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## **Abstract**

**Background:** Trait impulsivity is central to all forms of externalizing psychopathology, including problematic substance use. The Cambridge Gambling task (CGT) is a popular neurocognitive task used to assess impulsivity in both clinical and healthy populations. However, the traditional methods of analysis in the CGT do not fully capture the multiple cognitive mechanisms that give rise to impulsive behavior, which can lead to underpowered and difficult-to-interpret behavioral measures. **Objectives:** The current study presents the cognitive modeling approach as an alternative to traditional methods and assesses predictive and convergent validity across and between approaches. **Methods:** We used hierarchical Bayesian modeling to fit a series of cognitive models to data from healthy controls (78 male/46 female) and individuals with histories of substance use disorders (Heroin: 55 male/20 female; Amphetamine: 48 male/25 female; Polysubstance: 80 male/18 female; final total across groups  $N = 370$ ). Using Bayesian model comparison, we identified the best fitting model, which was then used to identify differences in cognitive model parameters between groups. **Results:** The cognitive modeling approach revealed differences in quality of decision making and impulsivity between controls and individuals with substance use disorders that traditional methods alone did not detect. Crucially, convergent validity between traditional measures and cognitive model parameters was strong across all groups. **Conclusion:** The cognitive modeling approach is a viable method of measuring the latent mechanisms that give rise to behavior in the CGT, which allows for stronger

## 1. Introduction

Impulsivity is a multidimensional, heritable trait that confers liability to progression along the externalizing spectrum, which spans from attention-deficit/hyperactivity disorder in early childhood on toward substance use disorders in adulthood (Beauchaine and McKulty, 2013; Beauchaine et al., 2017). Defined as a preference for immediate over delayed rewards, action taken without forethought, and/or deficient self-control, many behavioral tasks are used to make inferences on trait impulsivity given individuals' behavior. Recently, the use of behavioral tasks in combination with neuroimaging technologies such as functional magnetic resonance imaging (fMRI) has revealed neurocognitive mechanisms that give rise to risky and impulsive behaviors (Robbins et al., 2012). Importantly, neuroimaging work has made it clear that impulsive behavior is *equifinal*, arising from interactions among multiple cognitive mechanisms (Turner et al., 2018). Traditional methods of analyzing behavioral task data rely on behavioral summary statistics, which ignore the equifinal nature of impulsive behavior and may subsequently decrease power in detecting differences in quality of decision making and impulsive behavior among groups.

Cognitive modeling is an alternative to traditional summary methods which allows us to identify separable effects of *latent cognitive variables* that are difficult to observe directly from behavioral data alone (e.g. Ahn and Busemeyer, 2016; Busemeyer and Stout, 2002). Therefore, cognitive modeling has the potential to more reliably estimate individual decision-making differences and subsequently increase statistical power to detect subtle differences across groups. Yechiam et al. (2005), for instance, used a cognitive model of the Iowa Gambling Task to identify distinct “cognitive profiles” of several clinical groups, such as individuals with bilateral

lesions in ventromedial prefrontal cortices, patients with Huntington's or Parkinson's disease, and various subtypes of substance users (cannabis, cocaine, alcohol, and polysubstance users), despite traditional analyses finding little evidence of group differences. Many others have followed suit (Ahn and Busemeyer, 2016; Busemeyer and Stout, 2002; Neufeld, 2015; Stout et al., 2004; for a review of cognitive modeling, see Ahn and Busemeyer, 2016; Ahn et al., 2016), and it has become clear that cognitive modeling can lead to greater predictive validity and reliability than traditional methods (Wiecki et al., 2015; that is, parameter estimates from cognitive models tend to be more reliable and correlate better with expected, external measures, as cognitive models tend to base their estimates on the entire data set for each participant, as opposed to simply analyzing the means; Busemeyer and Diederich, 2010). Here, we adopt the cognitive modeling approach and apply it to the Cambridge Gambling Task (CGT; Rogers et al., 1999), which is commonly used for neuropsychological assessment in clinical populations (Kräplin et al., 2014; Lawrence et al., 2009; Sørensen et al., 2017; Wu et al., 2017), including for individuals with substance use disorders (e.g., Ahn et al., 2016; Ahn et al., 2017; Baldacchino et al., 2015; Lawrence et al., 2009; Passetti et al., 2008). To our knowledge, this is the first study to develop a cognitive model for the CGT. Below, we describe the CGT and common methods of summarizing CGT performance before explaining the cognitive modeling approach.

The CGT was developed to assess decision-making and risk-taking outside a learning context. In the CGT, a yellow token is hidden in one of ten boxes that appear on a computer screen. Some of the boxes are red, others are blue, and the ratio between red and blue boxes can range from one red (nine blue) to nine red (one blue) boxes, with each (red, blue)-pair having an equal chance of occurring. The CGT proceeds in two stages, where the participant: (1) predicts the color of the box hiding the token, and then (2) bets a proportion of their accumulated points

(see Fig. 1) based on the certainty of their decision. The bet proportions are fixed across trials (.05, .25, .5, .75, .95) and presented sequentially with delays such that participants must wait to select their preferred bet. Importantly, the presentation order is varied across conditions, where ascending and descending blocks present the lowest (5%) and highest (95%) bet first, respectively.

Traditionally, performance on the CGT is probed with multiple indices that capture different facets of decision-making. The CGT has a number of behavioral measures, including: (1) quality of decision-making, (2) impulsivity/delay aversion, (3) risk-taking, (4) deliberation time, (5) risk adjustment, and (6) overall proportion bet. First, *quality of decision-making* (QDM) is the percentage of trials where a participant chooses the color with more boxes. The first study using the CGT revealed a dissociation between amphetamine and opiate users such that amphetamine, but not opiate users, showed lower QDM relative to healthy controls (Rogers et al., 1999). However, later studies using more participants showed that QDM is similar across individuals with opiate and amphetamine use disorders (Vassileva et al., 2014; Wilson and Vassileva, 2018). For example, studies consistently show that individuals with alcohol use disorder have QDM scores that are indistinguishable from healthy controls (e.g., Bowden-Jones et al., 2005; Lawrence et al., 2009; Monterosso et al., 2001; Zois et al., 2014), which could be due to no true difference or measurement error. Altogether, because QDM findings are inconsistent, the external and predictive validity of QDM is unclear<sup>1</sup>.

Second is *impulsivity* (IMP), or *delay aversion*, which is operationalized as the difference in average betting ratios chosen across ascending and descending conditions (  $E[A - D]$ ; Rogers et al. 1999), but only counting the optimal trials (i.e., where the participant chose the color that

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<sup>1</sup> We address the limitations of the CGT in the “Discussion” section.

had the greater number of boxes). Large, negative differences between conditions correspond to more rapid, impulsive betting. For example, someone betting impulsively would be expected to choose the “sooner” bets (those shown first) more frequently than the “later” bets. Therefore, IMP can be considered an index of *intertemporal choice* (Dai and Busemeyer, 2014) and is conceptually similar to measures of impulsivity derived from delay discounting tasks (DDT; Green and Myerson, 1993; Mazur, 1987; Myerson and Green, 1995; Reynolds, 2006). Importantly, individuals with substance use disorders consistently show steeper discounting of future rewards relative to healthy controls (Bickel et al., 2007; Dai et al., 2016; Green and Myerson, 1993; Johnson et al., 2015; Mazur, 1987; Miedl et al., 2014; Myerson and Green, 1995; Reynolds, 2006; Wilson and Vassileva, 2018), a pattern that has also been observed in IMP with problem gamblers (e.g., Kräplin et al., 2014; Lawrence et al., 2009; Zois et al., 2014; but see Monterosso et al., 2001). Like QDM, however, findings regarding differences in IMP among individuals with substance use disorders are inconsistent. For example, alcohol dependence has been linked to higher IMP relative to controls (Lawrence et al., 2009), but other studies have revealed no differences (Czapla et al., 2016; Monterosso et al., 2001; Zois et al., 2014).

We hypothesize that the inconsistent findings using QDM and IMP result, in part, because they ignore the equifinal nature of behavior (i.e. the underlying mechanisms). Subsequently, summary statistics like QDM and IMP are dependent on multiple cognitive mechanisms, which may each contribute to measurement error. Using IMP as an example, compare an individual who selects (A) all 5% bets, versus someone selecting (B) all 95% bets. Here, both individuals are given the same IMP score despite showing clear differences in risk valuation (i.e. A is risk-averse while B is risk-seeking). Because risk sensitivity, temporal

discounting, and other cognitive mechanisms all jointly determine which bet proportion participants choose (see eq.'s 4–8 in the Supplementary Materials [SM]<sup>2</sup>), impulsivity as traditionally measured in the CGT is inherently confounded with the effects of other cognitive mechanisms. As in temporal discounting paradigms, IMP should provide an independent measure of how much the expected utility is discounted with each delayed bet. Cognitive modeling allows for the specification of such a measure, which should provide a more precise measurement of IMP that can better differentiate groups.

*Risk taking* (Risk) is a third measure often used to assess decision-making in the CGT. However, unlike QDM and IMP, Risk is often operationalized in different ways across studies. For example, some studies define Risk as the average betting ratio only on those trials where participants choose the optimal color (Risk+; e.g. Kräplin et al., 2014; Zois et al., 2014); others as the average betting ratio across all trials (Risk-Avg; e.g. Bowden-Jones et al., 2005; Lawrence et al., 2009); and still others as the average betting ratio for trials in which participants choose the less optimal color (Risk-, e.g. Baldacchino et al., 2015). Based on our literature review, we chose to operationalize risk-taking as the Risk+ definition above due to its more widespread use (which is typically called 'Risk Taking').

As mentioned earlier, there are a number of additional measures from the CGT: Deliberation time is the time between the beginning of the trial and the bet choice; overall proportion bet records the average betting ratio chosen across all trials, including non-optimal trials and trials in which the proportion of red and blue boxes are the same; risk adjustment

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<sup>2</sup> Supplementary material can be found by accessing the online version of this paper at <http://dx.doi.org> and by entering doi: ...

captures the tendency for participants to bet more when the odds are in their favor. While we also assess the models' capabilities to capture trends in these data, as well as report on these data, these additional measures are not as common, or are very similar to the first three measures, and so our focus will be mostly on the three main measures of QDM, Risk+, and IMP.

Because parameters from cognitive models are often difficult to estimate, we use hierarchical Bayesian analysis (HBA; Kruschke, 2014; Wagenmakers et al., 2018). HBA has many advantages over traditional approaches (e.g., maximum likelihood estimation) and is particularly well-suited for estimating group-level effects while accounting for individual-level variation. Specifically, HBA uses estimates of participant-level parameters to inform the group-level estimate, which leads to pooling of information across subjects within each group. Once estimated, group-level parameters can then be directly compared to identify differences in cognitive mechanisms between groups (Kruschke, 2014; Wagenmakers et al., 2018)

In sum, the explanatory power of the behavioral CGT data in isolation is quite restricted, and here we offer a cognitive model of the CGT as a partial solution. We develop models based on both expected utility theory (von Neumann and Morgenstern, 1944) and prospect theory (Kahneman and Tversky, 2013/1979) and use Bayesian model comparison to determine which model is most generalizable. Further, we evaluate the predictive validity of the model relative to traditional behavioral summary methods (QDM, IMP, Risk+, etc.) by comparing model parameters and traditional measures of CGT behavior across control participants and multiple groups of *currently abstinent* individuals with a history of pure amphetamine, heroin, and polysubstance use disorders. Additionally, we examine convergent validity between traditional methods and our proposed alternatives. We expect to find steeper delay discounting of bets in all substance using groups relative to healthy controls. Further, we hypothesize that cognitive

modeling will reveal differences in the groups' parameter estimates that many of the behavioral measures will fail to detect.

## **2. Methods**

### **2.1 The Cognitive Models**

The full mathematical details for each model are relegated to the SM (see S1 Table for complete mathematical details<sup>3</sup>), but we provide a verbal description here. All models assume that a color and bet is chosen based on its “strength” (i.e., expected utility) relative to all other options (von Neumann and Morgenstern, 1944). Comparing the strength of each possible choice to the summed strength of all choices (Luce’s choice rule; Busemeyer and Stout, 2002) yields a probability of choosing that particular option, so that our models output *probability mass functions* for each stage of the task. In particular, the probability of each betting option, the more difficult option to predict, is proportional to the expected utility of that option, which itself is conditional on the color choice. The color choice will be based on distortions of the available color data during the task, where the distorted values are our “expected utilities” in this context.

We tested a total of 12 different models that varied assumptions regarding the color and bet utility functions. Below, we limit our discussion to what we term the Cumulative model (CM), which provided the best fit to the data relative to competing models and was deemed more

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<sup>3</sup> Supplementary material can be found by accessing the online version of this paper at <http://dx.doi.org> and by entering doi: ...



generalizable<sup>4</sup>. The CM model bases the expected utility on the *accumulated* points up to the given trial (i.e. based on what the resulting “Point” value would be in Fig. 1), contrasting models such as in Prospect Theory which assume that only the potential outcomes determine an option’s expected utility<sup>5</sup>. Additionally, we assume that both the color choice probabilities and bet proportions are used to determine the expected utility for each option (the assumed dependence). Formally, we can represent the expected utility of each option as:

$$EU(X |Color) = P(Color Chosen Wins) \cdot Utility(Points After Gain) \\ + P(Opposite Color Wins) \cdot Utility(Points After Loss)$$

That is, the expected utility of option X, given the chosen color, is a weighted average of the utility of the cumulative point values across gaining or losing points on the current trial. We assume that the Expected Utility (EU) is computed for each of the possible bets, the strength of which determines the probability of bet choice.

The CM model has a total of 5 free parameters that are estimated for each participant/group. First, subjective probabilities for the color choice are captured by  $\alpha$  ( $0 \leq \alpha \leq 5$ ), which “distorts” the objective probabilities of the coin being under a red versus blue box.

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<sup>4</sup> The best-fitting model described here is termed “Model 12 (M12)” in the SM. Supplementary material can be found by accessing the online version of this paper at <http://dx.doi.org> and by entering doi: ...

<sup>5</sup> Note that we tested variants of this idea from Prospect Theory, which we now call the Immediate Models (IM), and the CM provided the best fit (See SM; Supplementary material can be found by accessing the online version of this paper at <http://dx.doi.org> and by entering doi: ...).

Higher values for  $\alpha$  indicate underweighting of low and overweighting of high probabilities, which leads to more optimal color choices. A value of  $\alpha = 1$  indicates objective probability weighting. Therefore, probability distortion is our proposed cognitive mechanism that captures what is traditionally referred to as QDM. To account for biases that participants might have toward one color (which can introduce noise in estimation of probability distortion), we also include a color bias parameter  $c$  ( $0 \leq c \leq 1$ ) where values closer to 1 indicate a bias for red. Note that SEU uses the “distorted” color probabilities to generate color choice probabilities, but the objective color probabilities are used to weight the bets in the EU equation above<sup>6</sup>.

For calculating the value of gaining/losing a bet, we include a utility parameter  $\rho$  ( $0 \leq \rho < +\infty$ ) which governs how risk-sensitive participants are to losses relative to gains. Specifically, we set  $\rho = 1$  for gains and freely estimated  $\rho$  for losses, which allows for  $\rho$  to capture variations in loss sensitivity that can lead to risk-seeking ( $\rho > 1$ ) or risk-averse ( $\rho < 1$ ) behavior. Therefore,  $\rho$  should align with traditional measures of Risk (i.e., Risk+<sup>7</sup>).

