Homework 4: Macroeconomics 210C

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06/05/2024

PROBLEM 1

1. Productivity Shocks in the Three Equation Model

The log-linearized NK model boils down to three equations:

$$\hat{y}_t = -\sigma[\hat{i}_t - E_t\{\hat{\pi}_{t+1}\}] + E_t\{\hat{y}_{t+1}\}$$
$$\hat{\pi}_t = \kappa(\hat{y}_t - \hat{y}_t^{flex}) + \beta E_t\{\hat{\pi}_{t+1}\}$$
$$\hat{i}_t = \phi_\pi \hat{\pi}_t + v_t$$

with $\hat{y}_t^{flex} = \frac{1+\varphi}{\gamma+\varphi}\hat{a}_t$.

For this part assume that $v_t = 0$ and that $\hat{a}_t = \rho_a \hat{a}_{t-1} + \epsilon_t$.

(a) Using the method of undetermined coefficients, solve for $\hat{y}_t$ and $\hat{\pi}_t$ as a function of $\hat{a}_t$.

$$\hat{y}_t = -\sigma[\hat{i}_t - E_t\{\hat{\pi}_{t+1}\}] + E_t\{\hat{y}_{t+1}\}$$

$$\hat{\pi}_t = K(\hat{y}_t - \hat{y}_t^{flex}) + \beta E_t\{\hat{\pi}_{t+1}\} \quad ; \quad \hat{y}^{flex} = \frac{1+\varphi}{\gamma+\varphi}\cdot \hat{a}_t$$

$$\hat{i}_t = \phi_\pi \cdot \hat{\pi}_t + v_t$$

Consider: $v_t = 0$ (shock to interest rate) ; $\hat{a}_t = \rho_a \cdot \hat{a}_{t-1} + \varepsilon_t$ (shock to TFP)

white noise $E[\varepsilon_t] = 0$

Method of undetermined coefficients:

shock $\hat{a}_t$ to $\hat{y}_t$

guess policy function: $\hat{y}_t = \Psi_{ya}\cdot \hat{a}_t$

$\hat{\pi}_t = \Psi_{\pi a}\cdot \hat{a}_t$

$E_t[\hat{a}_{t+1}] = E_t[\rho_a \cdot \hat{a}_t + \varepsilon_{t+1}] = \rho_a \cdot \hat{a}_t$

$E_t[\hat{\pi}_{t+1}] = E_t[\Psi_{\pi a}\cdot \hat{a}_{t+1}] = \Psi_{\pi a}\cdot \rho_a \cdot \hat{a}_t$

$E_t[\hat{y}_{t+1}] = E_t[\Psi_{ya}\cdot \hat{a}_{t+1}] = \Psi_{ya}\cdot \rho_a \cdot \hat{a}_t$

Interest rate: $\hat{i}_t = \phi_\pi \cdot \hat{\pi}_t + \cancel{v_t}^{\,0}$

$\Rightarrow \hat{i}_t = \phi_\pi \cdot \Psi_{\pi a}\cdot \hat{a}_t$

Output: $\hat{y}_t = -\sigma[\hat{i}_t - E_t\{\hat{\pi}_{t+1}\}] + E_t\{\hat{y}_{t+1}\}$

$\Psi_{ya}\cdot \cancel{\hat{a}_t} = -\sigma[\phi_\pi \cdot \Psi_{\pi a}\cdot \cancel{\hat{a}_t} - \Psi_{\pi a}\cdot \rho_a \cdot \cancel{\hat{a}_t} + \Psi_{ya}\cdot \rho_a \cdot \cancel{\hat{a}_t}]$

$\Psi_{ya}(1-\rho_a) = -\sigma \phi_\pi \cdot \Psi_{\pi a} + \sigma \Psi_{\pi a}\cdot \rho_a$

$$\Psi_{ya} = \frac{\sigma \Psi_{\pi a}(\rho_a - \phi_\pi)}{(1-\rho_a)}$$

Inflation: $\hat{\pi}_t = K(\hat{y}_t - \hat{y}_t^{flex}) + \beta E_t\{\hat{\pi}_{t+1}\}$

