UNIT 4: DYNAMICS

Dynamics:- is a branch of mechanics concerned with the motion of bodies under the action of forces.

The four fundamental forces are:

1. Gravitational forces:- *between objects* which act over a long distance; ***attractive forces***.
2. Electromagnetic forces:- *between electric charges* which act on a charged particles; ***attractive*** and ***repulsive***
3. Strong nuclear forces:- *between subatomic particles* to form nuclei; ***bind protons and neutrons together***
4. Weak nuclear forces:-that arise in certain radioactive decay processes and responsible for some nuclear reactions; ***act between elementary particles***.

In classical physics, we are concerned only with gravitational and electromagnetic forces.

Newton's Laws Of Motion

1. **First law (the law of inertia)** states:-*An object at rest will remain at rest* and *an object in motion will continue its motion in a straight line with constant velocity unless it experiences a net external force*.

Inertia:-is the property of a body to resist (oppose) any change in its state of *rest* or *motion*.

The *resistance* (*inertia*) increases with the increase of mass and decreases with the decrease of mass.

This implies that **mass is the measure of inertia**.

2. **Second Law**:-The *acceleration* of an object is *directly proportional to the net force acting on it* and *inversely proportional to its mass*.

$$a=\frac{F_{net}}{m} \qquad \text{OR} \qquad F_{net}=ma$$

Note that if $F_{net}=0$, $a=0$, which corresponds to the equilibrium situation, Newton's first law, where V is constant.

3. **Third law**:- When one body exerts a force on another, the second body exerts on the first a force which is equal in magnitude, opposite in direction and in the same line of action.

Action and reaction are always equal and opposite. For every action, there is equal and opposite reaction.

Note:- The forces of which the third law states are always applied on two different bodies.

Questions

1. Passengers standing in a bus fall or tend to fall

   a) backward when the bus suddenly starts moving, b) forward when the bus suddenly stops. Explain the reason.
2. Why should safety belts be used when driving a car?
3. Can winnowing, to separate grains from husk, be explained in terms of the law of inertia? Explain.
4. What is the proper way to tighten a loosen head on a hammer?
5. "If a horse pulls a cart, the cart will equally pull the horse back." These forces are equal and opposite but we see the cart moving after the horse. What is the reason?

   A) There is fallacy in the quoted statement.

   B) Newton's third law is violated.

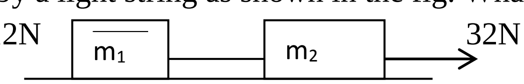
   C) The backward pull of the cart should be less in magnitude.

   D) Motion may start because both stated forces are not acting on the same object.

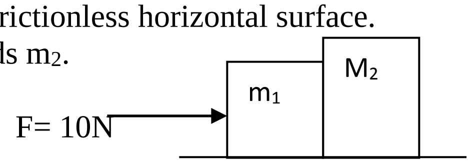
6. A monkey, armful of pieces of stone, is sliding on a horizontal frictionless table. She is dangerously getting to the edge of the table. If she understands Newton's laws of motion, what should she do to save herself from falling?
   A) She should press downward on the table to produce friction.
   B) She should put the pieces of stone on the table and sit on them.
   C) She should throw the pieces of stone in the forward direction with the appropriate amount.
   D) She should jump upwards and leave the table.
7. Two forces, $\vec{F}_1 = (2i+3j - 4k)$ N and $\vec{F}_2 = (4i - j+8k)$ N are exerted on a 2 kg mass. What is acceleration of the mass? **Ans.** $\vec{a} = (3i+j+2k)$ m/s^2.
8. A force applied on m_1 accelerates it at a_1=2.5 m/s^2. The same force applied on m_2 accelerates it at a_2=5 m/s^2. What is the ratio of m_1 to m_2? **Ans.** m_1:m_2=2:1

<u>Exercise</u>

1. A force on m_1 accelerates it at 6 m/s^2. The same force accelerates m_2 at 2 m/s^2.
   a) What is the mass m_2 in terms of m_1? **Ans.** $m_2=3m_1$.
   b) What would be the acceleration of the mass $m=m_1+m_2$ under the action of the same force? **Ans.** a=1.5m/s^2.
2. A force $\vec{F} = (20i+21j)$ N acts on a 29 kg mass. What is
   a) acceleration of the mass? **Ans.** $\vec{a} = \frac{(20i+21j)}{29}$ m/s^2. b) the magnitude of the acceleration? **Ans.** a=1 m/s^2.
3. Three forces, $\vec{F}_1 = i + 2j - k$, $\vec{F}_2 = 2i + 4j + 3k$ and $\vec{F}_3 = i - 3j - 2k$ act on the center of a body of mass 4 kg. Calculate the
   a) resultant force. **Ans.** $\vec{F}_{res} = 4i + 3j$. b) acceleration of the mass. **Ans.** $\vec{a} = (i + 0.75j)$ m/s^2
4. If a net force acting on a 10 kg object has components F_x=15 N and F_y=20 N, what is
   a) acceleration of the object? **Ans.** $\vec{a} = (1.5i+2j)$ m/s^2. b) magnitude of acceleration? **Ans.** 2.5 m/s^2.
5. A certain force gives a mass m_1 an acceleration of 8m/s^2, and a mass m_2 an acceleration of 24 m/s^2. What acceleration would it give the two when they are fastened together? **Ans.** a = 6 m/s^2
6. A 3 kg block, initially at rest, starts to move under the action of a horizontal force of 24 N. What is velocity of the object at the end 2 sec? **Ans.** V= 16 m/sec.
7. Two blocks of masses m_1 = 3 kg and m_2=5 kg are connected by a light string as shown in the fig. What would be the tension in the string connecting the masses? **Ans.** T=12N



