# Homework 1: Macroeconomics 210C

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# 1. Complementarity of Money and Consumption

Suppose the utility function in our classical monetary model is now

$$U(X_t, L_t) = \frac{X_t^{1-\gamma} - 1}{1-\gamma} - \chi \frac{N_t^{1+\varphi}}{1+\varphi}$$

where $X_t$ is a composite of consumption and money,

$$X_t = \left[(1-\underset{\alpha}{\vartheta})C_t^{1-\nu} + \underset{\alpha}{\vartheta}\left(\frac{M_t}{P_t}\right)^{1-\nu}\right]^{\frac{1}{1-\nu}}$$

$1/\varphi$: Frisch elasticity of labor supply

$\nu$: elasticity of money demand

$\gamma$: constant relative risk aversion (CRRA)

$\sigma$: $1/\gamma$ Intertemporal Elasticity of Substitution (IES)

I will use $\alpha$ instead of $\vartheta$

$$\max_{\{C_{t+s}, N_{t+s}, B_{t+s}, M_{t+s}\}} E_t\left\{\sum_{s=0}^{\infty} \beta^s U(X_t, L_t)\right\}$$

$$\max \quad E_t\left\{\sum_{s=0}^{\infty}\beta^s\left(\frac{\left[(1-\alpha)C_{t+s}^{1-\nu} + \alpha\left(\frac{M_{t+s}}{P_{t+s}}\right)^{1-\nu}\right]^{\frac{1-\gamma}{1-\nu}} - 1}{1-\gamma} - \chi\frac{N_{t+s}^{1+\varphi}}{1+\varphi}\right)\right\}$$

$TR_t$: transfers

$PR_t$: rebated profits

$Q_t$: gross nom. int. rate between $t$ and $t+1$

$$\text{s.t.}: \quad P_t C_t + B_t + M_t \leq W_t \cdot N_t + Q_{t-1} \cdot B_{t-1} + M_{t-1} + (TR_t + PR_t)$$

(budget contraint) from $t$ to $t+s$

(a) Derive the first order conditions for this economy.

$$\mathcal{L} = E_t\left\{\sum_{s=0}^{\infty}\left\{\beta^s\left(\frac{\left(\underbrace{\left[(1-\alpha)C_{t+s}^{1-\nu} + \alpha\left(\frac{M_{t+s}}{P_{t+s}}\right)^{1-\nu}\right]^{\frac{1}{1-\nu}}}_{X_{t+s}}\right)^{1-\gamma} - 1}{1-\gamma} - \chi\frac{N_{t+s}^{1+\varphi}}{1+\varphi}\right)\right\}\right.$$

$$\left\{\frac{1}{1-\nu} - 1 = \frac{1}{1-\nu} - \frac{(1-\nu)}{1-\nu} = \frac{+\nu}{1-\nu}\right\}$$

$$\left. + \lambda_{t+s}\left(W_{t+s} \cdot N_t + Q_{t-1+s} \cdot B_{t-1+s} + M_{t-1+s} + (TR_{t+s} + PR_{t+s}) - P_{t+s}C_{t+s} - B_{t+s} - M_{t+s}\right)\right\}$$

FOC: $\{C_{t+s}\}$: $$E_t\left\{\beta^s \cdot X_{t+s}^{-\gamma} \cdot \left[\frac{1}{\cancel{1-\nu}}\underbrace{\left[(1-\alpha)C_{t+s}^{1-\nu} + \alpha\left(\frac{M_{t+s}}{P_{t+s}}\right)^{1-\nu}\right]^{\left\{\frac{1}{1-\nu}\cdot(\nu)\right\}}}_{X_{t+s}}((1-\alpha)\cdot(\cancel{1-\nu})\,C_{t+s}^{-\nu}\right)\right\} = \lambda_{t+s} \cdot P_t$$

$$E_t\left\{\beta^s \cdot X_{t+s}^{\nu-\gamma} \cdot (1-\alpha) \cdot C_{t+s}^{-\nu}\right\} = E_t\left\{\lambda_{t+s} \cdot P_{t+s}\right\}$$

$$\beta^s \cdot X_{t+s}^{\nu-\gamma} \cdot (1-\alpha) \cdot C_{t+s}^{-\nu} = \lambda_{t+s} \cdot P_{t+s}$$

Consumption

$\{N_{t+s}\}$: $$E_t\left\{\beta^t(-\chi N_{t+s}^{\varphi}\right\} + E_t\left\{\lambda_{t+s} W_{t+s}\right\} = 0$$

$$\beta^s \cdot \chi N_{t+s}^{\varphi} = \lambda_{t+s} \cdot W_{t+s}$$

Labor

$\{B_{t+s}\}$: $$E_t\left\{-\lambda_{t+s}\right\} + E_{t+s}\left\{\lambda_{t+s+1} \cdot Q_{t+s}\right\} = 0$$

$$\lambda_{t+s} = E_{t+s}\left\{\lambda_{t+s+1} \cdot Q_{t+s}\right\}$$

Bonds

$\{M_{t+s}\}$: $$E_t\left\{\beta^s \cdot X_{t+s}^{\nu-\gamma} \cdot \alpha\left(\frac{M_{t+s}}{P_{t+s}}\right)^{-\nu} \cdot \frac{1}{P_{t+s}} - \lambda_{t+s}\right\} + E_{t+s}\left\{\lambda_{t+s+1}\right\} = 0$$

$$\beta^s \cdot X_{t+s}^{\nu-\gamma} \cdot \alpha\left(\frac{M_{t+s}}{P_{t+s}}\right)^{-\nu} \cdot \frac{1}{P_{t+s}} = \lambda_{t+s} - E_{t+s}\left\{\lambda_{t+s+1}\right\}$$

Money

(b) Under what conditions does this economy predict that money is neutral? Explain why.

Consider the set up for firms:

Firms produce output $Y_t$ constant return to scale with labor $N_t$ (no capital): $Y_{t+s} = A_{t+s} \cdot N_{t+s}$

Firms maximize profits: $\max_{N_{t+s}} \; Y_{t+s} - \frac{W_{t+s}}{P_{t+s}} N_{t+s}$

$$\Rightarrow \max_{N_{t+s}} \; A_{t+s} \cdot N_{t+s} - \frac{W_{t+s}}{P_{t+s}} N_{t+s}$$

F.O.C: $\{N_{t+s}\}$: $A_{t+s} = \frac{W_{t+s}}{P_{t+s}}$

Then, we can get the labor supply combining FOC for $\{N_{t+s}\}$ and $\{C_{t+s}\}$.

$$\left.\begin{aligned} \beta^s \cdot X_{t+s}^{\nu-\gamma} \cdot (1-\alpha) \cdot C_{t+s}^{-\nu} &= \lambda_{t+s} \cdot P_{t+s} \\ \beta^s \chi N_{t+s}^{\varphi} &= \lambda_{t+s} \cdot W_{t+s} \end{aligned}\right\} \cancel{\beta^s} \cdot \chi N_{t+s}^{\varphi} = \cancel{\beta^s} \cdot X_{t+s}^{\nu-\gamma} \cdot (1-\alpha) \cdot C_{t+s}^{-\nu} \frac{1}{P_{t+s}} \cdot W_{t+s}$$

$$\frac{W_{t+s}}{P_{t+s}} = \frac{\chi N_{t+s}^{\varphi}}{(X_{t+s}^{\nu-\gamma} \cdot (1-\alpha) \cdot C_{t+s}^{-\nu})}$$

$\Rightarrow$ Labor supply: $\frac{W_{t+s}}{P_{t+s}} = \frac{\chi N_{t+s}^{\varphi}}{(X_{t+s}^{\nu-\gamma} \cdot (1-\alpha) \cdot C_{t+s}^{-\nu})}$

$$\frac{W_{t+s}}{P_{t+s}} = \frac{\chi N_{t+s}^{\varphi}}{((1-\alpha) \cdot C_{t+s}^{-\nu})} \quad (1)$$

(it does not depend on Money when $\nu = \gamma$)

(so labor, consumption and production does not depend on money when equilibrium).

