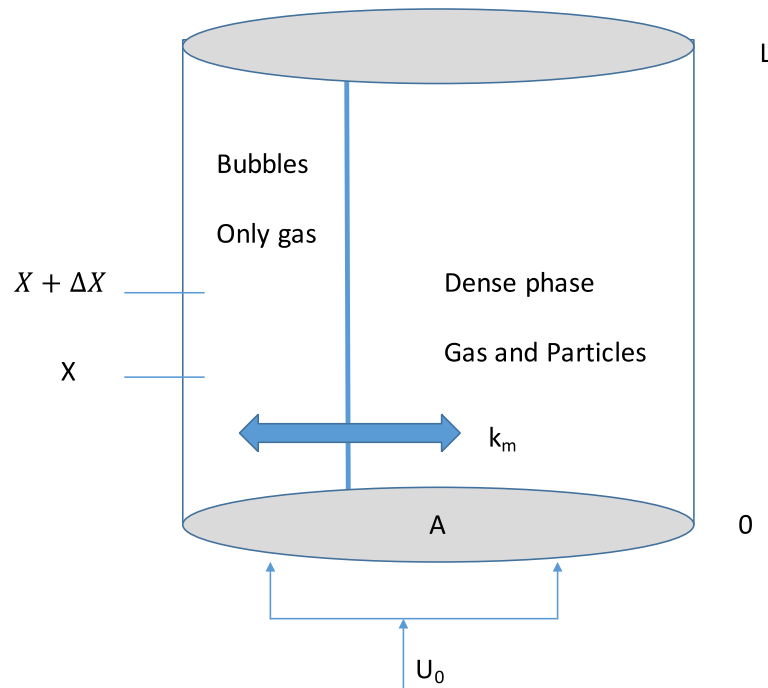


Two phase van Deemter model for bubbling fluidized beds

The bubbling bed is divided conceptually into a bubble phase and a dense phase (also called emulsion phase). In the simplest approach we assume that there are no solids in the bubbles. See the figure below for a graphical representation.



First a few definitions:

The bubble fraction (f_b or δ) is defined as:

$$f_b = \frac{m^3 \text{ gas in bubbles}}{m_{bed}^3}$$

Here m_{bed}^3 is the volume (height occupied by the bubbling bed = $A \cdot \text{Height of the bed}$)

The fraction of gas in the dense phase is:

$$f_d = \frac{m^3 \text{ gas in dense phase}}{m_{bed}^3}$$

The solids fraction is:

$$f_s = \frac{m^3 \text{ solids in bed}}{m_{bed}^3}$$

It holds that:

$$f_b + f_d + f_s = 1$$

The porosity of the dense phase is defined as:

$$\varepsilon_d = \frac{m^3 \text{ gas in dense phase}}{m_{dense \text{ phase}}^3} = 1 - \frac{f_s}{f_s + f_d}$$

The porosity of the whole reactor/bed is defined as:

$$\varepsilon_r = \frac{m^3 \text{ gas bed}}{m_{bed}^3} = 1 - f_s$$

For the linear velocity we take the symbol V and for the superficial velocity we use the symbol U . Note that the superficial velocity can be defined on basis of the whole bed or just the dense phase. An overall mass balance, assuming constant density, yields:

$$U_o = f_b V_b + f_d V_d$$

In this equation U_o is the superficial velocity (based on the whole bed) and V_b and V_d are the linear velocities of the bubbles and the gas in the dense phase respectively.

V_d can be written as:

$$V_d = \frac{U_d}{\varepsilon_d}$$

In which U_d is the superficial velocity of the dense phase (based on the area of the dense phase).

f_d can be written as:

$$f_d = (1 - f_b) \varepsilon_d$$

Filling in the two equation above in the mass balance gives:

$$U_o = f_b V_b + (1 - f_b) U_d$$

If we assume that the velocity in the dense phase is U_{mf} (a reasonable assumption) the equation becomes:

$$U_o = f_b V_b + (1 - f_b) U_{mf}$$

Or rewritten:

$$f_b = \frac{U_o - U_{mf}}{V_b - U_{mf}}$$

Now let's consider that a chemical reaction requiring a heterogeneous catalyst (the solid phase) is carried out in the bubbling fluidized bed. For simplicity we consider the reaction:

$A \rightarrow P$ and the rate of disappearance of A is described by R_A which has units:

$$R_A \left[\frac{\text{mol } A}{\text{kg catalyst} \cdot \text{s}} \right]$$

We are going to set-up mole balances for A in the bubble phase (C^b) and A in the dense phase (C^d). I dropped the reference to component A, so from now on C is the concentration of component A (in mole/m³)

