UNIT 3: KINEMATICS

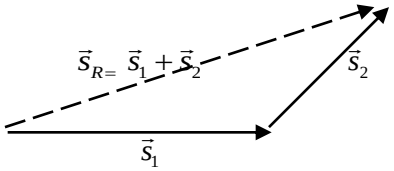
Kinematics:- is *a branch of mechanics* which *studies motion of bodies without dealing with the force that causes the motion.*

Frame of reference:- a rigid framework or coordinate system that can be used to define the position of a body in space.

Distance:- is the total length travelled by a body. It is a scalar quantity.

Displacement:- is the shortest path between the initial and final position of the body.

Note that distance travelled by a body is not zero, while its displacement may be zero.



The **average velocity of a particle is defined as the particle's displacement divided by the time interval during which that displacement occurs:**

$$\vec{V}_{av}=\frac{\Delta\vec{s}}{\Delta t}$$

The SI unit of average velocity is meters per second (m/s).

The **average speed** of a particle, a scalar quantity, is defined as **the total distance traveled divided by the total time interval required to travel that distance**:

$$\text{Average speed} = \frac{total\ dis\tan ace}{total\ time}$$

The SI unit of average speed is the same as the unit of average velocity: meters per second (m/s).

Example

1. Under which of the following conditions is the magnitude of the average velocity of a particle moving in one dimension smaller than the average speed over some time interval?
   A) A particle moves in the $+x$ direction without reversing.
   B) A particle moves in the $-x$ direction without reversing.
   C) A particle moves in the $+x$ direction and then reverses the direction of its motion.
   D) There are no conditions for which this is true.
2. A car moves 150km north in a time interval of 2hrs. What is the average velocity of the car?
   Ans. $\vec{V}_{av}$ =75km/hr north
3. A bus travels 48km due north in 3hrs. It then travels 20km due east in 1hr. What is the average velocity of the bus? **Ans.** $\vec{V}_{av}$ =13km/h at 22.6^0 E of N

Exercise

1. A car travels 120km due east in 3hrs. It then travels 5km south in 1hr. What is its average velocity?
   Ans. S_R=32.5kmhr at 23^0 S of E
2. Show that the average speed of the total motion of a body completing a distance S_1 moving with speed V_1, and a distance S_2 moving with speed V_2 is given by: $V_{av} = V_1V_2\dfrac{(S_1+S_2)}{S_1V_2+S_2V_1}$
3. A woman runs half way her destination at 10km/h and walks the rest of her way at 5km/h. What is her average speed for the entire trip? **Ans.** V_{av} = 6.67km/h
4. A car initially at rest and at the origin has moved a distance of 5, 10, 15, 20, 25, 30, 40, 50, 60, 70 meters during the 1, 2, 3, . . . 10 seconds periods respectively. What is the average speed during the first 5 seconds and the second half period from t = 6 sec to t =10 sec in m/sec respectively?
   A) 4, 10 B) 4, 9 C) 5, 9 D) 5, 10

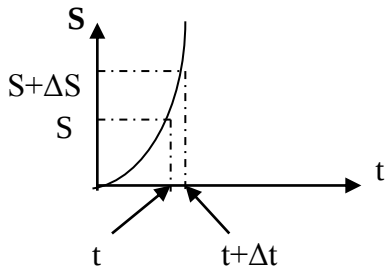
5. A car travels along a straight line at a constant speed of 20 m/s for a distance *d* and then another distance *d* in the same direction at a constant speed of 36 km/hr. What is the average speed for the entire trip?

A) 23 km/h B) 48 km/h C) 54 km/h D) 60 km/h

Instantaneous velocity

The instantaneous velocity of a particle at a given instant of time is given as

$$V_{inst} = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t}$$



Example

1. The position of an object as a function of time is given by $S(t) = 2t^2 + 5t$. Calculate the instantaneous velocity of the object at

a) t=1sec **Ans**. $V_{inst} = 9m/sec$, b) t= 2sec **Ans**. $V_{inst} = 13m/sec$, c) t = 3sec **Ans**. $V_{inst} = 17m/sec$

2. The displacement of a particle is given by $S = 2t^2 + t + 10$. What is

a) the average speed for the first 10 sec? **Ans**. $V_{av} = 21m/sec$

b) the instantaneous velocity at

i) t= 1sec? **Ans**. $V_{inst} = 5m/sec$, ii) t=2sec? **Ans**. $V_{inst} = 9m/sec$, *iii*) t = 3sec. **Ans**. $V_{inst} = 13m/sec$

Questions from national exams

1. The position of a particle moving along the x-axis is given by $x = 3t^2 - 5t$. What is the speed of the particle at t = 2sec?

A) 2m/sec B) 4m/sec C) 5m/sec D) 7m/sec

2. The position of a particle of mass 5kg moving along x-axis is given by $x = 2t^2 - 6t + 3$. What is the kinetic energy of the particle at t =2 sec?

A) 160 J B) 62.5 J C) 40 J D) 10 J

3. A force acts on a 3kg particle in such a way that the position of the particle as function of time is given by $x = 3t - 4t^2$. What is the work done by the force during the first 4seconds?

A) 1218 J B) 1228 J C) 1248 J D) 1258J

4. If a single constant force F acts on a particle of mass m and the particle starts from rest, what is the instantaneous power delivered by the force at time t?

A) $\frac{2F}{m}t$ B) $\frac{F^2}{2m}t$ C) $\frac{F^2}{m}t$ D) $\frac{3F^2}{m}t$

5. The coordinate of a particle in meters is given by $x(t) = 25t - 3t^3$. At what value t will the particle become momentarily at rest?

A) 2.78 sec B) 1.67 sec C) 0.6 sec D) 0.36 sec

Instantaneous acceleration

Instantaneous acceleration, $a_{inst,}$ at a given instant of time is given by:

$$\vec{a}_{inst} = \lim_{\Delta t \to 0} \frac{\Delta V}{\Delta t}$$

1.The velocity of a particle is given by $V= 3t^2 + 2t$. Find a_{inst} at

a) t=1sec . Ans. $a_{inst} = 8m/s^2$, c) t = 2sec. Ans. $a_{inst} = 14m/s^2$, e) t = 3sec. Ans. $a_{inst} = 20m/s^2$.

b) t= 1.5sec. Ans. $a_{inst} = 11m/s^2$, d) t = 2.5 sec. Ans. $a_{inst} = 17m/s^2$,

2. Calculate the instantaneous acceleration of a particle whose velocity is given by $V = – 3t^2 + 4t$: at

a) t = 1sec. Ans. $a_{inst} = – 2m/s^2$ c) t = 3sec. Ans. $a_{inst} = – 14m/s^2$

b) t= 2sec. Ans. $a_{inst} = – 8m/s^2$ d) t = 4sec. Ans. $a_{inst} = – 20m/s^2$.

Questions from national exams

1. The position of a particle moving along the x-axis is described by $x=3t^2 - 4t$. What is the acceleration of the particle?

A) $2m/s^2$ B) $4 m/s^2$ **C)** $6 m/s^2$ D) $8 m/s^2$

2. If the x and y coordinates of a particle at any time t is given by $x=7t+4t^2$ and y=5t,what is the acceleration of the particle at t=5s?

A) Zero **B)** $8 m/s^2$ C) $20 m/s^2$ D) $40 m/s^2$

3. The angular position of a swinging door is described by $\theta = 5+10t + 2t^2$. What is the angular speed and acceleration of the door at t=3s?

A) 22 rad/sec, 4 rad/s^2 B) –17rad/sec, $4rad/s^2$ C) 22 rad/sec, $12rad/s^2$ D) 12rad/sec, $3rad/s^2$

4. The x and y coordinates of a particle at any time t is given by $x= 9t + 5t^2$ and $y=5t + 8$.
The acceleration of the particle at t = 5sec is:

A) Zero **B)** $10m/s^2$ C) $14 m/s^2$ D) $20 m/s^2$

Average acceleration

The average acceleration of a particle whose velocity varies with time is given by:

$$\vec{a}_{av} = \frac{\vec{v} - \vec{u}}{t - t_0} \quad \text{or} \quad \vec{a}_{av} = \frac{\Delta \vec{v}}{\Delta t}$$

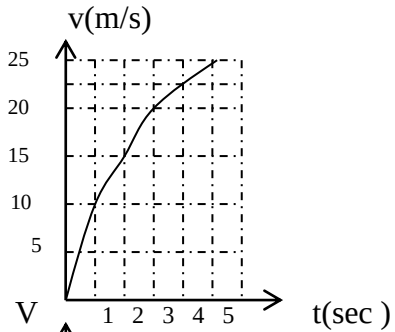
Example

1. The velocity-time graph of a particle moving in a straight line is shown in fig. below. What is a_{av} in the interval between:

a) 0 sec and 1 sec? **Ans**. $a_{av}=10m/s^2$

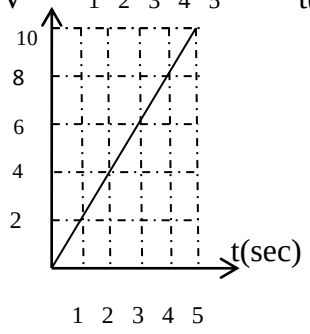
b) 1 sec and 3 sec? **Ans**. $a_{av}= 5m/s^2$

c) 3 sec and 5 sec? **Ans**. $a_{av}= 2.5m/s$



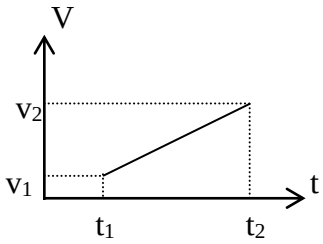
2. What is the average acceleration of an object shown in the fig. ?

Ans. $a_{av}= 2m/s^2$



3. The velocity-time history of motion of a particle is shown in fig. below. What is the average acceleration of the particle? (Given: $v_2 = 1.5v_1$ and $t_2 = 3t_1$)

A) $\frac{v_1}{2t_1}$ B) $\frac{v_1}{3t_1}$ C) $\frac{2v_1}{3t_1}$ D) $\frac{v_1}{4t_1}$



Uniformly accelerated rectilinear motion

Uniformly accelerated: the magnitude of acceleration is constant.