8. Two blocks of messes m_1=2kg and m_2= 3kg are kept side by side on a frictionless horizontal surface. A horizontal force of magnitude 10N is exerted on m_1 pushing it towards m_2. What is the force exerted by
   a) m_2 on m_1 ? **Ans.** $F_{1,2}$ = 6N b) m_1 on m_2 ? **Ans.** $F_{2,1}$ = 6N



Questions from national exam

1. If a force F results an acceleration A when acting on a mass M, then doubling the mass and increasing the force four times will result in an acceleration of

   A) ½ A B) 2A C) A D) 4A

2. A certain force is expressible as $\vec{F} = 10i + \mathbf{b}j$. What is the value of **b** if the magnitude of the force is 26 N?

   A) 16 N B) 24 N C) 20 N D) 28 N

3. A resultant force of magnitude **F** acts on an object of mass **m** and as a result the object attains an acceleration of magnitude **a**. If the magnitude of the driving force is halved and the mass of the object is tripled, what will be the magnitude of the acceleration of the object in this new circumstance?

   A) ½ **a** B) 1/6 **a** C) ¼ **a** D) 1/3 **a**

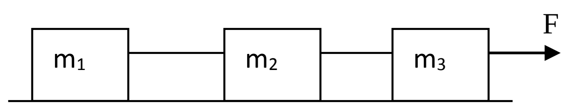
4. A 5kg block is pulled along a horizontal frictionless floor by a cord that exerts a force of F=12 N at an angle of 30^0 above the horizontal. If the force F is slowly increased, then what are the values of the force and acceleration of the block just before it is lifted off the floor, respectively?

   A) 116N, 21m/s^2 B) 100N, 20m/s^2 C) 116N, 20m/s^2 D) 100N, 21m/s^2

5. Neglecting friction, find the tension in the thread connecting m_1 and m_2.

   A) $\dfrac{m_1F}{m_1+m_2+m_3}$ C) $F - \dfrac{m_3F}{m_1+m_2+m_3}$

   B) $\dfrac{m_2F}{m_1+m_2+m_3}$ D) $\dfrac{(m_2+m_3)F}{m_1+m_2+m_3}$



6. Which of the following reasons is correct about the difficulty to walk in sand than on hard ground?

   A) The action of the feet on the sandy ground is greater than the reaction of the sandy ground on the feet.
   B) The action of the feet on the sandy ground is less than the reaction of the sandy ground on the feet.
   C) The hard ground can exert more reaction than the action of the man to keep him moving.
   D) The feet can't put in sufficient action which results in insufficient reaction to make walking difficult.

7. Which one of the following statements is true about action and reaction forces referred to in Newton's third law of motion?

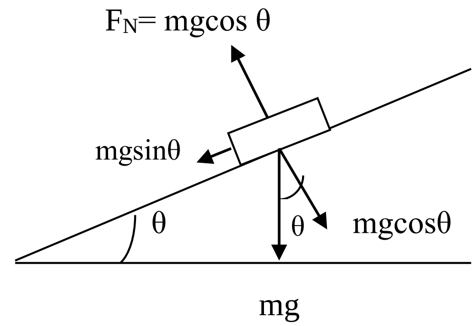
   A) They act on the same body.
   B) They act upon two different bodies.
   C) They are equal in magnitude but need not have the same line of action.
   D) They are not equal in magnitude but have the same line of action.

8. What is the direction to which a fish must push the water with its fins in order to propel eastward?

   A) Eastward B) Upward C) Westward D) Downward

Motion on an inclined plane

When a block of mass *m* is placed on a smooth (frictionless) inclined plane that makes an angle θ with the horizontal,



- *The force that pulls the block down the plane is*
  $F_{\parallel} = mg\sin\theta$
- *Acceleration of the block is*
  $a = g\sin\theta$
- *The force that presses the block with the inclined plane is*
  $F_{\perp} = mg\cos\theta$
- *The normal force is*
  $F_N = mg\cos\theta$

Example

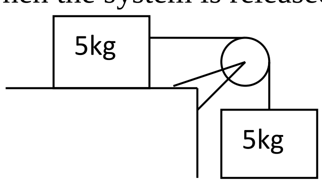
1. A block, initially at rest, slides down a frictionless plane at 37^0 with the horizontal. How long will it take the block to slid a distance of 3 m? **Ans**. t = 1 sec.
2. A block is projected with 3 m/sec up a frictionless plane at 30^0 with the horizontal. How far will the block move up the plane before coming to rest? **Ans**. S = 0.9 m
3. A 75 cm long inclined plane makes an angle of 37^0 with the horizontal. If a block is released from rest at the top of the inclined plane, what is the speed of the block at the bottom of the plane? **Ans**. V=3 m/sec