Labor demand: $\frac{W_{t+s}}{P_{t+s}} = A_{t+s}$ (2)

Market clearing condition for output: $Y_{t+s} = C_{t+s}$ (3) $\quad (Y_{t+s} = A_{t+s} \cdot N_{t+s})$ (4)

Combining the conditions: $A_{t+s} = \frac{\chi N_{t+s}^{\varphi}}{((1-\alpha)(A_{t+s} \cdot N_{t+s})^{-\nu})} \Rightarrow A_{t+s}((1-\alpha)(A_{t+s} \cdot N_{t+s})^{-\nu}) = \chi N_{t+s}^{\varphi}$

$$\frac{(1-\alpha) \cdot A_{t+s}^{1-\nu}}{\chi} = \frac{N_{t+s}^{\varphi}}{N_{t+s}^{-\nu}}$$

$$\frac{(1-\alpha) \cdot A_{t+s}^{1-\nu}}{\chi} = (N_{t+s})^{\varphi+\nu}$$

$$\Rightarrow N_{t+s} = \left(\frac{(1-\alpha) \cdot A_{t+s}^{1-\nu}}{\chi}\right)^{\frac{1}{\varphi+\nu}}$$

, which does not depend on money so real variables does not depend on nominal variables.

(Monetary neutrality: real outcomes are independent of the price level and unaffected by nominal variables). //

(c) Solve analytically for the steady state of the model (as far as you can), assuming $A = 1$.

For the steady state: $C_t = C_{t+1} = C$ $\quad Y_t = Y_{t+1} = Y$ $\quad M_t = M_{t+1} = M$ $\quad P_t = P_{t+1} = P$

$N_t = N_{t+1} = N$ $\quad B_t = B_{t+1} = B$ $\quad W_t = W_{t+1} = W$ $\quad \lambda_t = \lambda_{t+1} = \lambda$

From the firm problem, given $A=1$, we have: $Y_{t+s} = \cancel{A_{t+s}}^{1} \cdot N_{t+s}$ , and also: $\frac{W_{t+s}}{P_{t+s}} = \cancel{A_{t+s}}^{1}$

$Y_{t+s} = N_{t+s}$ $\qquad W_{t+s} = P_{t+s}$

$Y = N$ $\qquad W = P$

From labor supply: $\frac{W_{t+s}}{P_{t+s}} = \frac{\chi N_{t+s}^{\varphi}}{\left(X_{t+s}^{\nu-\gamma} \cdot (1-\alpha) \cdot C_{t+s}^{-\nu}\right)} \Rightarrow (1-\alpha) \cdot X^{\nu-\gamma} C^{-\nu} = \chi N^{\varphi}$

$$N = \left(\frac{(1-\alpha) \cdot X^{\nu-\gamma} C^{-\nu}}{\chi}\right)^{\frac{1}{\varphi}}$$

Combining FOC Consumption and FOC Bonds: $\lambda_{t+s} = \frac{\beta^{s} \cdot X_{t+s}^{\nu-\gamma} \cdot (1-\alpha) \cdot C_{t+s}^{-\nu}}{P_{t+s}}$

$$\lambda_{t+s+1} = \frac{\beta^{s+1} \cdot X_{t+s+1}^{\nu-\gamma} (1-\alpha)\, C_{t+s+1}^{-\nu}}{P_{t+s+1}}$$

$$\lambda_{t+s} = E_{t+s}\{\lambda_{t+s+1} \cdot Q_{t+s}\} \Rightarrow \frac{\beta^{s} \cdot \cancel{X_{t+s}^{\nu-\gamma}} \cdot \cancel{(1-\alpha)} \cdot \cancel{C_{t+s}^{-\nu}}}{\cancel{P_{t+s}}} = E_{t+s}\left\{\frac{\beta^{s+1} \cdot \cancel{X_{t+s+1}^{\nu-\gamma}}\, \cancel{(1-\alpha)}\, \cancel{C_{t+s+1}^{-\nu}}}{\cancel{P_{t+s+1}}} \cdot Q_{t+s}\right\}$$

$$\beta^{s} = \beta^{s+1} \cdot Q_{t+s} \Rightarrow 1 = \beta \cdot Q \Rightarrow Q = \frac{1}{\beta}$$

Combining FOC consumption and FOC money: $\beta^{s} \cdot X_{t+s}^{\nu-\gamma} \cdot \alpha \left(\frac{M_{t+s}}{P_{t+s}}\right)^{-\nu} \cdot \frac{1}{P_{t+s}} = \lambda_{t+s} - E_{t+s}\{\lambda_{t+s+1}\}$

$$\beta^{s} \cdot X_{t+s}^{\nu-\gamma} \cdot \alpha \left(\frac{M_{t+s}}{P_{t+s}}\right)^{-\nu} \cdot \frac{1}{P_{t+s}} = \frac{\beta^{s} \cdot X_{t+s}^{\nu-\gamma} \cdot (1-\alpha) \cdot C_{t+s}^{-\nu}}{P_{t+s}} - E_{t+s}\left\{\frac{\beta^{s+1} \cdot X_{t+s+1}^{\nu-\gamma} (1-\alpha)\, C_{t+s+1}^{-\nu}}{P_{t+s+1}}\right\}$$

$$\beta^{s} \cdot X^{\nu-\gamma} \alpha \cdot \left(\frac{M}{P}\right)^{-\nu} \cdot \frac{1}{P} = \frac{\beta^{s} \cdot X^{\nu-\gamma} (1-\alpha) \cdot C^{-\nu}}{P} - \frac{\beta^{s+1} \cdot X^{\nu-\gamma} \cdot (1-\alpha) \cdot C^{-\nu}}{P}$$

$$\cancel{\beta^{s}} \cdot \cancel{X^{\nu-\gamma}} \cdot \alpha \left(\frac{M}{P}\right)^{-\nu} \frac{1}{\cancel{P}} = \frac{\cancel{\beta^{s}} \cancel{X^{\nu-\gamma}} (1-\alpha) \cdot C^{-\nu} (1-\beta)}{\cancel{P}}$$

$$\alpha \left(\frac{M}{P}\right)^{-\nu} = (1-\alpha)\, C^{-\nu} (1-\beta) \Rightarrow \left(\frac{M}{P}\right)^{-\nu} = \left(\frac{(1-\alpha)(1-\beta)}{\alpha}\right)^{1} C^{-\nu} \quad /(\,)^{-\frac{1}{\nu}}$$

$$\Rightarrow \left(\frac{M}{P}\right)^{\nu} = C^{\nu} \frac{\alpha}{(1-\alpha)(1-\beta)} \Rightarrow \left(\frac{M}{P}\right) = \left(\frac{\alpha}{(1-\alpha)(1-\beta)}\right)^{\frac{1}{\nu}} \cdot C$$

