Homework #1

1.1

1.1.1)

a.) Not a proposition

b.) The soup is hot

c.) The patient does not have diabetes

1.1.2)

a.) $n \wedge m$

b.) $t \wedge m$

c.) $n \vee m$

1.1.3)

a.) true

B.) false

c.) True

1.2

1.2.1)

a.) $T \wedge F \equiv F$

b.) $T \vee F \equiv T$

c.) $F \vee \neg F \equiv F \vee T \rightarrow T$

1.2.2)

a.) $p \wedge q$

b.) $p \wedge \neg q$

c.) $\neg(p \vee q)$

1.2.3) $p = T$, $q = T$, $s = F$

a.) $T \vee \neg T \equiv T \vee F \equiv T$

b.) $(T \wedge T) \vee F \equiv T \vee F \equiv T$

c.) $T \wedge (T \vee F) \equiv T \wedge T \equiv T$

1.2.4)

a.) $\neg p \oplus q$:

p	q	¬p	¬p ⊕ q
T	T	F	T
T	F	F	F
F	T	T	F
F	F	T	T

b.) $\neg(p \vee q) = \neg p \wedge \neg q$

p	q	p ∨ q	¬(p ∨ q)
T	T	T	F
T	F	T	F
F	T	T	F
F	F	F	T

1.3

1.3.1) $\rightarrow$ ::= implies

a.) if february has 30 days $\rightarrow$ 7 is an odd number

$\equiv F \rightarrow T \equiv T$

b.) If January has 31 days $\rightarrow$ 7 is an even number

$\equiv T \rightarrow F \equiv F$

1.3 cont.

1.3.2) Inv: $\neg P \to \neg Q$ Contra pos: $\neg Q \to \neg P$ Conv: $Q \to P$

a.) Inverse: If she didn't finish her homework, then she didn't go to the party

Contrapositive: If she didn't go to the party, then she didn't do her homework

Converse: If she went to the party, then she finished her homework

b.) Inverse: If he didn't train for the race, then he didn't finis the race

Contrapositive: If he didn't finish the race, then he didn't train for the race

Converse: If he finished the race, then he traind for the race

1.3.3)

a.) Inverse: If 3 is not a prime number, then 5 is an odd number

↳ Truth: $F \to T \equiv T$

Contrapositive: If 5 is an odd number, then 3 is not a prime number

↳ Truth: $T \to F \equiv F$

Converse: If 5 is an even number, then 3 is a prime number

↳ Truth: $F \to T \equiv T$

B.) Inverse: If $7 > 5$ then $5 > 3$

↳ Truth: $T \to T \equiv T$

Contrapositive: If $5 > 3$, then $7 > 5$

↳ Truth: $T \to T \equiv T$

Converse: If $7 < 5$ then $5 < 3$

↳ Truth: $F \to F \equiv F$

1.3 cont pt.2

1.3.4)

A.)

p	q	$\neg p \land q$	$(\neg p \land q) \to p$
T	T	F	T
T	F	F	T
F	T	T	F
F	F	F	T

B.)

p	q	$p \to q$	$q \to p$	$(p \to q) \to (q \to p)$
T	T	T	T	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

1.3.5)

A.) $\neg j \to c$

B.) $c \to \neg j$

1.3.7)

A.) $p \to (s \land y)$

B.) $(s \lor y) \to p$

1.3.10) $p \equiv T$, $q \equiv F$, $r \equiv ?$

A.) $p \to (q \land r) \equiv T \to (F \land r) \equiv T \to F \equiv F$

B.) $(p \lor r) \to r \equiv (T \lor r) \to r \equiv r \to r \equiv T$

1.4

1.4.1)

A.)

p	q	$(p \lor q) \lor (q \to p)$
T	T	T
T	F	T
F	T	T
F	F	T

⇒ tautology since all tru

B.)

p	q	$p \land \neg q$	$(p \to q) \leftrightarrow p \land \neg q$
T	T	F	F
T	F	T	F
F	T	F	F
F	F	F	F

↳ contradiction, since all F

1.4.2)

A.)

