Homework #2

1.9

1.9.1)

A.) $M(1,1)$ is true $\therefore$ it is a proposition

B.) $\forall y\ M(x,y)$ is not a proposition since x is free $\therefore$ has no value in domain

C.) $\exists x\ M(x,3)$ is a proposition. It is also True because $M(1,3) \vee M(2,3) \vee M(3,3) \equiv T \vee T \vee F \equiv T$

1.9.2)

A.) $\exists x \forall y\ P(x,y) \equiv$ False

B.) $\exists x \forall y\ Q(x,y) \equiv$ True

C.) $\exists y \forall x\ P(x,y) \equiv T$

1.9.3)

A.) $\forall x \exists y\ (xy > 0) \equiv$ False because, when $y = 0$ inequality doesn't hold

B.) $\exists x \forall y\ (xy = 0) \equiv$ True when $x = 0$

1.9.4)

A.) $\forall x \exists y \exists z\ P(y,x,z) \xrightarrow{neg} \neg \forall x \exists y \exists z\ P(y,x,z)$

$\equiv \exists x \forall y \forall z \neg P(y,x,z)$

B.) $\forall x \exists y\ (P(x,y) \wedge Q(x,y)) \xrightarrow{neg} \neg \forall x \exists y\ (P(x,y) \wedge Q(x,y))$

$\equiv \exists x \forall y \neg (P(x,y) \wedge Q(x,y))$

$\equiv \exists x \forall y\ (\neg P(x,y) \vee \neg Q(x,y))$

C.) $\exists x \forall y\ (P(x,y) \to Q(x,y)) \xrightarrow{neg} \neg \exists x \forall y\ (P(x,y) \to Q(x,y))$

$\equiv \forall x \exists y \neg (P(x,y) \to Q(x,y))$

$\equiv \forall x \exists y \neg(\neg P(x,y) \vee Q(x,y))$

$\equiv \forall x \exists y\ (\neg\neg P(x,y) \wedge \neg Q(x,y))$

$\equiv \forall x \exists y\ (P(x) \wedge \neg Q(x,y))$

1.10

1.10.1)

A.) $\forall x \forall y \; M(x,y) \equiv$ False since $M(2,2) \equiv$ False $\therefore$ statement doesn't hold.

B.) $\forall x \forall y \; ((x \neq y) \rightarrow M(x,y)) \equiv$ True since whever $x \neq y$ $M(x,y)$ is True

C.) $\exists x \exists y \; \neg M(x,y) \equiv$ True when $M(2,2) =$ False $\therefore \neg F \equiv T$

1.10.2)

A.) $\forall x \exists y \; (x+y=0) \equiv$ False since y is one particular # any x value that is not $-y$ will make $x+y \neq 0$

B.) $\exists x \forall y \; (x+y=0) \equiv$ False since x is one particular # any y value that is not $-x$ will make $x+y \neq 0$

C.) $\exists x \forall y \; (xy=y) \equiv$ True since $x=1$ makes $1y=y \rightarrow y=y$ $\therefore$ computing the statement

1.10.4)

A.) $\exists x \exists y \; (x/y < 1)$

B.) $\forall x \; ((x>0) \rightarrow (\frac{1}{x} > 0))$

C.) $\exists x \exists y \; (x+y=xy)$

1.11

1.11.1)

A.

p	q	$(p \vee q) \wedge p$	$(p \vee q) \wedge p \Rightarrow q$
T	T	T	T
T	F	T	F
F	T	F	T
F	F	F	T

Since all of last column is not T, arg is invalid

1.11 cont

1.11.1)

B.)

p	q	$(p \vee q) \wedge (p \leftrightarrow q)$	$(p \vee q) \wedge (p \leftrightarrow q) \to p$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

Since all true in last column, arg is valid

C.)

p	q	$(p \wedge q) \to (p \leftrightarrow q)$
T	T	T
T	F	T
F	T	T
F	F	T

Since all true in last column, arg is valid.

