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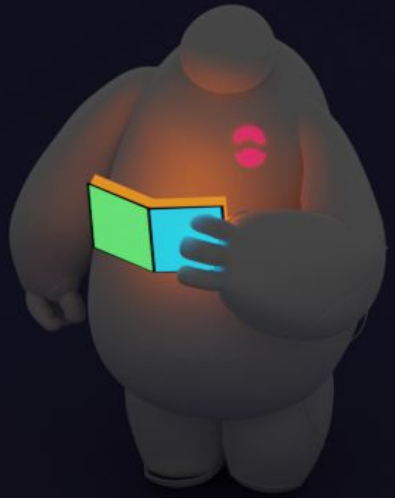
MACHINE LEARNING

Learning from Data



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|00 INTRODUCTION

Pattern Recognition
Study **Correlations** Between **Data Domains**
Tune a **Parametric Model** with **Data**

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|00 INTRODUCTION

SUPERVISED

Learn parameters from
Labeled Data

Regression Classification

UNSUPERVISED

Learn parameters from
Unlabeled Data

Clustering Latent Space

REINFORCEMENT

Autonomous Agent Learning
from **Experience**

Q-Learning

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01|

LINEAR ALGEBRA

The Last Space Bender

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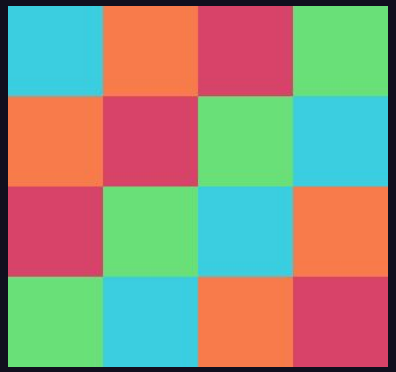
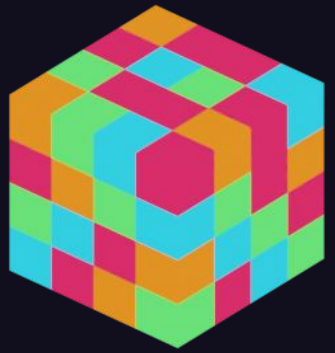
|01 LINEAR ALGEBRA

VECTOR

MATRIX



TENSOR



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|01 LINEAR ALGEBRA

VECTOR

$$\vec{v} = \begin{pmatrix} v_x \\ v_y \end{pmatrix}$$

LENGTH

$$\|\vec{v}\| = \sqrt{v_x^2 + v_y^2}$$

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|01 LINEAR ALGEBRA

VECTOR

ADDITION / SUBTRACTION

$$\vec{v} = \begin{pmatrix} v_x \\ v_y \end{pmatrix}$$

$$\vec{u} = \begin{pmatrix} u_x \\ u_y \end{pmatrix}$$

$$\vec{v} + \vec{u} = \begin{pmatrix} v_x + u_x \\ v_y + u_y \end{pmatrix}$$

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|01 LINEAR ALGEBRA

VECTOR

$$\vec{v} = \begin{pmatrix} v_x \\ v_y \end{pmatrix}$$

$$\vec{u} = \begin{pmatrix} u_x \\ u_y \end{pmatrix}$$

SCALAR (DOT) PRODUCT

$$\vec{v}.\vec{u} = v_x u_x + v_y u_y = |v|\,|u|\,cos\theta$$

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|01 LINEAR ALGEBRA

VECTOR

$$\vec{v} = \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix}$$

$$\vec{u} = \begin{pmatrix} u_x \\ u_y \\ u_z \end{pmatrix}$$

CROSS PRODUCT

$$\vec{v} \times \vec{u} = \begin{pmatrix} v_y u_z - v_z u_y \\ v_z u_x - v_x u_z \\ v_x u_y - v_y u_x \end{pmatrix}$$

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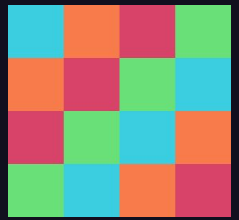
|01 LINEAR ALGEBRA

MATRIX



ADDITION / SUBTRACTION

$$A = \begin{pmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \end{pmatrix}$$

$$B = \begin{pmatrix} b_{1,1} & b_{1,2} \\ b_{2,1} & b_{2,2} \end{pmatrix}$$

$$A + B = \begin{pmatrix} a_{1,1} + b_{1,1} & a_{1,2} + b_{1,2} \\ a_{2,1} + b_{2,1} & a_{2,2} + b_{2,2} \end{pmatrix}$$

