of the model to the tail of the law. Thanks to this model, we can calculate risk measures and estimates to measure the market risk of a stock (or even a portfolio).

2. Presentation of the dataset

In this study, the dataset will correspond to the share price of Louis Vuitton Moet Hennessy (*LVMH*). This stock is listed on the French CAC 40 index. It's natural to have extreme value in equities : depending on the company's results, management announcements or the macro-economic aspect.

The LVMH stock is very popular in professional portfolios, and is characterized recently by high volatility. It seems worthwhile to study the behavior of the distribution tail in order to predict and measure potential maximum losses.

We use publicly available market data from 2000 to the present day. Then, we separate the data into quarters, in order to identify a set of extreme values and carry out our study on these.

3. Problematic

In this work, we want to know: what is the maximum expected loss on the LVMH share at the 95% confidence level?

To answer these questions, we first use the block maxima technique and POT method to estimate distribution tails, and then calculate various metrics to quantify maximum losses.

II. Block Maxima Method

1. Presentation of Extreme Value Theory

Let’s consider the sequence of independent and identically distributed (iid) realizations $X_1, X_2, \dots, X_n$ of the random variable X. In our case, these will be various observations of stock returns over time at regular intervals (daily). To characterize the worst-case behavior, we are interested in the limiting law of the maxima of the random variables. We will consider the sequence of maxima M_n such that $M_1 = X_1$ and $M_n = \max(X_1, \dots, X_n)$ for $n \geq 2$. *EVT* is interested in limit laws that model the probability distribution of maxima when they are normalized and centered (like *CLT*). The limiting laws of maxima are given by the **Fisher-Tippett Theorem** :

If there exist series (a_n) and (b_n) such that $\forall\, n > 0$,$a_n > 0$

$$P\left(\frac{M_n - b_n}{a_n} \leq x\right) \rightarrow F_{b,a,\xi}(x)$$

Where $F_{b,a,\xi}$ is non degenerate and $F_{b,a,\xi}$ belongs (a the scale parameter, b the location parameter, γ shape parameter) :

- if $\xi = 0$ → distribution of Gumbel

- if $\xi > 0$ → distribution of Fréchet

- if $\xi < 0$ → distribution of Weibull

2. Presentation of Block Maxima Method

The Block Maxima approach considers the division of the measurement sequence into blocks of size c. The sequence $X_1, X_2, \dots, X_n$ is divided into C blocks, from the first block of $X_1, X_2, \dots, X_c$ to the last block $X_{(C-1)c+1}, \dots, X_n$. The block size c is a measurement period (here a quarter). The maximum value of each block extracted, giving the sequence of maxima $M_1, M_2, \dots, M_C$. The experimental sequence of maxima obtained does not allow us to observe all possible values from the theoretical distribution of maxima. However using this sample of maxima, we are able to apply the usual methods of parameter estimation (a,b,ξ)such as maximum likelihood or moment estimation. The aim is to ensure that the theoretical law correspond as closely as possible to the experimental distribution of the sequence of maxima $M_1, M_2, \dots, M_C$.

$$H_\xi(x) = \begin{cases} \exp\left(-\left(1 + \xi\dfrac{x-b}{a}\right)^{-\frac{1}{\xi}}\right) & avec\ \xi \neq 0 \\ \exp\left(-\exp\left(-\dfrac{x-b}{a}\right)\right) & avec\ \xi = 0 \end{cases}$$

3. Data preparation

We will use the *getSymbols* function to download the LVMH stock price data since 2000 until today, with the ticker of the share (here it’s *MC.PA*) and the source (*yahoo*) as parameters.

To ensure that we have correctly downloaded the daily data, we display the daily closing price for close prices for LVMH share. We calculate daily closing price returns to obtain stationary data. Here is a histogram of the returns.