Exercise

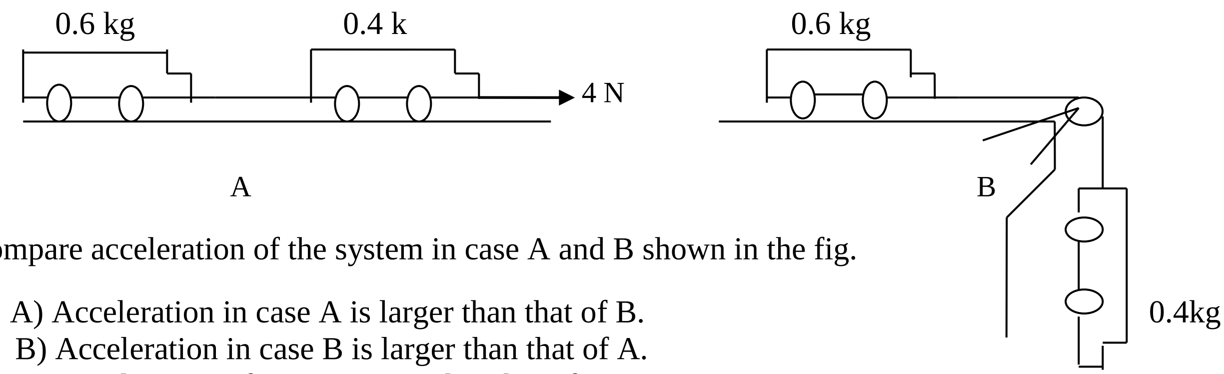
1. A 10 m long inclined plane has a height of 2 m. What force parallel to the plane is required to move a 20 kg block up the plane at a constant speed? **Ans**. F = 40 N
2. A 0.5 kg mass starts from rest and moves down an inclined plane a distance of 1 m in 0.5 sec. What is the net force on the mass acting along the plane? **Ans**. F_{net} = 4 N
3. A block slides down a frictionless inclined plane that makes an angle θ with the horizontal starting from rest. The distance versus time taken are given below. What is the angle θ? **Ans**. $\theta = 30^0$

S(m)	0	2.5	10	22.5	40
t(sec)	0	1	2	3	4

4. A block is projected up a 37^0 frictionless inclined plane with a speed of 6 m/sec. What is the distance the block move up along the plane before coming to rest? **Ans**. S=3 m
5. Calculate the acceleration and tension in the cord when the system is released. **Ans**. a = 5 m/s^2, T = 25 N



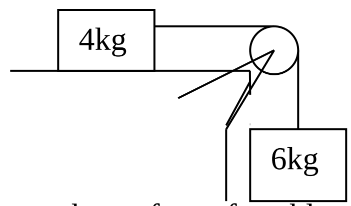
6. A block starts from rest and slides down a frictionless plane inclined at an angle of 30^0 above the horizontal. How long will it take the block to acquire a speed of 10m/sec? **Ans**. t = 2 sec.

7. 

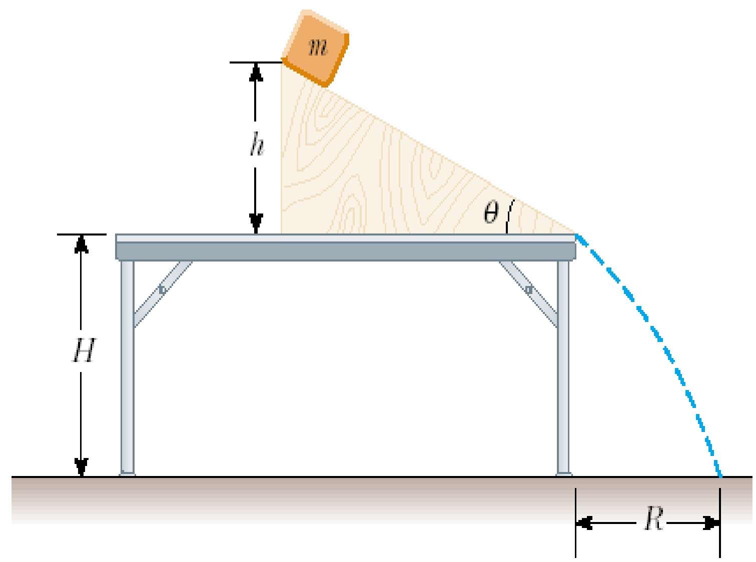
Compare acceleration of the system in case A and B shown in the fig.

A) Acceleration in case A is larger than that of B.
B) Acceleration in case B is larger than that of A.
C) Acceleration of case A is equal to that of B.
D) The acceleration cannot be compared from the given information.

