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Semana 4

Fatoração LU

Seção 6.2

1- $A = \begin{bmatrix} 5 & 2 & 3 \\ 3 & 1 & 4 \\ 1 & 1 & 2 \end{bmatrix} \begin{matrix} 5 & 2 \\ 3 & 1 \\ 1 & 1 \end{matrix}$

a-) $\Delta 1 = 5$

$\Delta 2 = 5 - 6 = -1$

$\Delta 3 = (10 + 8 + 9) - (12 + 20 + 3) = 27 - 35 = -8$

Satisfaz as condições de decomposição LU.

b-) $um = 1$

Linha 1 da matriz U

$j = 1 = U_{11} = A_{11} - 0 = 5$

$j = 2 = U_{12} = A_{12} - 0 = 2$

$j = 3 = U_{13} = A_{13} = 0 = 3$

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Coluna 1 matriz L

$i = 1 = L_{11} = 1$

$i = 2 = L_{21} = \frac{A_{21} - 0}{U_{11}} = \frac{3}{5}$

$i = 3 = L_{31} = \frac{A_{31} - 0}{U_{11}} = \frac{1}{5}$

$m = 2$

Linha 2 matriz U

$j = 2 = U_{22} = A_{22} - (L_{21} U_{12}) = 1 - (\frac{3}{5} \cdot 2) = 1 - \frac{6}{5} = -\frac{1}{5}$

$j = 3 = U_{23} = A_{23} - (L_{21} U_{13}) = 4 - (\frac{3}{5} \cdot 3) = 4 - \frac{9}{5} = \frac{11}{5}$

Coluna 2 matriz L

$L_{22} = 1$

$i = 3 = L_{32} = \frac{A_{32} - (L_{31} U_{12})}{U_{22}} = \frac{1 - (\frac{1}{5} \cdot 2)}{-\frac{1}{5}} = \frac{3}{5} \cdot \frac{-5}{1} = -3$

$m = 3$

Linha 3 da matriz U

$j = 3 = U_{33} = A_{33} - (L_{31} U_{13} + L_{32} U_{23}) = 2 - (\frac{1}{5} \cdot 3 + (-3 \cdot \frac{11}{5}))$

$= 2 - (\frac{3}{5} - \frac{33}{5}) = 2 - (-\frac{30}{5}) = 2 - (-6) = 8$

Coluna 3 da matriz L

$L_{33} = 1$

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$$A = \begin{bmatrix} 5 & 2 & 3 \\ 3 & 1 & 4 \\ 1 & 1 & 2 \end{bmatrix} \quad \text{N} \quad L = \begin{bmatrix} 1 & 0 & 0 \\ 3/5 & 1 & 0 \\ 1/5 & -3 & 1 \end{bmatrix} \quad U = \begin{bmatrix} 5 & 2 & 3 \\ 0 & -1/5 & 11/5 \\ 0 & 0 & 8 \end{bmatrix}$$

c-) $\det L = 1$; $\det U = 5 \cdot (-\frac{1}{5}) \cdot 8 = -8$

$\det A = 1 \cdot (-8) = -8$ //

~~Questão~~ 7.4

$$\begin{cases} 7x_1 + 2x_2 - 3x_3 = -12 \\ 2x_1 + 5x_2 - 3x_3 = -20 \\ x_1 - x_2 - 6x_3 = -26 \end{cases}$$

a-)

	Multiplicadores	Coeficientes			Operação
L_1		7	2	-3	
L_2	$m_{21} = 2/7$	2	5	-3	
L_3	$m_{31} = 1/7$	1	-1	-6	
L_2		m_{21}	$21/7$	$-15/7$	$L_2 = L_2 - \frac{2}{7} h_1$
L_3	$m_{32} = -\frac{9}{31}$	m_{31}	$-9/7$	$-39/7$	$L_3 = L_3 - \frac{1}{7} h_1$
L_3		m_{31}	m_{32}	$-\frac{192}{21}$	$L_3 = L_3 - \frac{9}{31} L_2$

$$U = \begin{bmatrix} 7 & 2 & -3 \\ 0 & 31/7 & -15/7 \\ 0 & 0 & -\frac{192}{31} \end{bmatrix} \quad L = \begin{bmatrix} 1 & 0 & 0 \\ 2/7 & 1 & 0 \\ 1/7 & -\frac{9}{31} & 1 \end{bmatrix}$$

b-) $\det A = 1 \times \left(7 \cdot \frac{31}{7} \cdot \frac{(-192)}{31} \right) = -192$ //

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c- $\begin{bmatrix} 1 & 0 & 0 \\ 2/7 & 1 & 0 \\ 1/7 & -9/31 & 1 \end{bmatrix} \cdot \begin{bmatrix} 7 & 2 & -3 \\ 0 & 3/7 & -15/7 \\ 0 & 0 & -192/31 \end{bmatrix}$

$$\begin{bmatrix} 7 & 2 & -3 \\ 2 & \frac{4}{7}+\frac{3}{7} & \frac{6}{7}-\frac{15}{7} \\ 1 & \frac{2}{7}-\frac{9}{7} & -\frac{3}{7}+\frac{135}{217}-\frac{192}{31} \end{bmatrix} = \begin{bmatrix} 7 & 2 & -3 \\ 2 & 5 & -3 \\ 1 & 1 & -6 \end{bmatrix}$$

d-) Ly = b por substituição sucessiva

$$\begin{bmatrix} 1 & 0 & 0 \\ 2/7 & 1 & 0 \\ 1/7 & -9/31 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} -12 \\ -20 \\ -26 \end{bmatrix} = \left[-12 \;\; -\frac{116}{7} \;\; -\frac{902}{31}\right]^t$$

$$\begin{cases} y_1 = -12 \\ \frac{2}{7}y_1 + y_2 = -20 \\ \frac{1}{7}y_1 - \frac{9}{31}y_2 + y_3 = -26 \end{cases}$$