1.12

1.12.2)

A.) $\begin{array}{l} p \to q \; (1) \\ q \to r \\ \neg r \\ \hline \neg p \end{array} \equiv \begin{array}{l} p \to r \; (2) \\ \neg r \\ \hline \neg p \end{array} \equiv \begin{array}{l} (3) \\ \neg p \\ \hline \neg p \end{array}$

(1) given
(2) Hypothesis syllogism
(3) modus tollens

B.) $\begin{array}{l} p \to (q \wedge r) \; (1) \\ \neg q \\ \hline \neg p \end{array} \equiv \begin{array}{l} p \to q \wedge r \; (2) \\ \neg q \vee \neg r \\ \hline \neg p \end{array} \equiv \begin{array}{l} p \to q \wedge r \; (3) \\ \neg(q \wedge r) \\ \hline \neg p \end{array} \equiv \begin{array}{l} (4) \\ \neg p \\ \hline \neg p \end{array}$

(1) given
(2) addition
(3) De morgan's law
(4) Modus Tollens

1.12 cont

1.12.2)

C.) $(p \wedge q) \to r$ ①, $\neg r$, q, $\therefore \neg p$

$$\begin{array}{c} (p\wedge q)\to r \\ \neg r \\ \underline{q} \\ \neg p \end{array} \equiv \begin{array}{c} \neg(p\wedge q) \\ \underline{q} \\ \neg p \end{array} \equiv \begin{array}{c} \neg p \vee \neg q \\ \underline{q} \\ \neg p \end{array} \equiv \begin{array}{c} \neg p \vee \neg q \\ \underline{\neg\neg q} \\ \neg p \end{array} \equiv \begin{array}{c} \underline{\neg p} \\ \neg p \end{array} \checkmark$$

1. given 2. modus tollens 3. De morgan's Law 4. Double N'gation 5. Disjunctive Syllogism

1.13

1.13.1)

B.) F(x): x owns a ferrari | x = people who live in the city

S(x): x has gotten a speeding ticket

$$\begin{array}{l} F(\text{linda}) \\ \underline{\forall x\,(F(x)\to S(x))} \\ \therefore S(\text{linda}) \end{array} \equiv \begin{array}{c} F(\text{linda}) \\ \underline{F(\text{linda})\to S(\text{linda})} \\ S(\text{linda}) \end{array} \equiv \begin{array}{c} \underline{S(\text{linda})} \\ S(\text{linda}) \end{array}$$

1. given 2. Universal instantiation 3. Modus ponens

1.13.3)

A.)

	P	Q
a	F	T
b	F	T

$\forall x\,(p(x) \to Q(x))$ is true since Q(x) is always true

$\exists x \neg p(x)$ is true since both a & b make it true

$\exists x \neg Q(x)$ is false

$\therefore$ arg is invalid!

1.13.4)

A.) $\exists x\,(p(x) \wedge Q(x)) \xrightarrow[\text{property}]{\text{distributive}} \exists x\, P(x) \wedge \exists_x Q(x)$, which is equal to the conclusion $\therefore$ arg. is valid!

2.1

2.1.1)

A. n = -1 is odd. when k = -1 $2(-1)+1 = 1$

B. n = -101 is odd. when k = -51 $2(-51)+1 = -101$

2.1 cont

2.1.3)

A.) $n = .25 = \frac{1}{4}$

B.) $N = -5 = -\frac{5}{1}$

2.1.5)

A.) neither

B.) neither

2.2

2.2.1)

A.) False. To show a universally quantified statement is true, the predicate must hold in all values w/in the domain.

B.) False, to prove an existential statement, you need to show it holds for all cases not just the one provided

2.2.2)

A.) $n=0$, $(0+1)^2 > 0^3 \equiv 1 > 0$ T
$n=1$, $(1+1)^2 > 1^3 \equiv 2^2 > 1 \equiv 4 > 1$ T
$n=2$, $(2+2)^2 > 2^3 \equiv 4^2 > 8 = 16 > 8$ T

$\therefore$ Since all true, by exhaustion, it holds

2.2.3)

B.) False. If $n=2$, $n^2 = 4$, which is divisible by 4. 2 is not.

2.2.5)

A.) True, let $x = 2$ & $y = 2$, $\frac{1}{x} + \frac{1}{y} = \frac{1}{2} + \frac{1}{2} = 1$, which is an integer! $\therefore$ statement holds

2.3

2.3.1)

A.) Let a, b, c be integers

Since $a^3 | b$, there exists an integer k s.t. $b = k \cdot a^3$

Also since $b^2 | c$, there exists an integer j st $c = b^2 \cdot j$

$\therefore$ $c = j \cdot (k \cdot a^3)^2 = j k^2 a^6$

↳ since both j & k are integers, $j \cdot k^2$ are also integers

$\therefore$ $a^6 | c$

2.3.2)

A.) the arbitrary value k is used multiple times despite it being different values in each usage. Using different constant provides clarity

B.) Assuming m is an integer in $x\,z = m \cdot w\,y$ is invalid as it is a ratio that may be an integer

2.3.3)

A. Giving an example is not enough to prove a theorem.

B.) We never state the reason why n & m are in the form $2x + 1$. This is the general statement for odd numbers

2.4

2.4.1)

B.) Let m and n be odd integers in $\mathbb{N}$.