$\Psi_{\pi a}\cdot \cancel{\hat{a}_t} = K(\Psi_{ya}\cdot \cancel{\hat{a}_t} - \frac{1+\varphi}{\gamma+\varphi}\cdot \cancel{\hat{a}_t}) + \beta \cdot \Psi_{\pi a}\cdot \rho_a \cdot \cancel{\hat{a}_t}$

$\Psi_{\pi a}(1-\beta\rho_a) = K\cdot \Psi_{ya} - K\cdot \frac{1+\varphi}{\gamma+\varphi}$

$\Psi_{\pi a}(1-\beta\rho_a) - K\cdot \frac{\sigma \Psi_{\pi a}(\rho_a - 1)}{(1-\rho_a)} = -K\cdot \frac{1+\varphi}{\gamma+\varphi}$

$\Psi_{\pi a}\left(1-\beta\rho_a - \frac{K(\rho_a-\phi_\pi)}{(1-\rho_a)}\right) = -K\cdot \frac{1+\varphi}{\gamma+\varphi}$

$\Psi_{\pi a} = -K\cdot \frac{1+\varphi}{\gamma+\varphi}\cdot \frac{(1-\rho_a)}{(1-\beta\rho_a)(1-\rho_a) - K(\rho_a - \phi_\pi)}$

$$\Rightarrow \Psi_{\pi a} = \frac{K(\rho_a-1)(1-\varphi)}{(\gamma+\varphi)[(1-\beta a)(1-\rho_a) - K(\rho_a-\phi_\pi)]}$$

Therefore: $\hat{y}_t = \Psi_{ya}\cdot \hat{a}_t \quad \Rightarrow \quad \hat{y}_t = \frac{\sigma \Psi_{\pi a}(\rho_a-\phi_\pi)}{(1-\rho_a)}\cdot \hat{a}_t$

$\hat{\pi}_t = \Psi_{\pi a}\cdot \hat{a}_t$

$$\Rightarrow \hat{\pi}_t = \frac{K(\rho_a-1)(1-\varphi)}{(\gamma+\varphi)[(1-\beta a)(1-\rho_a) - K(\rho_a-\phi_\pi)]}\cdot \hat{a}_t$$

(b) Plot the impulse response function for $\hat{y}_t, \hat{\pi}_t, \hat{y}_t^{flex}, \hat{y}_t - \hat{y}_t^{flex}, \hat{i}_t, \mathbb{E}_t\hat{r}_{t+1}, \hat{n}_t, \hat{a}_t$ to a one unit shock to $\hat{a}_t$. Use the following parameter values:

$\beta = 0.99, \sigma = 1, \kappa = 0.1, \rho_a = 0.8, \phi_\pi = 1.5$

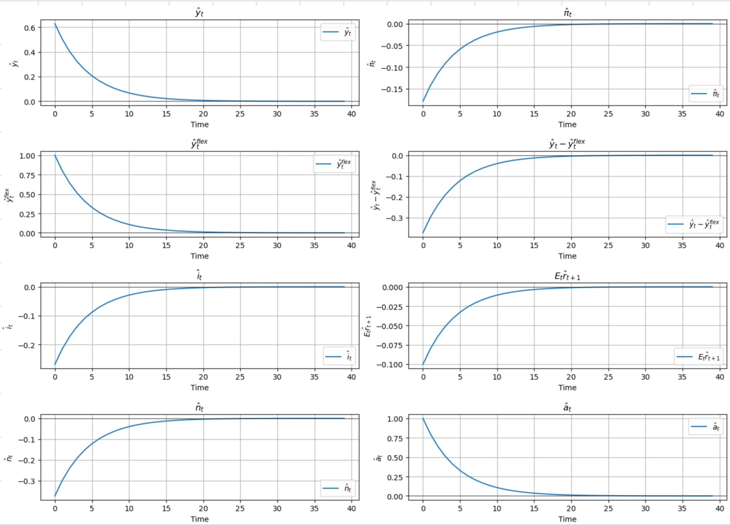
Let's find $E_t[\hat{r}_{t+1}]$ and $\hat{n}_t$ as functions of $\hat{a}_t$.