To capture the tendency to select immediate over delayed bets, the expected utility is then “discounted” for the time delay for each bet option. Specifically, the CM assumes that waiting for an option diminishes its subjective value linearly. The slope of this linear descent is given by  $\beta$  ( $0 \leq \beta < +\infty$ ), which is interpreted here as the impulsivity parameter (IMP) and is akin to the discounting rate measured in delay discounting paradigms. Greater descent in value due to time

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<sup>6</sup> We tested subjective versus objective probability weighting for the EU, and the CM model with objective bet weighting provides the best fit across all groups.

<sup>7</sup> We also tested variants of this loss aversion mechanism; details can be found in the SM.

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delays (higher  $\beta$ ) indicates higher impulsivity. Finally, the discounted expected utilities for each bet option are exponentiated, scaled with  $\gamma$  ( $0 \leq \gamma < +\infty$ ), and then compared to the utility of other bet options to generate a probability of selecting a given bet (known as *Luce's choice rule*, Bussemeyer & Stout, 2002). Higher (lower) values for  $\gamma$  indicate that participants are making more deterministic (random) choices with respect to their model-predicted expected value for each option. Table 1 summarizes each of the CM model's parameters and their respective psychological interpretations.

## 2.2 Experimental Details

Because the experimental details have been explained elsewhere (Ahn and Vassileva, 2016; Vassileva et al., 2014; Wilson and Vassileva, 2018), we only touch on some highlights here. More information can be found in the SM<sup>8</sup>.

First, our final sample consisted of 124 healthy controls<sup>9</sup> (i.e., those not having a lifetime diagnosis of substance dependence) and three groups of participants with a history of substance-dependence: (a) a “pure” heroin dependent group ( $N=75 + 3^{10}$ ); (b) a “pure” amphetamine

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<sup>8</sup> Supplementary material can be found by accessing the online version of this paper at <http://dx.doi.org> and by entering doi: ...

<sup>9</sup> Some controls had a history of cannabis use, and some with low IQ were incorrectly allowed into the study. Sensitivity analyses with the best-fitting model related to how these particular controls affect the results can be found in the SM. Supplementary material can be found by accessing the online version of this paper at <http://dx.doi.org> and by entering doi: ...

<sup>10</sup> The additional numbers for each  $N$  indicate the number of participants in the final sample for whom we have CGT behavioral data but missing demographic data.

dependent group (N=73 + 3); and (c) a polysubstance dependent group (N=98 + 5). Eighteen participants across the four groups needed to be excluded for reasons we elaborate below. The majority of substance dependent participants were in *protracted abstinence*, i.e. having a lifetime diagnosis of substance dependence but not meeting diagnostic criteria for dependence for a year or longer (heroin: 78%; amphetamine: 59%; polysubstance: 64%). All participants were recruited in Sofia, Bulgaria, as part of a larger study on impulsivity among substance users. All participants provided informed consent. The demographic data for the final sample is found in Table 2; more information on the procedures can be found in (Ahn and Vassileva, 2016; Vassileva et al., 2014) and in the SM<sup>11</sup>.

We administered the CGT using the CANTAB® battery (2016) to all participants. As mentioned before, we excluded 18 participants due to clear signs of boredom (details in the SM<sup>12</sup>): (i) Controls = 7; (ii) Heroin = 5; (iii) Amphetamine = 2; (iv) Polysubstance = 4. Our initial sample consisted of 437 participants; 18 of whom needed to be removed due to clear signs of boredom, and an additional 37 control participants were removed because, after further

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<sup>12</sup> Supplementary material can be found by accessing the online version of this paper at <http://dx.doi.org> and by entering doi: ...

inspection, they were found to be ineligible for study<sup>13</sup>. This leaves us with a final sample of  $N = 370 + 11$  participants (11 of whom have missing demographic data).

### **3. Results**

#### **3.1 Behavioral Results**

We performed an ANOVA on each of the behavioral measures available to us from the CGT: (1) QDM, (2) IMP/delay aversion, (3) Risk+/Risk-Taking, (4) Deliberation Time, (5) Risk Adjustment, and (6) Overall Proportion Bet.

The following measures were *not* found to be significant: QDM, with  $F(3, 366) = 0.65$ ,  $ns$  ( $p = 0.58$ ); Delay Aversion/IMP, with  $F(3, 366) = 2.39$ ,  $ns$  ( $p = 0.7$ ); Deliberation time, with  $F(3, 366) = 1.02$ ,  $ns$  ( $p = 0.38$ ); Overall Proportion bet, with  $F(3, 366) = 2.51$ ,  $ns$  ( $p = 0.6$ ); and Risk Adjustment, with  $F(3, 366) = 0.72$ ,  $ns$  ( $p = 0.54$ ).

However, Risk+ (Risk-Taking) was found to be significant, with  $F(3, 366) = 2.89$ ,  $p = 0.04$ . A Bonferroni correction, with 3 comparisons and an alpha-level set at 0.05, revealed that only the comparison between controls and polysubstance users was significant,  $t(220) = -2.64$ ,  $p = 0.0089$ .

Thus, most of the behavioral measures suggest that there is minimal evidence of any differences among the groups on the task, with the exception of Risk+ finding a significant difference only when comparing controls and polysubstance users.

#### **3.2 Cognitive Modeling Results**

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<sup>13</sup> More detail on this can be found in the section “Previous Analyses” in the supplement.

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Details on the procedures used to fit, check and determine the best model are included in the SM<sup>14</sup>. Briefly, we fit twelve competing models to each group separately, resulting in both group- and individual-level posterior estimates for each model and group (see S1 Table for mathematical details<sup>15</sup>). We checked convergence to target distributions using both graphical and quantitative measures (Gelman and Rubin, 1992), and then used the leave-one-out information criterion to determine which model described the data best while penalizing for model complexity (Vehtari et al., 2017).

Additionally, we conducted posterior predictive simulations to check how well the CM model could describe each groups' choice patterns across different color ratios and conditions. Fig. 2 shows that the CM model provided an excellent fit to both the color choice and bet proportions across conditions in the control group. Results were similar for the other groups (see S2 and S3 Tables for mean squared error measures across all models and groups<sup>16</sup>). (Note that the model follows the hypothesized effect of color ratio on betting choice.) Because the CM model provided an accurate account of all groups' color choice and betting behavior, we used it to infer substance-specific differences in cognitive mechanisms between groups.

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<sup>14</sup> Supplementary material can be found by accessing the online version of this paper at <http://dx.doi.org> and by entering doi: ...

<sup>15</sup> Supplementary material can be found by accessing the online version of this paper at <http://dx.doi.org> and by entering doi: ...

<sup>16</sup> Supplementary material can be found by accessing the online version of this paper at <http://dx.doi.org> and by entering doi: ...

Fig. 3 shows the group-level posterior estimates for the CM for each group. Note that posterior distributions reflect uncertainty in the parameter estimates, where areas of higher density reflect more probable parameter values. To compare groups in a Bayesian manner, we computed differences between group-level parameters (see Fig. 4A and 4B). For brevity, we only report comparisons where the 95% highest density interval (HDI) of the difference between group parameters excludes/nearly excludes 0 (Kruschke, 2014). However, we do not endorse binary interpretations of “significant differences” using this threshold and instead refer the reader to the graphical comparisons to judge whether parameter differences are meaningful. Overall, we found large differences between controls and substance users. Specifically, all substance using groups showed strong evidence for larger  $\beta$  parameters compared to the healthy controls, which is indicative of higher impulsivity (i.e. greater discounting of time delay when placing bets) in the substance using groups. Additionally, amphetamine users, and polysubstance users to a lesser extent, showed evidence for less probability distortion (lower  $\alpha$ ) than healthy controls, which reflects a willingness to make less optimal bets (analogous to QDM). Further, polysubstance users, and amphetamine users to a lesser extent, showed greatly reduced sensitivity to loss (lower  $\rho$ ) relative to controls, which leads to taking *more* risky bets. (This follows since reduced sensitivity makes losses less painful, and hence the person will be more willing to take riskier bets<sup>17</sup>.) Importantly, a difference in  $\rho$  between polysubstance users and controls is consistent with our findings using traditional measures of Risk+.

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<sup>17</sup> While in the context of the CM model, the parameter estimates are consistent with what was expected (i.e., that substance users are less sensitive to losses), the manner in which the risk sensitivity parameter has been implemented in our model makes its interpretation opposite to

In addition to identifying differences between healthy controls and substance users, the CM model differentiated well among substance using groups (Fig. 4B). For example, amphetamine users showed strong evidence of lower probability distortion of color choices ( $\alpha$ ) relative to heroin users, possibly indicating a quantitatively worse decision-making process in the color choice. (Specifically, amphetamine users appear to probability match to a somewhat greater degree than heroin users, where probability matching is the less optimal strategy in this situation; Shanks et al., 2002.) Additionally, pure substance users (i.e. pure amphetamine and pure heroin users) showed more risk-sensitivity to losses ( $\rho$ ) relative to polysubstance users. Notably, there were no strong differences in impulsivity ( $\beta$ ) among substance using groups. Finally, heroin users showed a smaller bias for the red color choice ( $c$ ) relative to polysubstance users. That is, the heroin users had a greater propensity to choose blue over red than polysubstance users. While not theoretically important, this result underscores the importance of modeling choice biases that could obscure inferences on more meaningful model parameters<sup>18</sup>. In fact, such biases may be partly responsible for inconsistent findings using traditional measures of QDM and IMP. Further, our finding that  $\rho$  differed between substance using groups shows

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traditional measures of risk: in our model,  $\rho < 1$  actually means *risk seeking*, since the parameter's interpretation is tied to loss sensitivity. We discuss this unorthodoxy more below.

<sup>18</sup> After running an independent-samples *t*-test against the two groups' rates of choosing red, we observed  $t(180) = -1.90, p = 0.06, ns$ . Additionally, we found that the mean rate of choosing red for the two groups were 0.4753 and 0.4901 for the heroin and polysubstance use groups, respectively. This leads us to conclude that this result should be interpreted with caution, and any serious consideration of this result should await further replication.



that the CM model is more sensitive to differences in risk sensitivity compared to traditional measures in the CGT (Risk+).

Finally, to test convergent validity between QDM, IMP, and Risk+ to similar corresponding measures offered by the CM model, we computed the posterior means for each participant's  $\alpha$ ,  $\beta$ , and  $\rho$  parameters and compared them to each participant's QDM, IMP, and Risk+ scores, respectively. Fig. 5 shows the Pearson's correlations between cognitive model parameters and traditional behavioral summary measures. Notably, correlations were high across all groups and measures (all  $|r|s \geq .75$ ), suggesting that the CM model captures facets of decision-making similar – but not identical – to those inferred using traditional summary statistics derived from the CGT. Importantly the CM maintained strong convergent validity with traditional measures while also identifying group differences to which traditional measures were not sensitive.

#### **4. Discussion**

We have presented a modeling perspective on measuring latent cognitive mechanisms that give rise to choice behavior in the Cambridge Gambling Task (CGT). Specifically, we developed a suite of cognitive models that made different assumptions about how individuals make choices on the CGT, and we used hierarchical Bayesian modeling and Bayesian model comparison to identify the model that provided the best fit to empirical data collected from both healthy controls and multiple groups of individuals with histories of substance use disorders. Finally, we probed the best-fitting model to determine which cognitive mechanisms varied

across groups. The model that performed the best (CM model; coded as M12 in the SM<sup>19</sup>) used a slowly-growing (i.e. logarithmic), cumulative utility function, and it demonstrated expected differences among the groups in its proxy measure of impulsivity ( $\beta$ ). In particular, we found that all the substance dependent groups had *higher* estimated  $\beta$ 's than the healthy controls, an indication of greater impulsivity (i.e. greater discounting of delayed bets) within these groups than in controls. The CM model further demonstrated differences among substance using groups that are consistent with prior literature. For example, we found that amphetamine use was linked to lower probability distortion ( $\alpha$ ) relative to heroin use, which indicates less optimal color choices, and is consistent with elevated sensation seeking predicting amphetamine rather than heroin use (Ahn and Vassileva, 2016).

The models developed in this article were a preliminary attempt to model the CGT, and so there is certainly room for improvement. For instance, it may be that our proxy measure of impulsivity is too crude. It is arguable that our proxy measure conflates related but distinct constructs such as sensation-seeking, novelty-seeking, and various types of trait impulsivity, to name a few (Magid et al., 2007). Possible improvements could see how these different forms of impulsivity could be implemented into the model. In addition, because the functional form of the bet utility function of the CM is specific to loss sensitivity, we caution readers and those interested in employing the model that the CM risk aversion parameter should not be interpreted in the same manner as risk aversion traditionally measured in economic tasks. In particular, the risk aversion parameter is: (1) opposite to that of traditional measures of risk aversion, and (2)

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<sup>19</sup> Supplementary material can be found by accessing the online version of this paper at <http://dx.doi.org> and by entering doi: ...

estimated using an unorthodox function – which provided the best fit to all groups in the current study. Therefore, those interested in traditional measures of risk aversion should not rely wholly on our CGT model (more on this can be found in the SM<sup>20</sup>).

There are some limitations to this study that should be kept in mind. First, the design of the CGT complicates not only any design for a model, but also the interpretation of results from such a model. The design allows multiple cognitive components to decision-making to be assessed at once, in a fairly realistic gambling simulation; however, this possibly comes at the cost of “watering down” any possible assessment of these components individually. An example of this possibility is our measurements of risk aversion mentioned earlier; the unorthodox results and model suggest that the CGT may not give a reliable and valid measure of risk aversion as it is defined in the broader psychological and economic literature. If we accept that cognitive models lead to more reliable and valid results, then these comments easily apply to any traditional measures of the CGT as a corollary. Indeed, as the CGT engages a variety of cognitive processes that may not be reliably assessed, this may partially explain the inconsistent results in the literature. While the CGT may arguably give a more ecologically valid assessment of decision-making processes, this comes at the cost of lower reliabilities for any measures derived from it.

Finally, these results came from a rare group of *former* predominantly mono-dependent substance users, currently in protracted abstinence, which may explain in part our unorthodox results from the risk aversion analysis. It has been suggested that long-term recovery from

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<sup>20</sup> Supplementary material can be found by accessing the online version of this paper at <http://dx.doi.org> and by entering doi: ...

addiction may involve developing capacities such as unusually strong impulse control, that may exceed those of individuals who have never been addicted (Humphreys and Bickel, 2018). However, it is unclear if factors additional to substance use condition could have driven the results we observed, e.g. the gender imbalance in our sample (mostly male) or individual differences in history of substance use. Additionally, some controls (34) had a history of former cannabis use, which may also partially account for the non-significant results in the traditional analyses. Nonetheless, these results potentially testify to the long-term effects of chronic substance use on decision-making capabilities; while it is certainly possible that diminished decision-making capabilities could have caused the substance use in our sample, our experimental results are inadequate to determine any direction of causality. However, our results are congruent with a meta-analysis of the long-term effects of alcohol use disorder (Starvo et al., 2013) and opioid use disorder (Biernacki et al., 2016). Our sample being mostly in protracted abstinence may also explain why we only found a marginally non-significant difference among groups in the raw data, whereas others have found significant differences (Kräplin et al., 2014; Lawrence et al., 2009; Zois et al., 2014). Indeed, cognitive functioning has been shown to improve gradually with abstinence, at least for alcohol users (Starvo et al., 2013). Therefore, future research should determine if our results generalize to groups of active substance users.

