Order Reduction of 2nd order ODEs

Equation

$$\frac{d^2c}{d\theta^2} = \Lambda^2 c^m \qquad 1$$

<u>Order Reduction:</u>

$$y(1) = c \qquad 2$$

$$y(2) = \frac{dc}{d\theta} = \frac{dy(1)}{d\theta} \qquad 3$$

Where, y is just an arbitrary letter and can also be p, q, s, etc.

So we are going from one 2nd order equation to two 1st order equations. The equation that defines the order reduction is:

$$\frac{dy(1)}{d\theta} = y(2) \qquad 4$$

Substituting eq. 4 → eq. 1 gives:

$$\frac{d^2c}{d\theta^2} = \frac{d}{d\theta}\left(\frac{dc}{d\theta}\right) = \frac{d}{d\theta}\left(\frac{dy(1)}{d\theta}\right) = \frac{dy(2)}{d\theta} = \Lambda^2 y(1)^m \qquad 5$$

The set of first order differential equations to be solved becomes:

The order reduction defining equation i.e. eq. 4 → $\boldsymbol{\frac{dy(1)}{d\theta} = y(2)}$

The substituted equation i.e. eq. 5 → $\boldsymbol{\frac{dy(2)}{d\theta} = \Lambda^2 y(1)^m}$

Boundary Conditions:

BC		MATLAB Code	Explanation
$@\theta = 0$	$\frac{dc}{d\theta} = 0$	$0 = y0(2)$	$y(2) = \frac{dc}{d\theta}$
$@\theta = 1$	$c = c_s$	$0 = y1(1) - c_s$	$y(1) = c$