From market clearing condition for output: $Y = C \Rightarrow N = \left(\frac{(1-\alpha)\, X^{\nu-\gamma} N^{-\nu}}{\chi}\right)^{\frac{1}{\varphi}}$

$$N = \left( \frac{(1-\alpha)\, X^{v-y} N^{-v}}{\mathcal{X}} \right)^{\frac{1}{\varphi}} \Rightarrow N = \left( \frac{(1-\alpha)\, X^{v-y}}{\mathcal{X}} \right)^{\frac{1}{\varphi}} \cdot N^{\left(\frac{v}{\varphi}\right)\cdot -1} \Rightarrow N \cdot N^{\frac{v}{\varphi}} = \left( \frac{(1-\alpha)\, X^{v-y}}{\mathcal{X}} \right)^{\frac{1}{\varphi}}$$

$$N^{\frac{\varphi+v}{\varphi}} = \left( \frac{(1-\alpha)\, X^{v-y}}{\mathcal{X}} \right)^{\frac{1}{\varphi}} \Rightarrow N = \left( \left( \frac{(1-\alpha)\, X^{v-y}}{\mathcal{X}} \right)^{\frac{1}{\varphi}} \right)^{\frac{\varphi}{\varphi+v}} \Rightarrow N = \left( \frac{(1-\alpha)\, X^{v-y}}{\mathcal{X}} \right)^{\frac{1}{\varphi+v}} = C$$

So, until now we have:

- $Y = N = C$
- $C = \left( \frac{(1-\alpha)\, X^{v-y}}{\mathcal{X}} \right)^{\frac{1}{\varphi+v}}$
- $Q = \frac{1}{\beta}$
- $W = P$
- $\left(\frac{N}{P}\right) = \left( \frac{\alpha}{(1-\alpha)(1-\beta)} \right)^{\frac{1}{v}} \cdot C$

We also know:

$$X_t = \left[ (1-\alpha)\, C_{t+s}^{1-v} + \alpha \left( \frac{N_{t+s}}{P_{t+s}} \right)^{1-v} \right]^{\frac{1}{1-v}} \Rightarrow X = \left[ (1-\alpha)\, C^{1-v} + \alpha \left( \frac{N}{P} \right)^{1-v} \right]^{\frac{1}{1-v}}$$

$$X = \left[ (1-\alpha) \cdot C^{1-v} + \alpha \left[ \left( \frac{\alpha}{(1-\alpha)(1-\beta)} \right)^{\frac{1}{v}} \cdot C \right]^{1-v} \right]^{\frac{1}{1-v}}$$

$$X = \left( (1-\alpha)\, C^{1-v} + \alpha \cdot \left( \frac{\alpha}{(1-\alpha)(1-\beta)} \right)^{\frac{1-v}{v}} \cdot C^{1-v} \right)^{\frac{1}{1-v}}$$

$$X = \left( C^{1-v} \left( (1-\alpha) + \alpha \cdot \left( \frac{\alpha}{(1-\alpha)(1-\beta)} \right)^{\frac{1-v}{v}} \right) \right)^{\frac{1}{1-v}}$$

$$X = C \cdot \left( (1-\alpha) + \alpha \cdot \left( \frac{\alpha}{(1-\alpha)(1-\beta)} \right)^{\frac{1-v}{v}} \right)^{\frac{1}{1-v}} \qquad 1 + \frac{1-v}{v} = \frac{v+1-v}{v} = \frac{1}{v}$$

$$X = C \cdot \left[ (1-\alpha) + \alpha^{1/v} \left( (1-\alpha)(1-\beta) \right)^{\frac{v-1}{v}} \right]^{\frac{1}{1-v}}$$

Then, we get X from:

$$C = \left( \frac{(1-\alpha)\, X^{v-y}}{\mathcal{X}} \right)^{\frac{1}{\varphi+v}} \Rightarrow C^{\varphi+v} = \frac{(1-\alpha)\, X^{v-y}}{\mathcal{X}} \Rightarrow \left( \frac{\mathcal{X}\, C^{\varphi+v}}{(1-\alpha)} \right)^{\frac{1}{v-y}} = X$$

Finally:

$$\left( \frac{\mathcal{X}\, C^{\varphi+v}}{(1-\alpha)} \right)^{\frac{1}{v-y}} = C \cdot \left[ (1-\alpha) + \alpha^{1/v} \left( (1-\alpha)(1-\beta) \right)^{\frac{v-1}{v}} \right]^{\frac{1}{1-v}}$$

$$C^{\frac{\varphi+v}{v-y}} \cdot \left( \frac{\mathcal{X}}{(1-\alpha)} \right)^{\frac{1}{v-y}} = C \cdot \left[ (1-\alpha) + \alpha^{1/v} \left( (1-\alpha)(1-\beta) \right)^{\frac{v-1}{v}} \right]^{\frac{1}{1-v}}$$

$$\left\{ \frac{\varphi + v - v + y}{v-y} = \frac{\varphi + y}{v-y} \right\} \qquad C^{\frac{\varphi+v}{v-y} - 1} = \left( \frac{\mathcal{X}}{(1-\alpha)} \right)^{\frac{-1}{v-y}} \left[ (1-\alpha) + \alpha^{1/v} \left( (1-\alpha)(1-\beta) \right)^{\frac{v-1}{v}} \right]^{\frac{1}{1-v}}$$

$$C^{\frac{\varphi+y}{v-y}} = \left( \mathcal{X}^{\frac{-1}{v-y}} \cdot (1-\alpha)^{\frac{1}{v-y}} \left[ (1-\alpha) + \alpha^{1/v} \left( (1-\alpha)(1-\beta) \right)^{\frac{v-1}{v}} \right]^{\frac{1}{1-v}} \right)$$

$$\Rightarrow C = \left( \mathcal{X}^{\frac{-1}{v-y}} \cdot (1-\alpha)^{\frac{1}{v-y}} \left[ (1-\alpha) + \alpha^{1/v} \left( (1-\alpha)(1-\beta) \right)^{\frac{v-1}{v}} \right]^{\frac{1}{1-v}} \right)^{\frac{v-y}{\varphi+y}}$$

Therefore, the steady state is:

- $Y = N = C \quad (1)$
- $Q = \frac{1}{\beta} \quad (2)$
- $W = P \quad (3)$
- $C = \left( \chi^{\frac{-1}{\nu - y}} \cdot (1-\alpha)^{\frac{1}{\nu - y}} \left[ (1-\alpha) + \alpha^{1/\nu} \left( (1-\alpha)(1-\beta) \right)^{\frac{\nu - 1}{\nu}} \right]^{\frac{1}{1-\nu}} \right)^{\frac{\nu - y}{\varphi + y}} \quad (4)$
- $\left(\frac{M}{P}\right) = \left( \frac{\alpha}{(1-\alpha)(1-\beta)} \right)^{\frac{1}{\vartheta}} \cdot C \quad (5)$
- $Y = \frac{W}{P} \cdot N \quad (6)$

(d) Based on your steady state equations describe an algorithm for how to solve for the steady state.

※1: Set the parameters values for: $\varphi, \alpha, \nu, \chi, \beta, \gamma$

※2: Calculate $C$ by plugging the parameters in the equation (4) for Consumption

※3: Calculate $Y$ and $N$ using the equation (1), which represents market clear condition

※4: Calculate $P$ (and $W$) using the zero profit condition for the firm; equation (6)

※5: Calculate $M$ using the equation (5).