## **5. Conclusion**

Cognitive modeling offers a powerful, valid alternative to traditional summary measures of behavioral performance on the CGT. We used hierarchical Bayesian modeling to develop and test multiple cognitive models on CGT data from healthy controls and several groups of substance users in protracted abstinence. Further, we used Bayesian model comparison to determine the best fitting, most generalizable model. To our knowledge, this is the first cognitive

model developed specifically for the CGT. We added the best fitting model to *hBayesDM*, an R package that will allow interested researchers to easily apply the CM model to data collected from the CGT (Ahn et al., 2017).

## **Appendix A. Supplementary Data**

Supplementary data associated with this article can be found at: DOI [10.17605/OSF.IO/YNGT6](https://doi.org/10.17605/OSF.IO/YNGT6) on the Open Science Framework.

## References

- Ahn, W. Y., Dai, J., Vassileva, J., Busemeyer, J. R., Stout, J. C. 2016. Computational modeling for addiction medicine: from cognitive models to clinical applications. *Prog. Brain Res.* 224, 53–65
- Ahn, W.-Y., Busemeyer, J. R. 2016. Challenges and promises for translating computational tools into clinical practice. *Curr. Opin. Behav. Sci.* 11, 1–7.
- Ahn, W.-Y., Haines, N., Zhang, L. 2017. Revealing neurocomputational mechanisms of reinforcement learning and decision-making with the hBayesDM package. *Comp. Psychiatry.* 8, 429–453.
- Ahn, W.-Y., Vassileva, J. 2016. Machine-learning identifies substance-specific behavioral markers for opiate and stimulant dependence. *Drug Alcohol Depend.* 161, 247–257.
- Baldacchino, A., Balfour, D. J. K., Matthews, K. 2015. Impulsivity and opioid drugs: differential effects of heroin, methadone and prescribed analgesic medication. *Psychol. Med.* 45, 1167–1179.
- Beauchaine, T. P., McNulty, T. 2013. Comorbidities and continuities as ontogenic processes: Toward a developmental spectrum model of externalizing psychopathology. *Dev. Psychopathol.* 25, 1505–1528.
- Beauchaine, T. P., Zisner, A. R., Sauder, C. L. 2017. Trait Impulsivity and the Externalizing Spectrum. *Annu. Rev. Clin. Psychol.* 13, 343–368.
- Bickel, W. K., Miller, M. L., Yi, R., Kowal, B. P., Lindquist, D. M., Pitcock, J. A. 2007. Behavioral and neuroeconomics of drug addiction: competing neural systems and temporal discounting processes. *Drug Alcohol Depend.* 90, S85-91.

- Biernacki, K., McLennan, S. N., Terrett, G., Labuschagne, I., Rendell, P. G. 2016. Decision-making ability in current and past users of opiates: A meta-analysis. *Neurosci. Biobehav. Rev.* 71, 342–351.
- Bowden-Jones, H., McPhillips, M., Rogers, R., Hutton, S., Joyce, E. 2005. Risk-taking on tests sensitive to ventromedial prefrontal cortex dysfunction predicts early relapse in alcohol dependency: a pilot study. *J. Neuropsychiatry Clin. Neurosci.* 17, 417–420.
- Busemeyer, J. R., Stout, J. C. 2002. A contribution of cognitive decision models to clinical assessment: decomposing performance on the Bechara gambling task. *Psychol. Assess.* 14, 253.
- Cognition, C. 2016. CANTAB®[Cognitive assessment software]. *All Rights Reserved Wwww. Cantab. com.*
- Czapla, M., Simon, J. J., Richter, B., Kluge, M., Friederich, H.-C., Herpertz, S., Mann, K., Herpertz, S. C., Loeber, S. 2016. The impact of cognitive impairment and impulsivity on relapse of alcohol-dependent patients: implications for psychotherapeutic treatment. *Addict. Biol.* 21, 873–884.
- Dai, J., Busemeyer, J. R. 2014. A probabilistic, dynamic, and attribute-wise model of intertemporal choice. *J. Exp. Psychol. Gen.* 143, 1489.
- Dai, J., Gunn, R. L., Gerst, K. R., Busemeyer, J. R., Finn, P. R. 2016. A random utility model of delay discounting and its application to people with externalizing psychopathology. *Psychol. Assess.* 28, 1198.
- Gelman, A., Rubin, D. B. 1992. Inference from iterative simulation using multiple sequences. *Statist. Sci.* 7, 457–472.

- Green, L., Myerson, J. 1993. Alternative frameworks for the analysis of self control. *Behavior and Philosophy*, 21, 37–47.
- Humphreys, K., Bickel, W. K. 2018. Toward a neuroscience of long-term recovery from addiction. *JAMA Psychiatry*. 75, 875 – 876.
- Johnson, M. W., Johnson, P. S., Herrmann, E. S., Sweeney, M. M. 2015. Delay and probability discounting of sexual and monetary outcomes in individuals with cocaine use disorders and matched controls. *PLoS One*, 10, e0128641.
- Kahneman, D., Tversky, A. 2013. Prospect theory: An analysis of decision under risk. In MacLean, L. C., Ziemba, W. T. (Eds.) *Handbook of the fundamentals of financial decision making: Part I*. World Scientific, Singapore, pp. 99–127.
- Kräplin, A., Dshemuchadse, M., Behrendt, S., Scherbaum, S., Goschke, T., Bühringer, G. 2014. Dysfunctional decision-making in pathological gambling: pattern specificity and the role of impulsivity. *Psychiatry Res.* 215, 675–682.
- Kruschke, J. 2014. *Doing Bayesian data analysis: A tutorial with R, JAGS, and Stan*. Academic Press, Cambridge.
- Lawrence, A. J., Luty, J., Bogdan, N. A., Sahakian, B. J., Clark, L. 2009. Problem gamblers share deficits in impulsive decision-making with alcohol-dependent individuals. *Addict.* 104, 1006–1015.
- Magid, V., MacLean, M. G., Colder, C. R. 2007. Differentiating between sensation seeking and impulsivity through their mediated relations with alcohol use and problems. *Addict. Behav.* 32, 2046–2061.
- Mazur, J. E. 1987. An adjusting procedure for studying delayed reinforcement. *Commons, ML.; Mazur, JE.; Nevin, JA*, 55–73.



- Miedl, S. F., Büchel, C., Peters, J. 2014. Cue-induced craving increases impulsivity via changes in striatal value signals in problem gamblers. *J. Neurosci.* 34, 4750–4755.
- Monterosso, J., Ehrman, R., Napier, K. L., O'Brien, C. P., Childress, A. R. 2001. Three decision-making tasks in cocaine-dependent patients: Do they measure the same construct? *Addict.* 96, 1825–1837.
- Myerson, J., Green, L. 1995. Discounting of delayed rewards: Models of individual choice. *J. Exp. Anal. Behav.* 64, 263–276.
- Neufeld, R. W. J. 2015. Mathematical and computational modeling in clinical psychology. In Busemeyer, J. R., Townsend, J. T., Wang, Z., Aidels, A. (Eds.) *The Oxford Handbook of Computational and Mathematical Psychology*. Oxford, UK, pp. 341.
- Passetti, F., Clark, L., Mehta, M. A., Joyce, E., King, M. 2008. Neuropsychological predictors of clinical outcome in opiate addiction. *Drug Alcohol Depend.* 94, 82–91.
- Reynolds, B. 2006. A review of delay-discounting research with humans: relations to drug use and gambling. *Behav. Pharmacol.* 17, 651–667.
- Robbins, T. W., Gillan, C. M., Smith, D. G., de Wit, S., Ersche, K. D. 2012. Neurocognitive endophenotypes of impulsivity and compulsivity: towards dimensional psychiatry. *Trends Cogn. Sci.* 16, 81–91.
- Rogers, R. D., Everitt, B. J., Baldacchino, A., Blackshaw, A. J., Swainson, R., Wynne, K., Baker, N. B., Hunter, J., Carthy, T., London, M., Deakin, J. F. W., Sahakian, B. J., Robbins, T. W. 1999. Dissociable deficits in the decision-making cognition of chronic amphetamine abusers, opiate abusers, patients with focal damage to prefrontal cortex, and tryptophan-depleted normal volunteers: evidence for monoaminergic mechanisms. *Neuropsychopharmacol.* 20, 322–339.

- Shanks, D. R., Tunney, R. J., McCarthy, J. D. 2002. A Re-examination of probability matching and rational choice. *J. Behav. Dec. Making*. 15, 233 – 250.
- Sørensen, L., Sonuga-Barke, E., Eichele, H., van Wagneningen, H., Wollschlaeger, D., Plessen, K. J. 2017. Suboptimal decision making by children with ADHD in the face of risk: Poor risk adjustment and delay aversion rather than general proneness to taking risks. *Neuropsychol.* 31, 119.
- Stavro, K., Pelletier, J., Potvin, S. 2013. Widespread and sustained cognitive deficits in alcoholism: a meta-analysis. *Addict. Biol.* 18, 203–213.
- Stout, J. C., Busemeyer, J. R., Lin, A., Grant, S. J., Bonson, K. R. 2004. Cognitive modeling analysis of decision-making processes in cocaine abusers. *Psychon. Bull. Rev.* 11, 742–747.
- Turner, B. M., Rodriguez, C. A., Liu, Q., Molloy, M. F., Hoogendijk, M., McClure, S. M. 2018. On the neural and mechanistic bases of self-control. *Cereb. Cortex*, 29, 732–750.
- Vassileva, J., Paxton, J., Moeller, F. G., Wilson, M. J., Bozgunov, K., Martin, E. M., Gonzalez, R., Vasilev, G. 2014. Heroin and amphetamine users display opposite relationships between trait and neurobehavioral dimensions of impulsivity. *Addict. Behav.* 39, 652–659.
- Vehtari, A., Gelman, A., Gabry, J. 2017. Practical Bayesian model evaluation using leave-one-out cross-validation and WAIC. *Stat. Comput.* 27, 1413–1432.
- Von Neumann, J., Morgenstern, O. 1944. Theory of Games and Economic Behavior. Princeton University Press, Princeton, New Jersey, USA.
- Wagenmakers, E.-J., Marsman, M., Jamil, T., Ly, A., Verhagen, J., Love, J., Selker, R., Gronau, Q. F., Šmíra, M., Epskamp, S., Matzke, D., Rouder, J. N., Morey, R. D. 2018. Bayesian

inference for psychology. Part I: Theoretical advantages and practical ramifications.

*Psychon. Bull. Rev.* 25, 35–57.

Wiecki, T. V., Poland, J., Frank, M. J. 2015. Model-based cognitive neuroscience approaches to computational psychiatry: clustering and classification. *Clin Psychol Sci.* 3, 378–399.

Wilson, M. J., Vassileva, J. 2016. Neurocognitive and psychiatric dimensions of hot, but not cool, impulsivity predict HIV sexual risk behaviors among drug users in protracted abstinence. *Am. J. Drug Alcohol Abuse.* 42, 231–241.

Wilson, M. J., Vassileva, J. 2018. Decision-Making Under Risk, but Not Under Ambiguity, Predicts Pathological Gambling in Discrete Types of Abstinent Substance Users. *Front. Psychiatry.* 9. 1 – 10.

Wu, M.-J., Mwangi, B., Bauer, I. E., Passos, I. C., Sanches, M., Zunta-Soares, G. B., Meyer, T. D., Hasan, K. M., Soares, J. C. 2017. Identification and individualized prediction of clinical phenotypes in bipolar disorders using neurocognitive data, neuroimaging scans and machine learning. *NeuroImage.* 145, 254–264.

Yechiam, E., Busemeyer, J. R., Stout, J. C., Bechara, A. 2005. Using cognitive models to map relations between neuropsychological disorders and human decision-making deficits. *Psychol. Sci.* 16, 973–978.

Zois, E., Kortlang, N., Vollstädt-Klein, S., Lemenager, T., Beutel, M., Mann, K., Fauth-Bühler, M. 2014. Decision-making deficits in patients diagnosed with disordered gambling using the Cambridge Gambling task: the effects of substance use disorder comorbidity. *Brain Behav.* 4, 484–494.

## TABLES FOR CGT PAPER

Table 1.

Parameter	Range of Values	Interpretation
$\alpha$	$0 \leq \alpha \leq 5$	Color probability weighting
$\rho$	$0 \leq \rho < +\infty$	Risk aversion
$c$	$0 \leq c \leq 1$	Bias for choosing RED
$\beta$	$0 \leq \beta < +\infty$	Impulsivity
$\gamma$	$0 \leq \gamma < +\infty$	Variability/noise in betting choice

**Table 1 Caption: The parameters in the model.** A compendium of the parameters in the models. The second column gives the permissible values for that particular parameter (note that  $\beta$  and  $\gamma$  are not bounded above). The final column gives the psychological interpretation of the parameter based on how its values affect the output of the models. See section 2 for more details.

Table 2:  
Demographics and Behavioral Measures for the Cambridge Gambling Task

Measure	Controls	Heroin	Amphetamine	Polysubstance
<i>N</i>	124 (+0)	75 (+3)	73 (+3)	98 (+5)
Age	26.10 (6.17)	30.13 (4.84)	24.53 (4.92)	27.28 (5.61)
Percent Male	62.90%	73.33%	65.75%	81.63%
Education	14.20 (2.88)	13.00 (2.48)	13.38 (2.25)	13.11 (2.49)
Last Alcohol	44.88 (188.07)	233.31 (557.45)	25.99 (82.70)	155.18 (262.63)
Last Drug	1013.2 (1340.0)	713.0 (940.9)	281.5 (1060.4)	344.8 (640.1)
Delay Aversion	.29 (.23)	.37 (.21)	.33 (.18)	.32 (.21)
Deliberation Time	2309.7 (691.8)	2291.4 (680.5)	2456.2 (792.5)	2419.2 (800.3)
Overall Proportion Bet	.52 (.14)	.54 (.13)	.56 (.12)	.57 (.14)
Quality of Decision Making	.89 (.12)	.86 (.15)	.87 (.11)	.87 (.13)
Risk Adjustment	1.08 (0.92)	.96 (.88)	.96 (.78)	.93 (.76)
Risk-Taking/Risk+ *	<b>.56 (.15)</b>	.58 (.14)	.60 (.13)	<b>.61 (.14)</b>

\* ANOVA reported  $p < .05$ .