Rectilinear motion: straight line motion.

Equations of Uniformly accelerated motion

$A = \frac{v-u}{t}$, $V_{av} = \frac{u+v}{2}$, $S = (\frac{u+v}{2})t$, $S = ut + \frac{1}{2}at^2$, $S = \frac{v^2-u^2}{2a}$

Example

1. An object is moving with a uniform acceleration of $4m/s^2$ in a straight line. How far will it move as it changes its speed from 10m/sec to 30m/sec? **Ans**. S= 100m
2. A car moves with constant acceleration of $1.5m/s^2$ starting from rest. What is its displacement in 8sec? **Ans**. S=48m
3. A boy moving at constant acceleration of $2m/s^2$ comes to rest in 4sec. What is its
   a) initial speed? **Ans**. u=8m/sec, b) displacement? **Ans**. S=16m
4. A car accelerates uniformly from rest to 60m/sec in 30 sec. How far does it go during this time? **Ans**. S= 900m
5. A body accelerates uniformly from 12m/sec to 16m/sec while covering a distance of 70m. Calculate
   a) its acceleration. **Ans**. $a=0.8m/s^2$, b) the time taken. **Ans**. t=5 sec
6. Prove that the distance covered by a moving body during n^{th} sec is: $S_{nth} = u + a(t_n - 1/2)$
7. A moving train slows down uniformly from 40m/sec to 20m/sec in 5sec.What is the distance it covers during the fourth second? **Ans**. $S_{4th} = 26m$

Exercise

1. A runner, starting from rest, covers a 100m straight distance in 10sec. If his speed increases uniformly, what is his acceleration? **Ans**. $a=2m/s^2$
2. An object initially at rest, is uniformly accelerated to a speed of 5m/sec as it travels 10m. How much *further* would it travel before its speed becomes 10m/sec? **Ans**. S=30m
3. A car moving with constant acceleration covers the distance between two points 200m apart in 4sec. If the speed of the car at the first point is 20m/sec, what is its speed at the second point? **Ans**. V=80m/sec
4. A car initially at rest is uniformly accelerated along a straight road. The car passes two marks separated by 36m at 8sec and 10sec. What is the acceleration of the car? **Ans**. $a=2m/s^2$
5. A car initially at rest is accelerated uniformly to a speed of 20m/sec in 5sec. What is the distance covered by the car during the fifth second? **Ans**. $S_{5th}=18m$
6. A particle starts from rest and moves with constant acceleration of $2m/s^2$ for the first two seconds, and then moves for the next 4sec without accelerating. Finally it uniformly decelerates and comes to rest in 2sec. What is the total distance travelled by the particle? **Ans**. S=24m

7. A car starts from rest and travels for 4sec with a uniform acceleration of $2m/s^2$.The driver then applies brakes causing a uniform deceleration of $3m/s^2$ for 2sec. How far has the car moved during the braking period of 2sec?**Ans**. S=10m
8. A particle starts from rest and is uniformly accelerated at $1m/s^2$ until it attains a maximum speed. It continues at this maximum speed for 1min. The total distance travelled is 122m.
   a) When was the particle attained this maximum speed? **Ans**. t=2sec
   b) What is the maximum speed? **Ans**. V_{max}=2m/sec
9. An object with a constant acceleration covers the distance between two points which are 160m apart in 8sec. If the speed of the object as it passes the second point is 30m/sec. What is its
   a) speed at the first point ? **Ans**. u= 10m/sec, b) acceleration? **Ans**. a= $2.5m/s^2$
10. At what distance from the first point was the object in question 9 at rest? **Ans**. 20m
11. A car starts from rest at position x=0 and accelerates uniformly along a straight road in the +ve x-direction. At position x_1 it has velocity V_1. What will its velocity be at position $4x_1$? **Ans**. $2V_1$
12. At the instant traffic lights turn green, an automobile that has been waiting at the intersection starts ahead with a constant acceleration of $2m/s^2$. At the same instant a truck, travelling with a constant velocity of 10m/sec, overtakes and passes the automobile.
   a) How long does it take for the automobile to overtake the truck? **Ans**. t =10sec
   b) How far from its starting point does the automobile overtake the truck? **Ans**. S=100m
   c) How fast is the car travelling when it overtakes? **Ans**. V= 20m/sec
13. A train travelling along a straight rail changes its speed from 20m/sec to 10m/sec through a distance of 30m when brakes are applied uniformly. Assuming uniform deceleration, the distance it *further travel* until it comes to rest is:

   A) $\frac{1}{4}$ of the total distance B) $\frac{1}{3}$ of the total distance C) $\frac{1}{2}$ of the total distance D) $\frac{2}{3}$ of the total distance

Questions from national exam

1. A car travelling at a constant acceleration covers the distance between two points 180m apart in 6sec. If the speed of the car at the second point is 45m/sec, what is the acceleration of the car?
   A) $5m/s^2$ B) $10m/s^2$ C) $15m/s^2$ D) $20m/s^2$
2. An object moves in a straight line with a constant acceleration of $4\ m/s^2$. At a certain instant its velocity is 16m/sec. What is velocity of the object 2 sec later?
   A) 24 m/sec B) 28 m/sec C) 32 m/sec D) 36 m/sec
3. An object has a constant acceleration of $3.2m/s^2$. At a certain instant its velocity is 9.6m/sec. What is the ratio of its velocities at 2.5 sec later to 2.5 sec earlier?
   A) 111 **B)** 11 C) 11.1 D) 1.1
4. An athlete accelerates at $1.9m/s^2$ for 2.2 sec at the beginning of the race. What is speed of the athlete at the end of a 3000m race?
   A) 0.86 m/sec B) 1.16m/sec **C)** 4.18m/sec D) 9.20m/sec
5. A car with initial speed of 4m/sec starts accelerating in a straight line at $3.2m/s^2$ for 10 sec. If the brakes are then applied and it travels a *further 20m before stopping*. What is the acceleration in m/s^2?
   A) – 48.6 B) – 16.2 C) – 8.1 **D)** – 32.4
6. In a 100m race, Usain Bolt and Assafa Powel cross the finish line in a dead heat, both taking 10 sec. Accelerating uniformly, Usain Bolt took 3 sec and Assafa Powel took 2 sec to attain maximum speed, which they maintain for the rest of the race. What was the acceleration of Usain Bolt and Assafa Powel respectively?
   A) $4.32\ m/s^2$,$2.25\ m/s^2$ **B)** $3.92\ m/s^2$, $5.55\ m/s^2$ C) $4.92\ m/s^2$, $6.45\ m/s^2$ D) $4.92\ m/s^2$, $5.35\ m/s^2$

7. A car accelerates steadily from 0m/sec to 30m/sec in a distance **d** and time **t**. Another car takes a time **3t** to accelerate steadily from stationary to the same final velocity. What distance does the second car cover during the new acceleration?

A) 3*d* B) *d* C) *d*/3 D) *d*/9

8. A cart on an air track is moving at 0.5 m/sec when the air is suddenly turned off. The cart comes to rest after traveling 1m. The experiment is repeated, but now the cart is moving at 1m/sec when the air is turned off. How far does the cart travel before coming to rest?

A) 1m B) 2m C) 3m D) 4m

9. An object moving with uniform acceleration has a velocity of 12m/sec in the +ve x-direction when its x coordinate is 3cm. If its x coordinate 2 sec later is – 5cm, what is its acceleration?

A) $-12\ m/s^2$ B) $-13\ m/s^2$ C) $-16\ m/s^2$ D) $12\ m/s^2$

10. An object of mass **m** moving with initial speed of 15m/sec got an acceleration of 10 m/s^2 after a constant force **F** is applied to it. How long does it take for the object to reach 90m from initial position?

A) 3.22 sec B) 0.75 sec C) 3 sec D) 2.5 sec

11. A driver moving at a constant speed of 20m/sec sees an accident up ahead and hits the brakes. If the car decelerates at a constant rate of $-5\ m/s^2$, how far does the car go before it comes to a stop?

A) 120m B) 100m C) 40m D) 20m

12. A body moving with constant acceleration covers the distance between two points 60m apart in 5s. Its velocity as it passes the second point is 15m/s. What is the acceleration?

A) $3m/s^2$ B) $2.4m/s^2$ C) $1.8m/s^2$ D) $1.2m/s^2$

13. Cars A and B travel along a straight road as follows. Car A is located at position x = 5 km at clock reading t = 0 hr and maintains a constant speed of 30 km/hr in the + x- direction. Car B is located at x = 0 km at clock reading t = 0.5 hr (half hour after start of clock) and maintains a constant speed of 50 km/hr in the + x-direction. At what clock reading will car B overtake and pass car A?

A) 2.0 hr B) 1.5 hr C) 1.0 hr D) 0.5 hr

Freely falling bodies

Free fall is any motion of a body where gravity is the only force acting upon it.
The equations of uniformly accelerated motion in the previous section also hold true for freely falling bodies.

