Homework 6 + 7

6.4

6.4.1)

A.) $S \circ R = S(R(x))$ | $S(R(a)) = S(d) = \emptyset$ | $S(R(c)) = S(b) = \emptyset$

$S(R(b)) = S(c) = d$ and a | $S(R(d)) = S(b) = \emptyset$

$\therefore S \circ r = \{(b,d),(b,a)\}$

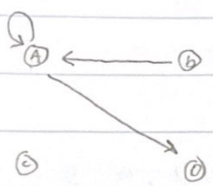
B.) $R \circ S = R(S(x))$ | $R(S(a)) = R(b) = c$ | $R(S(c)) = R(d) = b$

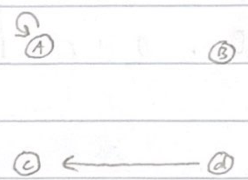
$= R(c) = b$ | $= R(a) = b$

$R(S(b)) = R(\emptyset) = \emptyset$ $\quad R(S(d)) = R(\emptyset) = \emptyset$

$\therefore R \circ S = \{(a,c),(a,b),(c,d),(c,b)\}$

6.4.3)

A.) 

B.) 

6.5

6.5.1)

A.) No, there is no walk length 2 in G^2

B.) yes, $b \to c \to f \to e$ is a walk len 3 in G^3

C.) No, no walk length 3 (G, G) in G^3

D.) yes, walk $(g \to f \to e \to c \to g)$ is a walk (G, G) in G^4

6.5.4)

A.) No, there is not a walk len 3 in G

B.) yes, $1 \to 2 \to 3$ is a closed length 3 walk in G

6.6

6.6.1)

A.) $G = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}$ $\quad G^2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$ $\quad G^3 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

6.6 cont

6.6.1)

A cont.) $G^{4} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}$ $\quad G^{T} = \begin{bmatrix} 0 & 2 & 1 & 1 \\ 0 & 1 & 2 & 1 \\ 0 & 1 & 1 & 2 \\ 0 & 2 & 1 & 1 \end{bmatrix}$

$G + \dots + G^{4}$

6.6.2)

A.) $A = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$

6.6.3)

A.) Looking at col 2 of A^3; vertices 2, 4, 5 can reach vert. 2 w/ len 3 walk

B.) The out degree of Vertex 4 is 5

6.7

6.7.1)

A.) Minimal elements: J, I, A, F

B.) Maximal elements: J H D G

C.) Comp. pairs. (A, D), (G, F), (D, B), (H, E)

6.8

6.8.1)

A.) The relation is a partial order since it is reflexive, asymmetric, and transitive by definition, but it is not a total order since some words are not comparable.

6.8 cont

6.8.2)

A.) $b \to d \to c \to f \to e \to a \to g$

$b \to c \to f \to e \to d \to a \to g$

6.9

6.9.2)

A.) $x \, D \, y \Leftrightarrow x \equiv y \pmod 4$:

$d(0) = \{44, 56, 4\}$
$d(1) = \{13, 17\}$
$d(2) = \{2, 34\}$
$d(3) = \{7, 99, 31\}$

Partitions of S (disjoint) → correspond to make equiv. classes of D

7.4

7.4.3)

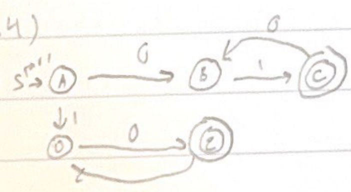
A.)

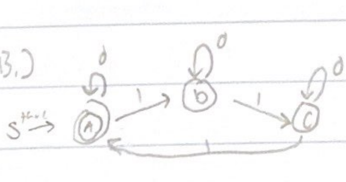
String	FSM 1 Final State	FSM 1 Accept?	FSM 2 Final State	FSM 2 Accept?
100011	B	NO	A	yes
0000	D	NO	A	yes
0010	D	NO	C	NO
1100	C	yes	D	NO

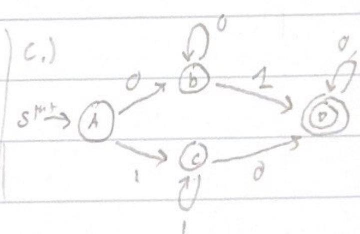
B.) Statement 2 describes the set of strings accepted by FSM 1. (has to start w/ 1 + end w/ 0)

C.) Statement 4 describes the set of strings accepted by FSM 2. (has to have 3 consecutive 0s somewhere)

7.4.4)

A.)


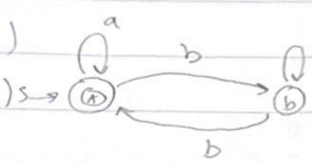
B.)


C.)


7.5

7.5.3)

A.)



B.)