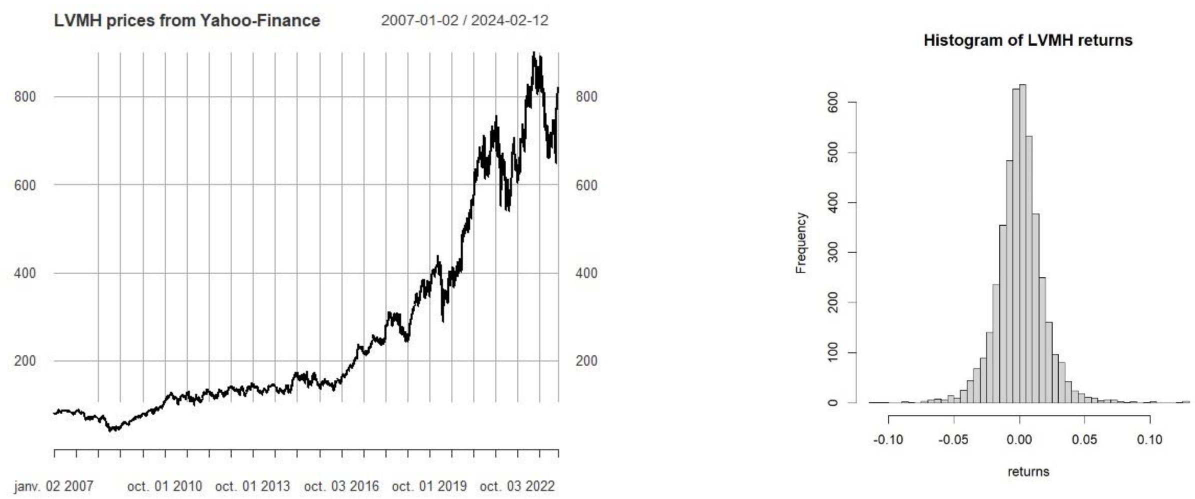


Figure 1 : Chart of LVMH share price and histogram of LVMH returns

In order to analyze and answer the initial problem, we are going to study negative returns to get an idea of maximum losses. For the rest of the assignment, we work with the absolute values of negative returns only. The condition *returns[returns <0]* selects only the daily losses observed.