8. Calculate
a) acceleration of the blocks. **Ans**. a = 6m/s^2
b) the tension in the string. **Ans**. T= 24N



9. A block of mass *m=2 kg* is released from rest at *h* = 0.5 m above the surface of a table, at the top of a 30^0 incline as shown in the figure below. The frictionless incline is fixed on a table of height H = 2 m.



a) Calculate the acceleration of the block as it slides down the incline. **Ans**. a = 5 m/s^2

b) What is

i) velocity of the block as it leaves the incline?
Ans. V = $\sqrt{10}$ m/s = 3.16 m/s

ii) the horizontal component of velocity as it leaves the incline? **Ans**. $V_x = \frac{\sqrt{30}}{2}$ m/s = 2.74 m/s

iii) the vertical component of velocity as it leaves the incline? **Ans**. $V_y = \frac{\sqrt{10}}{2}$ m/s = 1.58 m/s

c) How much time has elapsed

i) to slide on the incline?

Ans. $t_{slide} = \frac{\sqrt{10}}{5}$ sec = 0.632 sec

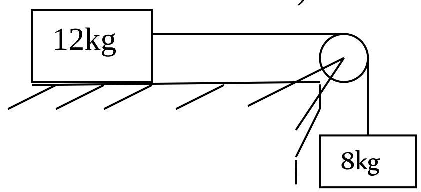
ii) to move from the base of the incline to hit the floor? **Ans**. t_{proj} = 0.494 sec

d) How far from the table will the block hit the floor? **Ans**. R = 1.35 m

e) Does the mass of the block affect any of the above calculations?

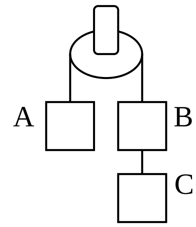
Questions from national exams

1. A block starts from rest and slides down a plane inclined at an angle of 30^0 above the horizontal. How long does it take the block to acquire a speed of 20 m/sec?
   A) 12 sec B) 8 sec C) 6 sec D**)** 4 sec
2. What is the tension in the connecting string?

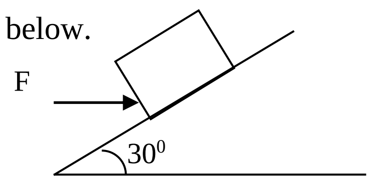


   A) 4N C) 16N
   B) 32N D**)** 48N

3. Three equal weights A, B, C of mass 2 kg each are hanging on a string which passes over a fixed light and Frictionless pulley as shown below. The tension in the string connecting weights B and C is:
   A) 26.6 N C**)** 13.3 N
   B) 20 N D) 3.3 N



4. Two masses 2 kg and 3 kg are attached to the end of a light string passing over a pulley, fixed at the top. What are the tension and acceleration in the string in terms of g?
   A**)** $12g/5$, $g/5$ B) $7g/8$, $g/5$ C) $21g/8$, $g/8$ D) $7g/8$, $g/8$
5. A 100kg box is pushed at a constant speed up the smooth 30^0 inclined plane shown below.



   What horizontal fore F is required? ($\sin 30^0=0.5$, $\cos 30^0=0.866$)
   A**)** 577.35N C) 1732.35N
   B) 1000.35N D) 1154.35N

Frictional force

Frictional force:- is a force between two surfaces in contact that always opposes the relative motion between them. The cause of friction is the roughness of the surfaces.

Advantages of friction:- it helps

- us to walk or to stop, hold things with our fingers.
- cars and bicycles to move or stop.
- car's and bicycle's brakes to work.
- to produce the heat necessary to start fire.
- to fasten pieces of wood together with nails or screws, etc.

Disadvantages of friction:-

- it opposes motion.
- it causes our shoes to wear out.
- it increases the work necessary to operate machine.
- it causes damage to machines.
- it causes machines to become less efficient.

Friction depends on

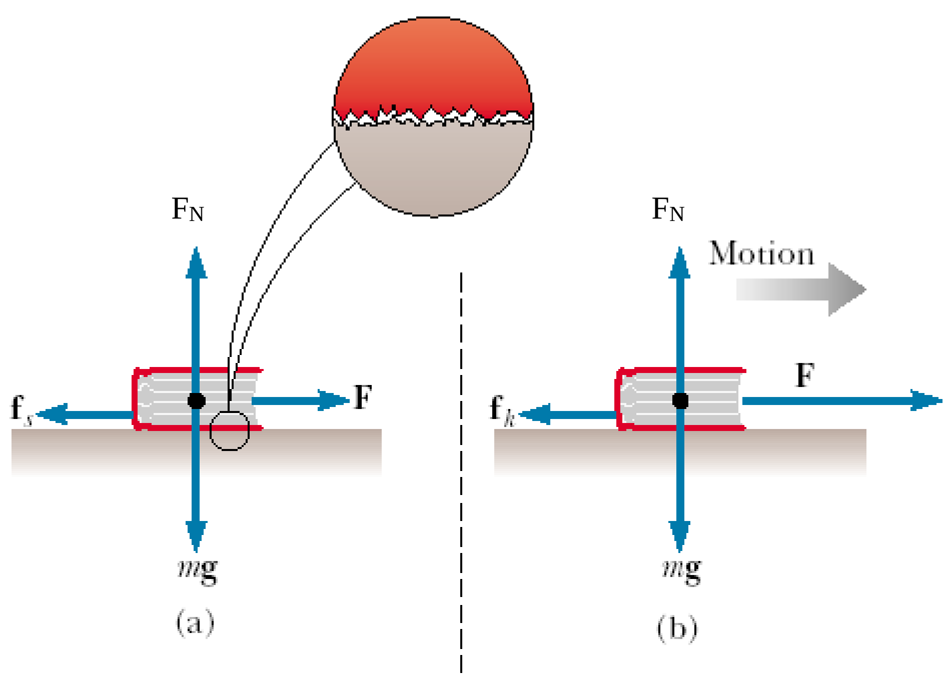
i) the nature of the two interacting surfaces. Eg. Friction between two pieces of wood is different from between wood & metal