① $E_t[\hat{r}_{t+1}]$: $R_{t+1} = i_t \dfrac{P_t}{P_{t+1}} \Rightarrow E[R_{t+1}] = E_t[i_t \cdot \dfrac{P_t}{P_{t+1}}] \Rightarrow E_t[\hat{r}_{t+1}] = \hat{i}_t - \hat{E}_t[\hat{\pi}_{t+1}]$

$\Rightarrow E_t[\hat{r}_{t+1}] = \hat{i}_t - \psi_{\pi a} \cdot \rho_a \cdot \hat{a}_t$

② $\hat{n}_t$: $y_t(i) = A_t \cdot N_t(i) \Rightarrow \hat{y}_t = \hat{a}_t + \hat{n}_t$

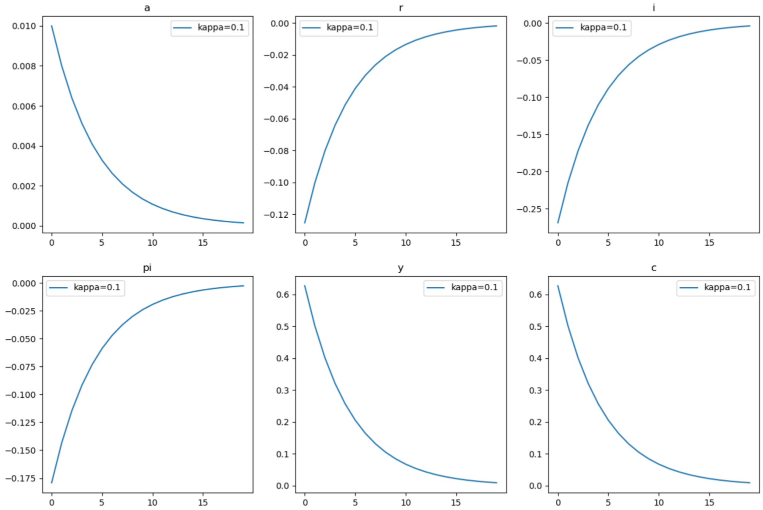
Then, we get the following graphs by setting: $\beta = 0.99$, $\sigma = 1$, $K = 0.1$, $\rho_a = 0.8$, $\phi_{\pi K} = 1.5$, assume $\psi = 0.5$



(c) Intuitively explain your results.

When we have a shock to the productivity (TFP), the production increase $\hat{y}_t$ by 0.6 and $\hat{y}_t^{flex}$ by 1, which means the firms that were able to adjust their prices had a higher increase in their productivity. Both production stabilize over time. On the other hand, the real interest rate decrease and also labor. One explanation for this employment decrease is the price stickiness for some portion of the firms, affecting real wages as well.

(d) Use the Jupyter notebook "newkeynesianlinear.ipynb" to check that your plots in (b) are correct.



The plots in (b) are correct.

PROBLEM 2

(a) The real reset price equation for the firm is,

$$p_t^* \equiv \frac{P_t^*}{P_t} = (1+\mu)E_t\left\{\sum_{s=0}^{\infty}\frac{\theta^s\Lambda_{t,t+s}Y_{t+s}(P_{t+s}/P_t)^{\epsilon-1}}{\sum_{k=0}^{\infty}\theta^k\Lambda_{t,t+k}Y_{t+k}(P_{t+k}/P_t)^{\epsilon-1}}\frac{W_{t+s}/P_t}{A_{t+s}}\right\}$$

Explain why this expression is not recursive.

This equation is not recursive because it's not linear and also depends on two summation, making more difficult to split the terms to make it recursive.