$v = u + gt$ $\qquad h = ut + \frac{1}{2}gt^2$

$h = (\frac{u+v}{2})t$ $\qquad h = \frac{v^2 - u^2}{2g}$

Example

1. A boy, standing on a bridge, drops a stone and hears a splash after 2sec.What is the height of the bridge?
Ans. h=20m.
2. A ball is released from a height of 80m. How long will it take to reach the ground? **Ans**. t = 4 sec
3. A ball is thrown down into a well at 5m/sec. How long will it take to reach *the water surface* 60m below?
Ans. t=3sec
4. An object is in a downward free fall. At one instant it is traveling at 30m/sec. What will be its speed after 2sec?
Ans. v=50m/sec

5. A ball is dropped from a balloon moving vertically upward with a speed of 20m/sec. If the balloon is 60m above the ground at the instant the ball is dropped, how long will it take the ball to reach the ground?
Ans. t = 6sec

Exercise

1. A ball is dropped from the top of a very tall building. One second later, another ball is thrown vertically downward with a speed of 20m/sec. How long will it take the second ball to overtake the first? **Ans**. t=0.5sec
2. If a ball travels three – fourth of its total path in the last second of its free fall from rest, the ball has been dropped from the height of _____m. **Ans**. h=20m
3. A ball is thrown vertically upward with an initial speed of 20m/sec from the top of a cliff which is 60m high.
   a) How long will it take the ball to strike the ground? **Ans**. t=6sec
   b) How high will it rise above the ground? **Ans**. 80m
   c) What will be the velocity of the ball when it strikes the ground? **Ans**. v= – 40m/sec

Questions from national exam

1. A ball is thrown vertically downward with an initial speed of 20m/sec from the height of 60m. What is the speed of the ball just before it strikes the ground?
   A) 20m/sec B) 30m/sec **C)** 40m/sec D) 50m/sec
2. A stone is thrown downward from the top of a tower with an initial speed of 15m/sec. If the stone hits the ground after 3sec, what is the height of the tower?
   A) 45m **B)** 90m C) 135m D) 6m
3. Hannah threw a rock straight upward from the top of a hill with a velocity 30m/sec. What is the time needed in seconds for the rock to return at the level of Hannah?($g=9.8m/s^2$)
   A) 6.1 B) 5.3 C) 4.2 D) 3.4
4. A boy releases a ball from the edge of a tall building. 2seconds later he throws a second ball with initial speed of 40m/sec downward. How many seconds after the second ball is thrown and at what point below the edge of the building will the second ball overtake the first?
   A) 1sec & 45m B) 1sec & 5m C) 3sec & 45m D) 3sec & 165m
5. A ball is dropped from a balloon moving vertically upward with a speed of 30m/sec. If the balloon is 80m above the ground at the instant the ball is dropped, how long does it take the ball to reach the ground?
   A) 4sec B) 6sec C**)** 8sec D) 10sec
6. A stone is dropped from the roof of a building and is found to reach the ground in 2.0sec. How high is the building?
   A) 40m **B)** 20m C) 10m D) 5m
7. A stone is thrown vertically upward. On its way up it passes point A with speed V and point B, 3m higher than A, with speed V/2. What is the speed V in m/sec and the maximum height reached by the stone above point B in meters respectively? **Ans**. 8.94m/sec, 1m
   A) 7.75, 0.5 B) 7.15, 0.75 C) 7.75, 0.75 D) 7.85, 1.15
8. A ball is thrown downward from the top of a tower. After being thrown, its downward:
   A) velocity will be uniform . C) acceleration will be greater than g.
   B) acceleration will be equal to g. D) acceleration will be smaller than g.

9. Which one of the following statements is correct?

A) An object moving toward the east cannot have an acceleration toward the west.

B) If the average velocity of an object is zero in some time interval, the average speed of the object for that time interval is also zero.

C) The velocity-time graph of an object moving with constant acceleration is parallel to the time axis.

D) An object having zero velocity can have acceleration different from zero.

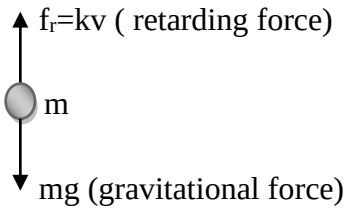
Terminal Velocity

You recall that air resistance (retarding force) is neglected in a free fall. When a solid object is falling from a very high altitude, air resistance opposes the motion. This retarding force increases proportionally with increase of velocity until the retarding force balances gravitational force, after which the velocity remains constant. *When this velocity is attained the object is* ***prevented from further increase of its velocity***. This velocity is known as *terminal velocity (*V_{ter}*).*

When the terminal velocity is attained

$kv = mg$

Where k is proportionality constant



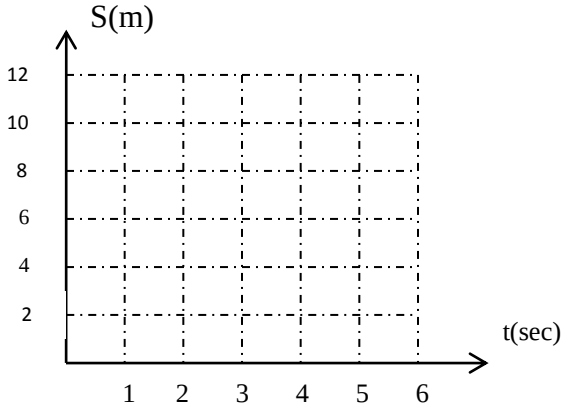
Example

Find V_{ter} of a marble , m=0.02kg and k=8Ns/m. **Ans** V_{ter}=0.025m/sec

Graphical representation of motion

Draw the **S**-t graph for the motion of an object whose data given below.

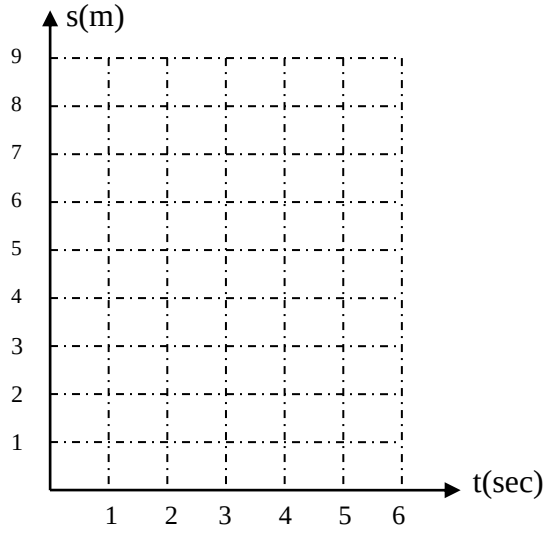
S(m)	0	2	4	6	8	10	12
t(sec)	0	1	2	3	4	5	6



For a uniform motion the **S**-t graph is straight line which indicates displacement is proportional to time.

1. a) Find the slope of **S**-t graph shown above. c) Calculate area under **V**-t graph.
   b) Draw **V**-t graph d) What do you conclude from the answer in c?

2. An object is accelerating at $0.5m/s^2$ starting from rest.
   Draw its **S**-t graph.



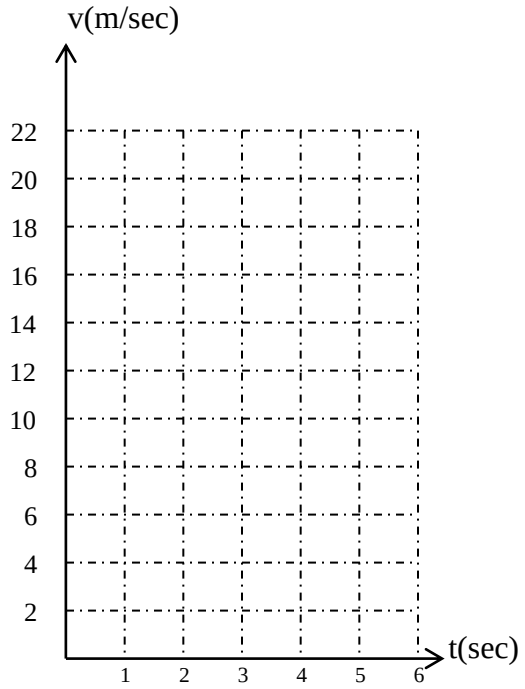
3. a) Draw the **V**-t graph for question 2 above.
   b) Calculate the slope
   c) What physical quantity does slope of **V**-t graph represent?
   d) Calculate area under **a**-t graph.
   e) What physical quantity does area under **a**-t graph represent

Example

1. Draw **V**-t graph and answer the following questions.

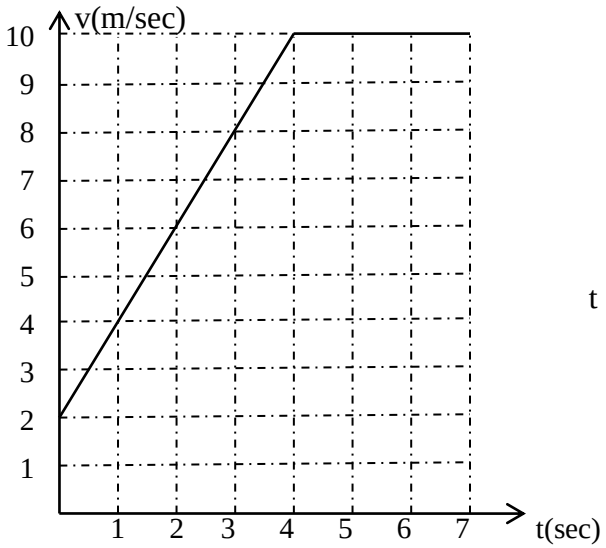
v(m/sec)	10	12	14	16	18	20	22
t(sec)	0	1	2	3	4	5	6

a) How fast is the particle moving at:
   t= 2.5sec? **Ans**. V_{inst}= 15m/sec
   t= 4.5sec? **Ans**. V_{inst}= 19m/sec
b) How far does it move between 2.5sec and 4.5sec?
   Ans. S= 34m
c) What is the total distance? **Ans**. S_{tot} = 96m
d) What is acceleration of the particle? **Ans**. $a=2m/s^2$



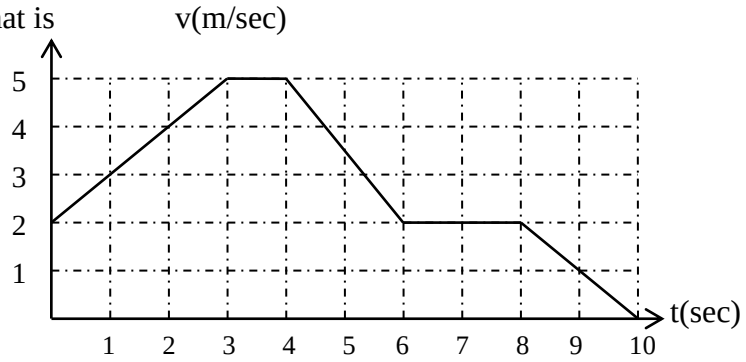
Exercise

The following **V**-t graph of the motion of a particle is for questions 1 and 2



1. Acceleration of the particle during the first 4sec and the next 3 sec are respectively. **Ans**. $2m/s^2$, 0
2. What is the total distance travelled by the particle? **Ans**. $S_{tot}=54m$

The following **V**-t graph is for questions 3 to 8. What is



3. the acceleration during the first 3sec? **Ans**. $a=1m/s^2$
4. the acceleration between 3sec and 4sec? **Ans**. a=0
5. the acceleration between 4sec and 6sec? **Ans**. $a= 1.5m/s^2$
6. the distance travelled in the interval 6sec to 10sec? **Ans**. 6m
7. the total distance travelled during 10sec? **Ans**. $S_{tot}=28.5m$
8. the speed of the object at t=5sec? **Ans**. $V_{inst}=3.5m/sec.$
9. The speed of a car decreases at a uniform rate from 30m/sec to 10m/sec in 4sec. Calculate the distance travelled during the first 2sec graphically. **Ans**. S=50m
10. What does the slope of a velocity-time graph represent?