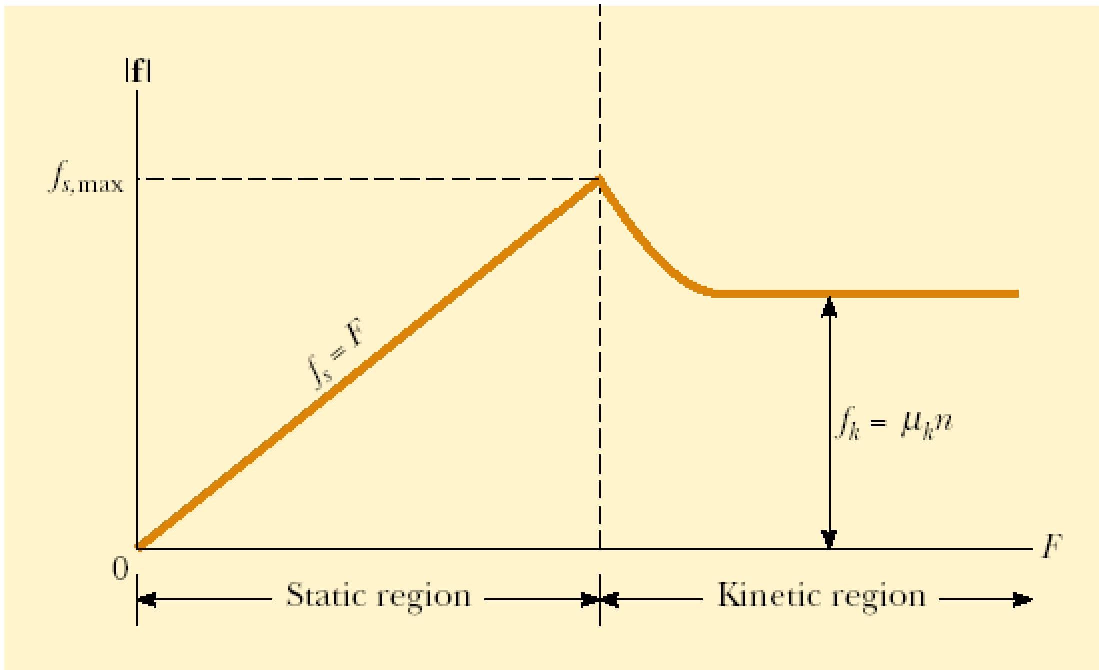
ii) the force keeping the surfaces in contact, which is the normal force F_N.

Note:- Friction does not depend on the surface area of contact.

Frictional forces are classified as

1. **Static friction (f_s)**:- an opposition force that arises when the surfaces are not in relative motion.
   $f_s = \mu_s F_N$ Where μ_s is coefficient of static friction which is between **0** and **1**.
2. **Kinetic friction (f_k)**:- an opposition fore that arises when one surface moves over the other.
   $f_k = \mu_k F_N$ Where μ_k is coefficient of kinetic friction which is between **0** and **1**
3. ***Rollin friction*:-** an opposition forces which when one body rolls over the other.
   Friction can be minimized by the use of wheels, bearings, rollers and lubricants.
   Rolling friction < kinetic friction < static friction. and
   $\mu_k < \mu_s$





(c)

The direction of the force of friction *f* between a book and a rough surface is opposite the direction of the applied force F. Because the two surfaces are both rough, contact is made only at a few points, as illustrated in the "magnified" view. (a) The magnitude of the force of static friction equals the magnitude of the applied force. (b) When the magnitude of the applied force exceeds the magnitude of the force of kinetic friction, the book accelerates to the right. (c) A graph of frictional force versus applied force.

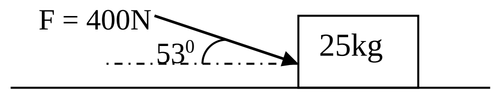
Coefficients of friction

	μ_s	μ_k
Steel on steel	0.74	0.57
Aluminum on steel	0.61	0.47
Copper on steel	0.53	0.36
Wood on wood	0.25 – 0.5	0.2
Glass on glass	0.94	0.4
Metal on metal (lubricated)	0.15	0.06
Ice on ice	0.1	0.03

Example

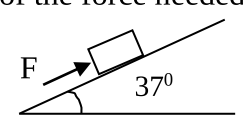
1. A 20N block rests on a horizontal surface. A balance connected to the block reads 8N just before it starts to slide. After the block starts moving a force of only 6N keep the block to move at constant speed. Find the coefficient of
   a) static friction. **Ans**. $\mu_s = 0.4$, b) kinetic friction. **Ans**. $\mu_k = 0.3$
2. A 5kg block is pulled horizontally at constant velocity. If $\mu_k = 0.25$, what is the force of kinetic friction? **Ans**. $f_k = 12.5N$
3. An object of mass 4kg is subjected to a forward force of 12N and retarding frictional force of 4N.What is acceleration of the object? **Ans**. $a = 2 m/s^2$
4. A 2kg block, pulled by a horizontal force of 6N, is accelerated at $2 m/s^2$. What is the retarding frictional force on the block? **Ans**. $f_k = 2N$