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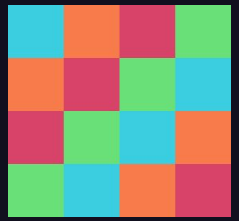
|01 LINEAR ALGEBRA

MATRIX



HADAMARD PRODUCT

$$A = \begin{pmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \end{pmatrix}$$

$$B = \begin{pmatrix} b_{1,1} & b_{1,2} \\ b_{2,1} & b_{2,2} \end{pmatrix}$$

$$A \odot B = \begin{pmatrix} a_{1,1} b_{1,1} & a_{1,2} b_{1,2} \\ a_{2,1} b_{2,1} & a_{2,2} b_{2,2} \end{pmatrix}$$

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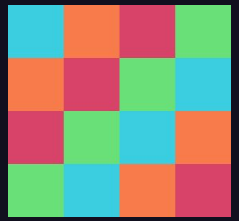
|01 LINEAR ALGEBRA

MATRIX



MATRIX MULTIPLICATION

$$A = \begin{pmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \end{pmatrix}$$

$$B = \begin{pmatrix} b_{1,1} & b_{1,2} \\ b_{2,1} & b_{2,2} \end{pmatrix}$$

$$A \cdot B = \begin{pmatrix} a_{1,1}b_{1,1} + a_{1,2}b_{2,1} & a_{1,1}b_{1,2} + a_{1,2}b_{2,2} \\ a_{2,1}b_{1,1} + a_{2,2}b_{2,1} & a_{2,1}b_{1,2} + a_{2,2}b_{2,2} \end{pmatrix}$$

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|01 LINEAR ALGEBRA

MATRIX

EYE

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

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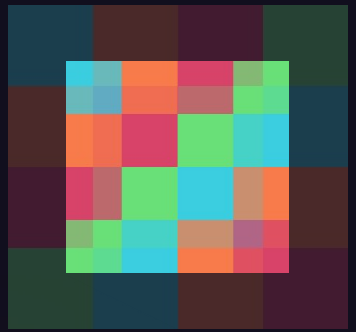
|01 LINEAR ALGEBRA

MATRIX



SCALE

$$\begin{pmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

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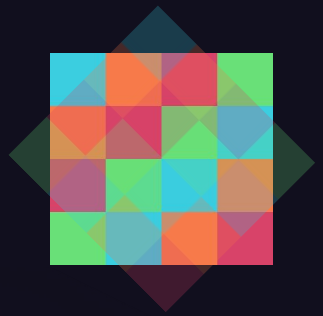
|01 LINEAR ALGEBRA

MATRIX



ROTATION

$$R_x(\theta) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ cos(\theta) & -sin(\theta) & 0 & 0 \\ sin(\theta) & cos(\theta) & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$R_y(\theta) = \begin{pmatrix} cos(\theta) & 0 & sin(\theta) & 0 \\ 0 & 1 & 0 & 0 \\ -sin(\theta) & 0 & cos(\theta) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$R_z(\theta) = \begin{pmatrix} cos(\theta) & -sin(\theta) & 0 & 0 \\ sin(\theta) & cos(\theta) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

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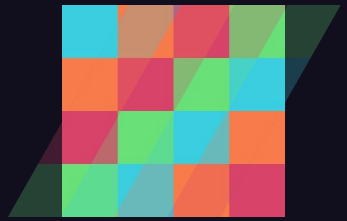
|01 LINEAR ALGEBRA

MATRIX



SHEAR

$$\begin{pmatrix} 1 & 0 & \lambda & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

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|01 LINEAR ALGEBRA

MATRIX

REFLECTION

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

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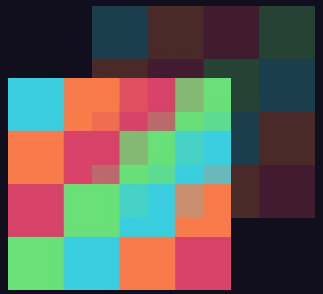
|01 LINEAR ALGEBRA

MATRIX



TRANSLATION

$$\begin{pmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

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|02

PROBABILITIES

Roll the Dice

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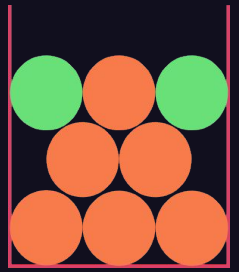
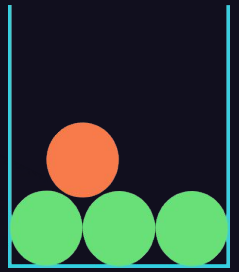