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

$\equiv$

p	q	$p \to q$	$q \to p$	$(p \to q) \land (q \to p)$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

1.4 cont

1.4.2)

B.)

p	q	S1: $\neg(p \to q)$	S2: $\neg p \vee q$
T	T	F	F
T	F	T	T
F	T	T	T
F	F	F	F

→ both statments (S1 + S2) are eqivalent

1.4.3)

A.)

p	q	S1: $p \to q$	S2: $q \to p$
T	T	T	T
T	F	F	T
F	T	T	F
F	F	T	T

S1 + S2 are not logically eqivalent

B.)

p	q	S1: $\neg p \to q$	S2: $\neg p \vee q$
T	T	T	T
T	F	T	F
F	T	T	T
F	F	F	T

S1 + S2 are not logically eqivalent

1.5

1.5.1)

A.)

$(p \to q) \wedge (q \vee p)$	
$(\neg p \vee q) \wedge (q \vee p)$	Implication
$(q \vee \neg p) \wedge (q \vee p)$	
$q \vee (\neg p \wedge p)$	Distributive Law
$q \vee (p \wedge \neg p)$	
$q \vee F$	Complement Law
q	Identity Law

B.)

$(\neg p \vee q) \to (p \wedge q)$	
$\neg(\neg p \vee q) \vee (p \wedge q)$	Implication
$(\neg\neg p \wedge \neg q) \vee (p \wedge q)$	De morgan's Law
$(p \wedge \neg q) \vee (p \wedge q)$	Double Negation
$p \wedge (\neg q \vee q)$	Distributive
$p \wedge T$	Negation
p	Identity

1.5 cont

1.5.1)

C.) $r \vee (\neg r \to p)$

$r \vee (\neg r \to p)$	
$r \vee (\neg\neg r \vee p)$	Implication
$r \vee (r \vee p)$	Double negation
$(r \vee r) \vee p$	Associative
$r \vee p$	Idempotent

1.5.2)

A.)

$\neg p \to \neg p \equiv \neg(\neg p) \vee \neg q$	Implication
$\equiv p \vee \neg q$	Double negation
$\equiv \neg q \vee p$	Commutative
$\equiv q \to p$	Implication

B.)

$p \wedge (\neg p \to q) \equiv p \wedge (\neg(\neg p) \vee q)$	Implication
$\equiv p \wedge (p \vee q)$	Double Negation
$\equiv (p \wedge p) \vee (p \vee q)$	Distributive
$= p \vee (p \wedge q)$	Idempotent
$= p$	Absorption

C.)

$(p \to q) \wedge (p \to r) \equiv (\neg p \vee q) \wedge (\neg p \vee r)$	Implication
$\equiv ((\neg p \vee q) \wedge \neg p) \vee ((\neg p \vee q) \wedge r)$	Distributive
$\equiv (\neg p) \vee ((\neg p \vee q) \wedge r)$	Absorption
$\equiv (\neg p) \vee (\neg p \wedge r) \vee (q \wedge r)$	Distributive
$\equiv (\neg p \vee (\neg p \wedge r)) \vee (q \wedge r)$	associative
$\equiv (\neg p) \vee (q \wedge r)$	Identity
$\equiv p \to (q \wedge r)$	Implication

1.5.3)

a.) $(p \wedge q) \to (p \vee r) \equiv \underbrace{\neg(p \wedge q) \vee (p \vee r)}_{\text{Implication}} \equiv \underbrace{(\neg p \vee \neg q) \vee (p \vee r)}_{\text{De'morgan's}}$

$\equiv \underbrace{(\neg p \vee p) \vee (\neg q \vee r)}_{\text{associative}} \equiv \underbrace{T \vee (\neg q \vee r)}_{\text{Inverse}} \equiv \underbrace{T}_{\text{null law}}$ $\therefore$ Tautology

1.5

1.5.3)

b.) $p \to (r \to p)$ | given
$p \to (\neg r \vee p)$ | Implication
$\neg p \vee (\neg r \vee p)$ | Implication
$(\neg p \vee p) \vee \neg r$ | Associative
$T \vee \neg r$ | inverse
T | null
$\therefore$ Tautology

c.) $\neg r \vee (\neg r \to p)$ | given
$\neg r \vee (\neg\neg r \vee p)$ | implication
$\neg r \vee (r \vee p)$ | Double negation
$(\neg r \vee r) \vee p$ | associative
$T \vee p$ | inverse
T | null
$\therefore$ Tautology

1.6

1.6.1)

A.) $p(3) ::=$ 3 is even, which is a proposition, since it can be true or false
↳ Truth value: false

b.) $\neg p(3) ::=$ 3 is not even, is a proposition
↳ Truth value: True

c.) $t(5, 32) ::= 2^5 = 32$, which is a proposition
↳ Truth value: true

1.6.2)

