Executive Summary

This report presents a comprehensive statistical analysis of the quality control process in a light bulb manufacturing plant. The study focuses on defect rates, product lifespan, and process efficiency using probability distributions and hypothesis testing. Key findings include defect probabilities, product reliability estimations, and recommendations for quality improvement.

Background and Core issue

Over View

The manufacturing Company produces 10.000 light bulbs daily, with regular quality inspections to ensure consistency. Defective bulbs must be identified and categorized to maintain high production Standards.

statistics that have been ⇒ gathered from plant quality Control department:

- The probability that a randomly chosen bulb is defective is 0.03
- A sample of 100 bulbs is randomly selected for inspection
- Historical data shows the defects follow specific probability distributions based on defect type

part 1: Discrete Random Variables and Distributions

Binomial Distribution

* question 1: probability that exactly 5 out of 100 inspected bulbs are defective

formula: $P(X=x) = \binom{n}{x} p^x (1-p)^{n-x}$ where: $n = 100$ (sample size)

$x = 5$ (defective bulbs)

$p = 0.03$ (probability of defect)

Calculation:

$$P(X=5) = \binom{100}{5} (0.03)^5 (0.97)^{95} \approx \underline{0.101}$$

⇒ The probability is 10.1% .

Implication: About 1 in 10 samples of 100 bulbs will have exactly 5 defects

*Question 2: probability of fewer than 3 defects

formula: Cumulative probability

$$P(X<n) = P(X=n-1)+P(X=n-2)+\dots+P(X=0)$$

formula for binomial distribution

$$P(X=x)=\binom{n}{x}p^{x}(1-p)^{n-x}$$

Calculation:

$$P(X<3) = P(X=0)+P(X=1)+P(X=2)$$

$$=\binom{100}{0}(0.03)^{0}(0.97)^{100}+\binom{100}{1}(0.03)^{1}(0.97)^{99}+\binom{100}{2}(0.03)^{2}(0.97)^{98}$$

$$\approx \underline{0.4196}$$

=> 41.96% probability

A high likelihood (≈ 42%) exists for fewer than 3 defects in a sample

Geometric Distribution

*Question 3: probability that the first defective bulb appears on 4th inspection

formula: $P(X=x)=(1-p)^{x-1}\times p$, where $x=4$

Calculation:

$$P(X=4)=(0.97)^{3}\times 0.03 \approx \underline{0.0273}$$

=> The probability is 2.73%

This means Early defects are rare; most defects appear in later testing

Hypergeometric distribution

*Question 4: probability of 3 defects in 10 random samples (from 100 bulbs, 10 defective)

formula: $P(X=x) = \dfrac{\binom{K}{x}\binom{N-K}{n-x}}{\binom{N}{n}}$ where $N = 100$ (total bulbs)
$K = 10$ (defective bulbs)
$n = 10$ (sample size)
$x = 3$ (defective in sample)

Calculation

$$P(X=3) = \frac{\binom{10}{3}\binom{90}{7}}{\binom{100}{10}} \approx \underline{\underline{0.0574}}$$

=> The probability is 5.74%

Implication: Small sample testing may miss defects due to low probability

Poisson Distribution

*Question 5: probability of observing exactly 5 defects in a hour (average rate $\lambda = 3$)

formula: $P(X=x) = \dfrac{e^{-\lambda}\lambda^{x}}{x!}$ where $x = 5$
$\lambda = 3$

Calculation:

$$P(X=5) = \frac{e^{-3} \times 3^{5}}{5!} \simeq \underline{\underline{0.1008}}$$

=> The probability is 10.08%

Conclusion: Defect clustering is expected occassionaly.

part 2: Continuous Random Variables and distributions

Normal distribution

* question 1: probability that a bulb lasts longer than 1,500 hours
  ($\mu = 1200$, $\sigma = 200$)

formula:

$$Z = \frac{X - \mu}{\sigma}$$

calculation:

$$Z = \frac{1500 - 1200}{200} = \underline{\underline{1.5}}$$

probality from Z-table: $P(X > 1500) = 1 - P(Z \leq 1.5) = 1 - 0.9332 = \underline{\underline{0.0668}}$

=> The probability is 6.68%

Implication: few bulbs exceed 1,500 hours, indicating room for lifespan improvement

* question 2: number of hours at which only 5% of bulbs have failed

z - value for 5%: $Z \approx -1.645$

formula: $X = \mu + Z \times \sigma$ where $\mu = 1200$, $\sigma = 200$, $Z = -1.645$

Calculation: $X = 1200 + (-1.645) \times 200 = 1200 - 329 = \underline{\underline{871}}$

Conclusion: 5% of bulbs fail before 871 hours.