Since m and n are arbitrary odd integers, we can utilize the general odd form to represent m and n:

$m = 2k + 1$, where k is an integer

$n = 2j + 1$, where j is an integer

↳ when adding $m + n$, $m + n = 2k + 2j + 2$. Since each term is a factor of 2, $m + n$ is even for all odd numbers. $\square$

2.4 Cont

2.4.2)

B.) Let x any y be rational #s with $y \neq 0$.
By the well ordering principle, there exists integers a + b S.T $x = \frac{a}{b}$ and there exists integers c + D S.T $y = \frac{c}{d}$, where b + $d \neq 0$ and $c \neq 0$ since y is non-zero.
b ∴ the quotient $\frac{x}{y} = \frac{ad}{bc}$ is rational since ad is a product of 2 integers ∴ an integer and bc is also an integer (non-zero) by the same principle

2.4.4)

A.) True. Let x and y be even integers, we can use the general form of even integers.
$X = 2k$, where k is an integer
$Y = 2j$, where j is an integer
∴ $x + y = 2k + 2j = 2(k + j)$
Since $x + y$ is in the form of even integer number ∴ $x + y$ is an even integer

B.) False. Counter example: Let $x = \frac{3}{4}$ + $y = \frac{5}{4}$, $x + y = \frac{8}{4} = 2$ which is even, but x + y are not integers!

2.5

2.5.1)

A.) Assume n is not odd ∴ even. Using the general form for even numbers: $n = 2k$, where k is an even integer.
∴ $n^2 = 4k^2 = 2(2k^2) = 2t$, where $t = 2k^2$. Since n^2 is a factor of 2, n^2 is even. By contrapositive, the statement holds

2.5 cont

2.5.2)

A.) contrapositive. If $x + y$ are integer such that $3 | x \Rightarrow 3 | xy$

proof:

Let $3 | x$ be true;

$\therefore x = 3k$, where k is an integer in $\mathbb{Z}$

$\therefore xy \equiv 3ky \equiv 3p$, where $p = ky \in \mathbb{Z}$

by this logic, $3 | xy$ due to the above logic

by contrapositive, since the contrapositive holds, the original statement holds

2.6

2.6.1)

B.) Assume $2 - \sqrt{2}$ is rational; since it is rational, there is a ratio of 2 integers that can represent it by well ordering principle

$\therefore 2 - \sqrt{2} = \frac{a}{b}$, where $a + b$ are integers

$\equiv 2 - \frac{a}{b} = \sqrt{2}$

Since $\sqrt{2}$ is irrational, there cannot be a ratio of integers that can represent an irrational number

$\therefore$ a contradiction occurs!, so $2 - \sqrt{2}$ must be irrational $\square$

2.7

2.7.1)

A.) case 1: $x > 0$	case 2: $x < 0$	case 3: $x = 0$
$(x > 0)^2 \equiv x^2 > 0$	$(x < 0)^2 \equiv x^2 > 0$	$(x = 0)^2 \equiv x^2 = 0$
$\therefore x^2 > 0 \Rightarrow x^2 \geq 0$	$\therefore x^2 > 0 \Rightarrow x^2 \geq 0$	$\therefore$ statement $x^2 \geq 0$ holds
statement holds	statement holds	

↳ By proof by cases, statement holds in all cases

2.7 cont.

2.7.1)

B) case 1: $n \geq 1$	case 2: $n \leq -1$	case 3: $n = 0$
$\therefore$ multiplying both sides by n,	$\therefore$ let $n = -n$	$n^2 \geq n \equiv 0^2 \geq 0$
$n^2 \geq 1 \cdot n$	$\therefore (-n)^2 \geq -n$	$\therefore 0 \geq 0$, which is true
$\therefore n^2 \geq n$, which holds general statement	$\equiv n^2 \geq -n$ which always holds	
	$\therefore$ holds general statement	

by proof by cases, since all cases are true, the statement $n^2 \geq n$ holds for all cases.