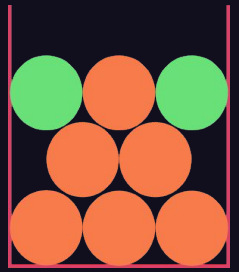


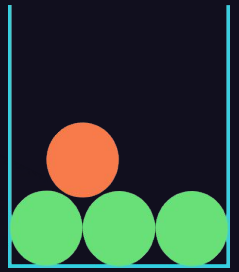
RANDOM VARIABLES

$$B = \begin{cases} r & \text{if Box is red} \\ b & \text{if Box is blue} \end{cases}$$

$$F = \begin{cases} a & \text{if Fruit is an apple} \\ o & \text{if Fruit is an orange} \end{cases}$$

PROBABILITIES





$$p(B = r) = \frac{\text{\# red boxes picked}}{\text{\# total boxes picked}} \in [0; 1]$$

$$p(B = r) + p(B = b) = 1$$

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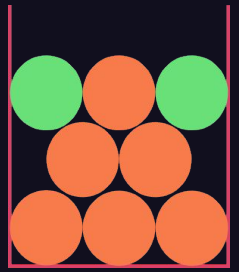

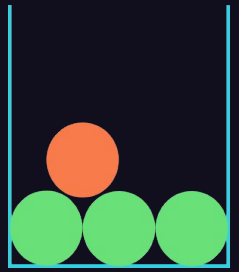

JOINT & CONDITIONAL

Sum Rule

Joint probabilties $p(B = r, F = a)$

Marginal probabilty $p(B = r) = p(B = r, F = a) + p(B = r, F = o)$

Product Rule

Conditional probabilties $p(B = r|F = o)$

$p(B = r, F = o) = p(F = o|B = r)\ p(B = r)$

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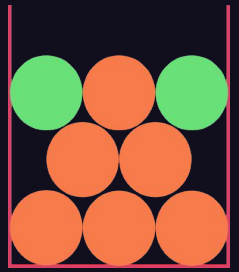
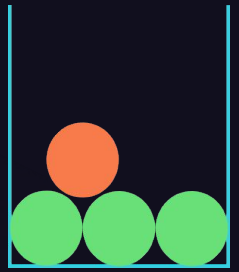
|02 PROBABILITIES





BAYES THEOREM

$$p(B=r|F=o) = \frac{p(F=o|B=r)p(B=r)}{p(F=o)}$$

$$p(B=r|F=o) = \frac{p(F=o|B=r)p(B=r)}{p(F=o|B=r)p(B=r)+p(F=o|B=b)p(B=b)}$$

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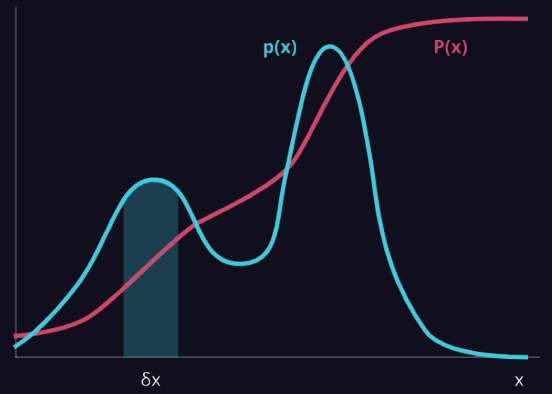
|02 PROBABILITIES



DENSITY

$$p(x \in (a,b)) = \int_a^b p(x)dx$$

$$\int_{-\infty}^{+\infty} p(x)dx = 1$$

$$p(x) \geq 0$$

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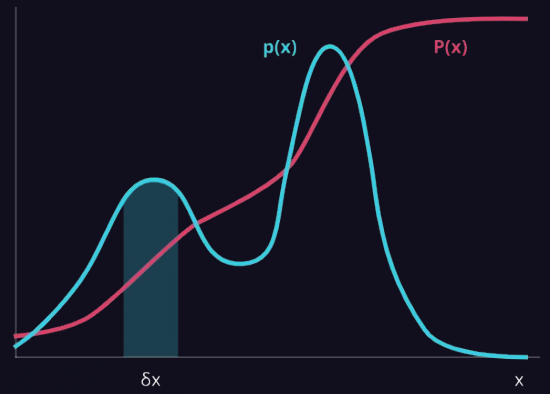