(b) We next show that we can write $B_t = E_t(F_{1t}/F_{2t})$, where both F_{1t}, F_{2t} are recursive. First, show that the denominator can be recursively written as,

$$F_{2t} \equiv \sum_{k=0}^{\infty}\theta^k\Lambda_{t,t+k}Y_{t+k}(P_{t+k}/P_t)^{\epsilon-1}$$
$$= Y_t + \theta\Pi_{t+1}^{\epsilon-1}\Lambda_{t,t+1}F_{2,t+1}$$

noting that $\Lambda_{t,t+k} = \Lambda_{t,t+1}\Lambda_{t+1,t+k}$ for all $k \geqslant 1$.

$$\Lambda_{t,t+K} = \beta E_t\left\{\frac{u'(C_{t+K})}{u'(C_t)}\right\} \quad ; \quad \beta_t = E_t\left\{\frac{F_{1t}}{F_{2t}}\right\}$$

$$F_{2,t+1} \equiv \sum_{K=0}^{\infty}\theta^K\Lambda_{t+1,t+K+1}\cdot Y_{t+K+1}\,(P_{t+K+1}/P_{t+1})^{\epsilon-1}$$

$$F_{2t} \equiv \sum_{K=0}^{\infty}\theta^K\,\Lambda_{t,t+K}\cdot Y_{t+K}\,(P_{t+K}/P_t)^{\epsilon-1}$$

$$F_{2t} = \theta^0\,\Lambda_{t,t}\cdot Y_t\,(P_t/P_t)^{\epsilon-1} + \sum_{K=1}^{\infty}\theta^K\,\Lambda_{t,t+K}\cdot Y_{t+K}\,(P_{t+K}/P_t)^{\epsilon-1}$$

$$= \theta^0\,\Lambda_{t,t}\cdot Y_t\,(P_t/P_t)^{\epsilon-1} + \sum_{K=1}^{\infty}\theta^K\,\Lambda_{t,t+1}\cdot\Lambda_{t+1,t+K+1}\;Y_{t+K}\,(P_{t+K}/P_t)^{\epsilon-1}$$

$$= Y_t + \sum_{K=0}^{\infty}\theta\cdot\theta^K\Lambda_{t,t+1}\cdot\Lambda_{t+1,t+K+1}\cdot Y_{t+K+1}\,(P_{t+K+1}/P_t)^{\epsilon-1}$$

$$= Y_t + \theta\cdot\Lambda_{t,t+1}\cdot\sum_{K=0}^{\infty}\theta^K\,\Lambda_{t+1,t+K+1}\cdot Y_{t+K+1}\,(P_{t+K+1}/P_t)^{\epsilon-1}\frac{(P_{t+1})^{\epsilon-1}}{(P_{t+1})}$$

$$= Y_t + \theta\,\Lambda_{t,t+1}\left(\frac{P_{t+1}}{P_t}\right)^{\epsilon-1}\underbrace{\sum_{K=0}^{\infty}\theta^K\,\Lambda_{t+1,t+K+1}\cdot Y_{t+K+1}\,(P_{t+K+1}/P_{t+1})^{\epsilon-1}}_{F_{2,t+1}}$$

$$= Y_t + \theta\cdot\Lambda_{t,t+1}\cdot\left(\frac{P_{t+1}}{P_t}\right)^{\epsilon-1}F_{2,t+1}\,, \qquad \Pi_{t+1} = \frac{P_{t+1}}{P_t}$$

$$= Y_t + \theta\,\Pi_{t+1}^{\epsilon-1}\,\Lambda_{t,t+1}\cdot F_{2,t+1}$$

(c) Second, show that the numerator can be recursively written as,

$$F_{1t} \equiv (1+\mu)\sum_{s=0}^{\infty} \theta^s \Lambda_{t,t+s} Y_{t+s} (P_{t+s}/P_t)^{\epsilon-1}\frac{W_{t+s}/P_t}{A_{t+s}}$$

$$= (1+\mu) Y_t \frac{W_t/P_t}{A_t} + \theta \Pi_{t+1}^{\epsilon} \Lambda_{t,t+k} F_{1,t+1}$$

$$= (1+\mu)\left(Y_t \frac{W_t/P_t}{A_t} + \theta \Pi_{t+1}^{\epsilon} \Lambda_{t,t+1} F_{1,t+1}\right)$$

noting that $\Lambda_{t,t+k}\Lambda_{t,t+1}\Lambda_{t+1,t+k}$ for all $k \geqslant 1$.