(e) How would you calibrate $\vartheta$ given knowledge of $\nu$? (I.e., what moments of the data would you use and how?)

To calibrate $\vartheta = \alpha$, we can use the following equations:

$$C = \left( \chi^{\frac{-1}{\nu - y}} \cdot (1-\alpha)^{\frac{1}{\nu - y}} \left[ (1-\alpha) + \alpha^{1/\nu} \left( (1-\alpha)(1-\beta) \right)^{\frac{\nu - 1}{\nu}} \right]^{\frac{1}{1-\nu}} \right)^{\frac{\nu - y}{\varphi + y}}$$

$$\left(\frac{M}{P}\right) = \left( \frac{\alpha}{(1-\alpha)(1-\beta)} \right)^{\frac{1}{\vartheta}} \cdot C$$

Then, given the values of others parameters and getting money and prices in SS, we can compute $\alpha$ using the above equations.

(f) Given knowledge of other parameters, how would you set $M$ such that $P = 1$ in steady state?

Money demand : $\frac{M_t^D}{P_t} = C_t \cdot \left( \left(1 - \frac{1}{Q_t}\right) \frac{(1-\alpha)}{\alpha} \right)^{\frac{-1}{v}}$

The Steady State is :

- $Y = N = C$
- $Q = \frac{1}{\beta}$
- $W = P$
- $C = \left( \chi^{\frac{-1}{v-y}} \cdot (1-\alpha)^{\frac{1}{v-y}} \left[ (1-\alpha) + \alpha^{1/v} \left( (1-\alpha)(1-\beta) \right)^{\frac{v-1}{v}} \right]^{\frac{1}{1-v}} \right)^{\frac{v-y}{\varphi + y}}$
- $\left(\frac{M}{P}\right) = \left( \frac{\alpha}{(1-\alpha)(1-\beta)} \right)^{\frac{1}{v}} \cdot C$

Money supply: $M_t^S$

In equilibrium : $M_t^D = M_t^S$

$\Big\}$ $M_t^S = P_t \cdot C_t \cdot \left( \left(1 - \frac{1}{Q_t}\right) \frac{(1-\alpha)}{\alpha} \right)^{\frac{-1}{v}}$

Then : $P_t = 1 \iff M_t^S = C_t \cdot \left( \left(1 - \frac{1}{Q_t}\right) \frac{(1-\alpha)}{\alpha} \right)^{\frac{-1}{v}} \iff SS: M^S = \left( \frac{\alpha}{(1-\alpha)(1-\beta)} \right)^{\frac{1}{v}} \cdot C$

$\Rightarrow$ I would set up M as follow: $M^S = C \cdot \left( \frac{\alpha}{(1-\alpha)(1-\beta)} \right)^{\frac{1}{v}}$

where : $C = \left( \chi^{\frac{-1}{v-y}} \cdot (1-\alpha)^{\frac{1}{v-y}} \left[ (1-\alpha) + \alpha^{1/v} \left( (1-\alpha)(1-\beta) \right)^{\frac{v-1}{v}} \right]^{\frac{1}{1-v}} \right)^{\frac{v-y}{\varphi + y}}$

## Appendix 1 :

FOC.

(1) $\beta^{s}\cdot X_{t+s}^{\nu-\gamma}\cdot(1-\alpha)\cdot C_{t+s}^{-\nu} = \lambda_{t+s}\cdot P_{t+s}$ Consumption

(2) $\beta^{s}\cdot \chi N_{t+s}^{\varphi} = \lambda_{t+s}\cdot W_{t+s}$ Labor

(3) $\lambda_{t+s} = E_{t+s}\{\lambda_{t+s+1}\cdot Q_{t+s}\}$ Bonds

(4) $\beta^{s}\cdot X_{t+s}^{\nu-\gamma}\cdot\alpha\left(\frac{M_{t+s}}{P_{t+s}}\right)^{-\nu}\cdot\frac{1}{P_{t+s}} = \lambda_{t+s} - E_{t+s}\{\lambda_{t+s+1}\}$ Money

From (1) consumption and (2) labor : $\boxed{\frac{W_{t+s}}{P_{t+s}} = \frac{\chi N_{t+s}^{\varphi}}{(X_{t+s}^{\nu-\gamma}\cdot(1-\alpha)\cdot C_{t+s}^{-\nu})}}$ Labor supply

From (1) consumption and (3) bonds : $\lambda_{t+s} = \frac{\beta^{s}\cdot X_{t+s}^{\nu-\gamma}\cdot(1-\alpha)\cdot C_{t+s}^{-\nu}}{P_{t+s}}$ ; $\lambda_{t+s+1} = \frac{\beta^{s+1}\cdot X_{t+s+1}^{\nu-\gamma}(1-\alpha)\, C_{t+s+1}^{-\nu}}{P_{t+s+1}}$

$$\lambda_{t+s} = E_{t+s}\{\lambda_{t+s+1}\cdot Q_{t+s}\} \Rightarrow \frac{\beta^{s}\cdot X_{t+s}^{\nu-\gamma}\cdot\cancel{(1-\alpha)}\cdot C_{t+s}^{-\nu}}{P_{t+s}} = E_{t+s}\left\{\frac{\beta^{s+1}\cdot X_{t+s+1}^{\nu-\gamma}\,\cancel{(1-\alpha)}\,C_{t+s+1}^{-\nu}}{P_{t+s+1}}\cdot Q_{t+s}\right.$$

$$C_t^{-\nu} = \frac{P_t\cdot\beta}{X_t^{\nu-\gamma}}\, E_t\left\{\frac{X_{t+1}^{\nu-\gamma}\cdot C_{t+1}^{-\nu}\cdot Q_t}{P_{t+1}}\right\}$$

$$\Rightarrow C_t^{-\nu} = \beta\cdot E_t\left\{\frac{P_t}{P_{t+1}}\cdot C_{t+1}^{-\nu}\left(\frac{X_{t+1}}{X_t}\right)^{\nu-\gamma}\cdot Q_t\right\} \quad \text{(Bonds)} \quad (5)$$

$$\boxed{1 = \beta\cdot E_t\left\{\frac{P_t}{P_{t+1}}\cdot\left(\frac{C_{t+1}}{C_t}\right)^{-\nu}\left(\frac{X_{t+1}}{X_t}\right)^{\nu-\gamma}\cdot Q_t\right\}}$$ Bonds demand

