MD2 - Prova #2

Relações de Recorrência

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1)

Tomando c = 1, temos:

$T(n) = T(\frac{n}{3}) + T(\frac{2n}{3}) + n$

2)

$T(1) = 1$

$T(n) = 2T(\frac{n}{3}) + n,\ n>1,\ n=3^k$

$T(n) = 2T(\frac{n}{3}) + n$

$T(n) = 2(2\ T(\frac{n}{3^2}) + \frac{n}{3}) + n = 2^2.T(\frac{n}{3^2}) + \frac{2}{3}n + n$

$T(n) = 2(2(2\ T(\frac{n}{3^3}) + \frac{n}{9}) + \frac{n}{3}) + n = 2^3.T(\frac{n}{3^3}) + (\frac{2}{3})^2\ n + (\frac{2}{3})\ n + n$

$T(n) = 2^k.\ T(\frac{n}{3^k}) + n.\sum_{i=0}^{k-1}(\frac{2}{3})^i$

Fazendo $n/3^k = 1$, temos $k = \log_3 n$

$T(n) = n^{\log_3 2} + 3n - 3\ n^{\log_3 2}$

$$\mathbf{T(n) = 3n - 2\ n^{\log_3 2}}$$

3)

$T(2) = 1$

$T(n) = 2T(\sqrt{n}) + 1$

$T(n) = 2T(\sqrt{n}) + 1$

$T(n) = 2(2\ T(\sqrt{\sqrt{n}}) + 1) + 1 =$

$T(n) = 2(2(2\ T(\sqrt{\sqrt{\sqrt{n}}}) + 1) + 1) + 1 =$

$T(n) = 2^k .\ T(n^{(1/2^k)}) + \sum_{i=0}^{k-1} 2^i$

Fazendo $n^{(1/2^k)} = 2$, *temos* $k = lg\ lg\ n$

$T(n) = lg\ n + lg\ n - 1$

$$\mathbf{T(n) = 2lg\ n - 1}$$

4)

$T(1) = 1.5$

$T(2) = 3$

$T(n) = 2T(n\text{-}1) – T(n\text{-}2)$

Equação característica $x^2 – 2x + 1 = 0$, $x = 1$

$T(n) = \alpha_1 r^n + n\, \alpha_2 r^n$

$T(1) = \alpha_1 + \alpha_2 = 1.5$

$T(2) = \alpha_1 + 2\, \alpha_2 = 3$

$\alpha_1 = 0, \alpha_2 = 1.5$

$\boldsymbol{T(n) = 1.5n}$

5)

T(1) = 2

$T(n) = T(\frac{n}{2}) + n$, n>1, $n=2^k$

$T(n) = T(\frac{n}{2}) + n$

$T(n) = T(\frac{n}{2^2}) + \frac{n}{2} + n$

$T(n) = T(\frac{n}{2^3}) + \frac{n}{4} + \frac{n}{2} + n$

$T(n) = T(\frac{n}{2^k}) + n.\sum_{i=0}^{k-1}(\frac{1}{2})^i$

Fazendo $n/2^k = 1$, temos $k = lg\ n$ (lg é log na base 2)

$T(n) = T(1) + n\ .\ (2 - 2\ .\ (1/2)^{lgn})$

$T(n) = 2 + 2n - 2n\ .\ n^0/n$

$$\boldsymbol{T(n) = 2n}$$