5. A 4kg block is accelerated at $1.5m/s^2$ when an 8N horizontal force is applied on it. At what rate would it accelerate if the force applied were 10N? **Ans**. a $=2m/s^2$.
6. A force **F**, acting on a block of mass **m** at an angle **θ** above the horizontal, moves the block along a horizontal surface with constant velocity. If the coefficient of sliding friction between the block and the surface is μ_k, show that the value of **F** is :- $F=\dfrac{\mu mg}{\cos\theta+\mu\sin\theta}$
7. If a block of mass **m** slides down an inclined plane with uniform velocity, show that the coefficient of kinetic friction is $\mu_k = \tan\theta$
8. A 5kg block slides down a plane at 37^0 to the horizontal. Find the acceleration of the block if:- a) the plane is frictionless. **Ans**. a= $6m/s^2$. b) the coefficient of friction is 0.2. **Ans**. a=$4.4m/s^2$
9. A block of mass **m** is placed on an inclined plane that makes an angle of 37^0 above the horizontal. If the block is in limiting equilibrium, find the coefficient of static friction. **Ans**. $\mu_s = 0.75$
10. Starting from rest, the box reaches a speed of 2m/s in 4sec. What is the coefficient of sliding friction? **Ans**. $\mu_k = 0.399$



Exercise

1. A 2kg block is moved along a level floor by a horizontal force of 20N. If μ_k=0.4, what is the acceleration of the block? **Ans**. a=$6m/s^2$
2. A 5kg block is placed on an inclined plane that makes an angle of 37^0 above the horizontal. Will the block slide or remain at rest if the coefficient of friction between the block and the plane is : a) 0.6 ? b) 0.75?Explain.
3. A 2.6kg block is pulled along a horizontal surface by a constant force that makes an angle of 37^0 above the horizontal. If the block moves with a constant speed, what is the magnitude of the applied force?(μ_k=0.4) **Ans**. F=10N
4. Suppose the constant speed of the block described in question 2 is 2m/sec. If the applied force is suddenly removed how far further does the block travel before coming to rest? **Ans**. S=50 cm
5. A 10kg block is pulled along a horizontal surface by a force of 50N at angle of 37^0 with the horizontal. If μ_k=0.4, what frictional force is acting on the block? **Ans**. f_k = 28N
6. What is the acceleration of the block described in question 4? **Ans**. a =$1.2m/s^2$
7. A 10kg object is moving under the action of a force along a floor at a constant speed of 4m/sec. If μ_k=0.2, what is the rate at which work is done by the force? **Ans**. P=80watts
8. A block of mass 2 kg is pushed up an inclined plane inclined at 37^o. The coefficient of kinetic friction between the block and the surface of the inclined plane is 0.25. Find the magnitude of the force needed to push the block so that it moves up at constant velocity. **Ans**. F= 16N



9. A force **F** acting at an angle of 37^0 above a horizontal on a block of mass 10kg accelerates it at $0.5m/s^2$ along a horizontal surface. If μ_k =0.5, what is the magnitude of the force? **Ans**. F=50N
10. A 21N force applied horizontally on a block resting on a horizontal surface accelerates it by $4m/s^2$. If μ_k=0.3, what is mass of the block? **Ans**. m=3kg

11. A block of mass **m** is pulled up along a plane inclined at an angle θ from the horizontal with a force **F**. If the block moves with a constant speed, the coefficient of sliding friction is given by:

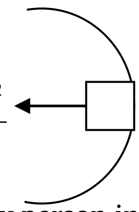
A) $\dfrac{F+mg\cos\theta}{mg\cos\theta}$ B) $\dfrac{F-mg\sin\theta}{mg\cos\theta}$ C) $\dfrac{F-mg\cos\theta}{mg\sin\theta}$ D) $\dfrac{F+mg\cos}{mg\sin\theta}$

12. A piece of marble is sliding inside a circular sheet of radius 0.8m. What is the minimum angular speed ω_{min} for the marble to be in a vertical equilibrium if μ_k=0.5?