Bubble phase:

It is assumed that the bubbles move in plug flow. The mass transfer between the bubble and dense phase is described by k_m :

$$k_m = \frac{\frac{\text{m}^3 \text{ gas exchanged between bubble and dense phase}}{\text{second}}}{m_{bed}^3}$$

k_m can also be defined on the bubble volume:

$$k'_m = \frac{\frac{\text{m}^3 \text{ gas exchanged between bubble and dense phase}}{\text{second}}}{m_{bubble}^3}$$

the forms of k_m can be easily transformed into each other by the bubble fraction (f_b)

We are going to set-up a differential balance between X and X+ ΔX (on the axial coordinate):

$$0 = Af_b V_b C_X^b - Af_b V_b C_{X+\Delta X}^b - A\Delta X k_m (C^b - C^d)$$

dividing by $A\Delta X$, taking the limit of $\Delta X \rightarrow 0$ and introducing the dimensionless length $\theta = \frac{X}{L}$ gives (L is the height of the bubbling bed):

$$\frac{dC^b}{d\theta} = -\frac{k_m L}{f_b V_b} (C^b - C^d)$$

Note that for a (very) fast chemical reaction $C^d=0$ and the conversion of a is solely determined by mass transfer between bubble and dense phase.

Dense phase:

The gas flow through the dense phase is described by the axial dispersion model. The differential balance between X and $X+\Delta X$ reads:

$$0 = Af_d V_d C_X^d - Af_d V_d C_{X+\Delta X}^d + A\Delta X k_m (C^b - C^d) + A(-D_e \frac{dC^d}{dx})_X - A(-D_e \frac{dC^d}{dx})_{X+\Delta X} + A\Delta X \rho_c (1 - f_b)(1 - \varepsilon_d) R_A$$

In this equation ρ_c is the density of the catalyst in $\text{kg/m}^3_{\text{catalyst}}$ and D_e an effective dispersion coefficient. A different formulation is:

$$0 = Af_d V_d C_X^d - Af_d V_d C_{X+\Delta X}^d + A\Delta X k_m (C^b - C^d) + A(-D_e \frac{dC^d}{dx})_X - A(-D_e \frac{dC^d}{dx})_{X+\Delta X} + A\Delta X \rho_c f_s R_A$$

dividing by $A\Delta X$, taking the limit of $\Delta X \rightarrow 0$ and introducing the dimensionless length $\theta = \frac{x}{L}$ gives:

$$0 = -\frac{f_d V_d}{L} \frac{dC^d}{d\theta} + \frac{D_e}{L^2} \frac{d^2 C^d}{d\theta^2} + k_m (C^b - C^d) + R_A \rho_c f_s$$

A limiting case:

Let's assume that there is no gas going through the dense phase (relatively to the bubble phase). This is a reasonable assumption for a bed of A/B particles operated at $U_0 > 0.2 \text{ m/s}$ (remember U_0 is $\text{max} \sim 1 \text{ cm/s}$). In this case $f_b V_b = U_0$ and $V_d = 0$. The balances above become:

Bubble phase:

$$\frac{dC^b}{d\theta} = -\frac{k_m L}{U_0} (C^b - C^d) = -N_T (C^b - C^d)$$

In which N_T is the number of transfer units ($N_T = \frac{k_m L}{U_0}$)

Dense phase:

For the dense phase we remove the convective term and divide by U_0/L

$$0 = \frac{D_e}{U_0 L} \frac{d^2 C^d}{d\theta^2} + \frac{k_m L}{U_0} (C^b - C^d) + \frac{R_A \rho_c f_s L}{U_0}$$

or written differently:

$$N_T = \frac{k_m L}{U_0} 0 = \frac{1}{N_E} \frac{d^2 C^d}{d\theta^2} + N_T (C^b - C^d) + \frac{R_A \rho_c f_s L}{U_0}$$

In which $N_E \left(\frac{U_0 L}{D_e} \right)$ are the number of mixing units.

In case the reaction is first order with $R_A = -K_r C^d$ the equation becomes:

$$0 = \frac{1}{N_E} \frac{d^2 C^d}{d\theta^2} + N_T (C^b - C^d) - N_R C^d$$

in which N_R is the number of reaction unit ($N_R = \frac{K_r \rho_c f_s L}{U_0}$).

When mixing in the dense phase is ignored the conversion (x) of a first order reaction in a bubbling fluidized bed becomes:

$$x = 1 - e^{\frac{-N_T N_R}{N_T + N_R}}$$

(you might want to derive this yourselves)