A) Distance B) Acceleration C) Displacement D) Speed

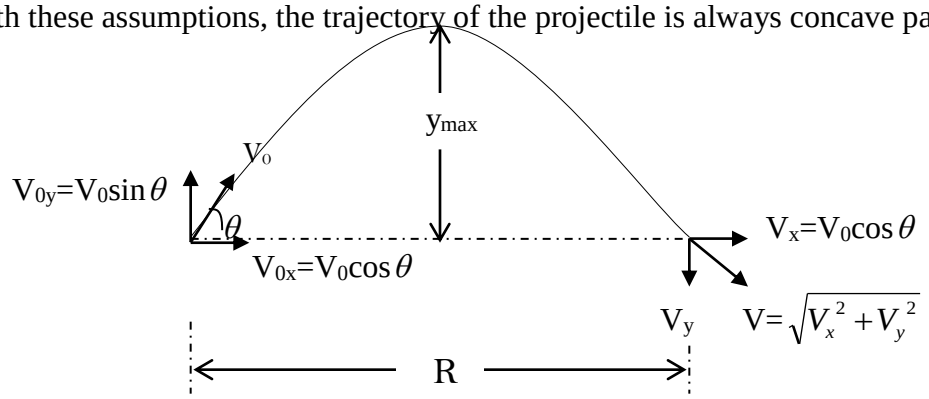
Projectile motion

In studying projectile motion, we assume that

i)the acceleration due to gravity, g, is constant and directed downward

ii) the effect of air resistance is negligible.

With these assumptions, the trajectory of the projectile is always concave parabola.



The horizontal component of the initial velocity remains constant

$V_{0x}=V_0\cos\theta$

The vertical component of the velocity is similar to that of vertical motion

$V_y=V_{0y}-gt$

$V_y=V_0\sin\theta - gt$. (The vertical component of velocity at time t)

Time taken by the projectile to reach maximum height

At the maximum height $V_y = 0$

Therefore $V_y= V_0\sin\theta - gt$ becomes

$0= V_0\sin\theta - gt$ or

$$t = \frac{V_0 \sin\theta}{g}$$

The magnitude of velocity is

$V=\sqrt{V_x^2+V_y^2}$

$V=\sqrt{(V_0\cos\theta)^2+(V_0\sin\theta-gt)^2}$

Prove that:

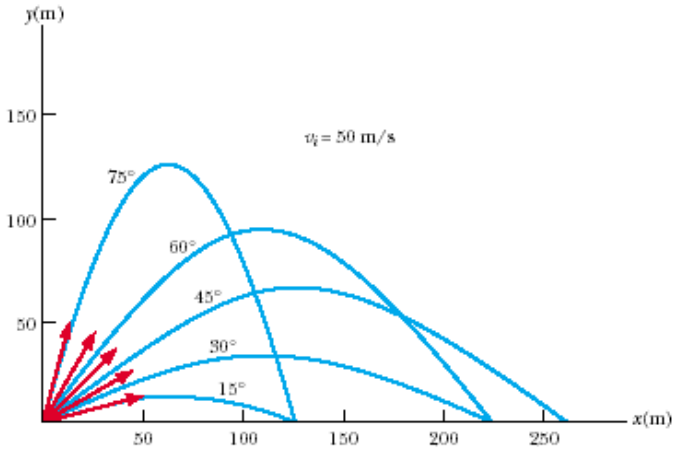
- the maximum height reached by a projectile is $y_{max}=\frac{V_0^2\sin^2\theta}{2g}$
- the total time of flight is $t_{tot}=\frac{2V_0\sin\theta}{g}$
- y is maximum for $\theta=90^0$

The range of the projectile is

$R=V_x.t_{tot}$

$= V_o\cos\theta\left(\frac{2V_0\sin\theta}{g}\right)$ $\Rightarrow R = \frac{V_0^2 2\sin\theta\cos\theta}{g}$ OR $R= \frac{V_0^2\sin 2\theta}{g}$

* A projectile launched from the origin with an initial speed of 50 m/s at various angles of projection. Note that complementary values of θ result in the same value of *R* (range of the projectile). Use g=10m/s^2



• For what value of θ will the range be maximum?
• Prove that the ranges for two complementary angles are equal.

The x and y components of the displacement at any instant of time are:

x= V_{0x}t
=($V_0\cos\theta$)t

OR $t = \frac{x}{V_0\cos\theta}$ . . . . . (1)

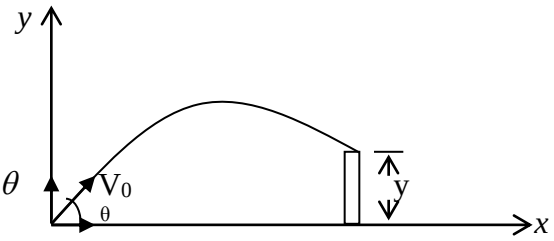
y=V_{0y}t – gt^2
y = ($V_0\sin\theta$)t – gt^2 . . . . . (2)

Substituting equation (1) into (2)we get

$$y = V_0\sin\theta\left(\frac{x}{V_0\cos\theta}\right) - \frac{gx^2}{2V_0^2\cos^2\theta}$$

OR

$$y = (\tan\theta)\,x - \frac{g}{2V_0^2\cos^2\theta}x^2$$



This equation is of a form y=a*x*-b*x*^{2} which is equation of a parabola concave down passing through the origin.

Example

1. A stone is thrown at an angle of 30^0 to the horizontal with a speed of 40m/sec. What is
   a) the time taken to reach maximum height? **Ans**. t= 2sec c) the range? **Ans**. R=80$\sqrt{3}$ m
   b) the maximum height reached? **Ans**. y_{max}=20m
2. A bullet is fired with an initial velocity of 600m/sec at an angle of 30^0 above the horizontal. Calculate
   a) total time of flight. **Ans**. t_{tot}=60sec, b) maximum height. **Ans**. y_{max}=4500m. c) range. **Ans**. R=18000$\sqrt{3}$ m
3. A ball is projected at an angle of 37^0 above the ground with a velocity of 80m/sec. What is
   a) total time of flight? **Ans**. t_{tot}=9.6sec c) the range? **Ans**. R=614.4m
   b) the maximum height? **Ans**. y_{max}=115.2m d) the magnitude of velocity at the end of 2sec? **Ans**. V=70m/sec.
4. A ball is projected at an angle of 37^0 above the horizontal and landed on the edge of a building that is 20m away. The top edge is 10m above the throw point. With what speed is the ball projected? **Ans**. V_0=25m/sec

Exercise

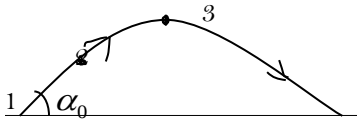
1. A ball is thrown with an initial velocity of 20m/sec at an angle of30^0 with the horizontal.
   a) How high will the ball rise? **Ans**. y_{max}=5m,
   b) How long will it take the ball before its speed is reduced to nine-tenth of the initial speed? **Ans**. t= 0.5sec
2. A ball is kicked with an initial velocity of 40m/sec at an angle of 37^0 above the ground. What is the height of the ball at the end of 2sec? **Ans**. h=28m
3. If you can throw a ball vertically up to a maximum height of 10m, to what maximum range can you throw along horizontal assuming the same initial speed? **Ans**. R_{max}=20m
4. The angle of departure of a particle is 30^0. How many times higher is the maximum height if the angle of departure is 60^0 for the same projectile? **Ans**. $h_2=3h_1$
5. If the initial velocity of a projectile were tripled, how would its maximum range be affected? **Ans**. $R'_{max}=9R_{max}$
6. A projectile is fired and covers a maximum range of R_{max}. Express y_{max} in terms of R_{max} for the same projection. **Ans**. $y_{max}=\frac{R_{max}}{4}$
7. If V_0=100m/sec and $\theta=60^0$, find $\vec{V}$ after
   a) 2sec. **Ans**. $\vec{V}=(50i+67j)$m/sec, b) 8sec **Ans**. $\vec{V}=(50i+7j)$m/sec, c) 12sec. **Ans**. $\vec{V}=(50i-33j)$m/sec
8. For what angle of projection will the range and maximum height be equal? **Ans**. $\theta=\tan^{-1}(4)$
9. A football player kicked a ball with a speed of 40m/sec. What should be the angle of projection for the ball to land on the foot of the second player at a distance of 80m from the first player? **Ans**. θ=15
10. A ball, thrown straight up, returns to the point of throw after 2sec. With what speed must a second ball be thrown at an angle of 30^0 with the horizontal to reach the same height as the first ball? **Ans**. V_0=20m/sec
11. What maximum height is reached by a ball thrown with a velocity $\vec{v}=(12i+16j)$m/sec? **Ans**. h_{max}=12.8m
12. If a boy throws a ball straight up into air, while running at
    a) a *constant velocity*, the ball will fall: A) behind him **B)** right at him C) in front of him
    b) a *constant acceleration* , the ball will fall: **A)** behind him B) right at him C) in front of him
    c) a *constant deceleration* , the ball will fall: A)behind him B) right at him **C)** in front of him