From (1) consumption and Money (4) : $\beta^{s}\cdot X_{t+s}^{\nu-\gamma}\cdot\alpha\left(\frac{M_{t+s}}{P_{t+s}}\right)^{-\nu}\cdot\frac{1}{P_{t+s}} = \lambda_{t+s} - E_{t+s}\{\lambda_{t+s+1}\}$

$$\beta^{s}\cdot X_{t+s}^{\nu-\gamma}\cdot\alpha\left(\frac{M_{t+s}}{P_{t+s}}\right)^{-\nu}\cdot\frac{1}{P_{t+s}} = \frac{\beta^{s}\cdot X_{t+s}^{\nu-\gamma}\cdot(1-\alpha)\cdot C_{t+s}^{-\nu}}{P_{t+s}} - E_{t+s}\left\{\frac{\beta^{s+1}\cdot X_{t+s+1}^{\nu-\gamma}(1-\alpha)\,C_{t+s+1}^{-\nu}}{P_{t+s+1}}\right\}$$

$$\frac{\beta^{s}\cdot X_{t+s}^{\nu-\gamma}\cdot(1-\alpha)\cdot C_{t+s}^{-\nu}}{P_{t+s}} = \beta^{s}\cdot X_{t+s}^{\nu-\gamma}\cdot\alpha\left(\frac{M_{t+s}}{P_{t+s}}\right)^{-\nu}\cdot\frac{1}{P_{t+s}} + E_{t+s}\left\{\frac{\cancel{\beta^{s+1}}\cdot X_{t+s+1}^{\nu-\gamma}\,\cancel{(1-\alpha)}\,C_{t+s+1}^{-\nu}}{P_{t+s+1}}\right\}$$

$$C_t^{-\nu} = \frac{\alpha}{(1-\alpha)}\left(\frac{M_t}{P_t}\right)^{-\nu} + \beta\, E_t\left\{\frac{P_t}{P_{t+1}}\cdot\left(\frac{X_{t+1}}{X_t}\right)^{\nu-\gamma}\cdot C_{t+1}^{-\nu}\right\} \quad \text{(Money)} \quad (6)$$

$$\frac{(6)}{(5)} \quad 1 = \frac{\frac{\alpha}{(1-\alpha)}\left(\frac{M_t}{P_t}\right)^{-\nu}}{\underbrace{\beta\cdot E_t\left\{\frac{P_t}{P_{t+1}}\cdot C_{t+1}^{-\nu}\left(\frac{X_{t+1}}{X_t}\right)^{\nu-\gamma}\cdot Q_t\right\}}_{C_t^{-\nu}}} + \frac{\cancel{\beta}\, E_t\left\{\frac{\cancel{P_t}}{\cancel{P_{t+1}}}\cdot\left(\frac{\cancel{X_{t+1}}}{\cancel{X_t}}\right)^{\cancel{\nu-\gamma}}\cdot\cancel{C_{t+1}^{-\nu}}\right\}}{\cancel{\beta}\cdot E_t\left\{\frac{\cancel{P_t}}{\cancel{P_{t+1}}}\cdot\cancel{C_{t+1}^{-\nu}}\left(\frac{\cancel{X_{t+1}}}{\cancel{X_t}}\right)^{\nu-\gamma}\cdot Q_t\right\}}$$

$$1 = \frac{\alpha}{(1-\alpha)}\left(\frac{M_t}{P_t}\right)^{-\nu}\cdot C_t^{\nu} + \frac{1}{Q_t} \Rightarrow \left(1-\frac{1}{Q_t}\right)\frac{(1-\alpha)}{\alpha}\cdot C_t^{-\nu} = \left(\frac{M_t}{P_t}\right)^{-\nu}$$

$$\Rightarrow \boxed{\frac{M_t}{P_t} = C_t\cdot\left(\left(1-\frac{1}{Q_t}\right)\frac{(1-\alpha)}{\alpha}\right)^{-\frac{1}{\nu}}}$$ Money demand

(g) Derive the log-linearized model.

**Equilibrium equations:**

| | | |
|---|---|---|
| (1) | $Y_t = A_t \cdot N_t$ | production function |
| (2) | $\frac{W_t}{P_t} = A_t$ | labor demand |
| (3) | $\frac{W_t}{P_t} = \frac{\chi N_t^{\varphi}}{(1-\alpha) C_t^{-\nu}} \cdot \frac{1}{X_t^{\nu-\gamma}}$ | labor supply |
| (4) | $Y_t = C_t$ | market clearing for output |
| (5) | $\frac{M_t}{P_t} = C_t \cdot \left( \left(1 - \frac{1}{Q_t}\right) \frac{(1-\alpha)}{\alpha} \right)^{-\frac{1}{\nu}}$ | money demand |
| (6) | $1 = \beta \cdot E_t \left\{ \frac{P_t}{P_{t+1}} \cdot \left(\frac{C_{t+1}}{C_t}\right)^{-\nu} \left(\frac{X_{t+1}}{X_t}\right)^{\nu-\gamma} \cdot Q_t \right\}$ | bonds demand |

**Loglinearization with respect SS:** $\hat{x} = \frac{x_t - x}{x} \approx \ln \frac{x_t}{x}$

| N° | Original equation | Loglinearized equation | | Hat notation |
|---|---|---|---|---|
| (1) | $Y_t = A_t \cdot N_t$ | $\log(Y_t) = \log(A_t) + \log(N_t)$ | $\frac{Y_t - Y}{Y} = \frac{A_t - A}{A} + \frac{N_t - N}{N}$ | $\hat{Y}_t = \hat{A}_t + \hat{N}_t$ |
| (2) | $\frac{W_t}{P_t} = A_t$ | $\log(W_t) - \log(P_t) = \log(A_t)$ | $\frac{W_t - W}{W} - \frac{P_t - P}{P} = \frac{A_t - A}{A}$ | $\hat{W}_t - \hat{P}_t = \hat{A}_t$ |
| (3) | $\frac{W_t}{P_t} = \frac{\chi N_t^{\varphi}}{(1-\alpha) C_t^{-\nu}} \cdot \frac{1}{X_t^{\nu-\gamma}}$<br>SS: $\frac{W}{P} = \frac{\chi N^{\varphi}}{(1-\alpha) C^{-\nu}} \cdot \frac{1}{X^{\nu-\gamma}}$ | $\log(W_t) - \log(P_t) = \log(\chi) + \varphi \log(N_t) - \log(1-\alpha) + \nu \log(C_t) - (\nu-\gamma) \log(X_t)$<br>$\log(W) - \log(P) = \log(\chi) + \varphi \log(N) - \log(1-\alpha) + \nu \log(C) - (\nu-\gamma) \log(X)$ | $\frac{W_t - W}{W} - \frac{P_t - P}{P} = \frac{\chi - \chi}{\chi} + \varphi\frac{(N_t - N)}{N_t} - \frac{(1-\alpha)-(1-\alpha)}{(1-\alpha)} + \nu\frac{(C_t - C)}{C} - \frac{(\nu-\gamma)(X_t - X)}{X}$ | $\hat{W}_t - \hat{P}_t = \varphi \hat{N}_t + \nu \cdot \hat{C}_t - (\nu-\gamma) \cdot \hat{X}_t$ |
| (4) | $Y_t = C_t$ | $\log(Y_t) = \log(C_t)$ | $\frac{Y_t - Y}{Y} = \frac{C_t - C}{C}$ | $\hat{Y}_t = \hat{C}_t$ |

$g(z) = f(x,y)$

$g'(z)\, z\, \hat{z}_t = f_x(x,y)\cdot x \cdot \hat{x}_t + f_y(x,y)\, y\, \hat{y}_t$

steady state

(5) $$\frac{M_t}{P_t} = C_t \cdot \left(\left(1-\frac{1}{Q_t}\right)\frac{(1-\alpha)}{\alpha}\right)^{-\frac{1}{v}} \Rightarrow \overbrace{M_t}^{g(M)} = \overbrace{P_t \cdot C_t \cdot \left(1-\frac{1}{Q_t}\right)^{-\frac{1}{v}}\left(\frac{1-\alpha}{\alpha}\right)^{-\frac{1}{v}}}^{f(P,C,Q)}$$

SS: $$\frac{M}{P} = C\left(\left(1-\frac{1}{Q}\right)\frac{(1-\alpha)}{\alpha}\right)^{-\frac{1}{v}}$$