**Caption Table 2: Participant demographics and Behavioral Indices.** Demographics, substance use, and behavioral data of the substance-using groups and controls. The statistics are given as: Mean (Standard Deviation). ‘Last Alcohol’ represents how many days since the participant had alcohol; similarly for ‘Last Drug.’ Delay Aversion (or, IMP in the text) is the difference in average betting ratio (including only the optimal trials) between the ascending and descending conditions. Deliberation time is the time from the start of the trial to the bet choice. Overall Proportion Bet is the average betting ratio across all trials, including non-optimal trials and trials on which the number of blue and red boxes were equal. Quality of Decision Making is

the proportion of trials across both conditions wherein the optimal color was chosen. Risk Adjustment captures the tendency for participants to bet more of their points when the odds are in their favor. Risk-Taking/Risk+ is the average of the betting ratios chosen across both conditions, and only counting trials where the optimal color was chosen. Note that the added numbers in parentheses in the *N* row represent the number of participants wherein CGT behavioral data are available but demographic data are missing. Controls have a non-zero value in 'Last Drug' because they were allowed to have used cannabis in the past, while still being eligible for study. Numbers in bold represent a significant difference after a Bonferroni correction.

FIGURES FOR THE CGT

Figure 1. Stages of the CGT

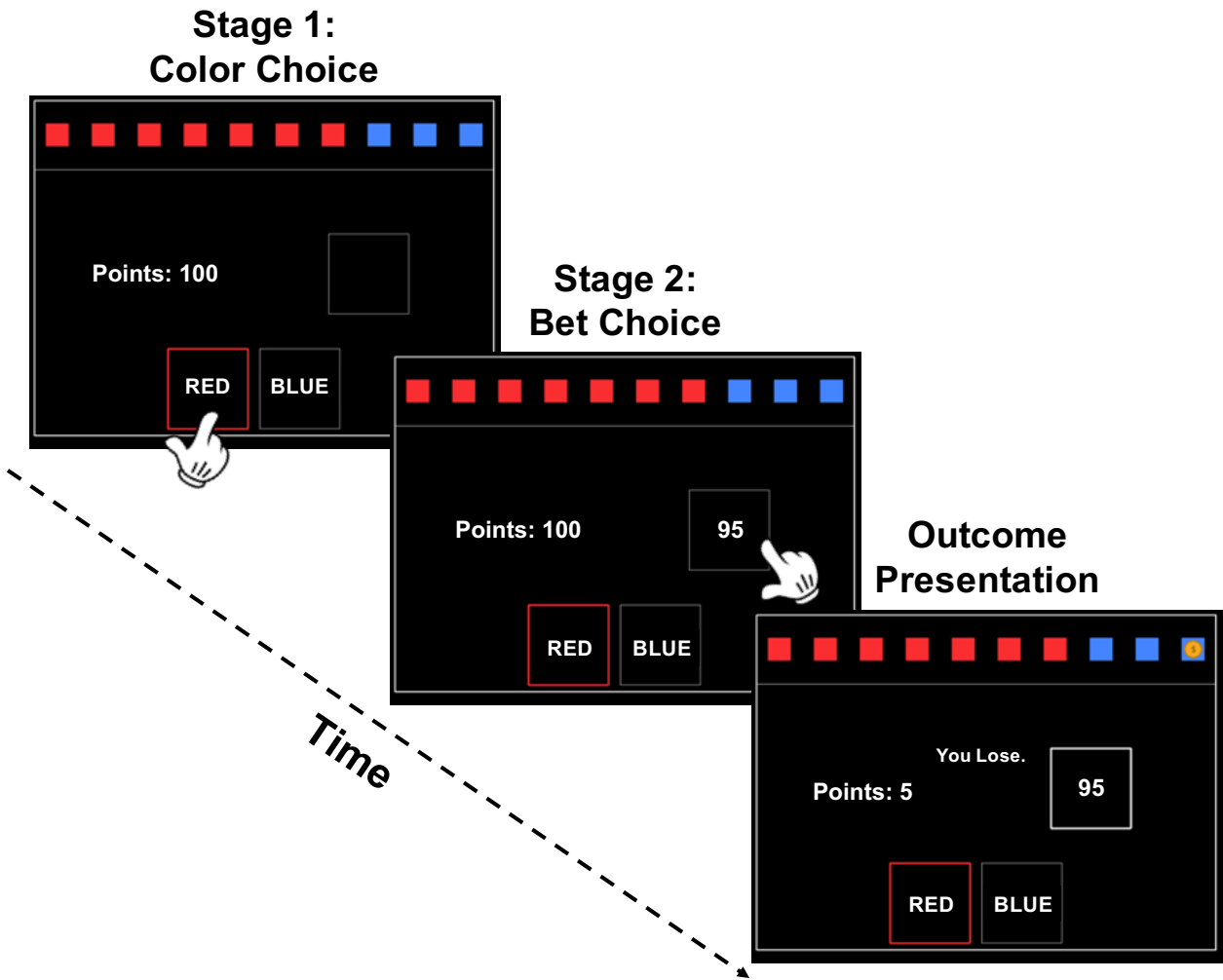


Figure 2. Comparison of Posterior Predictive Simulations versus Actual Average Choices.

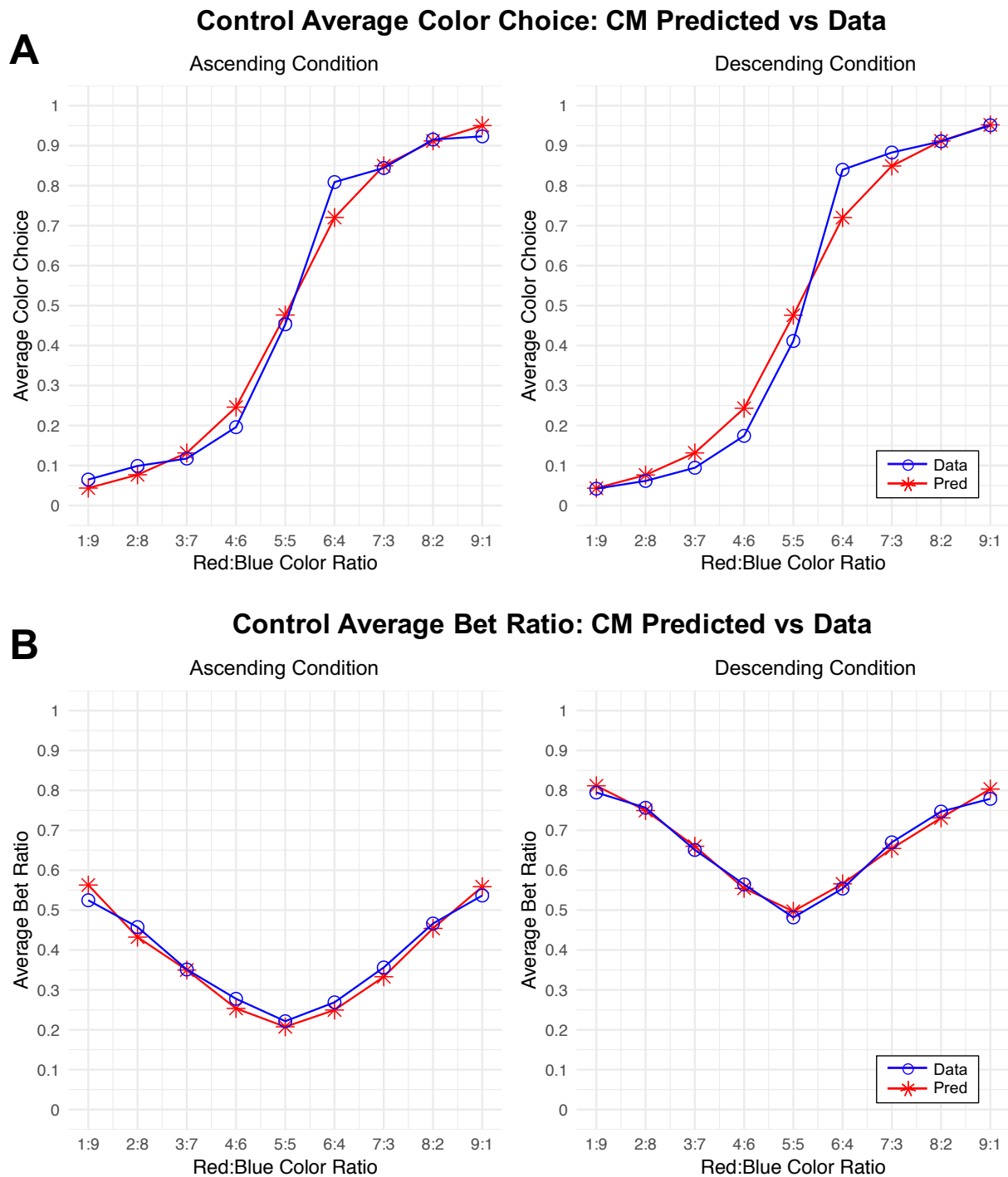




Figure 3. Posterior Distributions for the Means of Each Parameter Across Groups.