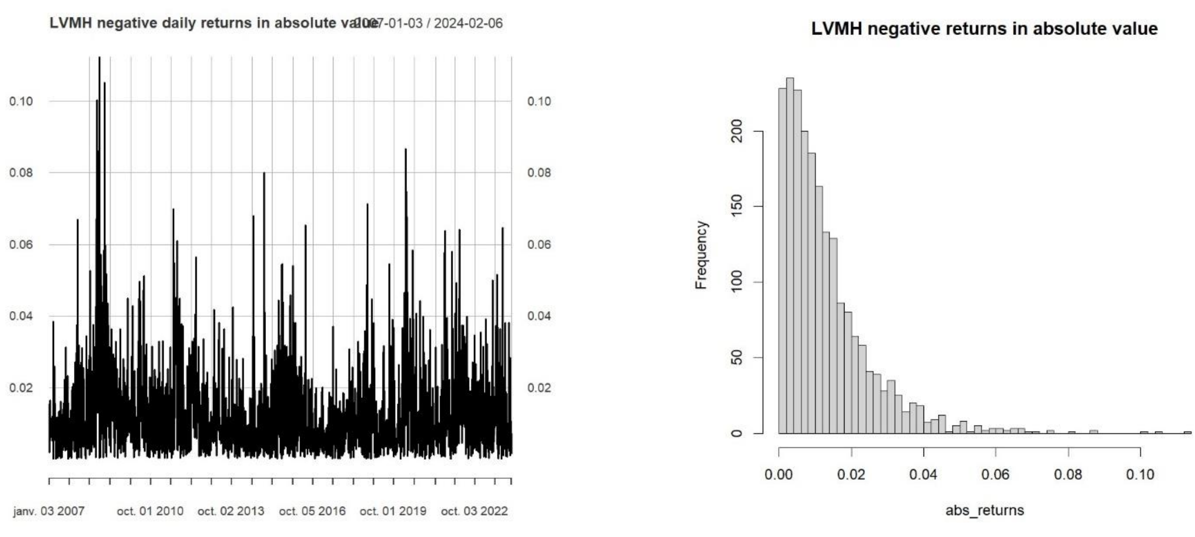


Figure 2 : LVMH negative returns in absolute value and his histogram

We calculate a few metrics (mean, variance, quantile 1, quantile 2 and kurtosis) to analyze our extreme losses. We also use a boxplot representation to provide visual support. In concrete terms, we can see that rare events (above the third quantile) are fairly evenly distributed between 1.7% and 11%. The average is 1.2%. We have a Kurtosis >> 0 which means that there is the presence of extreme values.

Indeed, it is legitimate to have extreme negative returns following news, rumors, publication of quarterly results... Our problem can therefore have a certain periodicity (trimonthly, for example) and looking at the minimum losses linked to each period makes sense to answer our question. So we will apply the block maxima method.

```
> cat("mean value correspond to ", mean, "\n")
mean value correspond to  0.01298878
> cat("the variance correspond to ", variance, "\n")
the variance correspond to  0.0001533843
> cat("kurtosis correspond to ", kurtosis, "\n")
kurtosis correspond to  8.487267
> cat("quantile à 0,25% =", quantile_1, "\n")
quantile à 0,25% = 0.00445865
> cat("quantile à 0,75% =", quantile_3, "\n")
quantile à 0,75% = 0.01734767
```

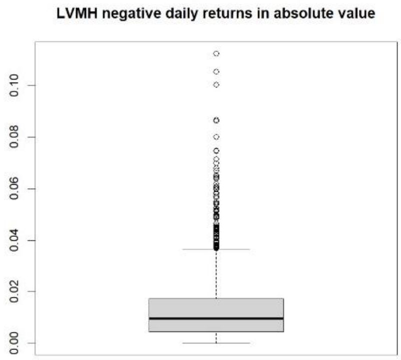


Figure 3 : Calculation of LVMH share metrics on extreme losses

4. Block Maxima

The Block Maxima will be applied on three-monthly blocks between the maximum losses on LVMH share returns between 2007 and 2024. There will therefore be 68 three-monthly blocks (17 years*4). The for loop selects the maximum of each block, so there will be 68 maximums in all. LVMH's negative returns are separated into three-monthly blocks and the maximum (worst) return over each period is selected.

```
num_blocks <- 68
returns_lengths <- length(returns)
block_size <- as.integer(returns_lengths/num_blocks)
block_maxima <- numeric(num_blocks)

for (i in 1:num_blocks) {
  start_index <- (i - 1) * block_size + 1
  end_index <- i * block_size
  block_maxima[i] <- min(returns[start_index:end_index])
}
block_maxima =abs(block_maxima)
```

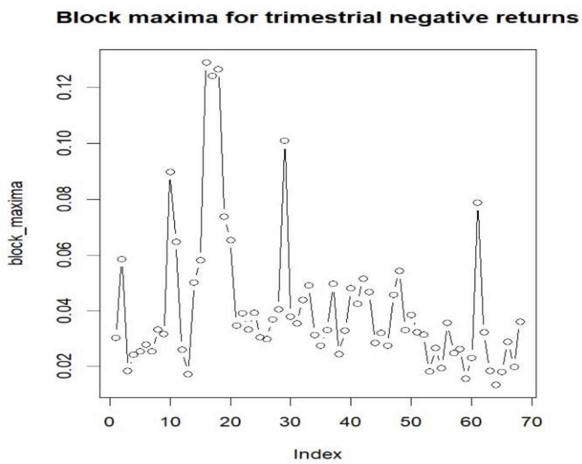


Figure 4 : Selection of the max for each quarter and graphical representation

The objective now is to find a distribution that will allow us to characterize the tail distribution of our negative returns. To do this, we will plot the histogram obtained by the block maxima method and plot in blue a first density estimate based on kernel estimation.

