Homework 3: Macroeconomics 210C

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1. Sticky Wage Model

Instead of assuming that prices are sticky for one period, we now assume that nominal wages are sticky for one period,

$$W_1 = W_0$$

The short-run equilibrium is

$$Y_1 = A_1 N_1 \quad \text{output}$$

$$W_1 = W_0 \quad \text{labor supply}$$

$$\frac{W_1}{P_1} = A_1 \quad \text{labor demand (MPL)}$$

$$Y_1 = C_1 \quad \text{market clearing for goods}$$

$$\frac{M_1}{P_1} = \zeta^{1/\nu}\left(1 - \frac{1}{Q_1}\right)^{-1/\nu} C_1^{\gamma/\nu} \quad \text{money demand}$$

$$1 = \beta E_1 \left\{ Q_1 \frac{P_1}{P_2} \frac{C_2^{-\gamma}}{C_1^{-\gamma}} \right\} \quad \text{euler equation}$$

$$\frac{W_t}{P_t} = \frac{\chi N_t^{\varphi}}{C_t^{-\gamma}}$$

The long-run equilibrium ($t \geqslant 2$) is

$$Y_t = A_t N_t \quad \text{output}$$

$$\frac{W_t}{P_t} = A_t \quad \text{labor demand (MPL)}$$

$$\frac{W_t}{P_t} = \frac{\chi N_t^{\varphi}}{C_t^{-\gamma}} \quad \text{labor supply}$$

$$Y_t = C_t \quad \text{market clearing for goods}$$

$$\frac{M_t}{P_t} = \zeta^{1/\nu}\left(1 - \frac{1}{Q_t}\right)^{-1/\nu} C_t^{\gamma/\nu} \quad \text{money demand}$$

$$1 = \beta E_t \left\{ Q_t \frac{P_t}{P_{t+1}} \frac{C_{t+1}^{-\gamma}}{C_t^{-\gamma}} \right\} \quad \text{euler equation}$$

(a) Are firms on their labor curve? Explain.

The sticky wage model is based on the idea that wages do not adjust inmediatly to changes in economic conditions. This stickiness can lead to periods where wages are either too high or too low relative to the equilibrium wage that would clear the labor market.

The labor demand curve represents the relationship between the real wage rate and the quantity of labor that firms are willing to hire. In the sticky wage model, firms are not necessarily on their labor demand curve. However, in this case, in both short and run equilibrium, firms are on their labor demand. $\frac{W_1}{P_1} = A_1$ and $\frac{W_t}{P_t} = A_t$, respectively.

(b) Are households on their labor supply curve? Explain.

The labor supply curve represents the relationship between the real wage rate and the quantity of labor that households are willing to supply. In the short run equilibrium we have sticky prices : $W_1 = W_0$, which replace the labor supply curve ($\frac{W_t}{P_t} = \frac{\chi N_t^{\varphi}}{C_t^{-\gamma}}$), so households are not in their supply curve. Households will supply all labor being demanded by firms regardless the real wage (inellastic at N_1). In the long run equilibriu, they are because the wages are not sticky.

(c) How does the labor market clear?

Firms demand labor by labor demand: $A_1 = \frac{W_1}{P_1}$, and market clear for goods: $Y_1 = C_1$

MPL ($\frac{W_1}{P_1} = A_1$) does not depend on N_1. Then, the choice of how many workers to hire at wage A_1 is to meet goods demand C_1 at fixed wage $W_0 = W_1 = A_1$: $N_1 = \frac{C_1(W_0)}{A_1}$. Then, households supply labor inelastically at W_0 to meet labor demand, where the equilibrium wage is A_1. //

(d) Solve for the long-run steady state.