A.) existential statement is false because:
$x + x = 1 \equiv 2x = 1 \equiv x = \frac{1}{2}$
↳ $x = \frac{1}{2}$ is not in domain $\therefore$ false

B.) existential statement is true because:
$x + 2 = 1 \equiv \boxed{x = -1}$

c.) existential statement is true because:
$x = 0$; $0^2 - 0 = 0 \neq 1$ ✓

1.6 con

1.6.3)

A.) $\exists x\ (x^3 = 2)$

B.) $\forall x\ (x^2 \geq 0)$

1.6.4)

a.) $\forall x\ P(x)$ is true because:

$P(a) = T$, $P(b) = T$, $P(c) = T$, & $P(d) = T$

b.) $\exists x\ P(x)$ is true because:

$P(a) = P(b) = P(c) = P(d) = T$, so any domain has a value that makes $P(x)$ true. ∴ such x exists

1.7

1.7.1)

A.) $\exists x\ Q(x)$ is True ∴ a proposition

↳ ex. $x = 9$, $3^2 = 9$, which is a perfect square

B.) $\forall x\ Q(x) \wedge \neg P(x)$ is False ∴ a proposition, $\exists x$ that is not a prime and not a perfect square in $\mathbb{R}$

↳ ex. $x = 6$, $P(6) = F$ $Q(6) = F$ ∴ $F \wedge \neg F \equiv F$

C.) $\forall x\ Q(x) \vee P(3)$ is true ∴ a proposition

↳ because $P(3) \equiv T$, any disjunction with 1 True will always be true.

1.7.2)

A.) $\exists x\ (E(x) \rightarrow T(x))$

B.) $\forall x\ (E(x) \rightarrow T(x))$

C.) $\forall x\ (T(x) \rightarrow E(x))$

1.7.3)

A.) $\exists x\ (\neg B(x))$

B.) $\forall x\ B(x)$

1.7 cont

1.7.4)

A.) $\exists x \; S(x)$

B.) $\forall x \; (\neg S(x) \land W(x))$

1.7.5)

A.) $\exists x \; (\neg O(x))$

B.) $\forall x \; (O(x) \rightarrow M(x))$

1.7.7)

A.) $\forall x \; (D(x) \lor N(x))$ is true since for all x $D(x)$, or $N(x)$ is at least true for one.

↳ Eng: All group company members either missed the deadline or is a new employee

B.) $\forall x \; ((x \neq \text{sam}) \rightarrow N(x))$ is true

↳ english: For all employees in the group, if the employee is not Sam, then they are a new employee

1.7.8)

A.) $\exists x \; (M(x) \land D(x))$ is False ∴ a proposition

↳ English: Of the five patients, there is a patient that had migraines and was given medication

B.) $\exists x \; M(x) \land \exists x \; D(x)$ is True ∴ a proposition

↳ English: Of the five patients, one patient took the medication and one patient had a migraine

1.8

1.8.1)

A.) $\neg \exists x \; P(x) \equiv \forall x \; \neg P(x)$

B.) $\neg \exists x \; (P(x) \lor Q(x)) \equiv \forall x \; \neg (P(x) \lor Q(x)) \equiv \forall x \; (\neg P(x) \land \neg Q(x))$

1.8 cont

1.8.2)

A.) $\forall x \; D(x)$

Negation: $\neg \forall x \; D(x)$

De morgan's law: $\exists x \; \neg D(x)$

English: One patient did not get the medication

B.) $\forall x \, (\neg P(x) \vee \neg D(x))$

Negation: $\neg \forall x \, (\neg P(x) \vee \neg D(x))$

De morgan's Law: $\exists x \, (P(x) \wedge D(x))$

English: One patient was given the medication and placebo

1.8.4)

A.) $\neg \forall x \, (P(x) \wedge \neg Q(x)) \equiv \exists x \, \neg (P(x) \wedge \neg Q(x))$

$\equiv \exists x \, (\neg P(x) \vee \neg\neg Q(x))$

$\equiv \exists x \, (\neg P(x) \vee Q(x))$

B.) $\neg \forall x \, (\neg P(x) \rightarrow Q(x)) \equiv \neg \forall x \, (\neg\neg P(x) \vee Q(x))$

$\equiv \exists x \, \neg (P(x) \vee Q(x))$

$\equiv \exists x \, (\neg P(x) \wedge \neg Q(x))$