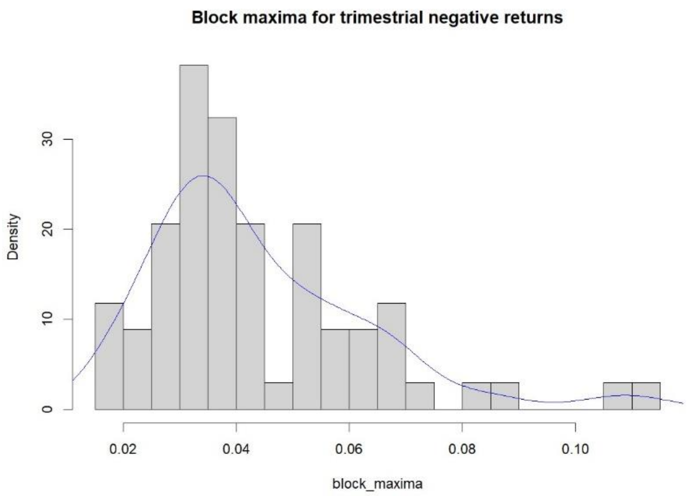


Figure 5 : First Kernel estimation of the tail distribution obtained

With the *gev* package available on the "*extRemes*" package, we estimate the coefficients ξ, μ and σ explained earlier in this report (cf II.1).

We obtain a $\xi > 0$, which means it's a rightward distribution with a thick distribution tail to the right. Here $\mu = 0.03$, this corresponds to localization i.e. the centering point. Dispersion is measured by $\sigma = 0.01$, meaning that the distribution is more concentrated.

```
Xi :  0.1036136
> cat("Mu : ",  Mu, " \n")
Mu :  0.03463091
> cat("Sigma : ",  Sigma, " \n")
Sigma :  0.01332779
```

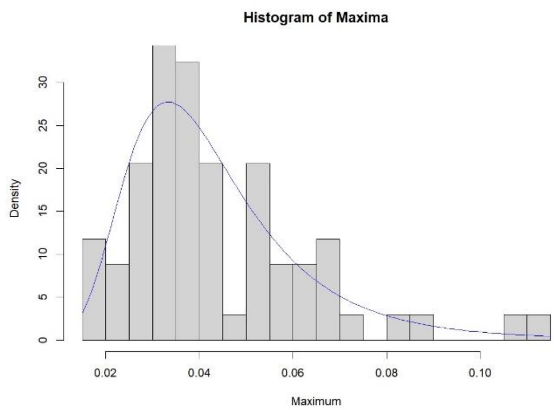


Figure 6 : GEV results on LVMH's negative returns

The estimated GEV is plotted in blue on our histogram, it follows the high bars of our histogram pretty well, which is a good sign. We use the "*dgev*" function, putting in the parameters ξ, μ and σ that we estimated. The ξ is positive, which means our model belongs to the domain of Fréchet's max of attraction.