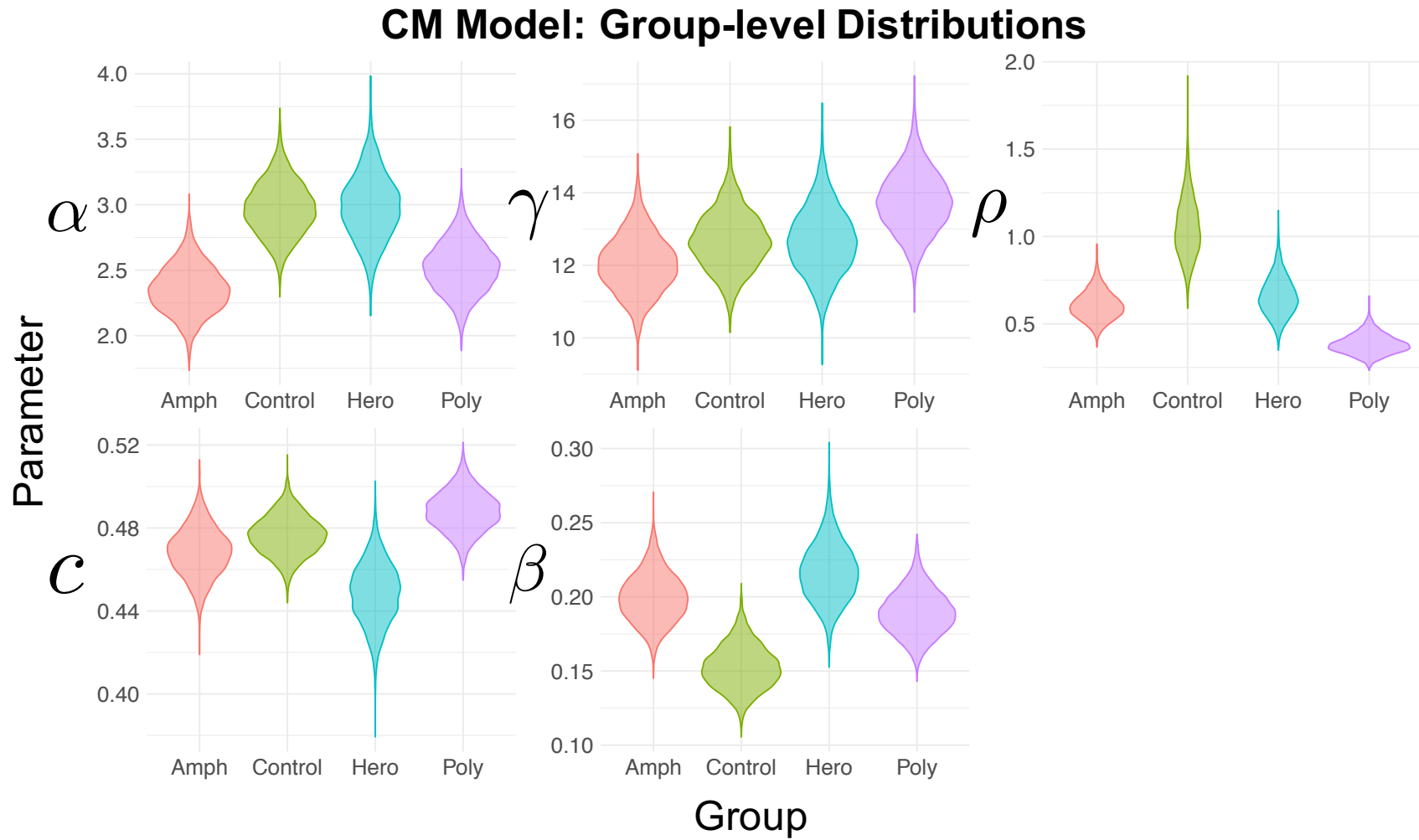


Figure 4. Group Differences in Posterior Estimates of Parameters Using Model 12

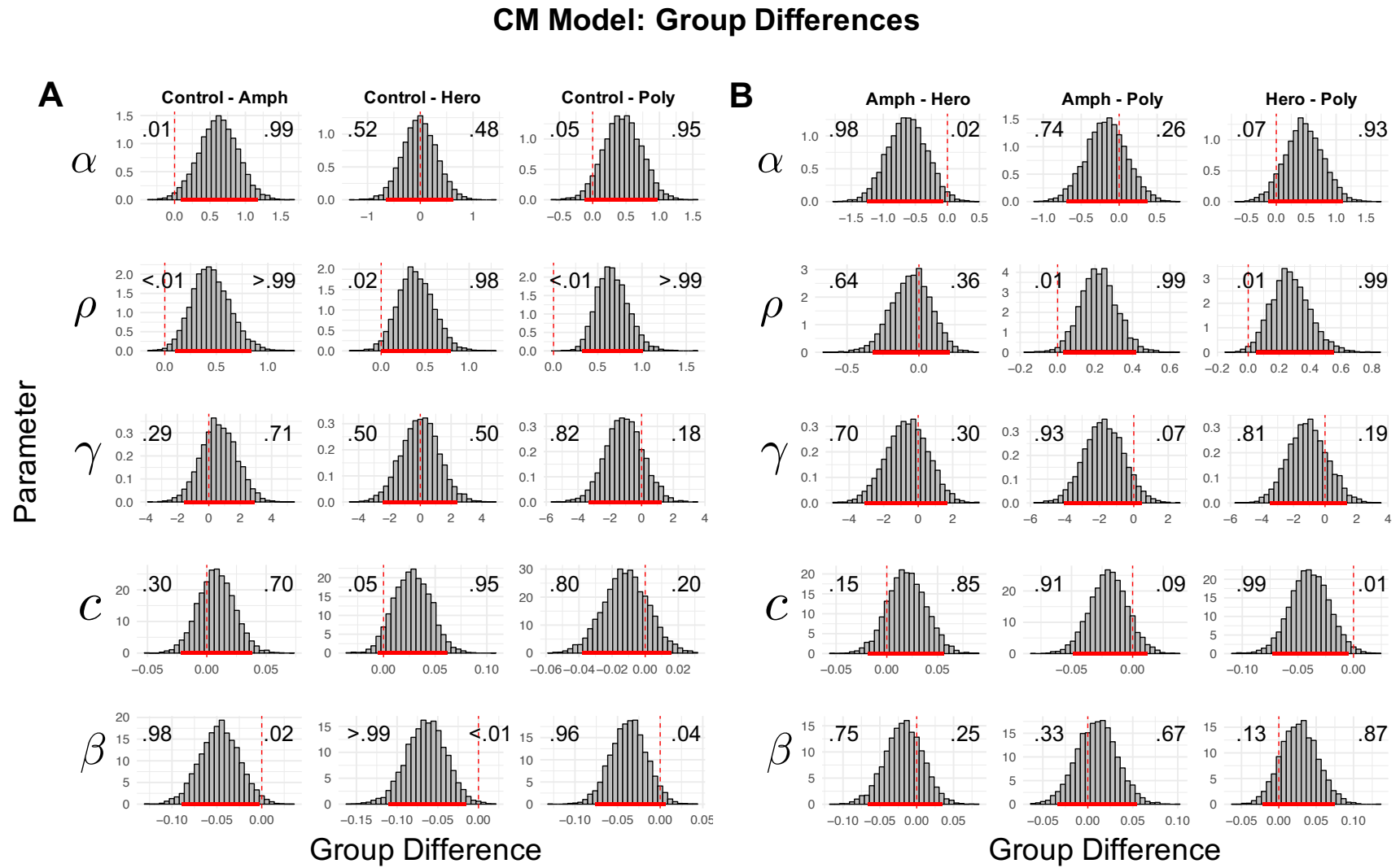
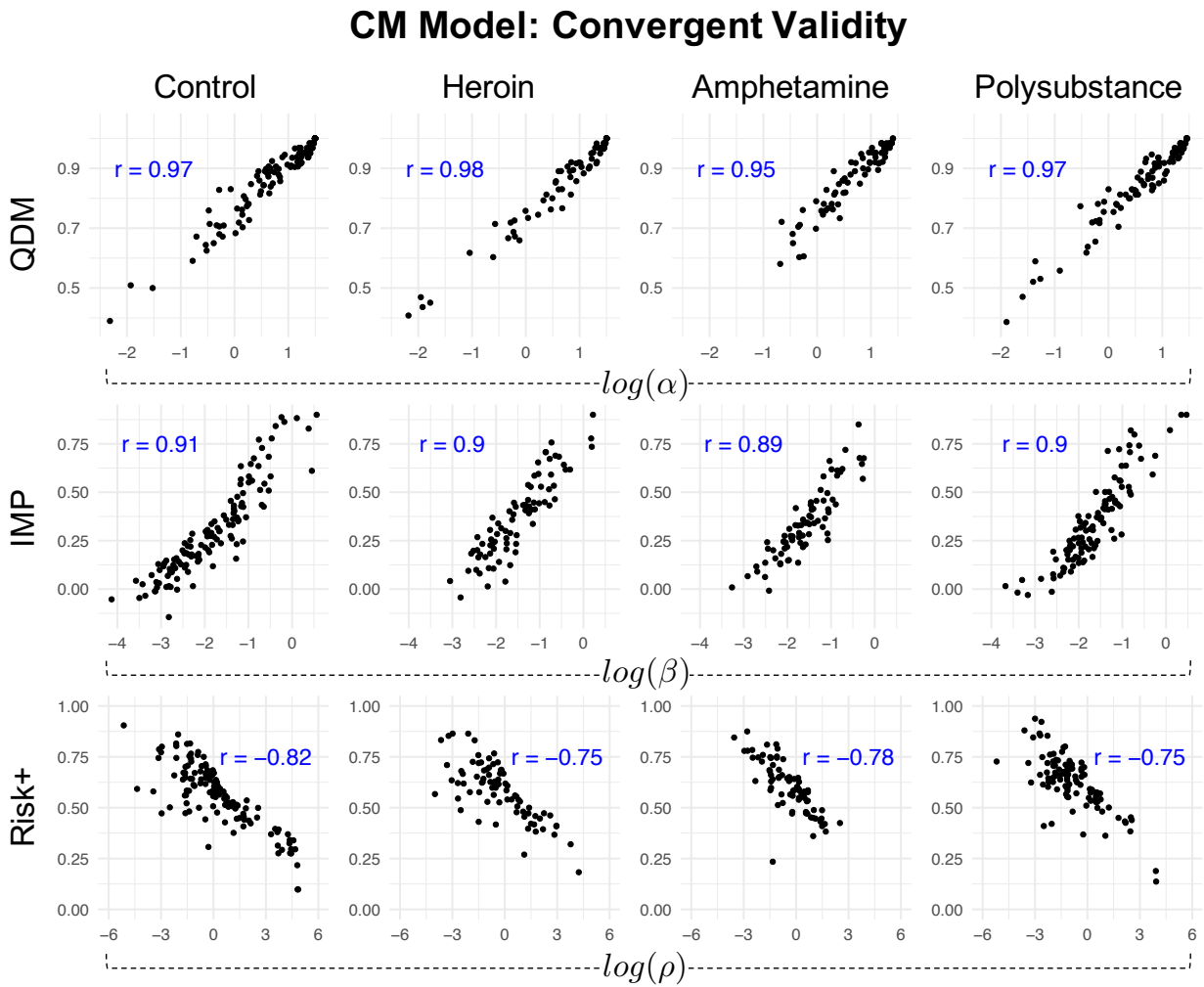


Figure 5. Convergent validity between traditional metrics and cognitive model parameters



**Figure 1 Caption:**

The CGT progresses through two stages before the outcome of the token (and subsequently of the chosen bet) is revealed. First, participants must guess which color hides the token. Once a color is chosen, bet proportions (.05, .25, .5, .75, .95) are shown in either ascending or descending order, where each bet is shown for 5 seconds in total. Note that the “bet box” shows the number of points that can be gained/lost for each bet proportion rather than the bet proportions themselves.

**Figure 2 Caption:**

Posterior predictive simulations generated from the best fitting model (CM with log valuation function; Model 12 in the Supplementary Text). Simulations from the model were generated for each individual after fitting the model, conditional on their actual choices. Resulting individual-level predictions were then averaged across participants within groups and plotted against the actual behavioral choices averaged across participants within groups. Note that these graphs depict only healthy control data, though results were consistent across groups. We refer the reader to S2 and S3 Tables for comparison of posterior predictive simulations across models and groups. **(A)** Simulations versus actual behavioral data for color choices. Note that a choice of blue was coded as 0, and a choice of red was coded as 1. Thus, averages close to zero indicate more choices of blue, and those closer to 1 indicate more choices of red. **(B)** Simulations versus actual behavioral data for bet choices. The y-axis here now represents the average *betting ratio* that was chosen versus predicted. Note that while the shapes for the ascending and descending cases are similar, the descending graph is shifted upward relative to the ascending graph—this difference indicates impulsivity and is captured by  $\beta$  in the CM model.

**Figure 3 Caption:**

The posterior densities (distributions) for the means of each parameter, color-coded by group.

Note that parameters are estimated from the CM model with a log valuation function (Model 12 in the Supplementary Text).

**Figure 4 Caption:**

(A) Posterior distribution of differences in estimated parameters between groups, using the best-fitting model (CM model with log valuation function; termed M12 in Supplementary Text). Each column shows the difference distributions between Controls and one of the substance-using groups. These differences are comparing the mean-level hyperpriors for each parameter. The solid red bars indicate the 95% Highest Density Interval (HDI) for each difference, meaning where 95% of the “mass” of the posterior distribution lies. Note that we do not endorse binary interpretations of significance using HDIs. The vertical dashed lines show where zero lies in each graph. The numbers overlaying each subplot indicate the proportion of the posterior distribution above and below 0, which can be interpreted as the model-estimated probability of a difference in parameter values between groups (where .50 indicates no evidence for a difference, and more peaked and diffuse distributions indicate more and less certainty in the difference, respectively). (B) Posterior distributions of the differences in cognitive model parameters between substance use groups. Solid and dashed lines and overlaying numbers are interpreted in the same way as for (A).

**Figure 5 Caption:**

Convergent validity between traditional measures of QDM, IMP, and Risk+ and corresponding cognitive model parameter alternatives. The  $\alpha$  parameter captures distortion of color choice probabilities, which leads to more or less optimal color choices and is analogous to traditional measures of QDM. The  $\beta$  parameter captures how much participants discount the utility of each bet with the passing of time and is analogous to traditional measures of IMP. The  $\rho$  parameter captures risk sensitivity, where values of  $\rho < 1$  indicate less sensitivity to losses than gains and values of  $\rho > 1$  have the opposite interpretation. Therefore,  $\rho < 1$  reflects risk-seeking behavior, and  $\rho > 1$  indicates risk-averse behavior. Note that the correlations between  $\rho$  parameters and Risk+ scores are negative because  $\rho$  is specific to losses, so the interpretation is reversed. The cognitive model parameters are log-transformed. All  $p$ -values  $\ll 0.001$ .

**Supplementary material for Romeu et al. “A computational model of the Cambridge Gambling Task with applications to substance use disorders”**

**Introduction**

In this supplementary materials section, we expand on developments in the main paper. The headings are as follows:

- (1) Mathematical Description of the models (p. 1)
- (2) Constant Relative Risk Model (p. 10)
- (3) Hierarchical Bayesian Modeling & Bayesian Model Comparison (p. 14)
- (4) Table of Models (p. 17)
- (5) Validity & Response Time Modeling (p. 21)
- (6) More Details on the Methods (p. 24)
- (7) Sensitivity Analysis (p. 27)
- (7) Extra References (p. 29)

**Mathematical Description of Models**

Here we present the details of the cognitive models we tested in the main text: the Cumulative model (CM) and the Immediate model (IM). The interpretation of each parameter will be somewhat repeated here, so that the reader may link the interpretation with the parameter’s action within the model.

We begin this section by first stating explicitly what we are predicting with these models. We can consider the “output” on each trial to be an ordered pair of the color choice on that trial and the betting ratio choice on that same trial. We notate this as:  $(k_t, b_t)$ , where we use  $k$  for color on trial  $t$  and  $b$  for the betting ratio on trial  $t$ . Our models should give us a probability that a particular ordered pair should appear. That is, for each trial, we want to predict:  $P(b_t \cap k_t)$ , or

the probability of observing both the bet and color choice on trial  $t$ . This is equivalent to predicting  $P(b_t | k_t)P(k_t)$ , which is the form more conducive to analysis (since we will use the color choice to predict the betting choice). That is, we want a joint probability distribution over the bet and color choices.

In what follows, we first present the general structure of the two models by walking through what we call the *cumulative model* (CM). Here we give the denser explanations for the parameters. Our second model, the *immediate model* (IM), is quite similar to the CM, so we only emphasize where they differ, the bulk of the explanation being exactly the same as the CM.

Within both the IM and CM forms of the model, we additionally tested: (1) power versus log valuation functions, and (2) the use of subjective versus objective color utilities/proportions in weighting the bet values. Because the utilities of the CM only have positive support (i.e. bet utilities cannot be negative) and each bet involves a potential gain and loss state (i.e. winning or losing money relative to the current amount), we further tested different settings for  $\rho$  in the CM to avoid difficulties in interpreting the  $\rho$  parameter (more details below).

### *The Cumulative Model*

The CM begins at the color choice on a given trial. The goal of the color choice model is to translate the “raw data” of the number of colored boxes a participant sees on a given trial into a probability weight in a meaningful way. We first establish some notation:

Let  $r \in [0, 1]$  denote the *proportion* of red boxes on display, with  $u = 1 - r \in [0, 1]$  as the proportion of blue boxes. These values we term the *objective* color values, the probability that a token will be hidden by color  $i$ , and we note that

$$P_{obj}(red) \equiv r = \frac{r}{r + u}. \quad (1)$$



Now, we introduce the *probability distortion*<sup>21</sup> parameter  $\alpha \in [0, 5]$  to the right-most side of equation (1) to allow for an inexact assessment of probability:

$$P(\text{red}) \equiv P(R) := \frac{r^\alpha}{r^\alpha + u^\alpha}. \quad (2)$$

Note that there is no distortion when  $\alpha = 1$ , and that as  $\alpha \rightarrow 0$ , color choice becomes completely random (i.e.,  $P(R) = P(B) = \frac{1}{2}$ ). As we let  $\alpha \rightarrow 5$ , then the probabilities become extreme. For example, with  $\alpha = 5$ ,  $r = .9$ , we have that  $P(R) \approx 1$ , so that the participant sees 9 red boxes as near certainty of being the better color to choose.

Lastly in the color choice model, we add a color bias parameter  $c \in [0, 1]$ . This parameter accounts for any bias for red over blue (if  $c = 1$ , red will be chosen with probability 1). Formally, and here we arrive at the form of the model used throughout the rest of the paper, we have that:

$$P(R) := \frac{cr^\alpha}{cr^\alpha + (1-c)u^\alpha}. \quad (3)$$

---

<sup>21</sup> This may not be the best name for the function of this parameter, but ‘distortion’ was chosen because in most of its domain of values, this is exactly what  $\alpha$  does. We can also consider  $\alpha$  as a choice variability parameter in the context of the color choice, as a higher value indicates choosing the higher-proportioned color with probability 1, and values near 0 indicate completely uniformly random choice. However, calling  $\alpha$  a choice variability parameter would introduce the possibility of confusion, since another important parameter later on is given the name ‘choice variability’, but in the context of the betting choices. This latter choice variability turns out to be more useful in analysis, and so we save that name for it. Thus, while not totally satisfactory, we leave the name as ‘distortion’ for  $\alpha$ .

(Here,  $R$  represents the event of choosing “red,” while  $r$  represents the proportion of red boxes on that trial.)

This form of the color choice model was inspired from that introduced in Gonzalez and Wu (1999); we note, though, that our introduction of a bias parameter is slightly different from how it is introduced in Gonzalez and Wu (1999).

Observe that the parameters give us a more detailed insight into *why* someone would choose a non-optimal color. The data of the colored boxes is distorted in the assessment, leading to non-optimal results, and this distortion could be from differing levels of appreciation of the objective values or from a bias for one color over the other.

Next, given the color choice predicted from the model, we then predict the betting ratio. The core of the CM is that the probability of a certain betting choice is dependent upon the expected value of the utilities of each outcome (win or loss) of a certain bet. These outcomes are then used to calculate a probability. We can start by giving the general formula:

$$EU(Bet | Color) := P(Color)u(X) + (1 - P(Color))u(Y), \quad (4)$$

where  $X$  and  $Y$  are values to be expanded upon soon,  $u(\bullet)$  is the utility function, and “Color” indicates the information given to us from the previous step in the overall model. That is to say, the  $P(Color)$  is simply the objective probability of the color that was actually chosen, and  $1 - P(Color)$  is the complement probability. For instance, if the participant chose “red,” then the equation would read:  $P(Red) * u(X) + P(Blue) * u(Y)$ , whereas if “blue” were chosen, then the equation would read:  $P(Blue) * u(X) + P(Red) * u(Y)$ , so that the color choice determines which utility is weighted by the objective probabilities of red and blue.

In the CM, we define:

$$\begin{aligned} X &:= C + Cb; \\ Y &:= C - Cb, \end{aligned} \tag{5}$$

where  $C$  is the “capital,” or the number of points, prior to the start of trial  $t$ , and  $b$  is the betting ratio,  $b \in \{.05, .25, .5, .75, .95\}$ . For instance,  $C = 100$  at the start of each block. In the model, we scale the capital so that 100 points in the task correspond to 1 point in the model (we do this for computational efficiency). Thus, the CM compares the utility of the *cumulative* change in capital (hence the name), and thus, since we can never have negative capital in this task, the utility function for the CM only accepts positive real numbers. One of the specific utility functions we use is the power function:

$$u_{CM}(x) := x^\rho, \tag{6}$$

where  $x \geq 0$ , and we set  $0 \leq \rho \leq 2$ . Such a utility function is common in decision modeling (see, e.g., Kahneman and Tversky, 1979; Luce, 1959; Wallsten et al., 2005). Under the power function of utility, a  $\rho$  value under 1 indicates risk-aversion (a concave utility function), whereas a value greater than 1 indicates risk-seeking (a convex utility function). Intuitively, we may think of the changes as follows: a smaller  $\rho$  decreases the rate of change in utility, so that the difference between two amounts will go to 0 as  $\rho \rightarrow 0$  (take the derivative of the power function and let  $\rho \rightarrow 0$ ).