Hint:- For the marble to be in a vertical equilibrium: f_k=mg

Ans. ω_{min} =5rad/sec

$F_N = \dfrac{mv^2}{R}$

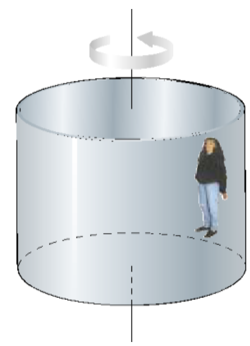


13. A large vertical cylinder that spins about its axis fast enough such that any person inside is held up against the wall when the floor drops away. The coefficient of static friction between person and wall is μ_s , and the radius of the cylinder is *R*. Show that

a) the minimum angular speed necessary to keep the person from falling is

$$\omega_{min} = \sqrt{\frac{g}{R\mu_s}}$$

b) the maximum period of revolution is $T = 2\pi\sqrt{\dfrac{R\mu_s}{g}}$



Questions from national exams

1. A 3kg block initially at rest is pulled along a level floor by a 25N force that makes an angle of 37^0 above the horizontal. If μ_k=0.4, what is the speed of the block at t=1.55 sec? **Ans**. V= 7.23m/sec

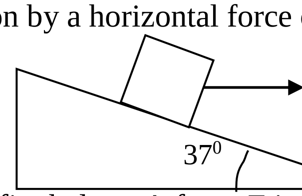
A)2 m/sec B)3 m/sec C) 6 m/sec D) 7 m/sec

2. A block is placed on a plane inclined at an angle of 37^0 above the horizontal. If the block slides down the plane with an acceleration of 4m/s^2, what is the coefficient of kinetic friction between the block and plane?

A) 0.2 B) 0.25 C) 0.3 D) 0.35

3. A block of mass 2kg placed on a rough inclined plane is acted on by a horizontal force of 10N as shown in the diagram. If μ_k=0.4, what is acceleration of the block?

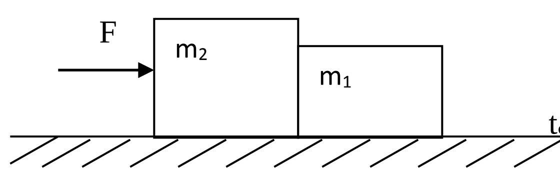
A) 2.8m/s^2 C) 6m/s^2
B) 6.8m/s^2 D) 8m/s^2



4. Two objects of masses m_1 and m_2 are arranged as shown in the fig. below. A force F is applied on the system in the direction shown. The coefficient of friction between the objects and the table is μ_k . What is the magnitude of the acceleration of the system?

A) $\dfrac{F}{m_1+m_2} + \mu_k g$ C) $\dfrac{F}{m_1+m_2} - \mu_k g$

B) $\dfrac{F}{m_1+m_2} + \dfrac{1}{\mu_k} g$ D) $\dfrac{F}{m_1+m_2} - \dfrac{1}{2} g$



5. A block is placed on an inclined plane. The plane is then raised until it makes an angle α with the horizontal. If the coefficient of friction is 0.3, what is the value of α to which the plane may be raised before the block begins to slide?

A) 45^0 B) 30^0 C) 22^0 D) $\tan^{-1}(0.3)$

6. A 1500kg car going at 30m/sec applies its brakes and skids to rest. If the frictional force between the skidding tyres and the road is 600N, how far does the car skid before coming to rest?

A) 112.5m B) 11.25m C) 112.5km D) 1.125km

7. A wooden box loaded with books is given a push on a concrete floor. An initial velocity of 8m/sec is imparted to it. How far does the box move in meters before coming to rest if μ_k=0.8? (g=9.8m/s^2)

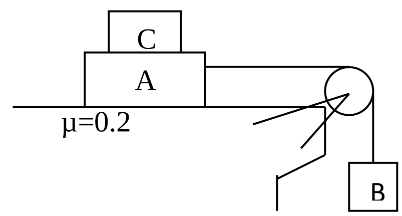
A) 5.07 B) 6.04 C) 4.08 D) 7.10

A block slides on a rough 45^0 incline. The coefficient of friction between the block and incline is μ_k. What is the ratio of acceleration when the block accelerates down the incline to the acceleration when the block is projected up the incline?

A) $\dfrac{\mu_k - 1}{\mu_k + 1}$ B) $\dfrac{\frac{\mu_k}{\sqrt{2}} - 1}{\mu_k + 1}$ C) $\dfrac{1-\mu_k}{1+\mu_k}$ D) $\dfrac{1+\mu_k}{1-\mu_k}$

9. Two masses A and B of 10kg and 5kg respectively are connected with a string passing over a frictionless pulley fixed at the corner of a table as shown.
What is the value of the minimum mass of C that may be placed on A to prevent it from moving?

A) 15kg C) 5kg
B) 10kg D) 0kg



10. A 4kg block is given an initial speed of 8m/sec at the bottom of 30^0 inclined plane. The frictional force that retards its motion is 15N. If the block is directed up the inclined plane, how far will it move before it stops?

A) 3.35m B) 3.66m C) 4.40m D) 4.67m

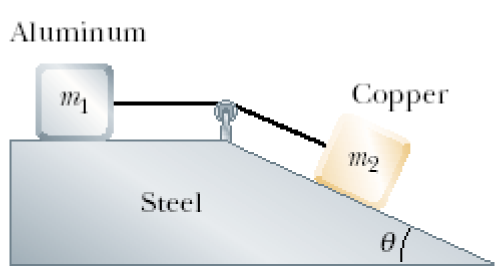
11. A woman driving a 2000kg car along a level road at 30m/sec takes her foot off the gas to see how far her car will roll before it slows to a stop. She discovers that it takes 150m. What is the average force of friction acting on the car?