* Question 3: Convert 1,500 hours to Z-score

Z-score: $Z = \frac{1500 - 1200}{200} = 1.5$

Interpretation: A bulb lasting 1,500 hours is 1.5 standard deviations above the mean

* Question 4: Normal approximation to binomial (probability of at least 10 defective bulbs)

- mean (μ): $n \times P = 100 \times 0.03 = \underline{3}$
- variance (σ^2): $n \times P \times (1-P) = \cancel{2.91}$ $100 \times 0.03 \times 0.97 = \underline{\underline{2.91}}$
- standard deviation (σ): $\sqrt{2.91} \approx 1.706$
- Continuity Correction: $P(X \geq 10) \approx P(X > 9.5)$

Z-score: $Z = \frac{9.5 - 3}{1.706} \approx 3.81$

=> probability: $P(Z > 3.81) \approx 0.00007$

Answer: the probability is 0.007 % (very low)

Exponential Distribution

* Question 5: probability the next defect occurs within 20 minutes ($\lambda = 1/30$ per minute)

formula: $P(X \leq x) = 1 - e^{-\lambda x}$, where $x = 20$, $\lambda = 1/30$

Calculation: $P(X \leq 20) = 1 - e^{-20/30} \approx 1 - 0.5134 = \underline{0.4866}$

=> The probability is 48.66%

Implication: Defects are likely within short intervals, suggesting frequent monitoring

part 3 ! Central limit theorem and Hypothesis testing

Centeral limit theorem

* Question 1 ; standard deviation of sample mean ($n=36$)

formula ! $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$

Calculation;

$$\sigma_{\bar{x}} = \frac{200}{\sqrt{36}} \approx 33.33$$

=> Answer 33.33 hours

Interpretation ! Larger samples reduce variability in mean estimates

* Question 2 ! probability sample mean life span < 1,150 hours

Z-score ! $Z = \frac{1150 - 1200}{33.33} \approx -1.5$

probability ; $P(Z<1.5) \approx 0.0668$, from Z-table

=> The probability is 6.68%
Consistent with part 2

Hypothesis testing

Question 3 ! Test if defect rate < 4% ($H_0 : P \geq 0.04$, $H_1 : P < 0.04$).

sample proportion ($\hat{p}$) ; $\frac{3}{100} = 0.03$

formula ! Test static ;
and
Calculation

$$Z = \frac{\hat{p} - P_0}{\sqrt{\frac{P_0(1-P_0)}{n}}} = \frac{0.03 - 0.04}{\sqrt{\frac{0.04 \times 0.96}{100}}} \approx -0.51$$

Cirtical Z-value (5% significance ! -1.645

=> Conclusion ! Since $-0.51 > -1.645$ fail to reject H_0.

=> No significant evidence to support defect rate < 4%

Question 4: Standard error for finite population ($N = 10{,}000$, $n = 100$)

Formula:

$$SE = \sqrt{\frac{N-n}{N-1}} \times \frac{\sigma}{\sqrt{n}}$$

Calculation:

$$SE = \sqrt{\frac{10000-100}{10000-1}} \times \frac{200}{\sqrt{100}}$$

$$= \sqrt{\frac{9900}{9999}} \times \frac{200}{10} \approx 19.9 \text{ hours}$$

=> Answer 19.9 hours

Implication: finite correction slightly reduces standard error

Part 4: Conclusions and Recommendations

Key findings

1. Defect rate: - 3% defect probability aligns with historical data
   - No statistical evidence supports reducing the defect rate below 4%
2. Product life span: - 6.68% of bulbs exceed 1,500 hours.
   - 5% fail before 871 hours, indicating early life failures
3. Process monitoring:
   - Defects cluster (Poisson: 10% chance of 5 defects/hour)
   - Exponential distribution suggests frequent defect occurrences.

Recommendations

- Increase Testing sample size for better defect detection
- Improve early life reliability
- Real-time Defect monitoring to reduce high defect intervals.