EXPECTATION & COVARIANCE

$$\mathbb{E}[f] = \int p(x) f(x) dx$$

$$var[f] = \mathbb{E}[(f(x) - \mathbb{E}[f(x)])^2]$$

$$var[f] = \mathbb{E}[f(x)^2] - \mathbb{E}[f(x)]^2$$

$$cov[x, y] = \mathbb{E}_{x,y}[xy] - \mathbb{E}[x]\mathbb{E}[y]$$

03|

OPTIMIZATION

One Method to Rule them All

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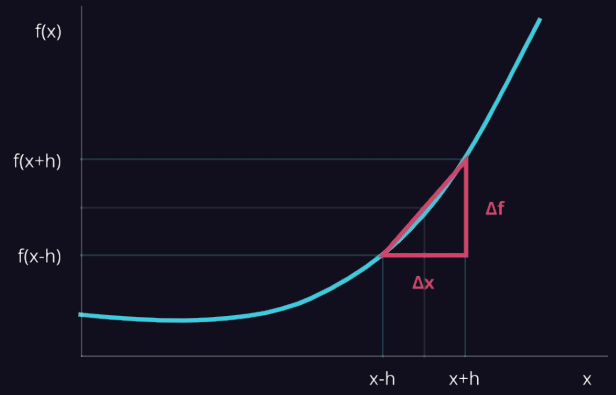
|03 OPTIMIZATION



DERIVATIVES

$$f'(x) = \lim_{h \to 0} \frac{\Delta f}{\Delta x} = \lim_{h \to 0} \frac{f(x+h) - f(x-h)}{2h}$$

First order Derivative

Direction of the Slope

Second order Derivative

Rate of Changes in the Slope

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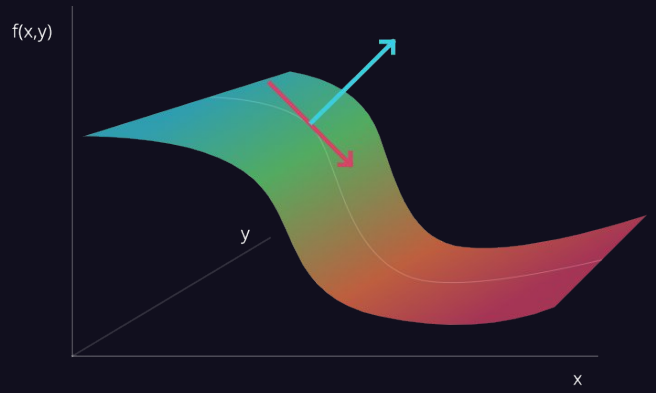
|03 OPTIMIZATION



GRADIENTS

$$\nabla f = \begin{pmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{pmatrix}$$

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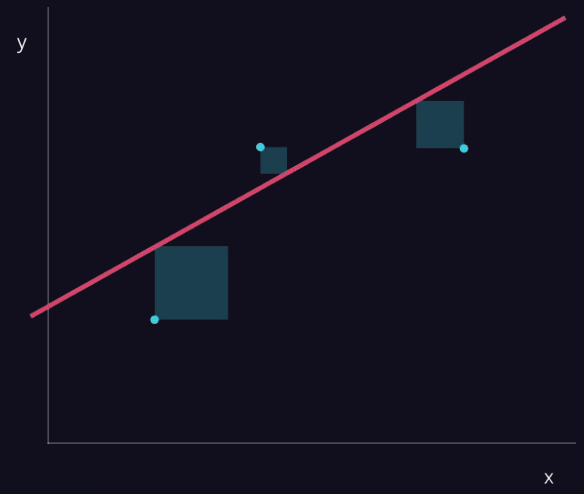
|03 OPTIMIZATION