The IM has the same utility function, but it must be extended to deal with negative values, since losses have a more prominent role in the IM. This adjustment is simply given as:

$$u_{IM}(x) := \begin{cases} x^\rho & x \geq 0 \\ -\delta |x|^\rho & x < 0 \end{cases}. \tag{7}$$

The function of  $\delta$  (which we bound as  $0 \leq \delta \leq 2$ ) is to incorporate loss aversion into the model; the parameter exaggerates the absolute value of the loss, so that a loss becomes more painful than a gain feels good.

Equation (7) gives the IM one more parameter than the CM, the *loss aversion* parameter  $\delta \geq 0$ . With a loss aversion of  $\delta > 1$ , we then have that, for  $x > 0$ ,  $|u(-x)| > u(x)$ , indicating that a loss of, say, \$50 is more painful than a gain of \$50 is enjoyable. For  $\delta < 1$ , we have the opposite effect, wherein, for  $x > 0$ ,  $|u(-x)| < u(x)$ , indicating that a gain is more enjoyable than a loss is painful. Obviously, a  $\delta = 1$  indicates that losses are judged equivalent in utility to their gainful counterpart, i.e.  $|u(-x)| = u(x), \forall x \in \mathbb{R}$ . Such a parameter is important, since prior work has demonstrated that losses loom larger than equivalent gains (Kahneman and Tversky, 1979). The IM then has six parameters: all those that the CM has, plus the loss aversion ( $\delta$ ) parameter.

As mentioned in the main text, we tried another version of the utility function, which here we call the log utility function. We tried this version because we were worried that the power function might grow too quickly for the number of points a participant could win from trial to trial. The log utility function is given by the following:

$$u_{CM}(x) := \log(1 + \rho x), \quad (8)$$

where  $\rho \geq 0$  is our risk aversion parameter, as before<sup>22</sup>. In a typical utility function (i.e. the power function discussed above), as  $\rho \rightarrow 0$ , the difference in utility between any two values

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<sup>22</sup> We add a “+1” to the argument of the log function in order to avoid the singularity of “log(0).”

While the actual number of points rarely got close to zero, we still have the possibility of effectively getting a 0 in the argument if our function were  $\log(\rho x)$ , since a low value for  $\rho$

outside a neighborhood of zero becomes negligible, and as  $\rho \rightarrow 2$ , the utilities have quadratic growth. The main purpose of this utility function is that the log function is known to grow slower than any polynomial utility function (which includes the standard power function). While these parameters do not change the concavity of the log function used here, as is the case for the power function, the parameter changes affect the *rate of change* of the log function, effectively giving us a similar interpretation of the parameter  $\rho$ , as in the power utility function.

To see this, note that:

$$\frac{d}{dx} \log(1 + \rho x) = \frac{\rho}{1 + \rho x} = \frac{1}{\rho^{-1} + x}. \quad (9)$$

Now, if  $\rho_1 < \rho_2$ , we then have:

$$\rho_1^{-1} > \rho_2^{-1} \Rightarrow x + \rho_1^{-1} > x + \rho_2^{-1} \Rightarrow (x + \rho_1^{-1})^{-1} < (x + \rho_2^{-1})^{-1}.$$

This means that the derivative of our utility function, thought of now as a function of  $\rho$  with  $x$  fixed, is a monotonically increasing function of  $\rho$ , so that higher  $\rho$ 's mean greater increases in utility over amount.

The IM version of the log function was chosen to mimic the form of the power version in the following way:

$$u_{IM}(x) := \begin{cases} \log(1 + \rho x) & x \geq 0 \\ -\delta \log(1 + \rho |x|) & x < 0 \end{cases} \quad (10)$$

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would make the function value blow up in absolute value. In order to be safe, and to make sure that the singularity is always avoided, we simply bound the function away from zero; the simplest way to do this is to add 1 to the argument. Then, having  $\rho x \approx 0$ , we have

$\log(1 + \rho x) \approx \log(1 + 0) = \log(1) = 0$ , so no singularity.

We have not, technically, changed the utility function; the CM is simply not defined for the bottom half of (10) since it cannot accept negative values (see Kahneman and Tversky, 1979).

The loss aversion parameter  $\delta$  is exactly as in the power version. Note that we add a 1 inside the log function so that our function is defined for all non-negative  $x$ . This also means that our log utility function will be non-negative for all non-negative  $x$ , and similarly will be non-positive for all non-positive  $x$  (and exactly 0 when  $x = 0$ ). Note that the best fitting model was the Cumulative Model (CM) with a log function for utility such that  $\rho = 1$  for gains and  $\rho \geq 0$  is a free parameter for losses (known as M12 in S1 Table).

We assume in the model that the order in which the bets are displayed affects their expected utilities. That is, for each bet, we assume that there is a *cost of time* to wait for each betting option to appear (recall that in a typical experiment, each betting ratio is displayed for at most five seconds). We assume that this wait time negatively affects the expected utility of a given bet, and so we subtract an assumed cost of time to the utility prior to assigning a probability to the bet choice.

Toward this end, we developed a “cost system” in the following way. We simply think of the cost system as a linear function of the *position* of the bet in the trial, and, when the slope of this linear function is 1, we arbitrarily anchor the first position at 0 and the last position at 1. Hence, the “cost” for the first bet is 0 (since it can be immediately chosen after a color choice) and the “cost” for the last option is 1. We evenly spaced the rest of the options so that we can represent our cost function as:

$$COST(bet\_position) := (position - 1) / 4. \quad (11)$$

We then introduce the slope of this function as the impulsivity parameter  $\beta \geq 0$ , so our cost function is as in (Equation 11), but now multiplied by  $\beta$ . In this way, since the maximal value is

1 for the cost function (before we introduce  $\beta$ ), we can actually identify the maximal cost with the slope of the function. Thus, our general cost function is given by:  $f(position) = \beta \cdot COST(position)$ .

We then arrive at the final utility we use to assign a probability:

$$Z(bet | Color) := EU(bet | Color) - \beta * f(bet\_position) \quad (12)$$

The  $\beta$  parameter is a *proxy measure of impulsivity*, wherein a larger  $\beta$  value implies a higher level of impulsivity, since a larger  $\beta$  indicates a larger cost of time to the participant (larger maximal cost). To transform these adjusted utilities into corresponding probabilities for the given betting ratios, we employ the soft-max rule<sup>23</sup> (sometimes called the “ratio-of-strengths” rule; see Busemeyer and Stout, 2002; Luce, 1959):

$$P(b_i | k_i) = \frac{\exp\{\gamma Z(b_i | k_i)\}}{\sum_j \exp\{\gamma Z(b_j | k_i)\}}; \quad (13)$$

that is, our soft-max rule states that the probability of a bet choice given the color choice is dependent upon the relative “strength” of the expected utility of that bet compared to the sum of expected utilities across all possible bets. In this way, the betting option with the highest expected utility is most likely to be chosen. In the above, we introduce a new parameter, the

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<sup>23</sup> We also tried a different error model where, instead of the exponential term in eq. 13 (i.e.  $\exp\{\gamma Z(b_i | k_i)\}$ ), we used a Fechnerian term that “contextualizes” utilities (i.e.  $\exp\{Z(b_i | k_i)/d/\gamma\}$ , where  $d = \max(Z(b_i | k_i)) - \min(Z(b_i | k_i))$ ; see Drichoutis and Lusk, 2014), as suggested by one reviewer. However, using M12 with this Fechnerian rule performed worse than M12 with Luce’s rule.

*choice consistency* (or choice variability, as referenced earlier) parameter  $\gamma \geq 0$ , where a larger  $\gamma$  indicates that the participant's choices are more or less deterministic (since a large value will have one expected utility "dominate" the others in strength). A choice consistency near zero indicates that the choices are made approximately uniformly at random.

In sum, for the CM we have five parameters:  $\alpha$  (probability distortion),  $c$  (color bias),  $\gamma$  (choice consistency),  $\rho$  (risk aversion/seeking), and  $\beta$  (impulsivity proxy measure). We then finally have the two pieces needed to predict the ordered pair of choices using the CM. Next, we highlight where the IM deviates from what has been presented above.

### *The Immediate Model*

The IM is identical to the CM, save for one exception. In Equation (5), take:

$$\begin{aligned} X_{IM} &:= Cb \\ Y_{IM} &:= -Cb \end{aligned} \tag{14}$$

That is, the IM compares the utilities of the immediate changes in wealth associated with each outcome of winning or losing the bet, in line with the predictions of prospect theory (Kahneman and Tversky, 1979). (Of course, this causes us to extend our utility function as described above.) That is, we consider the current capital  $C$  as the reference point, and the possible gains or losses as deviations from this reference point. We can categorize these two models by saying that the CM compares *cumulative* changes in wealth associated with each outcome, whereas the IM compares the *immediate* changes in wealth (hence the names).

### **Constant Relative Risk Aversion Model**

One reviewer suggested that we test a utility model described in Wakker (2008). Here we detail the model, some criticisms of the model that we have, and the results of implementing such a model in our Hierarchical Bayesian approach.



The model, which for simplicity we will call here the elasticity model (for reasons that will become apparent soon), is given by the following piecewise function (piecewise with respect to the parameter  $\rho$ ) :

$$u(x) := \begin{cases} x^\rho & \text{if } \rho > 0, \\ \log(x) & \text{if } \rho = 0, \\ -x^\rho & \text{if } \rho < 0. \end{cases}$$

Note that here,  $x \geq 0$ . This essentially means that the utility function should change as  $\rho$  changes, and the claim is that a power utility function ( $x^\rho$ ) “converges” to the log function as  $\rho \rightarrow 0$ . The argument for this comes from applying what we will call here the elasticity operator (a.k.a. the Arrow-Pratt coefficient of absolute risk aversion; Pratt, 1964) to the power utility function and to the simple  $\log(x)$  function:

$$E[u] := \frac{-u''(x)}{u'(x)}.$$

The operator  $E$  takes an at least twice-differentiable function and maps this function to another function; note that this “target” function may not even be continuous. Indeed, if  $u$  has any local extrema in the domain of interest, then  $E[u]$  will yield a discontinuous function. Now, if one applies the differential operator  $E$  to the power utility function, this yields:

$$E[x^\rho] = \frac{-\rho(\rho-1)x^{\rho-2}}{\rho x^{\rho-1}} = \frac{1-\rho}{x},$$

and if we take the limit as  $\rho \rightarrow 0$ , we see that  $E[x^\rho] \rightarrow \frac{1}{x} = E[\log(x)]$ . This would seem to suggest that the power utility functions, thought of as a *net* of functions (which is a generalization of sequences of functions), converges in this operator to the log function, and hence we should treat the power utility functions as the log function when  $\rho = 0$ .

The problem with this argument is that it implicitly uses the non-existent continuity of the differential operator to arrive at this conclusion (Rudin, 1991). Indeed, we are not even clear what function space  $E$  maps into (again, this operator can yield a discontinuous function, complicating matters), and without this knowledge, we cannot choose an appropriate norm with which this net will converge. To be more explicit, the implicit assumption is the following:

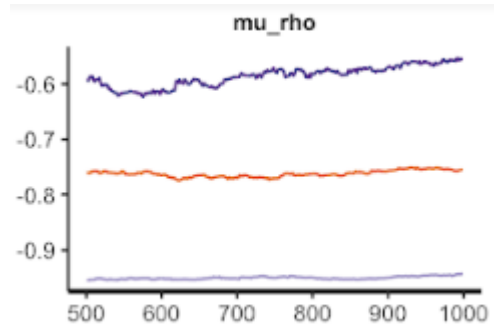
$$\lim_{\rho \rightarrow 0} E[x^\rho] = E[\lim_{\rho \rightarrow 0} x^\rho] = E[\log(x)],$$

which leads to the conclusion that  $x^\rho \rightarrow \log(x)$ , which itself assumes that the limit is unique, which we do not know beforehand (and have ample reason to suspect that the limit is in fact not unique; Rudin, 1991).

In short, there are a number of assumptions made in this claim that are unsubstantiated, and we thus conclude that the theoretical justification for the elasticity model (i.e. the utility model derived from the elasticity operator) is rather weak. Nonetheless, we tried an implementation of the elasticity model with our data, with a certain modification. Since the MCMC chain will almost never arrive at a  $\rho$  of zero, we need to allow the function to be a special kind of piecewise function as follows:

$$u(x) = \begin{cases} x^\rho & \text{if } \rho > \varepsilon \\ \log(x) & \text{if } |\rho| \leq \varepsilon \\ -x^\rho & \text{if } \rho < -\varepsilon \end{cases},$$

where we chose a generous  $\varepsilon = .05$  (this should be as close to zero as possible while also allowing a reasonable chance for the computer to recognize when to use the log function). Doing so led to the following chains:



This indicates that the chains were not able to adequately search through parameter space (they were effectively completely stationary), and thus we cannot trust the samples that we obtained for any analysis.

We additionally tested a re-parameterized version of the constant relative risk aversion utility described above, which follows the form:

$$u(x) = \frac{x^{1-\rho}}{1-\rho}.$$

To determine if such a utility function could explain the data better than the power or log functions described in Table S1, we replaced the bet valuation function of the best fitting model (M12) with the utility function above, and fit the model to the healthy control data. With the re-parameterized utility function, the MCMC sampler was better able to converge. We then used the leave-one-out information criterion (LOOIC; see Bayesian Model Comparison below) to determine if the above utility better captured the healthy control data relative to M12 in Table S1. We found that M12 had a lower LOOIC value ( $\text{LOOIC}_{\text{M12}} = 33,027$ ) relative to the same model with the above re-parameterized utility function ( $\text{LOOIC}_{\text{New}} = 33,729$ ). Importantly, the difference in LOOIC between the models shows that the log utility function in M12 better captures healthy controls' bet choices relative to the constant relative risk aversion utility above ( $\text{LOOIC}_{\text{M12}} - \text{LOOIC}_{\text{New}} = 33027 - 33729 = -702$ ;  $\text{SE}_{\text{Difference}} = 262$ ). Because models performed

consistently across groups (see S1 Fig), we did not further test the model in groups with a history of substance use.

### **Hierarchical Bayesian Modeling**

We used a standardized convention for parameterizing all tested models, which is described in great detail in our previous work (Ahn et al., 2017; Haines et al., 2018). Here, we briefly describe this convention, but we refer the reader to previous work for more details. We assumed that each individual-level distribution was drawn from a normally-distributed group distribution where prior distributions for the means and standard deviations of each group-level parameter followed  $\text{normal}(0,1)$  and  $\text{normal}(0,0.2)$  distributions, respectively. This parameterization equates to a diffuse prior over each group-level mean while constraining the variance across individual-level parameters for more efficient sampling. Bounded parameters were estimated in an unconstrained space and then inverse probit-transformed  $\in (0, 1)$  and scaled appropriately. Parameters with only positive support were estimated in an unconstrained space and exponentially transformed  $\in (0, \infty)$ . To minimize dependence between group-level means and standard deviations during MCMC sampling, which improves sampling efficiency, we also used non-centered parameterizations of the group-level means and standard deviations (Betancourt and Girolami, 2015).