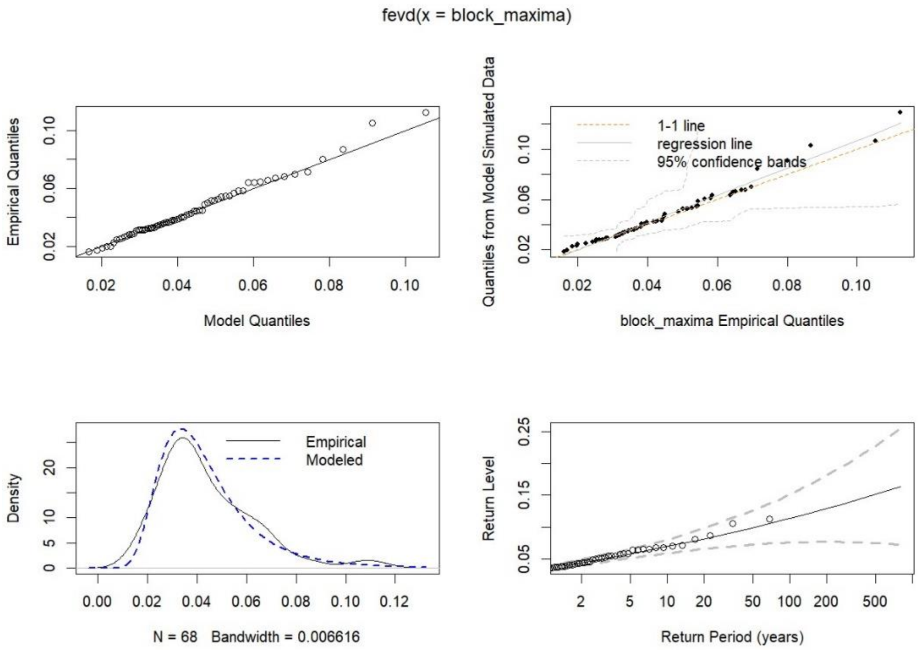


Figure 7 : Adjustment and analysis of the established GEV model

The use of the available *fevd* function tells us about the efficiency of our GEV model and its significance.

The probability plot (top left) is a graph representing the empiric quantiles versus the theoretical quantiles fitted to our GEV model estimated earlier. We note that for the early tail of the distribution, our quantiles correctly follow the estimated distribution. The most extreme quantiles are less represented by our model.

The second plot shows the trend between quantiles and data blocks. Extreme quantiles are far from our trend line.

The third plot shows the empirical distribution (with the kernel) against the GEV estimated distribution. We note that they are very similar, which confirms our choice of Fréchet's GEV to model our tail distribution.

The fourth plot represents yield levels as a function of yield periods, in order to analyze the occurrence of extreme values on different time scales. We can see that yield levels increase rapidly with return period: a high probability of occurrence of long-term extreme events.

5. Risk Measures

Now, to answer the problem, we are interested in risk measures. First, we simulate 1000 draws with the *rgev* function on our GEV model (with the three estimated parameters). We obtain the sample below:

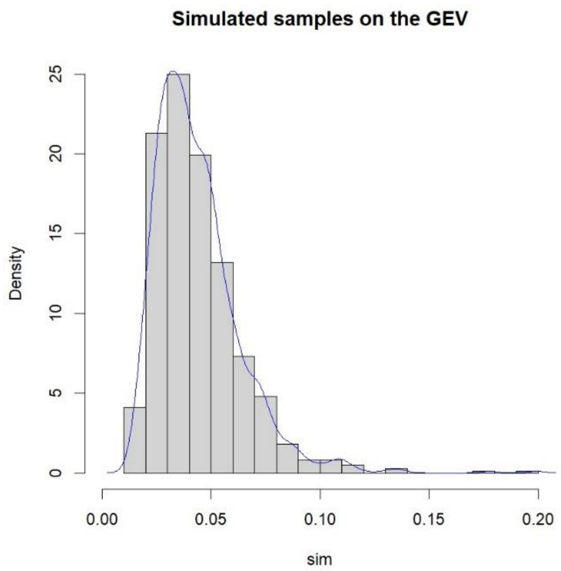


Figure 8 : Sample simulated using our GEV model

The **Value at Risk (VaR)** of a distribution is the maximum expected loss for a given confidence level. It corresponds to the quantile of the projected distribution of losses (in our case) over a time horizon.

It can be written as : $VaR(X;\alpha) = \inf\{x \in \mathbb{R}, P(X \leq x) \geq \alpha\} = F_X^{-1}(\alpha)$

To compute the 95% VaR on negative returns, we use the quantile function to find the right quantile corresponding to this level of probability. We obtain the VaR at 95% = 0.0785.

This means that in 95% of cases, given the estimated tail of the distribution, we will have negative returns above -7.85%. Or from another point of view, in extreme cases, my 5% worst returns will be below -7.85%. As we can see, VaR does not provide any information about the severity of losses generated by an event with a probability of less than 5%.