LEAST SQUARES

Objective Function

Minimize Squared Distances
from Expected Value

$$\hat{y} = ax + b$$

$$\sum_i (y_i - \hat{y}_i)^2 = 0$$

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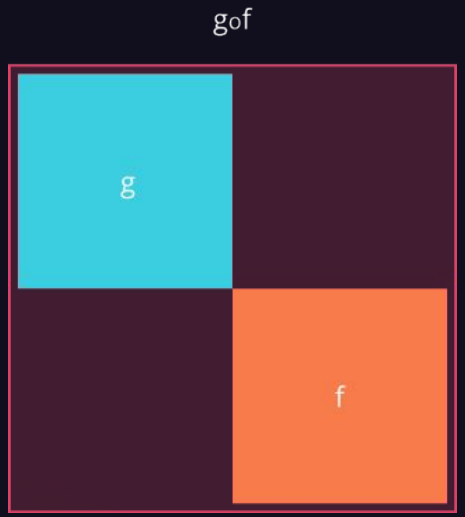
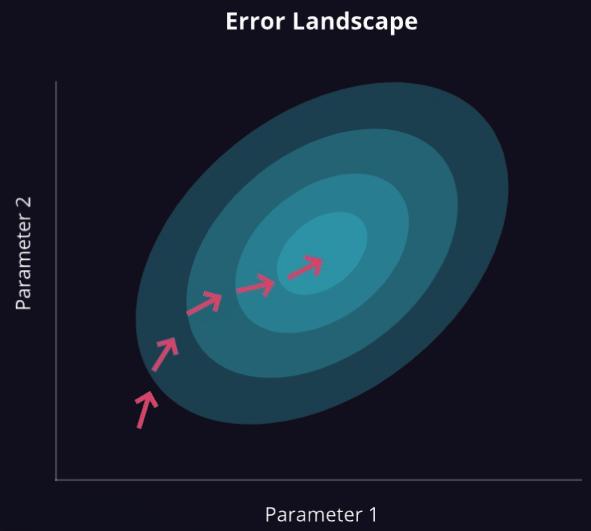


CHAIN RULE

$$h = g \circ f$$

$$h'(x) = g'(f(x))f'(x)$$

|03 OPTIMIZATION



GRADIENT DESCENT

Steps

1) **Forward Propagate** Through the Chain
2) Compute Output **Error**
3) **Backpropagate** Error Through the Chain
4) **Update** Weights w/ Learning Rate
5) **Repeat** Until Convergence Threshold

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|03 OPTIMIZATION

DATASET SPLIT

Training Set

Samples used to Fit/Train the Model

Validation Set

Samples used to provide an Unbiased Evaluation of the Model Becomes Biased during Training

Testing Set

Samples used to provide an Unbiased Evaluation of the Model after Training

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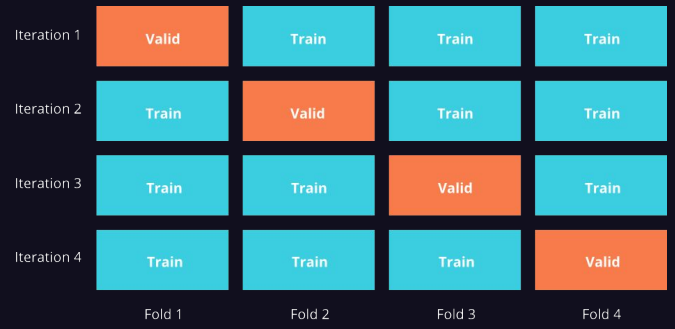
|03 OPTIMIZATION



CROSS-VALIDATION

Method

1) Split Dataset into **k-Folds**
2) **Train on k-1 Folds** and Validate w/ Last
3) **Repeat** k-times
4) Use **Ensemble** Method for **Inference** or **Retrain** on all Dataset

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|04

DIMENSIONALITY REDUCTION

Small Worlds are Filled with Things to See

Paul Safranek

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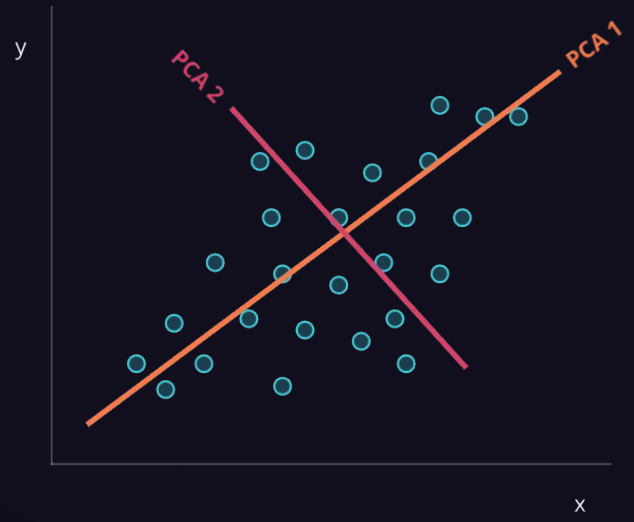
|04 DIMENSIONALITY REDUCTION



PCA

$$C = \begin{pmatrix} Var(x) & Cov(x,y) & Cov(x,z) \\ Cov(y,x) & Var(y) & Cov(y,z) \\ Cov(z,x) & Cov(z,y) & Var(z) \end{pmatrix}$$

$$\begin{matrix} v_1 & \lambda_1 \\ v_2 & \lambda_2 \\ v_3 & \lambda_3 \end{matrix} \quad \uparrow \begin{matrix} \text{Big} \\ \\ \text{Small} \end{matrix}$$