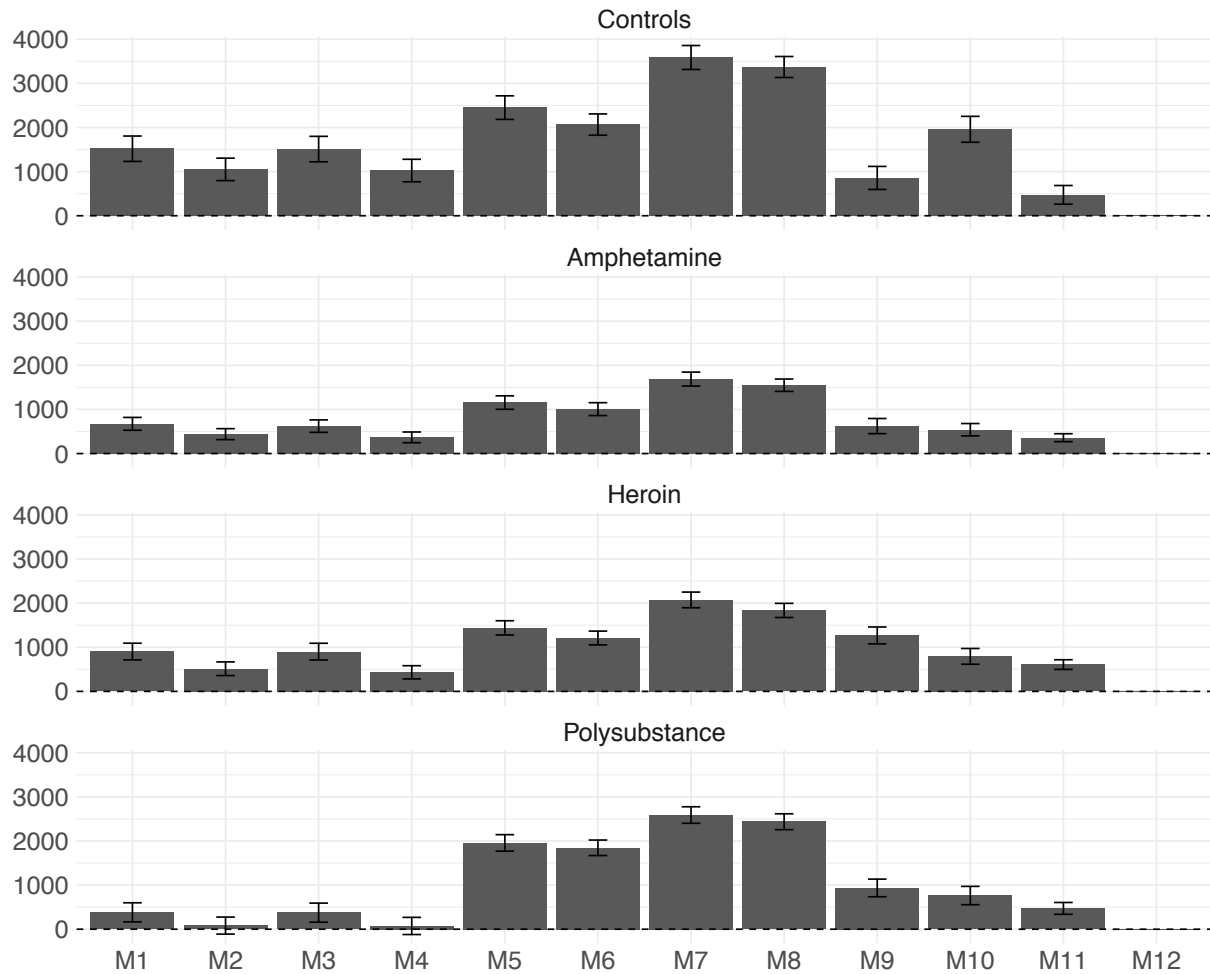
### **Bayesian Model Comparison**

We used the leave-one-out information criterion (LOOIC) to determine which of the competing models provided the best fit to each group's data while penalizing for model complexity. LOOIC is a fully-Bayesian information criterion which estimates how well a model can make out-of-sample (i.e. generalizable) predictions (Vehtari et al., 2017); it is analogous to Akaike and Bayesian ICs which are often used to compare models that are fit using maximum

likelihood estimation. We implemented LOOIC in the same manner as in previous studies (e.g., Ahn et al., 2017; Haines et al., 2018), where the log likelihood for each subject was summed across trials, resulting in an  $S \times N$  matrix (posterior samples by participants). We used the `loo` R package to compute LOOIC values for each model and group (13). S1 Fig. shows the relative LOOIC values for each model and group. Note that lower LOOIC values indicate better model fits. The CM with a log valuation function (described in the Introduction and referred to as “Model 12” in S1 Table) provided the best fit for all groups, so we used it for all further analyses. While LOOIC model comparison allowed us to identify the best model relative to competing models, it does not provide an absolute measure of model fit.

Table S1. A summary and comparison of the major features of each model that was tested. The first column references what part of the model/task the third column (which gives the explicit equations) refers to. The second column gives the model codes (for reference) and the final two columns give the free parameter names and their range/domain of definition. Note that every model used the same color choice model to predict the color choice (i.e., all models used the same “color utility” model); however, the models could differ in whether or not the probability predicted from this color utility model was used in calculating the expected utility, or whether the untransformed objective probabilities were used. In other words, models could differ in what equation was used under  $P(\textit{Chosen Color Wins})$  in the first equation in Section 2: “The Cognitive Models” to calculate the expected utility for each betting option, but all models used the same color utility model (Eq. 3 in this supplement) to *predict* the color choice. This simply reflects the models following the two-stage approach of the task: predict color choice first (where all models use the same color utility function), then use color data (i.e., objective probabilities or transformed probabilities) and points to gain/lose to predict the bet choice.

Component	Model(s)	Model Equation	Free Parameters	Range
Color Utility	M1–12	$P(R) := \frac{cr^\alpha}{cr^\alpha + (1-c)u^\alpha}$	$\alpha$ : Probability distortion  $c$ : Red bias	$[0, 5]$  $[0, 1]$
Bet Utility	M1–4  M5–12	$X_{IM} := Cb$ $Y_{IM} := -Cb$ $X_{CM} := C + Cb$ $Y_{CM} := C - Cb$		
Bet Valuation Function	M1–2  M3–4  M5–6  M7–8  M9–10  M11–12	$u_{IM}(x) = \begin{cases} x^\rho & x \geq 0 \\ -\delta x ^\rho & x < 0 \end{cases}$ $u_{IM}(x) = \begin{cases} \log(1 + \rho x) & x \geq 0 \\ -\delta \log(1 + \rho x) & x < 0 \end{cases}$ $u_{CM}(x) = \begin{cases} x^\rho & x \geq 0 \\ x^1 & x < 0 \end{cases}$ $u_{CM}(x) = \begin{cases} x^1 & x \geq 0 \\ x^\rho & x < 0 \end{cases}$ $u_{CM}(x) = \begin{cases} \log(1 + \rho x) & x \geq 0 \\ \log(1 + x) & x < 0 \end{cases}$ $u_{CM}(x) = \begin{cases} \log(1 + x) & x \geq 0 \\ \log(1 + \rho x) & x < 0 \end{cases}$	$\rho$ : Utility shape  $\delta$ : Loss aversion	$[0, \infty]$  $[0, \infty]$
Bet Probability Weighting	M1–12  Odd M's  Even M's	$EU(bet Color) := P(Color)u(X) + (1 - P(Color))u(Y)$ $P(Color) = \text{Color Utility}$ $P(Color) = \text{Objective Color Proportions}$		
Bet Time Cost Function	M1–12	$Z(bet Color) := EU(bet Color) - \beta \cdot COST(bet)$ $COST(bet) := \frac{(position - 1)}{4}$	$\beta$ : Impulsivity	$[0, \infty]$
Bet Choice Rule	M1–12	$P(b_i k_t) = \frac{e^{\gamma Z(b_i k_t)}}{\sum_j e^{\gamma Z(b_j k_t)}}$	$\gamma$ : Choice sensitivity	$[0, \infty]$



S1 Fig. The above graph demonstrates the relative differences in fit between M12 and all other models tested in this paper. Lower leave-one-out-information-criterion (LOOIC) values indicate better model fit, so positive values for the LOOIC difference above reflect poorer performance relative to the best-fitting model. M12 showed the best performance across all groups. Error bars indicate  $\pm 1$  SE of the difference between the best fitting model and the competing model.



Table S2. Sum of squared errors between posterior predictive simulations and true color choices across Red:Blue color proportions

<b>Model</b>	<b>SSE<sub>HC</sub></b>	<b>SSE<sub>Hero</sub></b>	<b>SSE<sub>Amph</sub></b>	<b>SSE<sub>Poly</sub></b>
M1	.075	.060	.078	.074
M2	.039	.032	.039	.041
M3	.076	.063	.083	.075
M4	.039	.032	.040	.041
M5	.073	.056	.074	.059
M6	.039	.032	.039	.041
M7	.056	.044	.062	.055
M8	.039	.032	.039	.041
M9	.234	.135	.192	.101
M10	.039	.032	.039	.041
M11	.306	.252	.224	.157
M12	.039	.032	.039	.041

*Note.* Sum of squared errors between mean of posterior predictive simulations of color choice and true behavioral data averaged within groups across red:blue color ratios (see Fig. 2A). The sum of squared error represents the discrepancy between the average prediction of the model in question and the averaged results from the behavioral data, for each group. Note that because most models shared the same color choice utility, equivalent performance across models should be expected.

Table S3. Sum of squared errors between posterior predictive simulations and true bet choices across Red:Blue color proportions

<b>Model</b>	<b>SSE<sub>HC</sub></b>	<b>SSE<sub>Hero</sub></b>	<b>SSE<sub>Amph</sub></b>	<b>SSE<sub>Poly</sub></b>
M1	.080	.070	.065	.057
M2	.059	.043	.044	.035
M3	.081	.069	.047	.056
M4	.060	.027	.029	.034
M5	.054	.068	.086	.115
M6	.042	.060	.073	.106
M7	.097	.105	.113	.129
M8	.063	.066	.081	.098
M9	.005	.019	.019	.013
M10	.042	.017	.010	.005
M11	.003	.006	.007	.009
M12	.006	.007	.005	.005

*Note.* Sum of squared errors between mean of posterior predictive simulations of bet proportion and true behavioral data averaged within groups across red:blue color ratios (see Fig. 2B). The sum of squared error represents the discrepancy between the average prediction of the model in question and the averaged results from the behavioral data, for each group. Unlike for color choice, considerable differences can be seen in the prediction of betting choices, wherein it can be seen that M12 performs the best across all groups (apart from M11 in the Heroin group). Note that, while these are averages, these averages are obtained by predicting individual choices *and then* averaging the result, which is *not* the same as only predicting the average (Estes, 1956). The former method is, in fact, a much stronger test of the model (Estes, 1956).

## Validity

A word on construct validity: it may be reasonably asked if our proxy measure of impulsivity actually measures the construct at all. A typical test of this is to demonstrate that the measure captures *expected* differences between groups, in the correct direction (Cronbach and Meehl, 1955). We, for instance, in line with results from delay discounting models, hypothesized that we would see an increase in impulsivity (as measured by  $\beta$  here – our psychometric test of impulsivity) in the substance-using groups compared to controls, and our model indeed demonstrated this expected difference. However, this is not a perfect test of construct validity: our measure capturing expected differences is a *necessary*, not a *sufficient*, condition for demonstrating construct validity (Cronbach and Meehl, 1955). That is, *not* showing the expected differences would invalidate the model; demonstrating the expected differences does not, however, completely validate the model. Certainly, more tests and more precise predictions should be conducted to further investigate the construct validity of the model<sup>24</sup>.

Additionally, a comment should be made about the validity of the risk-aversion estimates obtained from the best-fitting model (M12), which, contrary to expectations, were quite low for all groups, especially for the substance-using groups (see Busemeyer and Stout, 2002, and the references therein). Indeed, the posteriors over the mean estimates for all substance-using groups

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<sup>24</sup> In a forthcoming paper, we test this notion by linking the delay discounting task and CGT via cognitive modeling analysis of the parameters, in a hierarchical Bayesian fashion. We demonstrate that the tasks are not as related as intuition might have us believe; those results suggest that a standard delay discounting model (such as the hyperbolic model) will not fare well with the CGT (Kvam et al., in prep.).

had most of their probability densities accumulate at values less than 1, which usually indicates risk-aversion. In particular, as mentioned in the main text, the interpretation for the current implementation of loss sensitivity is opposite to what most economic utility models use. In our case, a lower value indicates that the participant is less sensitive to losses, which should lead to riskier decisions being made. This follows since a less punishing loss will be less aversive than it would otherwise be for a healthy control. This suggests that the CGT may not be the best task to assess risky behavior in isolation, and the results should certainly be corroborated using more traditional risk assessment tasks. (Recall that this study was part of a larger study with data on several neurocognitive tasks; none of the group differences, especially those between substance users and controls, was significant (Wilson and Vassileva, 2018). This may suggest a partial recovery of neurocognitive functioning with abstinence.) In other words, the parameter estimates from M12 do not follow the nomological network we would expect in the context of risk aversion and substance use (Cronbach and Meehl, 1955). For this reason, while we suggested that the risk-aversion parameter  $\rho$  could, in theory, be interpreted in a similar manner to interpretations afforded to more traditional models, the validity of this estimate should certainly be assessed in further research using the CGT, along with corroborating, traditional risk assessment tasks. The complication almost certainly is related to the complex design of the CGT more generally, as addressed in the Discussion section of the main paper, or could be due to our specific participants in protracted abstinence.

### **Response Time Modeling**

Since the CANTAB® administration of the CGT records response times – and indeed, these response times are included in our data set – one might wonder if a response time model could be applied to the CGT. Such a consideration is important, as response time models tend to

be some of the strongest process models in cognitive science (Bussemeyer and Diederich, 2010; Ratcliff and McKoon, 2008). However, some serious considerations need to be made, given the structure of the task. First, a traditional diffusion model (e.g., the Ratcliff Diffusion Model; Ratcliff, 1978; Ratcliff and McKoon, 2008) is unlikely to be a viable model of each trial, owing to the (very) long response times that could occur in the task (recall that any trial could last between 1 and 25 *seconds*). The Ratcliff Diffusion Model, the most popular and successful model in this vein, is known to have trouble fitting such long response times (Ratcliff and McKoon, 2008).