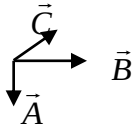
Questions from national exam

1. If a projectile is fired with an initial speed of 20m/sec at an angle of 30^0 above the horizontal, then what is the maximum height reached by the projectile?
   A) 8m B) 12m **C)** 5m D) 10m
2. A projectile is fired with an initial speed of 30m/sec at an angle of 30^0 to the horizontal. What are the maximum height (h_{max}) the projectile attains and the horizontal distance (D) the projectile falls? (g = 9.8m/s^2)
   A) h_{max}=11.5m, D=79.5m B) h_{max}=17.5m, D=75.5 m C) h_{max}=13.5m. D72.5m D) h_{max}=9.5m, D=83.5m
3. A ball is thrown with a speed of 20m/sec at an angle of 37^0 above the horizontal directly toward a pole. If the pole is 32m from the release point of the ball, how far above the release point does the ball hit the pole?
   A) 2m B**)** 4m C) 3m D) 5m
4. A footballer kicks a ball at an angle of 37^0 from the horizontal with an initial speed of 25m/sec. How long will the ball be in the air and how far down the field the ball land respectively ? (g = 9.8m/s^2)
   A) 5.35sec, 55.3m B) 3.24sec, 67.8m **C)** 3.07sec, 61.3m D) 4.62sec, 70.5m

5. A bullet is fired from a cannon at point 1 and follows the trajectory shown in the figure bellow. Four possible vectors are also shown in the figure; the letter $\vec{D}$ represents the zero vector. Which vector best represents the bullet's acceleration at point 2?

A) $\vec{A}$ C) $\vec{C}$
B) $\vec{B}$ D) $\vec{D}$





6. A projectile is fired at 60^0 with the horizontal. At the highest point of its path, the vertical component of its velocity is momentary zero. What is the magnitude of its acceleration at this point?

A) $20 m/s^2$ **B)** $10 m/s^2$ C) $5 m/s^2$ D) $0 m/s^2$

7. Two projectiles are fired from ground level at equal speed but different angles. One is fired at an angle of 30^0 and the other at 60^0. The projectile to hit the ground first will be the one fired at (neglect air resistance)

A) 60^0 C) both hit at the same time.
B) 30^0 D) cannot be determined from the given information.

8. A placekicker must kick a football from a point which is at a distance of 36.0m from the goal. When kicked, the ball leaves the ground with a speed of 20.0m/s at an angle of 53^0 to the horizontal. If the ball hits the crossbar of the goal at a height *h* and bounces back what will be the height of the crossbar?

A) 2.45 m B) 2.85 m **C)** 3.00 m D) 3.15 m

9. If a long distance jumper athlete leaves the ground at an angle of 37.0^0 above the horizontal surface at a speed of 10.0 m/s, how far does he jump in the horizontal direction?

A) 4.8 m B) 6 m **C)** 9.6 m D) 12 m

Projectile with a horizontal velocity

When we throw a body in air with horizontal initial velocity (V_0) from a height h, there are two motions at the same time.

•The horizontal motion:-is uniform because there is no horizontal resisting force.

$R = V_0 t$

•The vertical motion:- is uniformly accelerated motion because gravity acts on it.

$h = V_{0y}t + \frac{1}{2}gt^2$

$h = \frac{1}{2}gt^2$, because $V_{0y}=0$

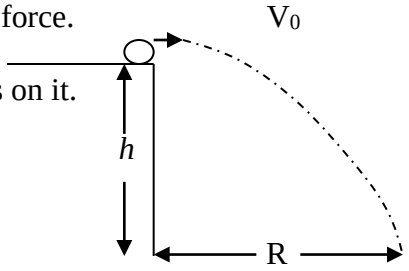
$V_y = V_{0y} + gt$

$V_y = gt$

The magnitude of velocity after time t is

$V = \sqrt{V_x^2 + V_y^2}$

$V = \sqrt{V_0^2 + (gt)^2}$



Example

1. A stone is thrown horizontally from a height of 5m with a velocity of 8m/sec.
   a) How long will it take to strike the ground? **Ans**. t=1sec b) What is its range? **Ans**. R=8m
   c) Determine the magnitude of the velocity at the instant it strikes the ground. **Ans**. $V = 2\sqrt{41}\,m$
2. An airplane flying at a speed of 400m/sec drops a box at a height of 4500m above the ground. What is the magnitude of the velocity at the instant it strikes the ground? **Ans**. V= 500m/sec

Exercise

1. A projectile is fired horizontally from the height of 45m above the ground. How long will it take to reach the ground? **Ans**. t=3sec
2. A ball is thrown horizontally from the top of a building with a speed of 12m/sec. What is the speed of the ball after 1/2sec? **Ans**. V= 13m/sec
3. A projectile is fired horizontally from a height of 80m. What is the vertical component of the velocity as it strikes the ground? **Ans**. V_v= 40m/sec
4. A ball, thrown horizontally with a speed of 16m/sec from the top of a building, strikes the ground at a distance of 48m from the base of the building. What is the height of the building? **Ans**. h=45m
5. Point C is 3000m east of A. Point B is 2000m above the ground directly above A. An airplane, flying at a velocity of 150m/sec horizontally due east, released a package while passing through point B. Will the package be dropped at point C? **Ans**.
6. A rifle is held horizontally at a height of 1.25m above the ground and the bullet is fired with 400m/sec. What is the horizontal distance travelled by the bullet before it hits the ground? **Ans**. R= 200m
7. A 0.2kg ball is thrown horizontally from the top of a building with a speed of 15m/sec. What is the kinetic energy of the ball after 2sec? **Ans**. K.E= 62.5J
8. A shooter aims his gun and shoots directly at a target placed 400m horizontally away from him. By how much will he miss his target if the muzzle velocity of the bullet is 400m/sec? **Ans**. 5m
9. A ball is thrown horizontally with a speed of 8m/sec from the top of a building. The ball strikes the ground 24m from the base of the building. What is the height of the building?

   A) 15m B) 30m C) 45m D) 60m

10. A ball rolls off the edge of a horizontal table top 4m high. It strikes the floor at a point 5m horizontally away from the edge of the table. How long was the ball in air in seconds?

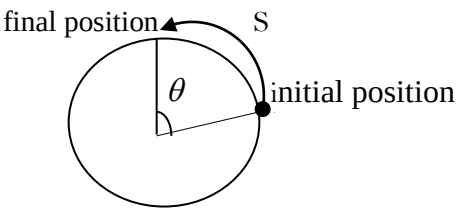
    A) $\frac{3}{\sqrt{5}}$ B) $\frac{1}{\sqrt{5}}$ C) $\frac{2}{\sqrt{3}}$ D) $\frac{2}{\sqrt{5}}$

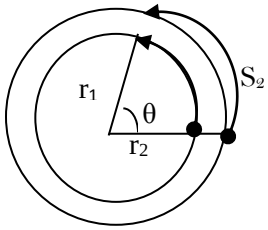
Uniform Circular Motion

Motion of an object in a circular path of radius *r* with uniform speed V is known as *uniform circular motion.*
In a uniform circular motion, the speed of the object remains constant while the direction changes constantly.

Tangential and angular displacement

- Tangential displacement(S):- is the arc length that a circulating object covers during a given time.
- Angular displacement (θ):- is the angle through which a body is rotated between initial and final positions during a given time.

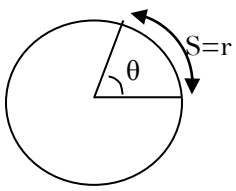




$S_2 > S_1$ but θ is the same for both

One radian (1rad) is the angle subtended at the center of a circle by an arc length equal to radius of the circle.

1rad= $\frac{s}{r}$, when S=r



Tangential and angular velocity

Tangential velocity (V):- is the rate of change of its tangential displacement. $V=\frac{S}{t}$

Angular velocity (ω):- is the rate of change of angular displacement. $\omega=\frac{\theta}{t}$, its unit is *rad/sec*

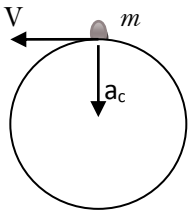
• *Show that* $V=\omega r$

Acceleration in a uniform circular motion

Acceleration in a uniform circular motion is due to continuous change in the direction of the velocity.

$a_c=\frac{V^2}{r}$

Show that: $a_c=\omega^2 r$ Or $a_c=\frac{4\pi^2}{T^2}r$



Example

1. A ball moves in a horizontal circle of radius 0.4m at a constant speed of 4m/sec. What is
   a) a_c ? b) ω? c) f? d) T? **Ans**. a) a_c= 40m/s^2, b) ω=10 rad/sec, c) f=5/π Hz, d) T=π/5 se
2. An object is moving at a speed of 10m/sec along a circular path of radius 0.1m. What is
   a) a_c? b) ω? c) f? d) T? **Ans**. a) a_c= 1000m/s^2, b) ω=100 rad/sec, c) f=50/ π Hz, d) T= π/50sec

Exercise

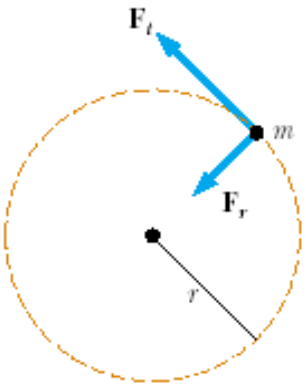
1. The angular speed of a particle is 8rad/sec. If it is moving along a horizontal circle of radius 0.5m, what is
   a) a_c ? **Ans**. a_c= 32m/s^2, b) V? **Ans**. V= 4m/sec, c) f? **Ans**. f= 4/ πHz, d) T? **Ans**. T= π/4sec
2. A particle moves in a circular path of radius 2m with radial acceleration of 18m/s^2. What is the angular speed of the particle ? **Ans**. ω=3rad/sec
3. A girl swings a ball in a horizontal circle 1.25m above the ground by a means of string 1.5m long. If the string suddenly breaks and the ball strikes the ground after travelling a horizontal distance of 6m, what was the centripetal acceleration of the ball while it was in circular motion? **Ans**. a_c=96m/s^2
4. If a car moves around a circular road of radius r at a constant speed
   A) its velocity changes and the acceleration is v^2/r. C) there is no velocity change since the speed is constant.
   B) there is no force on the car since its speed is constant. D) the force per unit mass on the car is towards the center and is v^2/r
5. A body is rotated with a uniform speed in a circle of radius 1m. What is the centripetal acceleration if it complete 12 rotations in 3sec?
   A) 36 π^{2}m/s^2 B) 64 π^{2}m/s^2 C) 72 π^{2}m/s^2 D) 144 π^{2}m/s^2
6. Uniform circular motion is a
   A) motion with constant velocity. C) motion with acceleration of constant magnitude.
   B) motion in one dimension. D) motion in which net force is constant.