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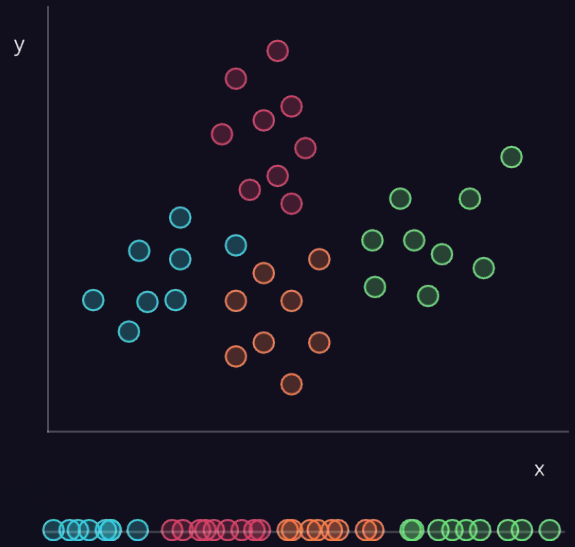
|04 DIMENSIONALITY REDUCTION



t-SNE

Steps

1) Use **Normal Distribution** to Estimate **Similarity** b/ Data Points
2) Create another Distribution (**t-distribution**) Capturing the **Same Similarity** Between Data Points using **Gradient Descent** on **KL-divergence**

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05|

SUPERVISED LEARNING

Keep Calm & Label them All

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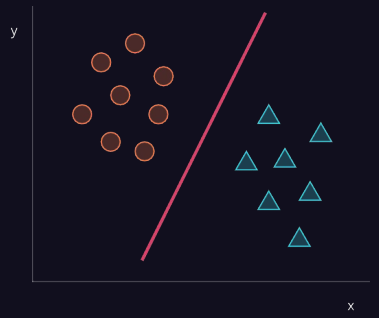
|05 SUPERVISED LEARNING

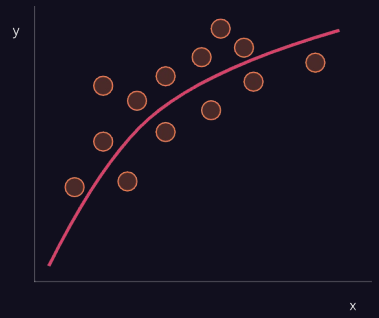
LEARN FROM LABELED DATA

$$Y = f(X)$$

Classification



Regression



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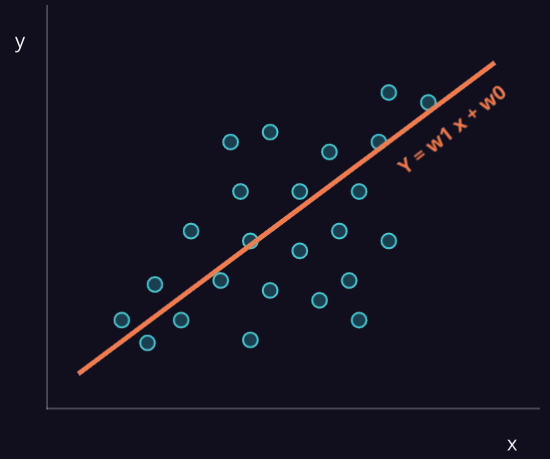
|05 SUPERVISED LEARNING



LINEAR REGRESSION

$$Y = f(W, X) = XW^t + \epsilon$$

$$\mathcal{L} = MSE(\hat{Y}, Y) = \frac{1}{N}\sum_i (w_i x_i - y_i)^2$$

$$\frac{\partial \mathcal{L}}{\partial W} = \frac{2}{N}(X^t XW - X^t Y) = 0$$

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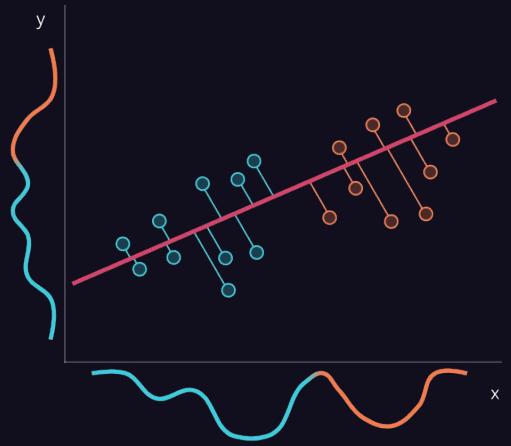