This is why we calculate the Expected Shortfall (ES), which represents the average of losses above the VaR. The formula is : $ES_\alpha(X) = TCE_\alpha(X) = E(X|X \geq VaR_\alpha(X))$

We obtain the ES at 95% = 0.1262, which means that on average the 5% of negative returns have a value of -12.6%. This gives us a clear indication of what we can lose on average in 5% of rare events.

We have plotted the representation of VaR and SE on the graph below:

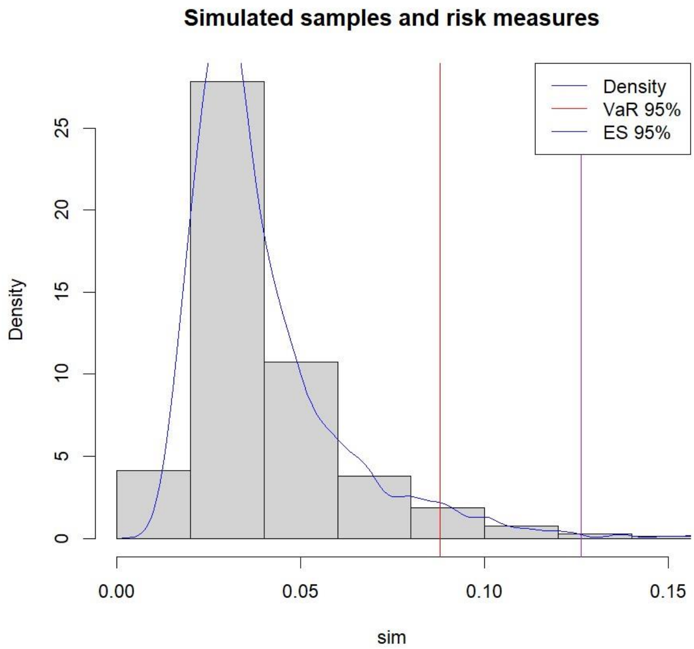


Figure 9 : Histogram of GEV distribution with VaR and ES calculations

III. Peaks Over Threshold approach (POT)

1. Presentation of POT

The Generalized Pareto distribution is considered a second approach to extreme value theory. It is equivalent to the distribution of all values of realizations $(Y_1, \ldots, Y_n)$ above a certain threshold (u for example). With X the random variable of the initial data and Y the random variable of X_i exceeding a certain predefined threshold u.

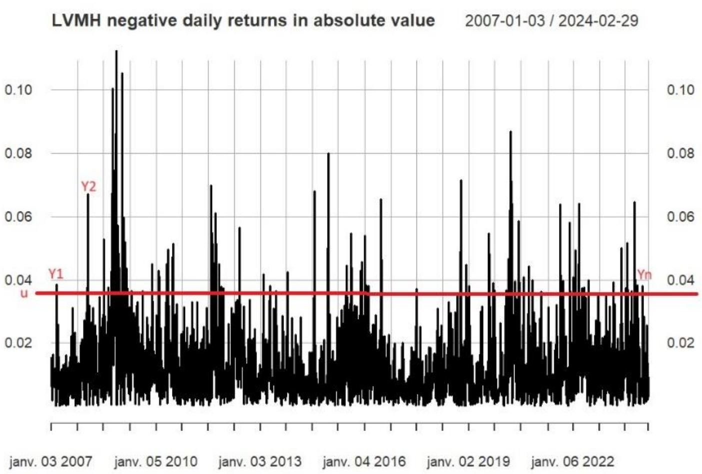


Figure 10 : Threshold u on LVMH negative returns

The generalized Pareto distribution is used to model observations exceeding a predefined threshold u. Let $F(x)$ be the distribution function and then the excess distribution above u is :

$$F_u(x) = P(X - u < x \mid X > u) = \frac{F(u+x) - F(u)}{\bar{F}(u)}$$

According to Pickands' theorem, we have : $\lim_{u \to \infty} F_u(x) = G_{\xi,\beta}(x)$

