

Order Reduction of 2nd order ODEs

Equation

$$\frac{d^2c}{d\theta^2} = \Lambda^2 c^m \quad 1$$

Order Reduction:

$$y(1) = c \quad 2$$

$$y(2) = \frac{dc}{d\theta} = \frac{dy(1)}{d\theta} \quad 3$$

Where, y is just an arbitrary letter and can also be p , q , s , etc.

So we are going from one 2nd order equation to two 1st order equations. The equation that defines the order reduction is:

$$\frac{dy(1)}{d\theta} = y(2) \quad 4$$

Substituting eq. 4 \rightarrow eq. 1 gives:

$$\frac{d^2c}{d\theta^2} = \frac{d}{d\theta} \left(\frac{dc}{d\theta} \right) = \frac{d}{d\theta} \left(\frac{dy(1)}{d\theta} \right) = \frac{dy(2)}{d\theta} = \Lambda^2 y(1)^m \quad 5$$

The set of first order differential equations to be solved becomes:

The order reduction defining equation i.e. eq. 4 \rightarrow

$$\frac{dy(1)}{d\theta} = y(2)$$

The substituted equation i.e. eq. 5 \rightarrow

$$\frac{dy(2)}{d\theta} = \Lambda^2 y(1)^m$$

Boundary Conditions:

BC	MATLAB Code	Explanation
$\theta = 0 \quad \frac{dc}{d\theta} = 0$	$0 = y0(2)$	$y(2) = \frac{dc}{d\theta}$
$\theta = 1 \quad c = c_s$	$0 = y1(1) - c_s$	$y(1) = c$

The diagram illustrates the mapping of boundary conditions from physical terms to MATLAB code. It consists of three columns: BC, MATLAB Code, and Explanation. The first row shows the boundary condition at $\theta = 0$ where the derivative of concentration is zero, the corresponding MATLAB code $0 = y0(2)$, and the explanation that $y(2)$ represents the derivative $\frac{dc}{d\theta}$. The second row shows the boundary condition at $\theta = 1$ where the concentration is c_s , the corresponding MATLAB code $0 = y1(1) - c_s$, and the explanation that $y(1)$ represents the concentration c . Arrows indicate the flow from the physical conditions to the MATLAB code and then to the explanation.