One possible approach might be to apply a diffusion-like model on each sequential choice option (i.e. on each betting option). However, the wait time of 5 seconds is still too large for the diffusion model in general (Ratcliff and McKoon, 2008). The fact that one of the response choices is inaction for 5 seconds (i.e. waiting to move on to the next option) also makes application of the traditional boundary-value diffusion models difficult (Ratcliff and McKoon, 2008). This becomes a problem if we want reliable estimates of our parameters and adequate fit of the model to the data. One possibility is to have each option shown for a much smaller time period (as in Zois et al., 2014; a 2 to 3-second time period would be about as much as a typical diffusion model could tolerate). To fix the problem of inaction as a choice response, the participant, upon presentation of a betting option, could push one button for “accept the option” and another for “reject and move to next option.” This would make choices on each betting option a speeded (~2-3 seconds) binary-choice response task – exactly what the diffusion model was designed for (Ratcliff and McKoon, 2008). A general model of the CGT which employs response times will inevitably have to resolve these issues somehow, either methodologically as

suggested here, or with a diffusion-like model that can handle longer response times.

Overcoming this difficulty will come with the reward of a powerful process model for the task.

### **More detail on Methods**

#### *Participants*

Participants were recruited from a larger study on impulsivity among substance users in Sofia, Bulgaria (for more details, see Ahn and Vassileva, 2016; Wilson and Vassileva, 2018). Participants were recruited using flyers posted in addiction clinics and relevant public areas in Sofia, Bulgaria. Prospective participants were screened using a telephone or an on-site interview and provided informed consent to participate.

Participants in the substance dependent groups were diagnosed as having a lifetime diagnosis of substance dependence for either heroin or amphetamines and were classified into one of the three groups: (“pure”) mono-dependent heroin users, (“pure”) mono-dependent amphetamine users, and polysubstance dependent users. The majority of substance dependent participants were in protracted abstinence at the time of testing (i.e., in “full sustained remission” for more than one year by DSM-IV criteria), as determined by the Structured Clinical Interview for DSM-IV disorders (SCID-IV; First et al., 1996). Local adults from the community with comparable demographics to the substance using groups and no history of substance dependence were recruited as controls.

Inclusion criteria for the larger study are quoted from Vassileva et al. (2014), p. 653 (and can also be found in Ahn and Vassileva, 2016): “Inclusion criteria included: 1) age between 18 and 50 years; 2) minimum of 8<sup>th</sup> grade education; 3) estimated IQ > 75; 4) no history of neurologic illness (including dementia secondary to substance abuse); 5) no history of penetrating head injury or closed head injury with a loss of consciousness > 30 min; 6) no history

of psychotic or mood disorders, or current use of psychotropic medication; 7) HIV seronegative status; 8) no history of dependence on both amphetamines and heroin or current dependence on any substance; and 9) negative breathalyzer test for alcohol and negative urine toxicology screen for opiates, cannabis, amphetamines, methamphetamines, benzodiazepines, barbiturates, cocaine, MDMA, and methadone.” However, criterion (8) is an earlier criterion that was in place before adding the polysubstance using group and no longer applies.

We additionally excluded participants who exhibited clear signs of boredom/non-participation during the task. We defined boredom as having both (1) a standard deviation of 0 in either condition in betting proportion choice, and (2) having a difference in betting proportion means greater than .20. Both criteria needed to be met in order to qualify for exclusion due to boredom. This way, we excluded those who wanted to rush through the task, but not those who were genuinely and extremely risk averse or risk seeking. (We tried other cutoff scores, but these reported here seemed to strike a nice balance between not excluding enough and excluding too many participants who may actually display risky or risk-averse behavior. These cutoff scores should be treated as experimental, as they happened in the current study to adequately distinguish bored participants who were clearly disengaged from those who demonstrated genuine risky or risk-averse behavior.) By group, these exclusions totaled: (i) Controls = 7; (ii) Heroin = 5; (iii) Amphetamine = 2; (iv) Polysubstance = 4. Table 3 in the main text gives the demographic breakdown of the final group of participants included in the current study. Our final sample after all exclusion criteria consisted of a total of  $N = 419$  individuals, with an  $N = 161$  for controls,  $N = 79$  for the Heroin group,  $N = 76$  for the Amphetamine group, and  $N = 103$  for the Polysubstance group.

### *Procedures*

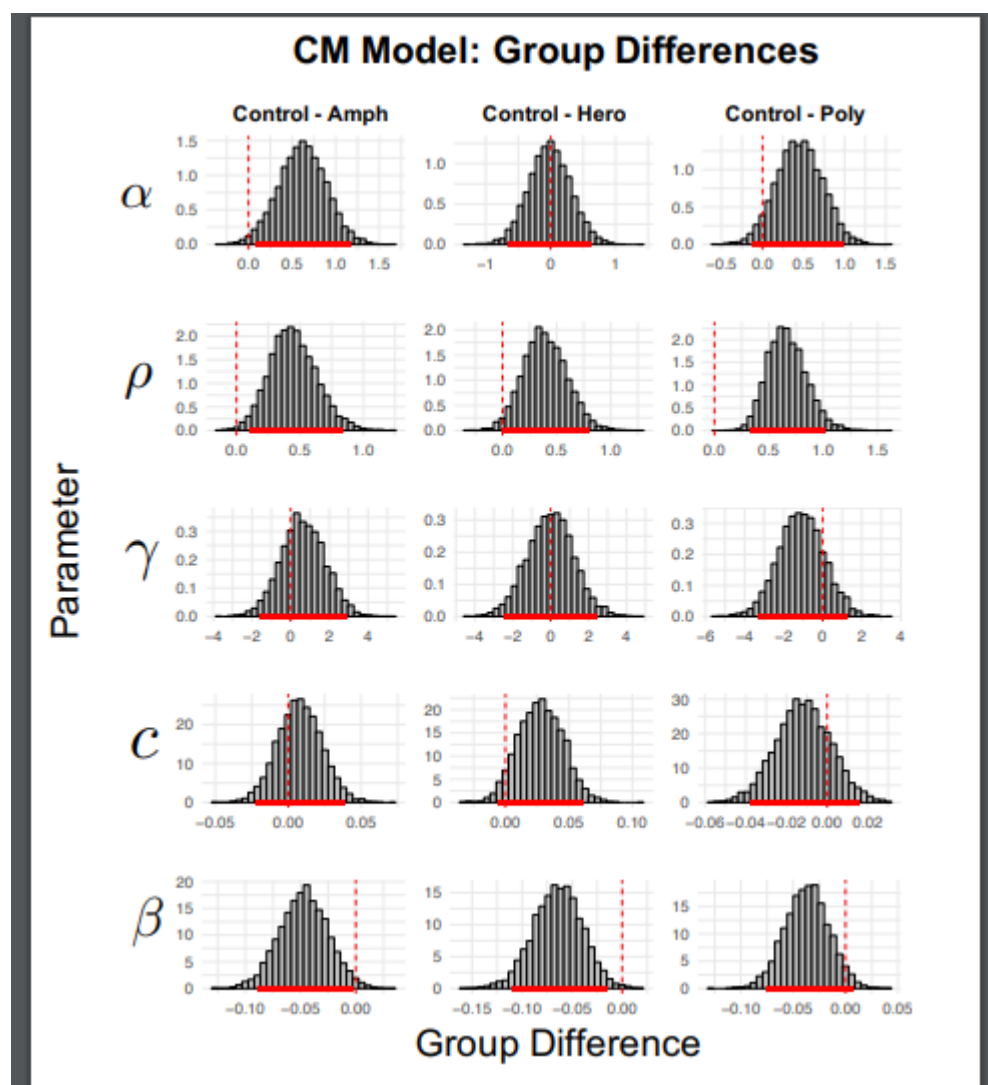
All self-report and interview measures were translated and back-translated to and from Bulgarian. The forward translation from English to Bulgarian was done by the last author of the current paper, a native Bulgarian speaker. The back-translation from Bulgarian to English was done by several Bulgarian psychiatrists and psychologists. The current study was a subset of a larger study on impulsivity that required two sessions of interviews and neurocognitive testing, each ~3.5 hours. To minimize attrition rates, the two sessions were done as close in time as possible. Both sessions were conducted at the Bulgarian Addictions Institute, with appropriate IRB approval from Virginia Commonwealth University and the Medical University in Sofia on behalf of the Bulgarian Addictions Institute. The Cambridge Gambling Task (CGT) was done on the second testing session for each participant.

The CGT version we administered was part of the CANTAB® battery (2016). Immediately prior to the CGT, each participant also completed the ‘Motor Screening Test’ (MOT) from the CANTAB®, designed to relax the subject and to introduce them to the computer and touch screen. The MOT also screens for vision, motor, and comprehension problems and provides mean latency and mean error rate as performance indices. At the start of the CGT, each participant performed four practice trials that did not count towards their final scores. We followed Rogers et al. (1999) in recording indecision: if, after the presentation of the last bet, no choice is made, then the task records the last bet shown as the chosen bet. More details on the CGT can be found in Rogers et al. (1999). Lastly, each block ended after either: (1) participants reached 1 point, or (2) after 9 trials per block, with ascending and descending conditions having 4 blocks each. Note that the order of ascending and descending conditions was counterbalanced across participants.



### Sensitivity Analysis of Control Participants

We found that 34 control subjects had a lifetime diagnosis of cannabis abuse or dependence, but were in protracted abstinence, and 3 controls with low IQ were incorrectly labeled as eligible for study. We performed a sensitivity analysis to observe how much the results would change for the controls if we removed these participants from analysis. As the following figure will attest (which should be compared with Fig. 3 in the main document), there is minimal change in the mean parameter estimates of the control group, with the exception of risk aversion. Removing the cannabis users and low IQ participants exaggerates the mean differences between controls and all substance using groups in terms of risk aversion, such that controls are found to have higher risk-seeking behavior (larger  $\rho'_S$ ). However, this result should be treated with caution, given our previous concerns about how risk aversion was estimated in this task, i.e. the unorthodox modeling of utility in the CGT.



### Supplementary References

- Aczél, J. 1966. *Lectures on functional equations and their applications* (Vol. 19). Academic press.
- Ahn, W.-Y., Haines, N., Zhang, L. 2017. Revealing neurocomputational mechanisms of reinforcement learning and decision-making with the hBayesDM package. *Comp. Psychiatry*. 8, 429–453.
- Ahn, W.-Y., Vasilev, G., Lee, S.-H., Busemeyer, J. R., Kruschke, J. K., Bechara, A., Vassileva, J. 2014. Decision-making in stimulant and opiate addicts in protracted abstinence: evidence from computational modeling with pure users. *Front. Psychol.* 5, 849.
- Ahn, W.-Y., Vassileva, J. 2016. Machine-learning identifies substance-specific behavioral markers for opiate and stimulant dependence. *Drug Alcohol Depend.* 161, 247–257.
- Betancourt, M., Girolami, M. 2015. Hamiltonian Monte Carlo for hierarchical models. *Current Trends in Bayesian Methodology with Applications*, 79, 30.
- Busemeyer, J. R., Diederich, A. 2010. *Cognitive modeling*. Sage.
- Busemeyer, J. R., Stout, J. C. 2002. A contribution of cognitive decision models to clinical assessment: decomposing performance on the Bechara gambling task. *Psychol. Assess.* 14, 253.
- Cognition, C. 2016. CANTAB®[Cognitive assessment software]. *All Rights Reserved Wwww. Cantab. Com.*
- Cronbach, L. J., Meehl, P. E. 1955. Construct validity in psychological tests. *Psychol. Bull.* 52, 281.
- Drichoutis, A. C., Lusk, J. L. 2014. Judging statistical models of individual decision making

- under risk using in-and out-of-sample criteria. *PloS one*. 9, e102269.
- Estes, W. K. 1956. The problem of inference from curves based on group data. *Psychol. Bull.* 53, 134.
- First, M. B., Gibbon, M., Spitzer, R. L., Williams, J. B. W. 1996. User's guide for the structured clinical interview for DSM-IV axis I Disorders—Research version. *New York: Biometrics Research Department, New York State Psychiatric Institute.*
- Gonzalez, R., Wu, G. 1999. On the shape of the probability weighting function. *Cogn. Psychol.* 38, 129–166.
- Haines, N., Vassileva, J., Ahn, W.-Y. 2018. The Outcome-Representation Learning Model: A Novel Reinforcement Learning Model of the Iowa Gambling Task. *Cogn. Sci.* 42, 2534–2561.
- Kahneman, D., Tversky, A. 2013. Prospect theory: An analysis of decision under risk. In MacLEAn, L. C., Ziemba, W. T. (Eds.) *Handbook of the fundamentals of financial decision making: Part I*. World Scientific, Singapore, pp. 99–127.
- Luce, R. D. 1959. *Individual choice behavior: A theoretical analysis*. Courier Corporation.
- Luce, R. D. 2000. *Utility of gains and losses: Measurement-theoretical and experimental approaches*. Psychology Press.
- Pratt, J. (1964). Risk Aversion in the Small and in the Large. *Econometrica*, 32, 122-136.  
doi:10.2307/1913738
- Ratcliff, R. 1978. A theory of memory retrieval. *Psychol. Rev.* 85, 59.
- Ratcliff, R., McKoon, G. 2008. The diffusion decision model: theory and data for two-choice decision tasks. *Neural Comp.* 20, 873–922.
- Rogers, R. D., Everitt, B. J., Baldacchino, A., Blackshaw, A. J., Swainson, R., Wynne, K.,

- Baker, N. B., Hunter, J., Carthy, T., London, M., Deakin, J. F. W., Sahakian, B. J., Robbins, T. W. 1999. Dissociable deficits in the decision-making cognition of chronic amphetamine abusers, opiate abusers, patients with focal damage to prefrontal cortex, and tryptophan-depleted normal volunteers: evidence for monoaminergic mechanisms. *Neuropsychopharmacol.* 20, 322–339.
- Rudin, W. 1991. *Functional Analysis*, 2<sup>nd</sup> Ed. McGraw-Hill: New York.
- Vassileva, J., Paxton, J., Moeller, F. G., Wilson, M. J., Bozgunov, K., Martin, E. M., Gonzalez, R., Vasilev, G. 2014. Heroin and amphetamine users display opposite relationships between trait and neurobehavioral dimensions of impulsivity. *Addict. Behav.* 39, 652 – 659.
- Vehtari, A., Gelman, A., Gabry, J. 2017. Practical Bayesian model evaluation using leave-one-out cross-validation and WAIC. *Stat. Comput.* 27, 1413–1432.
- Wallsten, T. S., Pleskac, T. J., Lejuez, C. W. 2005. Modeling behavior in a clinically diagnostic sequential risk-taking task. *Psychol. Rev.* 112, 862.
- Wilson, M. J., Vassileva, J. 2018. Decision-Making Under Risk, but Not Under Ambiguity, Predicts Pathological Gambling in Discrete Types of Abstinent Substance Users. *Front. Psychiatry.* 9. 1 – 10.
- Zois, E., Kortlang, N., Vollstädt-Klein, S., Lemenager, T., Beutel, M., Mann, K., Fauth-Bühler, M. 2014. Decision-making deficits in patients diagnosed with disordered gambling using the Cambridge Gambling task: the effects of substance use disorder comorbidity. *Brain Beh.* 4, 484–494.
- Forthcoming paper:**
- Kvam, P. D., Romeu, R. J., Turner, B. M., Vassileva, J., Bussemeyer, J. R. In preparation. Testing

the factor structure underlying behavior using joint cognitive models: Impulsivity in delay discounting and Cambridge gambling tasks.

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