Centripetal Force

The force acting on a body moving around a circular path is known as *centripetal force*. It acts along the radius of the circle pointing towards its center.

$F_c = F_r = \frac{mV^2}{r}$ or

$F_c = F_r = m\omega^2 r$



Example

1. What centripetal force that is necessary to keep a 2kg mass in a horizontal circle of radius 2m with a speed of 10m/sec. **Ans**. F_c=100N
2. What radial force acts on a 1kg particle that moves in a circular path of radius 1.5m with angular speed of 4rad/sec? **Ans**. F_c=24N
3. A 0.5kg ball is tied to a string 50cm long and rotated in a horizontal circle. If it takes 0.1 πsec for one complete revolution, what is the centripetal force acting on the ball? **Ans**. F_c=100N
4. A 0.5kg body is moving along a circular path of radius 2m with speed 4m/sec. What is the magnitude of the force acting on the body?

A) 2N B) 4N C) 6N D) 8N

Uniformly accelerated circular motion

As in a straight line motion, objects around a circular path can also be uniformly accelerated.

- The angular acceleration is

$\alpha = \frac{\omega - \omega_0}{t}$

- The tangential acceleration is

$a_t = \frac{v-u}{t} = \frac{\omega r - \omega_0 r}{t}$

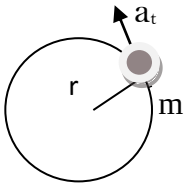
$= \frac{(\omega - \omega_0)}{t} r$

$a_t = \alpha t$

The resultant acceleration is

$a = \sqrt{a_t^2 + a_c^2}$

$a = \sqrt{(\alpha r)^2 + (\omega^2 r)^2}$ OR $a = r\sqrt{\alpha^2 + \omega^4}$



The analogy between uniformly accelerated circular motion and rectilinear motion:

Uniformly accelerated rectilinear motion	Uniformly accelerated circular motion
$V = u + at$	$\omega = \omega_0 + \alpha t$
$S = ut + \frac{1}{2} at^2$	$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$
$V^2 = u^2 + 2aS$	$\omega^2 = \omega_0^2 + 2\alpha\theta$
$V_{av} = \frac{(u+v)}{2}$	$\omega_{av} = \frac{(\omega_0+\omega)}{2}$
$S = \frac{(u+v)}{2}t$	$\theta = \frac{(\omega_0+\omega)}{2}t$

Example

1. A wheel, starting from rest, is rotating at 200rad/sec in 200sec. What is
   a) its angular acceleration? **Ans**. $\alpha = 1 rad/s^2$, b) the angle through which it turns? **Ans**. θ = 20000 rad
2. The wheel of bicycle rotates with constant angular acceleration of $2rad/s^2$. If it attains an angular speed of 100rad/sec in 30sec, what is its initial angular speed? **Ans**. $\omega_0 = 40 rad/sec$
3. A disk, initially at rest, starts to rotate at an angular acceleration of $5rad/s^2$. What is its angular speed at t=4sec? **Ans**. $\omega = 20 rad/sec$

Exercise

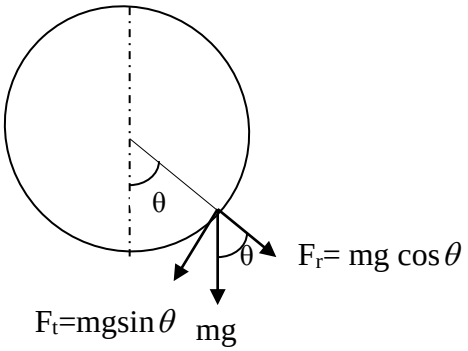
1. A wheel of an engine is uniformly rotating at an angular speed of 24rad/sec. If the engine is turned off, what is the angle through which the wheel rotate before coming to rest in 3sec? **Ans**. $\theta = 36 rad$
2. The wheel of a cycle is accelerated uniformly from rest to 10rad/sec in 2sec. What is the angular speed of the wheel at the end of 3sec? **Ans**. $\omega = 15 rad/sec$
3. A disk, initially at rest, rotates with constant acceleration through an angle of 22.5rad in 3sec. What is the angular speed at 4sec? **Ans**. $\omega = 20 rad/sec$
4. A wheel, initially at rest, is uniformly accelerated to a speed of 100rad/sec in 20sec. If the radius of the wheel is 0.3m, what is the tangential acceleration of a point at the rim of the wheel? ? **Ans**. $a_t = 1.5 m/s^2$
5. A disc rotating at 180rev/min slows down uniformly and comes to rest in 4sec. How many revolutions does it make before it stops? ? **Ans**. # of rev = 6
6. A car, moving at a speed of 17m/sec on a straight road, speeds up with an acceleration of $2m/s^2$ for 5 sec. If radius of the tyre is 0.5m, what is the approximate number of revolution of the wheel during this time? **Ans**. # of rev = 35
7. a) What is the tangential speed of a point on the earth at the equator? ($R_E = 6.4 \times 10^6 m$) **Ans**. v=465m/sec
   b) Suppose the earth starts to increase its tangential speed at a rate of $1.1m/s^2$, how long will it take before an object at the equator to be weightless? ($g = 10m/s^2$) **Ans**. t = 6850sec=1.9hr
8. A wheel starts from rest and rotates with constant angular acceleration to reach an angular speed of 12rad/sec in 3sec. What is the magnitude of the angular acceleration in rad/s^2 and the angle in rad through which it rotates in this time interval respectively?

   A) 4, 18 B) 3, 18 C) 18, 5 D) 18, 4

9. Which one of the following statements is true about the motion of a particle in a circular path?
   A) The centripetal acceleration is constant if the particle's speed is constant.
   B) The tangential acceleration can be perpendicular to the velocity vector of the particle.
   C) The centripetal acceleration is always in the direction perpendicular to the velocity vector of the particle.
   D) The acceleration is always perpendicular to the velocity vector of the particle.

Motion in a vertical circle

When a mass *m* attached to a cord of length *R* is whirled in a vertical circle, the motion is not uniform. The speed increases on the way down and decreases on the way up.



- The tangential force is

  F_t = mg sin θ

- The tangential acceleration is

  $a_t = \frac{F_t}{m} = \frac{mg\sin\theta}{m}$ ⟶ a_t = g sin θ

➢ The centripetal force is $F_c = \frac{mv^2}{R}$

There are two forces on the mass along the radial direction

- tension is toward the center of the circle and
- mgcos θ outward

❖ The resultant force which is equal to F_c is

T— mgcos θ = F_c OR | $T = \frac{mv_t^2}{R} + mg\cos\theta$ OR $T = m(\frac{v_t^2}{R} + g\cos\theta)$

T= F_c+mgcos θ

$a_c = \frac{F_c}{m}$ Or $a_c = \frac{T - mg\cos\theta}{m}$

- At the lowest point of the path, $\theta = 0^0$, $\sin0^0=0$, $\cos0^0=1$, $a_t = g\sin0^0 = 0$, $F_r = mg\cos0^0 = mg$

$T = \frac{mv_t^2}{R} + mg$

•• When the string is horizontal, $\theta = 90^0$, $\sin90^0=1$, $\cos90^0=0$, $a_t = g\sin90^0 = g$, $F_r = mg\cos90^0 = 0$

$T = \frac{mv_t^2}{R} + mg\cos90^0$ ⟹ $T = \frac{mv_t^2}{R}$

••• At the highest point, $\theta = 180^0$, $\sin180^0=0$, $\cos180^0=-1$, $a_t = g\sin180^0 = 0$, $F_r = mg\cos180^0 = -mg$

$T = \frac{mv_t^2}{R} - mg$

With motion of this sort, when the speed at the highest point is less than some minimum value V_{min}, the cord becomes slack (not taut) and the path is no longer circular. To find this minimum speed, set T=0 and $V_{min} = \sqrt{Rg}$

Example

1. Calculate the tangential force and the radial component of weight at
   a) $\theta = 0^0$ **Ans**. F_t=0, F_r =mg
   b) $\theta = 30^0$ **Ans**. F_t=0.5mg, F_r=0.866mg,
   c) $\theta = 60^0$, **Ans**. F_t=0.866mg, F_r= 0.5mg
   d) $\theta = 90^0$ **Ans**. F_t=mg, F_r=0
   e) $\theta = 180^0$, **Ans**. F_t=0,F_r=−mg

2. A 0.2kg mass swings in a vertical circle of radius 1m. If its tangential speed is $\sqrt{12}$ m/sec when the cord makes an angle of 30^0 with the vertical. What is
   a) the tangential and centripetal acceleration at this instant? **Ans**. a_t = 5m/sec, a_c=12m/sec
   b) the magnitude of the resultant acceleration? **Ans**. a=13m/s^2
   c) the tension in the string? . T=4.132N

