Quantifying Acne Severity Dynamics: A Bayesian and Information Theory Based Framework

By Nathaniel Wolff

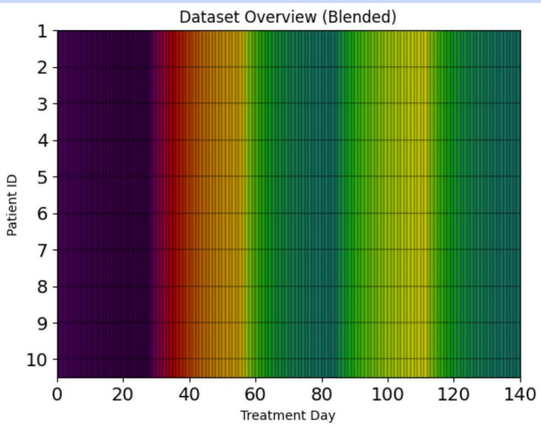
Motivation and Research Question:

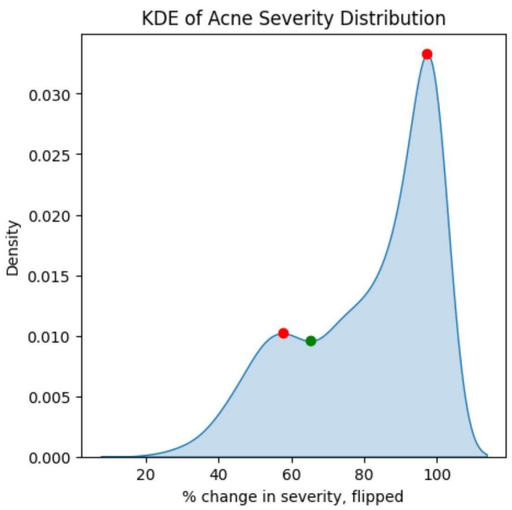
For all of the cost and effort of acne treatment, it is important to quantify how well acne treatment products lead to positive patient outcomes.

Central Question: Does repeated use of the same acne treatment lead to diminishing returns in acne severity improvement? Can we quantify those diminishing returns?

Dataset

A cohort of N = 10 simulated patients, each receiving alternating 28-day blocks of “Cream” versus “Antibiotics”. Daily acne severity score was recorded and normalized relative to patient’s average baseline intensity. The distribution of acne severity scores was trimodal with low skewness, motivating the binning of scores into three distinct severity change states.





Top: Heatmap of daily treatments, with facecolor blend representing treatment series.
Bottom: Kernel Density Estimate of normalized acne severities. Labeled points correspond to severity change states.

Methods

The Dirichlet distributions of acne severity change states were computed for each day of treatment. The information cost of not updating was determined by computing Kullback-Leibler Divergence between consecutive distributions. To identify treatment regimes with diminishing returns, a univariate spline curve was fitted to the cumulative KL Divergence, and segments where its first order derivative exceeded a given threshold were obtained. A linear regression model was fitted to each segment. To predict acne severity state change, a Nonlinear State Space model (below) was fitted with the obtained distributions. Hidden acne physiology was modeled using a 3-dimensional state vector (v_t) with nonlinear transition dynamics $F_\Theta + w_t$, where treatment inputs (u_t) reduce bacterial growth (B_t), inflammation (I_t), and sebum production (S_t). Observations (x_t) represent measured acne severity. Posterior State Trajectories were computed using an Extended Kalman Filter, and parameters were optimized with Expectation-Maximization.

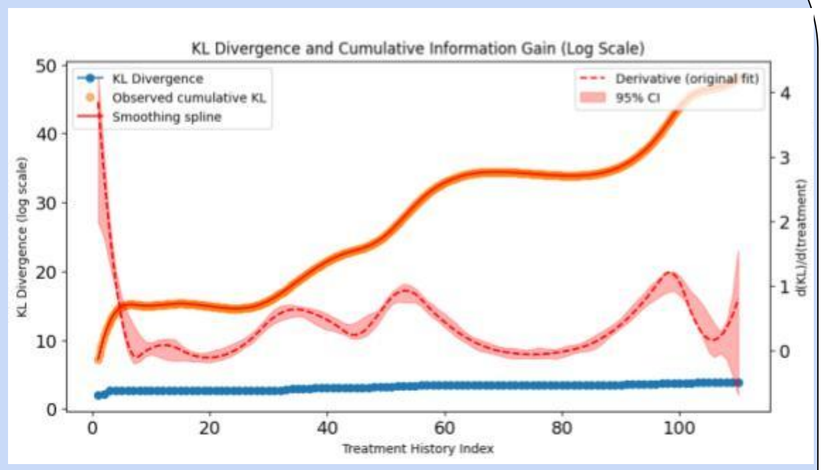
$$v_t = \begin{pmatrix} B_t \\ I_t \\ S_t \end{pmatrix}, u_t = \begin{pmatrix} days_{antibiotics} \\ cream_{used} \\ T(tstd) \end{pmatrix}$$

$$v_t = F_\theta(v_{t-1}, u_t \mid \theta) + w_t, \qquad w_t \sim N(0, R)$$

$$x_t = g(v_t) + \eta_t, \qquad \eta_t \sim N(0, R)$$

$days_{antibiotics}$	Days of antibiotics taken in current block up to day t.
$cream_{used}$	Either 0 or 1 if cream used on day t.
T(tstd)	E(ΔX|tstd), tstd refers to cumulative treatment to day t.

Results and Clinical Benefits



Segment	Start	Starting History	End	Ending History	Num Points	Slope	p-value	R^2
0	25	(('Baseline', 26),)	66	(('Baseline', 28), ('Antibiotics', 28), ('Cream', 11))	42	0.5373	2.534E-35	0.9793
1	83	(('Baseline', 28), ('Antibiotics', 28), ('Cream', 28))	104	(('Baseline', 28), ('Antibiotics', 28), ('Cream', 28), ('Antibiotics', 21))	22	0.6964	2.247E-13	0.9355

Top: Cumulative KL divergence, fitted spline, first derivative, and 95% confidence interval of bootstrapped first derivative.
Bottom: Table of Non-Diminishing Return Segments, with regression model parameters.

- Improvement is strongest in the first treatment block.
- Each repeated block of antibiotics yields non-diminishing returns, but cream blocks yield progressively smaller improvements.
- The model captures nonlinear coupling between bacterial load, inflammation, and sebum.
- The approach can generalize to patient-specific treatment personalization, offering significant clinical benefits.