$$F_{1t} \equiv (1+\mu)\sum_{s=0}^{\infty} \theta^s \Lambda_{t,t+s} \cdot Y_{t+s} (P_{t+s}/P_t)^{\epsilon-1}\frac{W_{t+s}/P_t}{A_{t+s}}$$

$$= (1+\mu) Y_t \cdot \frac{W_t/P_t}{A_t} + (1-\mu)\sum_{s=1}^{\infty} \theta^s \Lambda_{t,t+1} \cdot \Lambda_{t+1,t+s+1} Y_{t+s} (P_{t+s}/P_t)^{\epsilon-1}\frac{W_{t+s}/P_t}{A_{t+s}}$$

$$= (1+\mu) Y_t \cdot \frac{W_t/P_t}{A_t} + (1-\mu)\sum_{s=0}^{\infty} \theta^{s+1} \Lambda_{t,t+1} \cdot \Lambda_{t+1,t+s+1} Y_{t+s+1} (P_{t+s+1}/P_t)^{\epsilon-1}\frac{W_{t+s+1}/P_t}{A_{t+s+1}} \cdot \underbrace{\left(\frac{P_{t+1}}{P_{t+1}}\right)^{\epsilon-1} \cdot \frac{P_{t+1}}{P_{t+1}}}_{\left(\frac{P_{t+1}}{P_t}\right)^{\epsilon-1} \cdot \left(\frac{P_{t+1}}{P_t}\right)}$$

$$= (1+\mu) Y_t \cdot \frac{W_t/P_t}{A_t} + (1-\mu)\, \theta \cdot \Lambda_{t,t+1} \cdot \Pi^{\epsilon}_{t+1} \cdot F_{1,t+1} \quad //$$

$$\Pi^{\epsilon-1} \quad \Pi \quad = \Pi^{\epsilon}$$

(d) Show that (gross) inflation can implicitly be written as

$$1 = \theta \Pi_t^{\epsilon-1} + (1-\theta) p_t^{*1-\epsilon}$$

Gross Π: $1 = \theta \cdot \Pi_t^{\epsilon-1} + (1-\theta) p_t^{*\,1-\epsilon}$, $\quad \Pi_t = \frac{P_t}{P_{t-1}}$

$$P_t = \left(\theta \cdot P_{t-1}^{1-\epsilon} + (1-\theta) P_t^{*\,1-\epsilon}\right)^{\frac{1}{1-\epsilon}}$$

$$P_t^{1-\epsilon} = \theta P_{t-1}^{1-\epsilon} + (1-\theta) P_t^{*\,1-\epsilon}$$

$$1 = \theta \left(\frac{P_{t-1}}{P_t}\right)^{1-\epsilon} + (1-\theta)\left(\frac{P_t^*}{P_t}\right)^{1-\epsilon}$$

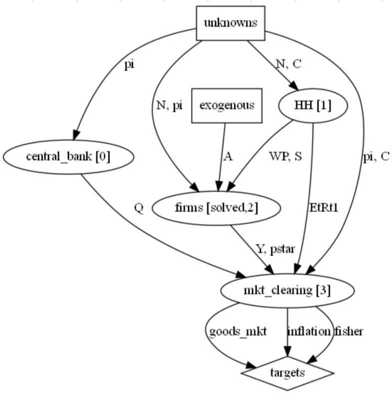
$$1 = \theta \cdot \Pi_t^{\epsilon-1} + (1-\theta) p_t^{*\,1-\epsilon}$$

(e) Explain intuitively how when $p_t^* > 1$, then $\Pi_t > 1$.