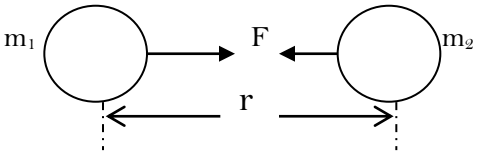
Exercise

1. A 0.1kg mass swings in a vertical circle of radius 1m. If its speed $V_t=\sqrt{5}$ m/sec. When the cord makes an angle of 60^0 with the vertical, calculate
   a) a_t and a_c at this instant. **Ans**. $a_t=5\sqrt{3}\ m/s^2$, $a_c=5m/s^2$
   b) the magnitude of the resultant acceleration. **Ans**. $a=10m/s^2$
   c) the tension in the cord. **Ans**. T= 1N
2. A ball attached to a 1m long string is rotated in a vertical circle. If the tangential velocity of the ball is 2m/sec when the string makes an angle of 18^0 with the vertical, what is the magnitude of the resultant acceleration? ($\sin 18^0=0.3$) **Ans**. $a=5m/s^2$
3. A girl is swinging a bucket attached to the end of a string 0.9m long in a vertical circle. What is the minimum speed needed at the top of the circle so that the bucket stays in a vertical circle? **Ans**. $V_{min}=3m/sec$
4. A woman is rotating a bucket of water in a vertical circle of radius 0.9 m. The mass of bucket and water is 5 kg. What is the bucket's minimum speed at the top of the circle if no water is to spill out?
   A) 0 B) 1 m/s C**)** 3 m/s D) 9 m/s

Newton's Law Of Universal Gravitation

The mutual force of attraction between two masses m_1 and m_2 is directly proportional to the product of the masses and inversely proportional to the square of the separation between their centers.

$$F_G=\frac{Gm_1m_2}{r^2}$$



Where $G=6.67\times10^{-11}Nm^2/kg^2$ universal gravitational constant

Example

1. Two men, standing 0.5m apart, have masses 70kg and 80kg. Calculate the force of attraction between them. **Ans**. $F=1.494\times10^{-6}N$
2. The gravitational force between two objects is $4\times10^{-9}N$ when they are 10m apart. For what separation would the force between the two objects be $10^{-7}N$? **Ans**. r = 2m

Exercise

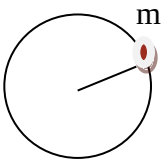
1. The gravitational force between two objects is $9\times10^{-9}N$ when they are 8m apart. What would be the force between the two objects when they are 12m apart? **Ans**. $F=4\times10^{-9}N$
2. Two objects of equal mass, whose centers are 10cm apart, exert a force of $6.67\times10^{-9}N$ on each other. What are the masses? **Ans**. $m_1=m_2=1kg$
3. The gravitational force exerted on a particle by spherical body of mass *m* and radius *r* is F. What force would be exerted on the same particle by another spherical body of mass *M* and radius *R*?
   A) $\frac{mr^2F}{MR^2}$ B**)** $\frac{Mr^2F}{mR^2}$ C) $\frac{mrF}{MR}$ D) $\frac{MrF}{mR}$
4. The gravitational force between two objects is $9\times10^{-9}N$ when they are 10m apart. For what separation would the force between the objects be equal to $4\times10^{-9}N$?
   A**)** 15m B) 20m C) 25m D) 30m
5. The planet Jupiter is about 318 times as massive as Earth, yet on its surface you would weigh only 2.4 times your weight on Earth. Why is this happened?
   A) Your mass is 132.5 times less on Jupiter.
   B) You are 132.5 times more weightless there.
   C) Jupiter is significantly farther from the sun.
   D**)** Jupiter's radius is about 11.5 times the Earth's radius.

Gravitational field strength

The gravitational field strength

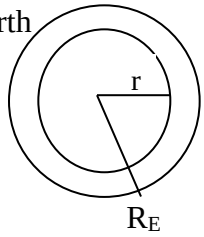
- on the surface of the earth is

$g = \frac{GM_E}{R_E^2}$ towards the center of the earth



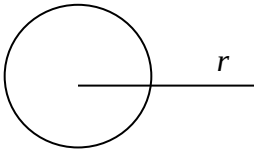
- at a distance $r < R_E$ is

$g = \frac{GM_E}{R_E^3} r$

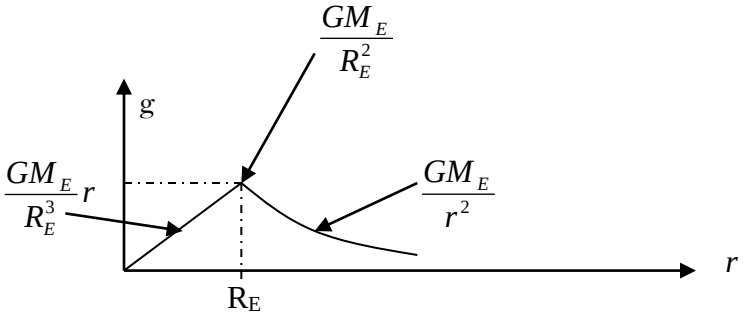


- at a distance $r > R_E$

$g = \frac{GM_E}{r^2}$



The graph of gravitational field strength against distance from the center of the earth



Example

$M_E = 5.97\times10^{24}$kg, $R_E = 6.37\times10^{6}$m.

1. Calculate g

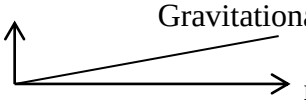
a)on the surface of the earth. **Ans**. g = 9.8m/s^2, b) at r =1/2R_E. **Ans**. g = 4.9m/s^2

c) at a height of 6000km above the surface of the earth. **Ans**. g=2.6 m/s^2

2. Calculate mass of neutron star of radius 8000km and g =10^5m/s^2. **Ans**. $M_s = 9.6\times10^{28}$kg

Questions from national exams

1. The graph represents the relationship between the gravitational force and mass for objects near the surface of the earth. The slope of the graph represents:



A**)** acceleration due to gravity. C) momentum of object

B) universal gravitational constant. D) weight of object.

2. A newly discovered planet has twice the mass and three times the radius of the earth. What is the free-fall acceleration at this surface in terms of the free-fall acceleration g at the surface of the earth?

A. $\frac{4}{3}g$ B) $\frac{3}{4}g$ C**)** $\frac{2}{9}g$ D) $\frac{2}{3}g$

3. An object weighs 80N on earth's surface. When it is moved to a point one earth radius above the earth's surface, it will weigh

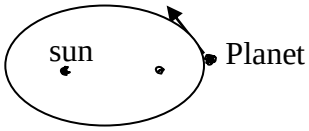
A**)** 20N B) 40N C) 80N D) 320N

4. At what altitude above the earth's surface would the acceleration due to gravity be g/3?

A) $\sqrt{3}\,R_E$ B**)** $(\sqrt{3}-1)\,R_E$ C) $3R_E$ D) $2R_E$

Kepler's Laws

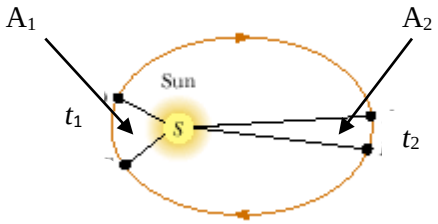
1. Kepler's First Law (*the law of orbits*):- all planets move in elliptical orbits with the sun at one focus. The orbits are very nearly circular for most of the planets.



2. Kepler's Second Law (*the law of areas)*:- a line joining any planet to the sun sweeps out equal areas in equal time intervals.

$t_1 = t_2$

$A_1 = A_2$



3. **Kepler's Third Law (*the law of periods*):-** the square of the period of any planet is proportional to the cube of the planet's mean distance from the sun. Kepler's third law follows from Newton's second law and the law of universal gravitation applied to a planet in a circular orbit.

$$m_p a_p = \frac{G m_s m_p}{r^2} \Longrightarrow \quad a_p = \frac{G m_s}{r^2} \;,\; \omega^2 r = \frac{G m_s}{r^2}$$

$$T^2 = \frac{4\pi^2}{G m_s} r^3$$

Find the period of an earth satellite in a circular orbit at an altitude of 230km. **Ans**. T=5336 sec

Escape Velocity

The minimum value of the initial speed u =V_{esc} such that the object will escape the earth's gravitational field is

$$V_{esc} = \sqrt{\frac{2GM_E}{R_E}} = \sqrt{2\left(\frac{GM_E}{R_E^2}\right)R_E}$$

$$= \sqrt{2gR_E}$$

Note that this equation can be applied to objects projected vertically from any planet of mass **M** and radius **R** .

$$V_{esc} = \sqrt{\frac{2GM}{R}} = \sqrt{2gR}$$

Satellites, orbital velocity and period

Anybody that rotates in orbit round other bodies of greater mass under the influence of mutual gravitation is known as satellite. The velocity of a satellite with which it revolves the earth just above its surface is known as *critical orbital velocity*. The critical orbital velocity can be obtained by equating the centripetal force on the satellite to the gravitational force of the earth exerts on the satellite in the orbit.

$F_c = F_G$

$$\frac{m_s V_c^2}{R_E} = \frac{GM_E m_s}{R_E^2} \Longrightarrow \quad V_c = \sqrt{\frac{GM_E}{R_E}}$$

and the period T is: $T = 2\pi\sqrt{\frac{r^3}{GM_E}}$

Example

1. Calculate the critical orbital velocity of a satellite about the earth. **Ans**. V_c=7906m/sec
2. Calculate the distance of a geosynchronous satellite from the center of the earth. **Ans**. r = 42,241,200m

Exercise

1. What is the height of a geosynchronous satellite above the earth's surface? **Ans**. h=36000km
2. What is the period of revolution of a satellite at a height of 1100km above the surface of the earth.

Ans. T= 6425 sec

Relative velocity

The rate at which body A changes its position with respect to a second body B is known as *the* ***relative velocity of A with respect to B*** **(V_{AB})**.