LINEAR DISCRIMINANT ANALYSIS

$$Y = f(W, X) = XW^t + \epsilon$$

$$Objective(W) = \frac{|\tilde{\mu_1} - \tilde{\mu_2}|}{\tilde{s_1^2} + \tilde{s_2^2}}$$

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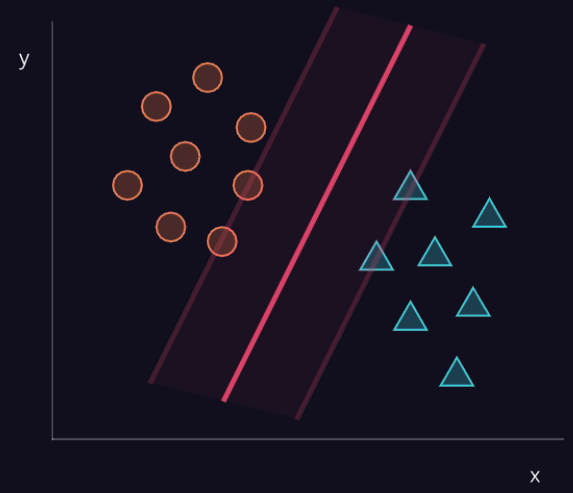

SUPPORT VECTOR MACHINE

$$Y_i(WX + b) - 1 \geq 0$$

$$margin = (X_+ - X_-)\frac{W}{\|W\|}$$

$$J(W) = \frac{1}{2}\|W\|^2 + C(\frac{1}{N}\sum_i max(0, 1 - y_i(wx_i + b)))$$

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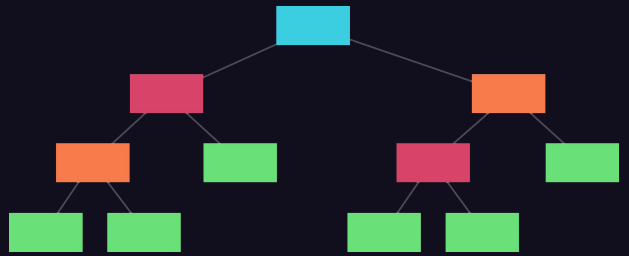
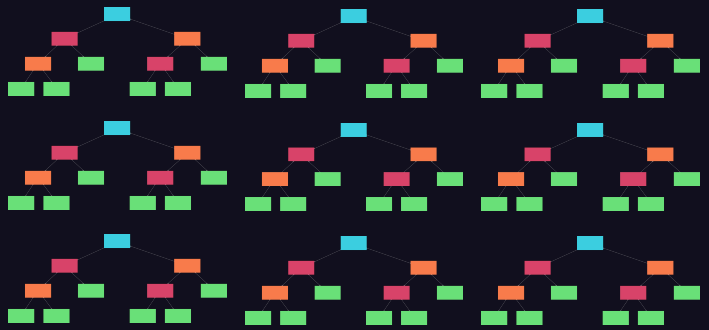


DECISION TREE

Steps

1) Compute all **Gini Impurity** Scores
2) **Lowest** Impurity Score **Becomes a Leaf**
3) Else **if Improvement** pick the Separation with the **Lowest Score**

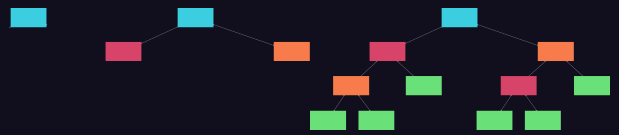
$$Gini(K) = \sum_i P_{i,K}(1 - P_{i,K}) = 1 - \sum_i P_{i,K}^2$$



RANDOM FOREST

Intuition

Forest of **Dense Decision Trees**
Like **Cross Validation**
Ensemble Methods give Better Results
Use Voting, Max or Average