$M_t: \quad 1\cdot \cancel{M}\cdot \hat{M}_t$

$P_t: \quad C\cdot\left(\left(1-\frac{1}{Q}\right)\frac{(1-\alpha)}{\alpha}\right)^{-\frac{1}{v}}\cdot P\cdot \hat{P}_t = \frac{M}{P}\cdot P\cdot \hat{P}_t = \cancel{M}\cdot \hat{P}_t$

$C_t: \quad P_t\cdot\left(\left(1-\frac{1}{Q}\right)\frac{(1-\alpha)}{\alpha}\right)^{-\frac{1}{v}}\cdot C\cdot \hat{C}_t = \frac{M}{C}\cdot C\cdot \hat{C}_t = \cancel{M}\cdot \hat{C}_t$

$Q_t: \quad \left(-\frac{1}{v}\right)\cdot P\cdot C\left(\left(1-\frac{1}{Q}\right)^{-\frac{1}{v}-1}\frac{(1-\alpha)}{\alpha}\right)^{-\frac{1}{v}}\left(\frac{1}{Q^2}\right)\cdot Q\cdot \hat{Q}_t = \left(-\frac{1}{v}\right)\frac{M}{\left(1-\frac{1}{Q}\right)}\cdot\frac{1}{Q}\hat{Q}_t$

$\left(\frac{Q-1}{Q}\right)^{-1}\cdot\frac{1}{Q} = \frac{1}{Q-1}$

$\Rightarrow$ Hat notation: $\hat{M}_t = \hat{P}_t + \hat{C}_t - \frac{1}{v}\cdot\left(\frac{\beta}{1-\beta}\right)\hat{Q}_t$, in SS: $\frac{1}{\beta} = Q \Rightarrow \frac{1}{\frac{1}{\beta}-1} = \frac{1}{\frac{1-\beta}{\beta}} = \frac{\beta}{1-\beta}$

(6) $$Q_t^{-1} = \beta\cdot E_t\left\{\frac{P_t}{P_{t+1}}\cdot\left(\frac{C_{t+1}}{C_t}\right)^{-v}\left(\frac{X_{t+1}}{X_t}\right)^{v-y}\right\}$$

SS: $$\frac{1}{Q} = \beta\, E_t\left\{\frac{P}{P}\cdot\left(\frac{C}{C}\right)^{-v}\left(\frac{X}{X}\right)^{v-y}\right\}$$

$\Rightarrow \frac{1}{Q} = \beta\cdot$

$g'(z)\, z\, \hat{z}_t = f_x(x,y)\cdot x \cdot \hat{x}_t + f_y(x,y)\, y\, \hat{y}_t$

$Q_t: \quad \frac{-1}{Q^2}\cdot Q\cdot \hat{Q}_t = -\frac{1}{Q}\cdot \hat{Q}_t = -\beta\hat{Q}_t$

$C_t: \quad -v\cdot\beta\cdot\frac{1}{C}\cdot C\cdot\hat{C}_t = -v\beta\hat{C}_t$

$C_{t+1}: \quad +v\cdot\beta\cdot\frac{1}{C^2}\cdot C\cdot C\cdot\hat{C}_{t+1} = +v\beta\cdot\hat{C}_{t+1}$

$P_t: \quad \beta\cdot\frac{1}{P}\, P\cdot\hat{P}_t = \beta\cdot\hat{P}_t$

$P_{t+1}: \quad -\beta\cdot\frac{1}{P^2}\cdot P\cdot P\cdot\hat{P}_{t+1} = -\beta\cdot\hat{P}_{t+1}$

$X_t: \quad -\beta(v-y)\cdot\frac{1}{X^2}\cdot X\cdot X\cdot\hat{X}_t = -\beta(v-y)\hat{X}_t$

$X_{t+1}: \quad \beta\cdot(v-y)\cdot\frac{1}{X}\cdot X\cdot\hat{X}_{t+1} = \beta(v-y)\hat{X}_{t+1}$

$\Rightarrow -\cancel{\beta}\hat{Q}_t = -v\cancel{\beta}\hat{C}_t + v\cancel{\beta}\hat{C}_{t+1} + \cancel{\beta}\hat{P}_t - \cancel{\beta}\hat{P}_{t+1} - \cancel{\beta}(v-y)\hat{X}_t + \cancel{\beta}(v-y)\hat{X}_{t+1}$

$\hat{Q}_t = v(\hat{C}_t - \hat{C}_{t+1}) + \hat{P}_{t+1} - \hat{P}_t + (v-y)(\hat{X}_t - \hat{X}_{t+1})$

**Equilibrium equations:**

(1) $Y_t = A_t \cdot N_t$ — production function

(2) $\frac{W_t}{P_t} = A_t$ — labor demand

(3) $\frac{W_t}{P_t} = \frac{\chi N_t^{\varphi}}{(1-\alpha) C_t^{-v}} \cdot \frac{1}{X_t^{v-y}}$ — labor supply

(4) $Y_t = C_t$ — market clearing for goods

(5) $\frac{M_t}{P_t} = C_t \cdot \left( \left(1 - \frac{1}{Q_t}\right) \frac{(1-\alpha)}{\alpha} \right)^{-\frac{1}{v}}$ — money demand

(6) $1 = \beta \cdot E_t \left\{ \frac{P_t}{P_{t+1}} \cdot \left(\frac{C_{t+1}}{C_t}\right)^{-v} \left(\frac{X_{t+1}}{X_t}\right)^{v-y} \cdot Q_t \right\}$ — bonds demand

**Steady State:**

- $C = \left( \chi^{\frac{-1}{v-y}} \cdot (1-\alpha)^{\frac{1}{v-y}} \left[ (1-\alpha) + \alpha^{1/v} \left( (1-\alpha)(1-\beta) \right)^{\frac{v-1}{v}} \right]^{\frac{1}{1-v}} \right)^{\frac{v-y}{\varphi+y}}$
- $\left(\frac{M}{P}\right) = \left( \frac{\alpha}{(1-\alpha)(1-\beta)} \right)^{\frac{1}{v}} \cdot C$

- $Y = N = C$
- $Q = \frac{1}{\beta}$
- $W = P$

**Loglinearized equations:**

(1) $\hat{y}_t = \hat{A}_t + \hat{N}_t$ — production function

(2) $\hat{W}_t - \hat{P}_t = \hat{A}_t$ — labor demand

(3) $\hat{W}_t - \hat{P}_t = \varphi \hat{N}_t + v \cdot \hat{C}_t - (v-y) \cdot \hat{X}_t$ — labor supply