The limiting distribution corresponds to the Generalized Pareto distribution. Surpluses therefore converge towards GPD. It is given by :

$$H(y) = \begin{cases} 1 - \left(1 + \xi \frac{y}{\sigma}\right)^{-\frac{1}{\xi}} & si\ \xi > 0 \\ 1 - \exp\left(-\frac{y}{\sigma}\right) & si\ \xi = 0 \end{cases}$$

The shape parameter is the same:

- ξ > 0, heavy tail
- ξ = 0, thin tail
- ξ < 0, bounded tail

2. Set Threshold

There are several methods for determining the threshold to identify extreme values. It's important to define it correctly, because if it's too small we won't have enough observations, and if it's too large we'll have a large bias. The aim is therefore to choose a good bias-variance compromise. It can also be interesting to choose it by observing our data.

The three best known are :

- The 90th percentile
- $k = \sqrt{n}$
- $k = \frac{n^{2/3}}{\log(\log(n))}$

3. POT method

In this section, we are also interested in negative returns (expressed as absolute values). First, we will estimate our threshold using the three formulas above and *abs_returns* as data.

```
## Method 2 : Threshold Method
cat(abs_returns)
Nb_values <- length(abs_returns)
Nb_90 <- ceiling(0.9 * Nb_values)
Nb_sqrt <- Nb_values - floor(sqrt(Nb_values))
k <- Nb_values - floor(Nb_values^(2/3)/log(log(Nb_values)))
cat("Nb_values :", Nb_values, "\n")
cat("Position quantile 90% :", Nb_90, "\n")
cat("Position racine(n) :", Nb_sqrt, "\n")
cat("Position log :",k, "\n")

Sorted_X <- sort(abs_returns)
Seuil_90    <- Sorted_X[Nb_90]
Seuil_sqrt  <- Sorted_X[Nb_sqrt]
Seuil_log   <- Sorted_X[Nb_log]

cat("Seuil quantile 90% :", Seuil_90, "\n")
cat("Seuil racine(n) :", Seuil_sqrt, "\n")
cat("Seuil log :", Seuil_log, "\n")
```

```
> cat("Seuil quantile 90% :", Seuil_90, "\n")
Seuil quantile 90% : 0.03997211
> cat("Seuil racine(n) :", Seuil_sqrt, "\n")
Seuil racine(n) : 0.002093802
> cat("Seuil log :", Seuil_log, "\n")
Seuil log : 0.0107389
```

Figure 11 : Estimation of thresholds

We can see that the three methods give us very different values. The rest of the assignment will use the quantile 90%, which seems to be a good choice between the three calculated values. We can now estimate the Generalized Pareto distribution model using the *gpd package* available on R, giving it our threshold and *abs_returns* values as parameters. We thus obtain these parameters, which seem to correspond correctly to the histogram under study:

```
Xi : 0.1007285
> cat("Beta :", Beta, "\n")
Beta : 0.01154069
> cat("Seuil Log :", quantile_90, "\n")
Seuil Log : 0.02836974
```

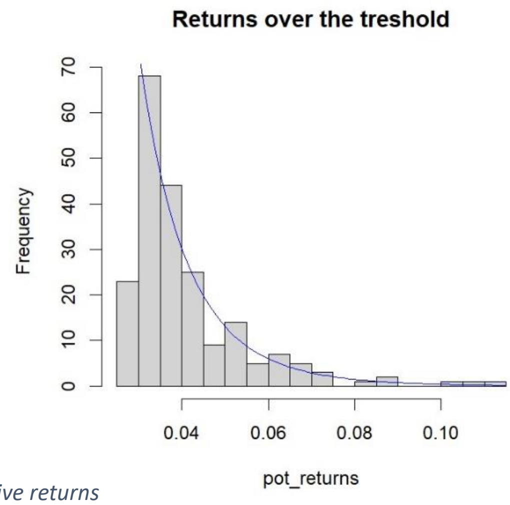


Figure 12 : GPD results on LVMH's negative returns

Thus $\xi > 0$ is a heavy-tailed Pareto distribution, and the representation of the estimated GPD distribution seems to follow our histogram. Extreme events occur quite frequently. The beta indicates that the amplitude of the excesses increases slightly with the values of the excesses.