Long-run: $t \geq 2$

$$
\begin{aligned}
&Y = A \cdot N && (1) && \Rightarrow \frac{Y}{N} = \frac{W}{P} = A \Rightarrow N = \frac{C}{A} \\
&\frac{W}{P} = A && (2) && \Rightarrow A = \chi \left(\frac{C}{A}\right)^{\varphi} \cdot C^{\gamma} \Rightarrow A^{1+\varphi}\frac{1}{\chi} = C^{\varphi+\gamma} \\
&\frac{W}{P} = \frac{\chi N^{\varphi}}{C^{-\gamma}} && (3) && \Rightarrow C = \left(\frac{1}{\chi} A^{1+\varphi}\right)^{\frac{1}{\varphi+\gamma}} \\
&Y = C && (4) && \\
&\frac{M}{P} = \varsigma^{1/\nu}\left(1-\frac{1}{Q}\right)^{-1/\nu} C^{\gamma/\nu} && (5) && \Rightarrow \frac{M}{P} = \varsigma^{1/\nu}(1-\beta)^{-1/\nu} Y^{\gamma/\nu} \\
&1 = \beta E\left\{Q \cdot \cancel{\frac{P}{P}} \cdot \cancel{\frac{C^{-\gamma}}{C^{-\gamma}}}\right\} && (6) && \Rightarrow \frac{1}{Q} = \beta
\end{aligned}
$$

Solution: $Y = C = \left(\frac{1}{\chi} A^{1+\varphi}\right)^{\frac{1}{\varphi+\gamma}}$ $\quad \frac{M}{P} = \varsigma^{1/\nu}(1-\beta)^{-1/\nu} Y^{\gamma/\nu}$ $\quad N = \frac{Y}{A}$

(e) Does the Classical Dichotomy hold in the long-run? Explain

Yes, Classical Dichotomy holds in the long run. Any change in M causes a proportional change in P, leaving Y, C, unchanged.

(f) Solve for output and the money market equilibrium in the short-run.

$$
\begin{aligned}
&Y_1 = A_1 \cdot N_1 && (1) && \Rightarrow \frac{Y_1}{N_1} = \frac{W_0}{P_1} = A_1 \Rightarrow P_1 = \frac{W_0}{A_1} \\
&\frac{W_1}{P_1} = A_1 && (2) && \\
&W_1 = W_0 && (3) && \Rightarrow \frac{M_1}{W_0} \cdot A_1 = \varsigma^{1/\nu}\left(1-\frac{1}{Q_1}\right)^{-1/\nu} Y_1^{\gamma/\nu} \\
&Y_1 = C_1 && (4) && \quad \frac{M_1}{W_0} = \frac{1}{A_1}\varsigma^{1/\nu}\left(1-\frac{1}{Q_1}\right)^{-1/\nu} Y_1^{\gamma/\nu} \\
&\frac{M_1}{P_1} = \varsigma^{1/\nu}\left(1-\frac{1}{Q_1}\right)^{-1/\nu} C_1^{\gamma/\nu} && (5) && \\
&1 = \beta E_1\left\{Q_1 \cdot \frac{P_1}{P} \cdot \frac{C^{-\gamma}}{C_1^{-\gamma}}\right\} && (6) && \Rightarrow Y_1 = C_1 = \left(\frac{1}{\beta \cdot Q_1} \cdot \frac{P}{P_1}\right)^{\frac{1}{\gamma}} C
\end{aligned}
$$

Solution. $Y_1 = C_1 = \left(\frac{A_1}{\beta \cdot Q_1} \cdot \frac{P}{W_0}\right)^{\frac{1}{\gamma}} C$ $\quad N_1 = \frac{Y_1}{A_1}$ $\quad \frac{M_1}{W_0} = \frac{1}{A_1}\varsigma^{1/\nu}\left(1-\frac{1}{Q_1}\right)^{-1/\nu} Y_1^{\gamma/\nu}$

(g) Does the Classical Dichotomy hold in the short-run?

In the short run, sticky prices and wages allow nominal changes, such as variations in the money supply, to influence real economic variables like output and employment (so neutrality of money does not hold). In this case, an increase in Money supply will lead to a reduction in the interest rate (Q_1), which will affect consumption (C_1), and therefore, it will affect output (Y_1), so here money neutrality does not hold since it affect the real economy. //

(h) Explain intuitively (in words) how an increase in the money supply affects output in the short-run.

① Higher M_t (money supply) reduces real interest rate: $R_{t+1} = Q_t \cdot \left(\frac{P_t}{P_{t+1}}\right)$

- Nominal rate $Q_t \downarrow$ to induce households to hold extra money supplied.
- Expected inflation is fixed because current prices P_t are sticky, which depends on W_0 and A_1. ($P_1 = \frac{W_0}{A_1}$)

If consume more today, I consume less in the future.