(4) $\hat{y}_t = \hat{C}_t$ — market clearing for output

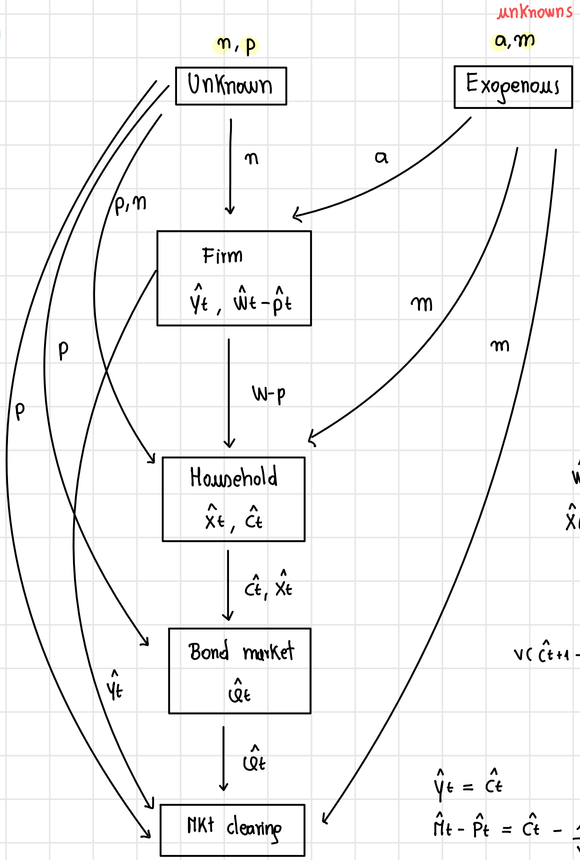
(5) $\hat{M}_t = \hat{P}_t + \hat{C}_t - \frac{1}{v} \cdot \left( \frac{\beta}{1 \beta} \right) \hat{Q}_t$ — money demand : marking clearing

(6) $v(\hat{C}_{t+1} - \hat{C}_t) = \hat{Q}_t + (\hat{P}_t - \hat{P}_{t+1}) + (v-y)(\hat{X}_{t+1} - \hat{X}_t)$ — bonds demand

$A = 1$

$\theta = \alpha$

## DAG




$$\hat{y}_t = \hat{A}_t + \hat{N}_t$$
$$\hat{w}_t - \hat{p}_t = \hat{A}_t$$

$$\hat{w}_t - \hat{p}_t = \varphi \hat{N}_t + v \cdot \hat{c}_t - (v-\gamma) \cdot \hat{x}_t$$
$$\hat{x}_t = (1-\theta) \cdot \left(\frac{c^*}{x^*}\right)^{1-v} \hat{c}_t + \theta \left(\frac{M^*}{p^*}\right)^{1-v} (\hat{m}_t - \hat{p}_t)$$

$$v(\hat{c}_{t+1} - \hat{c}_t) = \hat{Q}_t + (\hat{p}_t - \hat{p}_{t+1}) + (v-\gamma)(\hat{x}_{t+1} - \hat{x}_t)$$

$$\hat{y}_t = \hat{c}_t$$
$$\hat{m}_t - \hat{p}_t = \hat{c}_t - \frac{1}{v} \frac{\beta}{1-\beta} \cdot \hat{Q}_t$$

Plugging the loglinearize in the labor supply:

$$\hat{w}_t - \hat{p}_t = \varphi \hat{N}_t + v \cdot \hat{c}_t - (v-\gamma) \cdot \hat{x}_t$$
$$\hat{x}_t = (1-\theta) \cdot \left(\frac{c^*}{x^*}\right)^{1-v} \hat{c}_t + \theta \left(\frac{M^*}{p^*}\right)^{1-v} (\hat{m}_t - \hat{p}_t) \qquad \text{(definition)}$$

$$\hat{w}_t - \hat{p}_t = \varphi \hat{N}_t + v \cdot \hat{c}_t - (v-\gamma) \cdot (1-\theta) \cdot \left(\frac{c^*}{x^*}\right)^{1-v} \hat{c}_t + \theta \left(\frac{M^*}{p^*}\right)^{1-v} (\hat{m}_t - \hat{p}_t)$$

$$(v-\gamma) \cdot (1-\theta) \cdot \left(\frac{c^*}{x^*}\right)^{1-v} \hat{c}_t = \varphi \hat{N}_t + \theta \left(\frac{M^*}{p^*}\right)^{1-v} (\hat{m}_t - \hat{p}_t) - (\hat{w}_t - \hat{p}_t)$$

- gmmp: good market w/r to p
- mmx: money market w/r to p
- mmwp: money market with respect (w−p)

## Market clearings:

(1) $\phi_{gmy} = -I$

(2) $\phi_{gmc} = I$

(3) $\phi_{gmwp} = \phi_{gmq} = Z$

(4) $\phi_{mmc} = I$

(5) $\phi_{mmq} = -\frac{1}{V} \frac{\beta}{1-\beta} \cdot I$

(6) $\phi_{mmy} = \phi_{mmwp} = Z$

$$\hat{C}_t = \hat{Y}_t$$

$$(\hat{N}_t - \hat{P}_t) = \hat{C}_t - \frac{1}{V} \cdot \left(\frac{\beta}{1-\beta}\right) \hat{Q}_t$$

variables: $y, c, wp, q, n, p$

gm: good market

mm: money market

## Firms

(1) $\phi_{yn} = I$

(2) $\phi_{ym} = Z$

(3) $\phi_{yp} = Z$

(4) $\phi_{wpn} = Z$

(5) $\phi_{wpm} = Z$

(6) $\phi_{wpp} = Z$

$$\hat{Y}_t = \hat{A}_t + \hat{N}_t$$

$$\hat{W}_t - \hat{P}_t = \hat{A}_t$$

$$\hat{W}_t - \hat{P}_t = \varphi \hat{N}_t + V \cdot \hat{C}_t - (V - \gamma) \cdot \left[(1-\theta) \cdot \left(\frac{C^*}{X^*}\right)^{1-V} \hat{C}_t + \theta \left(\frac{N^*}{P^*}\right)^{1-V} (\hat{N}_t - \hat{P}_t)\right]$$

## Households

(1) $\phi_{cm} = \frac{1}{A} \theta \left(\frac{N^*}{P^*}\right)^{1-V} (V - \gamma) \cdot I - \frac{1}{A} \cdot \phi_{wpm}$

(2) $\phi_{cn} = \frac{1}{A} \varphi * I - \frac{1}{A} \phi_{wpm}$

(3) $\phi_{cp} = Z$

(4) $\phi_{x(\cdot)} = (1-\theta) \left(\frac{C^*}{X^*}\right) \phi_{c(\cdot)}$

$$\hat{C}_t = \overset{A}{\overline{\left[V - \gamma(1-\theta) \left(\frac{C^*}{X^*}\right)^{1-V} - V\right]}}^{-1} \left[\varphi \hat{n}_t + (V-\gamma)\theta \left(\frac{N^*}{P^* X^*}\right)^{1-V} (\hat{N}_t - \hat{P}_t) - (\hat{W}_t - \hat{P}_t)\right]$$

$$\frac{\delta \hat{C}_t}{\delta \hat{m}} = \frac{1}{A} \left(\theta \left(\frac{N^*}{P^* X^*}\right)^{1-V} (V - \gamma) - \phi_{wpm}\right)$$

$$\frac{\delta \hat{C}_t}{\delta \hat{m}} = \frac{1}{A} (\varphi - \phi_{wpm})$$

## Bonds:

(1) $\phi_{qm} = V \cdot [p] + \phi_{cm} - (V - \gamma) \cdot I_{p1} \cdot \phi_{xm}$

(2) $\phi_{qn} = V \cdot I_{p1} \cdot \phi_{cn} - (V - \gamma) \cdot I_{p1} \cdot \phi_{xn}$

(3) $\phi_{qp} = -I_{p1} + V I_{p1} \times \phi_{cp} - (V - \gamma) \cdot I_{p1} \cdot \phi_{xp}$

$$V(\hat{C}_{t+1} - \hat{C}_t) = \hat{Q}_t + (\hat{P}_t - \hat{P}_{t+1}) + (V - \gamma)(\hat{X}_{t+1} - \hat{X}_t)$$