4. Risk Measures

As before, we simulate a heavy-tailed Pareto distribution using the parameters estimated for our GPD distribution. With 1000 draws , we obtain this histogram :

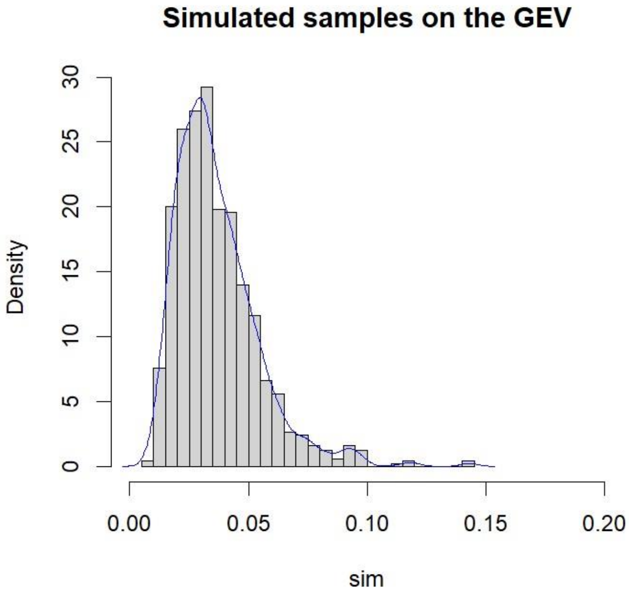
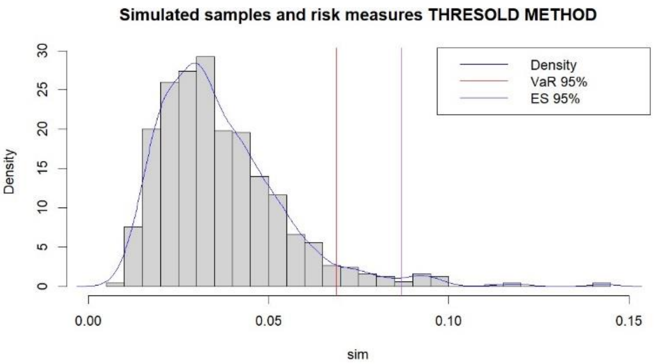


Figure 13 : Sample simulated using our GPD model

From these simulated samples, we will estimate the Value at Risk and the Expected Shortfall to measure the associated risks. The VaR will enable us to estimate the maximum amount of negative return we can expect in 95% of cases. And the ES will tell us the average of the worst 5% of negative returns that can be expected in the worst situations.




```
VaR at 95 % is: 0.06879903
>
>
> ES <- mean(sim[sim > VaR])
> cat("ES at", 100*p, "% is:", ES, "\n")
ES at 95 % is: 0.08682805
```

Figure 14 : Histogram of GPD distribution with VaR and ES calculations

In 95% of cases, when estimating our distribution tails with the POT method, we have negative returns above -6.8%. We can get an indication of the potential losses expected in 95% of situations.

We obtain an ES of -8.7%, i.e. the average loss below our VaR is equal to -8.7%, which gives a good indication of the worst 5% of situations.

IV. Conclusion

In conclusion, two methods were used in this report: block maxima and the POT method. We were able to estimate risk measures on both a GEV distribution and a GPD distribution. The block maxima method gives us an indication of the worst-case losses incurred on a quarterly basis. The Expected Shortfall is a good risk measure to give an indication of the supply to be prepared to lose in the event of a fall in LVMH's share price at 95% risk. It corresponds to -12.6% multiplied by the initial sum. The POT method, on the other hand, provides a long-term vision through the crises that LVMH has experienced. Based on this type of vision, we estimate a provision of -8.7% multiplied by the initial sum.