GRADIENT BOOSTING

Steps

1) Start from a Single Leaf of the **Average Prediction**
2) Build a **New Tree** to Predict the **Residual Error** from the Previous Tree and **Weight** its Contribution by a **Learning Rate**
3) Do **Until Convergence** Condition

|06

UNSUPERVISED LEARNING

In my Defense I was Left Unsupervised

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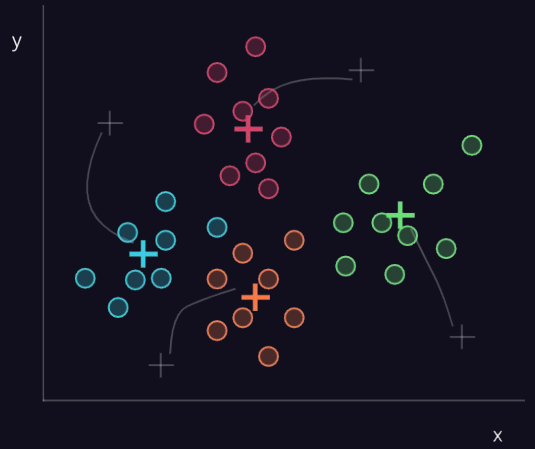
|06 UNSUPERVISED LEARNING



K-MEANS

Steps

1) **Randomly** Place **K Centroids**
2) **Assign** each Point to the **Closest Centroid**
3) **Move** Centroids to its **Cluster's Mean**
4) Do **Until Convergence** Condition

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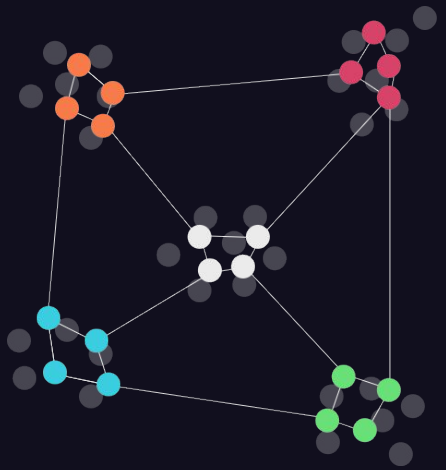

SELF-ORGANIZING MAP

Steps

1) **Randomly** Initialize the **Neurons**
2) Compute **Distance** from **Training Sample** and **Each Lattice**
3) The **Closest** is Declared as **Winner**
4) **Update** Winner and Neighbours **Weights**

$$W_i(s+1) = W_i(s) + \eta \cdot Neightbour(i,p) \cdot Distance(X - W_i(s))$$

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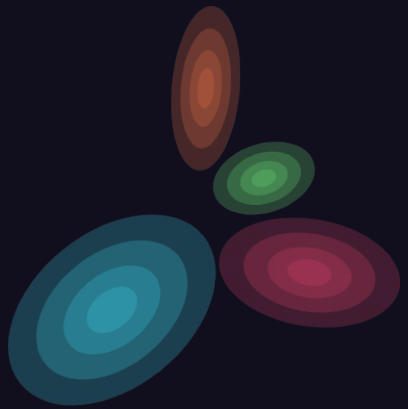

MIXTURE OF GAUSSIANS

$$p(x) = \sum_i \alpha_i \mathcal{N}(x|\mu_i, \Sigma_i)$$

Expectation Maximization

1) **Expectation**: Compute the **Likelihood** of each Datum **being part of Class k** with Initial Parameters

$$\gamma_{i,k} = \frac{\alpha_k \mathcal{N}(x_i|\mu_k,\Sigma_k)}{\sum_{j=1}^{K} \alpha_j \mathcal{N}(x_i|\mu_j,\Sigma_j)}$$

2) **Maximization**: **Update** each **Parameter** to **Maximize the Expectation**

$$\alpha_k = \sum_{i=1}^{N} \frac{\gamma_{i,k}}{N} \quad \mu_k = \frac{\sum_{i=1}^{N} \gamma_{i,k} x_i}{\sum_{i=1}^{N} \gamma_{i,k}} \quad \sigma_k = \frac{\sum_{i=1}^{N} \gamma_{i,k} (x_i - \mu_k)^2}{\sum_{i=1}^{N} \gamma_{i,k}}$$

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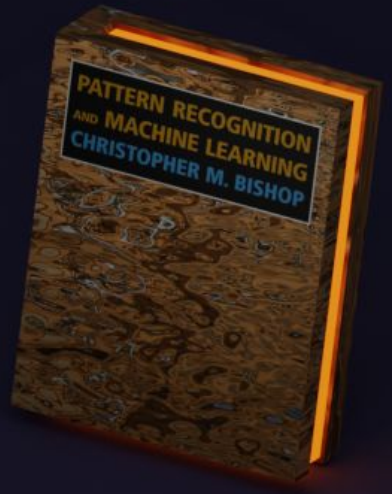

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REFERENCE

Not Sponsored

Pattern Recognition and Machine Learning

Christopher Bishop

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