$p_t^* > 1 \Rightarrow P_t^* > P_t \Rightarrow$ firms that adjusted their price chose a higher reset price than the aggregate price level in that moment.

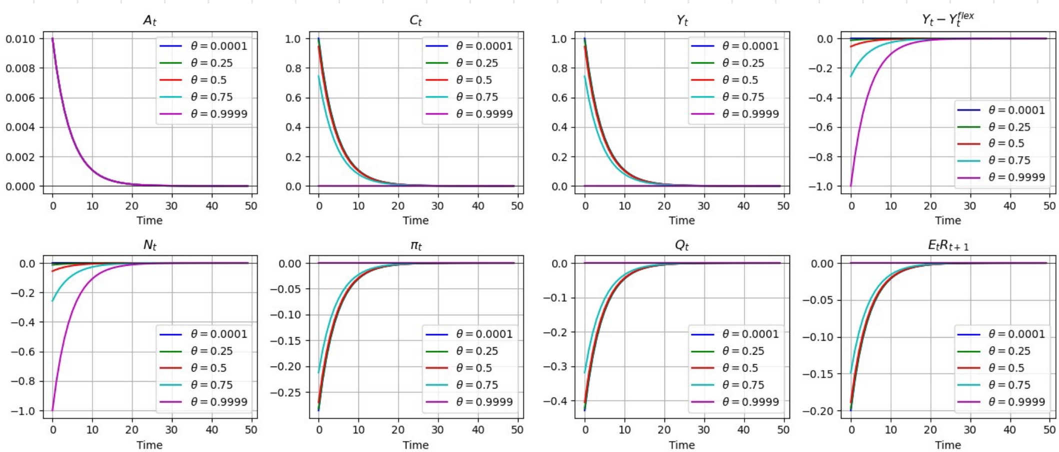
But this aggregate price level depends on the current / past price level, then: $P_t^* > P_t > P_{t-1} \Rightarrow P_t > P_{t-1} \Rightarrow \Pi_t = \frac{P_t}{P_{t-1}} > 1$ //

(f) Implement the non-linear NK using your recursive equations in Python using the Sequence Space Jacobian toolbox. For now, ignore the dispersion of labor in production and write the aggregate production function as $Y_t = A_t N_t$. Use the following parameters: $\beta = 0.99, \gamma = 1, \varphi = 1, \chi = 1, \epsilon = 10, \rho_a = 0.8, \phi_\pi = 1.5, \phi_y = 0$ where $A_t = (A_{t-1})^{\rho_a} e^{\epsilon_t^a}$. Productivity is the only shock. Price stickiness is specified below.



(g) Compute IRFs for $\theta \in \{0.0001, 0.25, 0.5, 0.75, 0.9999\}$ using a first order approximation to your non-linear equations.

Report the IRFs for consumption, the output gap, the level of output, employment, inflation, the mark-up, the nominal interest rate, and the ex-ante real interest rate. Your graph for each variable should contain all cases for θ, appropriately labelled.



(h) Intuitively explain how the impulse response functions depend on the value of θ.

Recall that θ measures de degree of price stickiness affecting the degree of non neutrality of the system of equations in equilibrium. For the sticky model decreases N_t since we don't have contemporaneous effect in production / consumption. Then, when we a degree of θ, a higher A_t will be less absorbed by R_{t+1}, and have less effect on C_t / Y_t.

(i) What would you expect to see from the same shock in an RBC model without capital? (No derivation should be necessary.)

$\theta \mapsto 0$, all firms adjust and we are back to the RBC model with flexible prices. So, a shock to A_1 will have 1:1 response in C_t, Y_t, R_{t+1}, and the output gap will be 0 ($Y_t = Y_t^{flex}$). Nominal variables don't respond.