$U: \ \{\hat{N}_t\},\ \{\hat{P}_t\}$

$Z: \ A = 1,\ \{\hat{m}_t\}$

$y: \ y,\ w-p,\ c,\ x,\ Q$

$$H(U, Z) = 0$$

$$dU = -H_U(\bar{U}, \bar{Z})^{-1}\, H_Z(\bar{U}, \bar{Z})\, dZ$$

$$H(U,Z) = \begin{pmatrix} \hat{C}_t - \hat{y}_t \\ \hat{C}_t - \frac{1}{V} \cdot \frac{\beta}{1-\beta} \cdot \hat{Q}_t - (\hat{N}_t - \hat{P}_t) \end{pmatrix} \begin{matrix} gm \\ mm \end{matrix}$$

$$H_U = \underbrace{\frac{dH}{dU}}_{2\times 2} = \underbrace{\frac{dH}{dy}}_{2\times 5} \cdot \underbrace{\frac{dy}{dU}}_{5\times 2} \qquad H_Z = \frac{dH}{dZ} = \frac{dH}{dY} \cdot \frac{dY}{dZ}$$

$$\frac{dH}{dY} = \begin{bmatrix} \Phi_{gm\,y} & \Phi_{gm\,wp} & \Phi_{gm\,c} & \Phi_{gm\,x} & \Phi_{gm\,Q} \\ \Phi_{mm\,y} & \Phi_{mm\,wp} & \Phi_{mm\,c} & \Phi_{mm\,x} & \Phi_{mm\,Q} \end{bmatrix}_{2\times 5}$$

$$\frac{dY}{dU} = \begin{array}{c} \begin{matrix} \quad n & \qquad p \end{matrix} \\ \begin{bmatrix} \Phi_{yn} & \Phi_{yp} \\ \Phi_{wpn} & \Phi_{wpp} \\ \Phi_{cn} & \Phi_{cp} \\ \Phi_{xn} & \Phi_{xp} \\ \Phi_{Qn} & \Phi_{Qp} \end{bmatrix}_{5\times 2} \end{array} \qquad \frac{dY}{dZ} = \begin{array}{c} m \\ \begin{bmatrix} \Phi_{ym} \\ \Phi_{wpm} \\ \Phi_{cm} \\ \Phi_{xm} \\ \Phi_{Qm} \end{bmatrix}_{5\times 2} \end{array}$$

1. $y: \ \hat{y}_t = \hat{N}_t$

$\Phi_{yn} = I$

$\Phi_{yp} = 0$

$\Phi_{ym} = 0$

2. $wp: \ \hat{w}_t - \hat{p}_t = \hat{A}_t$

$\Phi_{wpn} = 0$

$\Phi_{wpp} = 0$

$\Phi_{wpm} = 0$

$$\hat{W}_t - \hat{P}_t = \varphi \hat{N}_t + V \cdot \hat{C}_t - (V-y) \cdot \left[ (1-\theta) \cdot \left(\frac{C}{X}\right)^{1-V} \hat{C}_t + \theta \left(\frac{N}{P}\right)^{1-V} (\hat{N}_t - \hat{p}_t) \right]$$

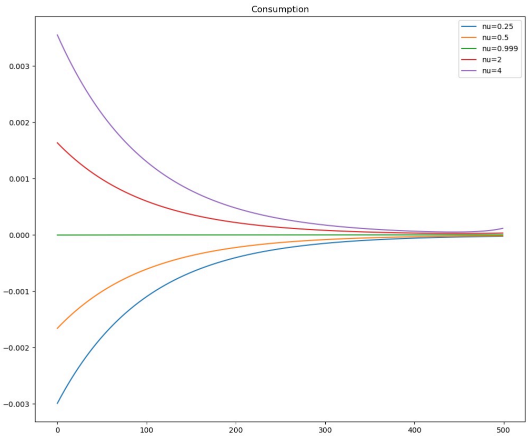
$$\hat{W}_t - \hat{P}_t = \hat{C}_t \left[ V - (V-y)(1-\theta) \cdot \left(\frac{C}{X}\right)^{1-V} \right] + \varphi \hat{N}_t - (V-y) \cdot \theta \left(\frac{N}{P}\right)^{1-V} (\hat{N}_t - \hat{p}_t)$$

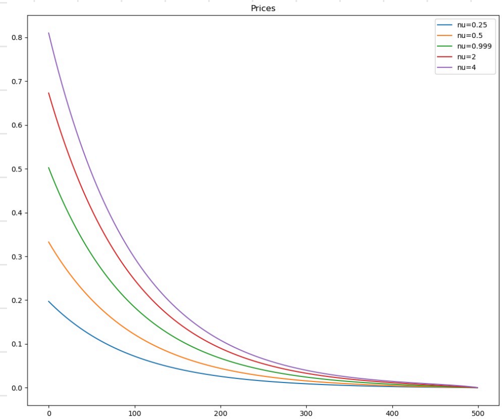
(h) Following your calibration strategy for each of $\nu \in \{0.25, 0.5, 1, 2, 4\}$, solve the model using sequence space methods using the following parameters:

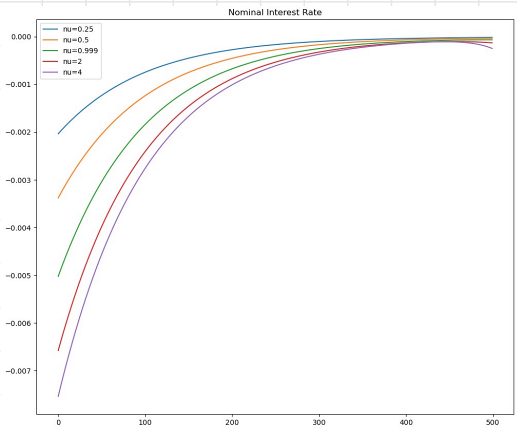
$$\gamma = 1, \varphi = 1, \chi = 1, \beta = 0.99, \rho_m = 0.99$$

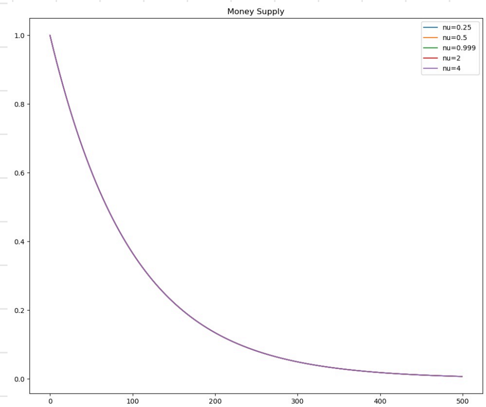
where $\hat{m}_t = \rho_m \hat{m}_{t-1} + \epsilon_t^m$.

Report the IRFs for consumption, prices, the nominal interest rate. Your graph for each variable should contain all five cases, appropriately labelled.




(i) Intuitively explain your results.

From the above graphs, we can see that when money supply increase, then prices increase and the interest rate decrease. Hence, the value of money decrease with respect consumption and the value of real bonds.

(j) If you had evidence that an increase in the money supply increases consumption, which values for $\nu$ can you rule out? Explain why.

From the graphs above, when we increase money supply in SS, increase consumption only when $\nu > 1$. Hence, we can rule out: $\nu = \{0.25, 0.5, 1\}$ //

(k) Make sure your code packet contains a file that produces your graphs with a single click. (It does not need to save the graphs.) Upload it to Github.

The code is on github already.