Final project

Submission deadline 28.02.2026

Conductive heat transfer in a two-dimensional square plate, $0 \le x, y \le 1$, containing a circular hole of radius $R = 0.2$ at the plate center (see **Fig.1**), is governed by the energy equation:

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) = \frac{\partial T}{\partial t}.$$

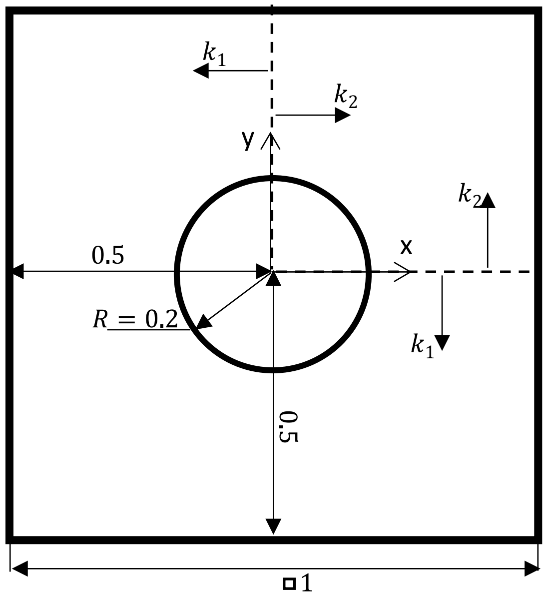

Fig. 1 Physical model, geometry and boundary conditions.

Part 1 –No hole in the plate.

1. Solve the steady state equation using the **Gauss-Seidel by-lines method** (sweeps in the x direction) for $k_1 = 10^{-3}$ and $k_2 = 100$, with the boundary conditions: $T(x = 0, y) = T(x = 1, y) = 1 - 4y^2$, $T_y(x, y = 0) = T_y(x, y = 1) = 0$. Use a 200×200 grid. **(15p)**
2. Repeat the solution of the previous item using a **multigrid method**. Use **Gauss-Seidel by-lines smoother** of one sweep in x direction followed by one sweep in y direction. Use the boundary conditions as in the previous item. **(15p)**
3. Repeat the item 2 but now use a single multigrid V cycle **as a left preconditioner for Matlab's built-in BiCgStab dolver**. **(15p).**
4. Compare the computational times required to reach a solution accurate to sixth decimal digits in items 2 and 3. Using numerical experiments, estimate the computational complexity (as a function of the grid size) for the methods in items 2 and 3. **(5p).**

Part 2 – Central circular hole ($R = 0.2$).

5. Solve the **unsteady** equation using the **direct LU** method for $k_1 = 10^{-3}$ and $k_2 = 100$ with the boundary conditions: $T(x = 0, y) = T(x = 1, y) = 1 - 4y^2$, $T_y(x, y = 0) = T_y(x, y = 1) = 0$, $T = 2$ on the hole boundary. Use 80×80 **body fitted triangular mesh** generated by the Matlab script provided with this project. Note

that the grid is generally **non-orthogonal**. Use time step equal to $\Delta t = 10^{-3}$. Report the temperature distribution at $t = 1, 5$ and at the steady state. **(20p)**

6. Solve the same unsteady problem for $k_1 = 10^{-3}$ and $k_2 = 100$ with the same boundary conditions, but now 200×200 **structured grid** and the **fully implicit direct forcing immersed boundary method** using a **Schur complement** formulation. Report the temperature field at $t = 1, 5$ and at steady state. **(20p)**
7. Perform simulations similar to item 6, but use an **explicit direct forcing immersed boundary method** and impose **thermally insulated** hole boundary condition (zero normal heat flux) on the hole surface. **(10p)**