Homework 3 & 4

3.5

3.5.1)

A.) Complement law

B.) Absorption law

C.) De morgan's law

3.5.2)

A.) $(\bar{A} \cap C) \cup (A \wedge C)$	Given
$(\bar{A} \cup A) \cap C$	Distributive
$U \cap C$	Compliment law
C	Identity

B.) $(B \cup A) \wedge (\bar{B} \cup A)$	Given
$(B \cap \bar{B}) \cup A$	Distributive
$\emptyset \cup A$	Compliment
A	Identity

3.6

3.6.2)

a.) $X \times Y = \{(x,52),(x,67),(+,52),(+,67),(\$,52),(\$,67)\}$

b.) $x^4 = \{(x,y,z,w) : x,y,z,w \in X\}$

3.6.6)

a.) $\{0x : x \in \{0,1\}^2\} \rightarrow \{0,1\}^2 = \{0,1) \cup \{0,1\} = \{00,01,10,11\}$

$\therefore \{0x\} = \{000, 001, 010, 011\}$

b) $\{0,1\}^0 \cup \{0,1\}^1 \cup \{0,1\}^2 = \{0,1\} \cup \{00,01,10,11\} \cup$ Any Const

$\therefore \{\epsilon, 0, 1, 01, 00, 10, 11\}$

3.7

3.7.1)

A.) $D = \{x \in z : 1 \leq x \leq 6\} = \{1,2,\ ,6\}$ | $A \cup B \cup C = \{1,2,\ ,6\}$

$\therefore A \equiv D$ $\therefore$ it is a partition of D

3.7 cont

3.7.1)

B.) $D = \{1, 2, 3, 4, 5, 6\}$ | $B \cup C = \{2, 3, 4, 5\}$

↳ since $D \neq B \cup C$ it is not a partition bc not entire is created

and is missing $\{1, 2, 6\}$

C.) $E = \{2, 3, 4, 5\}$ | $B \cup C = \{2, 3, 4, 5\}$

↳ $E \equiv B \cup C$ ∴ it is a partition of E

3.7.3)

A.) NO, A, B, + C do not form a partition of R. Specifically set A + B are not disjoined

B.) yes, $A \cup B, \cup D \equiv R$ ∴ it is a partition

C.) NO, B, D, + E do not form a partition of R. Specifically set D + E are not disjoined

4.5

4.5.2)

A.) $(f \circ g)(0) = f(g(0)) = (2^0)^2 = 1^2 = \boxed{1}$

B.) $(f \circ h)(52) = f(h(52)) = (\lceil \frac{52}{5} \rceil)^2 = (11)^2 = \boxed{121}$

C.) $(g \circ h \circ f)(4) = g(h(f(4))) = g(h(4^2)) = g(h(16))$

$= g(\lceil \frac{16}{5} \rceil) = g(4) = 2^4 = \boxed{16}$

4.5.5)

A.) Range of $g = \{2, 3\}$

B.) domain of $h \circ g = \{a, b, c\}$

C.) $h^{-1}(y) = 3$

4.6

4.6.1)

A.) $(6^k)^k = 6^{k^2}$

B.) $(6^{2k})^3 = 6^{2k \cdot 3} = 6^{6k}$

C.) $(6^{2^k})^2 = 6^{2 \cdot 2^k} = 6^{2^{k+1}}$

3.7 cont

3.7.1)

B.) $D = \{1,2,3,4,5,6\}$ | $B \cup C = \{2,3,4,5\}$

↳ since $D \neq B \cup C$ it is not a partition bc not entire set created and is missing $\{1,2,6\}$

C.) $\mathcal{E} = \{2,3,4,5\}$ | $B \cup C = \{2,3,4,5\}$

↳ $\mathcal{E} \equiv B \cup C$ ∴ it is a partition of $\mathcal{E}$

3.7.3)

A.) NO, A, b, & C do not form a partition of R. Specifically, Set A & B are not disjoined

b.) yes, $A \cup B, \cup D = R$ ∴ it is a partition

C.) NO, B, D, & E do not form a partition of R. Specifically Set D & E are not disjoined

4.5

4.5.2)

A.) $(f \circ g)(0) = f(g(0)) = (2^0)^2 = 1^2 = \boxed{1}$

B.) $(f \circ h)(52) = f(h(52)) = \left(\left\lceil \frac{62}{5} \right\rceil\right)^2 = (11)^2 = \boxed{121}$

C.) $(g \circ h \circ f)(4) = g(h(f(4))) = g(h(4^2)) = g(h(16))$

$= g\left(\left\lceil \frac{16}{5} \right\rceil\right) = g(4) = 2^4 = \boxed{16}$

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A.) Range of $g = \{2,3\}$

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4.6 cont

4.6.2)

A.) $\log_5 k + \log_5 7 = \log_5 k \cdot 2 = \log_5 2k$

B.) $2\log_5 k = \log_5 k^2$

C.) $\log_5 k - \log_5 7 = \log_5 \left(\frac{k}{7}\right)$

4.6.4)

A.) $\lceil \log_3 \text{rad} \rceil = \lceil 3^4 < x < 3^5 \rceil = 5$

B.) $\lfloor \log_3 536 \rfloor = \lfloor \log_3 3^5 < x < 3^6 \rfloor = 5$

C.) $\lfloor \log_3 2 \rfloor = \lfloor \log_3 (3^0 < x < 3^1) \rfloor = 0$