I) The relative velocities of two bodies moving independently

i) **when they are moving in the same direction:-**

• the relative velocity of A with respect to B is

$$\vec{V}_{AB} = \vec{V}_{Ag} - \vec{V}_{Bg}$$

B ⟶ $\vec{V}_{Bg}$ A ⟶ $\vec{V}_{Ag}$

• the relative velocity of B with respect to A is

$$\vec{V}_{BA} = \vec{V}_{Bg} - \vec{V}_{Ag}$$

ii) ***when they are moving in opposite directions:-***

• the relative velocity of B with respect to A is

$V_{BA}=V_{AB}=V_{Ag}+V_{Bg}$ $\vec{V}_{Ag}$ ⟵ A B ⟶ $\vec{V}_{Bg}$ OR A ⟶ $\vec{V}_{Ag}$ $\vec{V}_{Bg}$ ⟵ B

Example

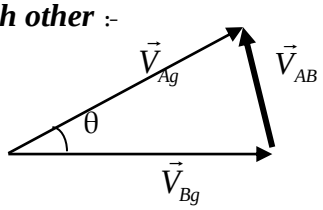
1. Car A, travelling with respect to ground at 60km/h on a straight level road, is ahead of car B travelling in the same direction at 80km/h with respect to ground. What is velocity of
   a) car B relative to A? **Ans**. V_{BA}=20km/h,
   b) car A relative to B? **Ans**. V_{AB}= – 20km/h. Car A appears to travel backward
2. Two cars, A and B, are moving with velocities of 25km/h and 30km/h, respectively. Find the velocity of car B relative to A if they are moving :
   a) in the same direction. **Ans**. V_{BA}=5km/h, b) in opposite direction. **Ans**. V_{BA}=55km/h
3. Cars A and B leave the same point at the same time with velocities of 75km/h east and 80km/h west respectively. How long will it take them to be 620km apart? **Ans**. t = 4hrs

iii) ***when they are moving at an angle*** θ ***to each other*** :-

• the velocity of A relative to B

$$\vec{V}_{AB} = \vec{V}_{Ag} - \vec{V}_{Bg}$$

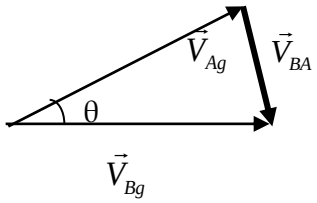
$$\left|\vec{V}_{AB}\right| = \sqrt{V_A^2 + V_B^2 - 2V_A V_B \cos\theta}$$



• the velocity of B relative to A is

$$\vec{V}_{BA} = \vec{V}_{Bg} - \vec{V}_{AG}$$

$$\left|\vec{V}_{BA}\right| = \sqrt{V_A^2 + V_B^2 - 2V_A V_B \cos\theta}$$



Example

1. Two aircrafts, A and B, are flying in still air with velocities of $\vec{V}_A$ =300km/h *north* and $\vec{V}_B$ =300km/h *east*. Calculate the velocity of

a) aircraft A relative to B. **Ans**. $\vec{V}_{AB} = 300\sqrt{2}$ km/h at 45^0 N of W.

b) aircraft B relative to A. **Ans**. $\vec{V}_{BA} = 300\sqrt{2}$ km/h at 45^0S of E

2. Boat A is moving at 6km/h east. Another boat B is moving at 10km/h 53^0 north of east. Calculate the velocity of

a) boat B relative to A. **Ans**. $\vec{V}_{BA}$ =8km/h ***north***. b) boat A relative to B. **Ans**. $\vec{V}_{AB}$ =8km/h ***south***

3. Car A is moving at 50km/h 30^0 west of north. Car B is moving at 30km/h east. Find the velocity of

a) car A with respect to B. **Ans**. $\vec{V}_{AB}$ =70km/h at θ =$\sin^{-1}\left(\frac{5\sqrt{3}}{14}\right)$ N of W

b) car B with respect to A. **Ans**. $\vec{V}_{BA}$ =70km/h at θ =$\sin^{-1}\left(\frac{5\sqrt{3}}{14}\right)$ S of E

Exercise

1. Aircraft A is flying at 500km/h 30^0 N of W. Another aircraft B is flying at 500km/h north. What is the relative velocity of

a)aircraft B relative to A? **Ans**. $\vec{V}_{BA}$ =500km/h 60^0 east of north.

b) aircraft A relative to B? **Ans**. $\vec{V}_{AB}$ =500km/h 60^0 west of south

2. Cars A and B are travelling at 30km/h west and 40km/h north respectively. What is velocity of

a) car A as seen by driver of car B? **Ans**. $\vec{V}_{AB}$ =50km/h 53^0 S of W

b) car B as seen by driver of car A? **Ans**. $\vec{V}_{BA}$ =50km/h 53^0 N of E

3. Boat A is traveling due east at 10km/h.

a) What must be the speed of the second boat B heading 30^0 N of E if it is always due north of boat A?

Ans. $V_{Bg}=\frac{20}{\sqrt{3}}$ km/h.

b) What is the relative velocity of boat B with respect to A? **Ans**. $\vec{V}_{BA} = \frac{10}{\sqrt{3}}$ km/h North.

5. A bird is flying toward north with a velocity of 40km/h and a train is moving with a velocity of 40km/h toward east. What is the velocity the bird noted by a man in the train?

A) $40\sqrt{2}$ km/h north east B) $40\sqrt{2}$ km/h south east C) $40\sqrt{2}$ km/h North West D) $40\sqrt{2}$ km/h south west

6. A small car and a bus move with velocities given by $\mathbf{V}_c = (3i - 2j)$ m/sec and $\mathbf{V}_b = (-5i - 4j)$ m/sec Respectively according to an observer on the ground. What is the velocity of the bus according to a car driver?

A) $(-2i + 6j)$ *m*/sec B) $(-2i - 6j)$ *m/sec* C) $(-8i + 2j)$ *m/sec* D) $(-8i - 2j)$ *m/sec*

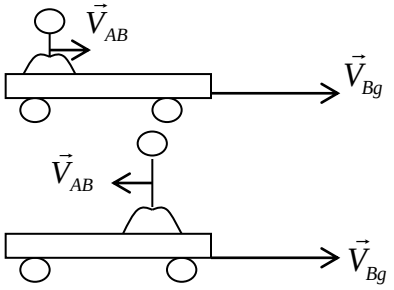
II) When a body is subjected to two velocities

i) If they are moving in the same direction

$\vec{V}_{Ag} = \vec{V}_{AB} + \vec{V}_{Bg}$

ii) If they are moving in opposite directions

$\vec{V}_{Ag} = \vec{V}_{AB} - \vec{V}_{Bg}$



Example

1. An airplane is flying at 480km/h east. What is the relative velocity of the plane with respect to the ground when a wind is blowing at

   a) 40km/h west? **Ans**. $\vec{V}_{pg}$ =440km/h east. b) 40km/h east? b) **Ans**. $\vec{V}_{pg}$ =520km/h east.

2. A boat travels at 8 km/h relative to water in a stream and the speed of the stream is 3 km/h. What is the relative velocity of the boat with respect to the ground when the boat travels:

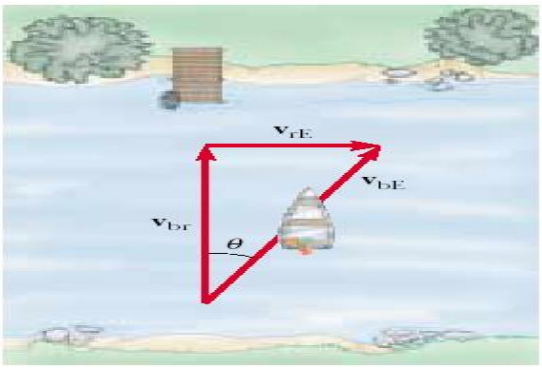
   a) upstream? **Ans**. $\vec{V}_{bg}$ =5 km/h upstream. b) downstream? **Ans**. $\vec{V}_{bg}$ =11 km/h downstream.

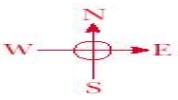
3. A boat is heading due north crosses a river with a velocity of 15km/h relative to the water. The river flows with a constant velocity of 8 km/h due East. Determine the velocity of the boat relative to a person standing on the bank of the river. **Ans**. V_{be} = 17 km/h in the direction $\theta=28^0$ East of North

4. A boat heading due north crosses a wide river with a speed of 8km/h relative to the water.

   The water in the river has a uniform speed of 6km/h due east relative to the Earth. Determine the velocity of the boat relative to an observer standing on either bank.

   Ans. The boat is moving at a speed of 10km/h in the direction 37^0 E of N relative to Earth.

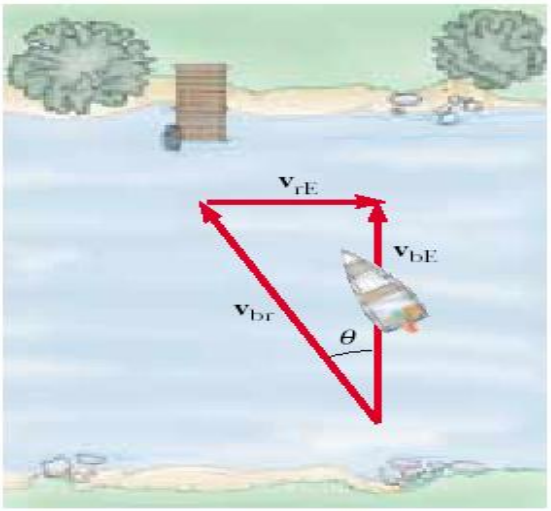


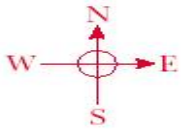


5. If the boat of the preceding example travels with the same speed of 8km/h relative to the river and is to travel due north, as shown in figure ,what should its heading be?

   Ans. $V_{bE} = 2\sqrt{7}km/h$

   = 5.3km/h at 48.5^0 W of N





4. Diredawa is approximately 300km east of Addis Ababa. A plane leaves Addis Ababa flying due east at 500km/h while the wind is blowing to the south at 100km/h. What is velocity of the plane with respect to ground?

   A) 510km/h north of east **B)** 510km/h south of east C) 490km/h north of west D) 490km/h south of west

5. A boat is traveling at 15km/h with respect to the water of a river in the upstream direction. The water itself is flowing at 6km/h with respect to the ground. A child on the boat walks from the rear seat to front seat at 5km/h with respect to the boat. What are the velocities of the boat and the child in km/h with respect to the ground respectively?

   A) 6, 12 B) 5, 9 C) 21, 12 D**)** 9, 14