② Lower real interest rate increases demand for output through intertemporal subititution:

- Low return on savings (better consuming than saving), so increase today's consumption relative to future consumption.
- But future consumption is fixed by the supply side as price are flexible (in long-run)
- So, there is an overall increase in consumption today ($\uparrow C_t \Rightarrow \uparrow Y_t \Rightarrow \uparrow N_t$), which is accomodated by firms hiring more workers at the fix wage $W_0 = W_1$ and producing more output)

(i) How does productivity affect output? Explain intuitively

$(P_1 \cdot A_1 = W_0)$ $\quad Y_1 = \frac{W_0 \cdot N_1}{P_1} = A_1$

Consider: $W_0 = P_1 \cdot A_1 \Rightarrow$ if the productivity increase, then the fix wage will increase as well.

$\frac{Y_1}{N_1} = \frac{W_0}{P_1} = A_1 \Rightarrow$ Keeping N_1 constant, a positive shock in the productivity will increase the output.

Increase productivity:

1. Higher output due to more productive worker.
2. Fixed wages lead to higher real incomes through increased profits.
3. Increased consumption due to higher real incomes.
4. Market clears as higher consumption matches higher output.

Conversely, when the productivity decrease, we will expect lower output.

(j) Derive the labor wedge. Is it procyclical or countercyclical?

$$(1-\tau_t^N) \equiv \frac{MRS_t}{MPL_t} = \frac{\chi N_t^{\varphi} \cdot C_t^{\gamma}}{(1-\alpha) Y_t / N_t}$$

MPL_t: mg. product of labor $\quad \delta Y_t / \delta N_t$

MRS_t : mg. rate of substitution $\quad MU_{Ct} / MU_{Lt}$

Preferences : $\max\limits_{\{C_{t+s}, N_{t+s}, B_{t+s}, M_{t+s}\}} E_t \{ \sum_{s=0}^{\infty} \beta^s U(C_t, L_t) \}$

$$U(C_t, L_t) = \frac{C_{t+s}^{1-\gamma}}{1-\gamma} + \xi \frac{(M_{t+s}/P_{t+s})^{1-\nu}}{1-\nu} - \chi \frac{N_{t+s}^{1+\varphi}}{1+\varphi}$$

$MU_{Lt} = -\chi N_t^{\varphi}$

$MU_{Ct} = C_t^{-\gamma}$

$$\Rightarrow MRS_t = \frac{\chi N_t^{\varphi}}{C_t^{-\gamma}} = \chi N_t^{\varphi} C_t^{\gamma} = \chi N_t^{\varphi} A_t^{\gamma} N_t^{\gamma}$$

$$Y_t = A_t \cdot N_t = C_t \quad \Rightarrow \quad MPL_t = \frac{\delta Y_t}{\delta N_t} = A_t = \frac{Y_t}{N_t}$$

$$\Rightarrow (1-\tau_t^N) = \frac{\chi N_t^{\varphi} A_t^{\gamma} N_t^{\gamma}}{A_t} = \chi N_t^{\varphi+\gamma} A_t^{\gamma-1}$$

$$\Rightarrow (1-\tau_t^N) = \chi N_t^{\varphi+\gamma} A_t^{\gamma-1}$$

In short-run : $(1-\tau_1^N) = \chi N_1^{\varphi+\gamma} A_1^{\gamma-1}$, where $\tau_1^N \neq 0$

In long-run : $(1-\tau_t^N) = \chi N_t^{\varphi+\gamma} A_t^{\gamma-1} = 1 \quad \Rightarrow \tau_t^N = 0$

$\tau_t^N = 1 - \frac{MRS_t}{MPL_t} = 1 - \chi N_t^{\varphi+\gamma} A_t^{\gamma-1} \Rightarrow MPL_t > MRS_t$ in Recessions. Then τ_t^N is countercyclical.

Recessions : in a downturn, sticky wages mean that firms cannot reduce wages to match lower demand for their products. As a result, the MPL falls, but wages remain high, increasing the labor wedge.

Expansions : during an economic upturn, even though demand for labor increases, sticky wages may not rise quickly enough, which helps reduce the labor wedge as firms can hire more workers without a proportionate increase in wages.

(k) What moments of the data would you use to discriminate between the predictions of the sticky price and the sticky wage model?

I would use :

- Standard deviation of inflation
- Standard deviation of real wages
- Cyclicality of real wages
- St. dev. of labor
- Standard deviation of output
- St. dev. of consumption
- St. dev. real interest rate
- St. dev. labor productivity