A) 9000N B) 6000N C) 3000N D) 400N

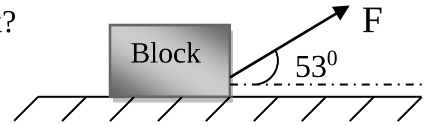
12. An aluminum block of mass m_1= 3kg and copper block of mass m_2= 5kg are connected by a light string over a frictionless pulley. They sit on a steel surface as shown in the fig below, where θ= 53^0. The coefficient of kinetic friction between the steel surface and each block is 0.3. If the blocks are released from rest what is the tension T in the string?

A) 29.40N C) 12.25N

B) 17.25N D) 8.60N



13. A force **F** of magnitude 20 N is applied to a block of mass 2 kg that lies on a rough, horizontal surface as shown in figure below. The coefficient kinetic friction between the block and surface is 0.4. What is the magnitude of the acceleration of the block?



A) 10 m/s^2 C) 4 m/s^2
B**)** 5.2 m/s^2 D) 2.8 m/s^2

Momentum

Linear momentum ($\vec{P}$) of a mass (m) moving at a velocity ($\vec{V}$) is

$$\vec{P} = m\vec{V}$$

Its direction is along $\vec{V}$ and its SI unit kg m/sec.

If the body is moving in an arbitrary direction in three dimensional space

$$\vec{P} = m(\vec{V}_x + \vec{V}_y + \vec{V}_z)$$

Example

1. A 1200kg car is moving at a velocity of 60m/sec east. A lorry of mass 18,000kg is moving with a velocity of 12m/sec east. Compare the momenta of the two vehicles.

Ans. $\vec{P}_{car}$ =7.2$\times 10^4$kg m/s East, $\vec{P}_{lorry}$=2.16$\times 10^5$ kg m/s East. Note that though the lorry has less velocity, its momentum is greater than that of the car due to its greater mass.

2. An object of mass 5kg is moving at 18m/s 37^0 east of north. Calculate:
   a) the momentum of the object. **Ans**. $\vec{P}$ =90kg m/s 37^0 east of north.
   b) the x and y components of momentum. **Ans**. $\vec{P}_x$ = 54 kg m/s, $\vec{P}_y$ =72 kg m/s.

Exercise

1. A car of mass 1200kg is moving with a velocity of 36km/h North. A truck of mass 4800kg is moving with a velocity of 9km/h South. Compare the momenta of the two vehicles.

   Ans. $\vec{P}_{car}$ =12,000kg m/s North. $\vec{P}_{Truck}$ =12,000kg m/s South.

2. A 2kg mass is moving at $\vec{V}$ = (3*i*+4*j*) m/s. Calculate momentum of the mass.

   Ans. $\vec{P}$ = (6*i*+8*j*) kg m/s, $|\vec{P}|$ =10kg m/s.

3. If a 4kg object has kinetic energy of 8J. What is its momentum? **Ans**. P=8kg m/s.
4. If the momentum of a body increases by 20%, then what is the percentage increase in its kinetic energy?
   A) 20 B**)** 44 C) 66 D) 88
5. If two particles have equal momenta, which one of the following statements is TRUE about their kinetic energies?
   A) Their kinetic energies are always equal.
   B) Their kinetic energies are never equal.
   C**)** Their kinetic energies are equal when their masses are equal.
   D) Their kinetic energies are equal when the speed of one of the particles is half the speed of the other and their masses are the same.

Impulse

Consider a body of mass m which changes its velocity from $\vec{u}$ to $\vec{V}$ during a time interval Δt. The net external force which acts during this time is

$$\vec{F} = m\vec{a}$$

$$= \frac{m\Delta\vec{V}}{\Delta t}$$

$$= \frac{\Delta\vec{P}}{\Delta t}$$

OR $\vec{I} = \vec{F}\Delta t = \Delta\vec{P}$

Impulse ($\vec{I}$):- is the product of the force and the time during which the force acts. It is equal to the change in the momentum of the body. The SI unit of Impulse ($\vec{I}$) is kg m/s = N s

Example

1. A tennis ball is thrown towards a wall. m_{ball} = 0.06kg, $\vec{u}_b$ =3m/s (– i) , $\vec{V}_b$ = 2m/s i, Δt = 0.05 sec. Calculate:

   a) $\vec{P}_i$, b) $\vec{P}_f$, c) $\vec{I}$. **Ans**. a) $\vec{P}_i$ = – 0.18 kg m/s i, b) $\vec{P}_f$ = 0.12kg m/s i, c) $\vec{I}$ = 0.30kg m/s i
2. A 1500kg car traveling at – 20m/s i crashes with a tree and comes to rest in 0.2sec. What is

   a) the impulse. **Ans**. $\vec{I} = 3\times10^4$ Ns i b) the force exerted on the car during the crash. **Ans**. $\vec{F} = 1.5\times10^5$N i
3. A 2kg brick moves at a speed of 6m/s. How large a force is needed to stop the brick in a time of 4×10^{-4}sec?

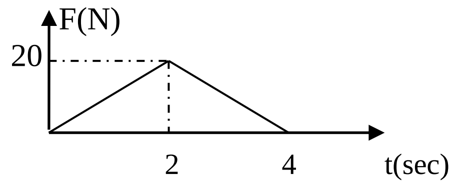
   Ans. F=3×10^4N
4. A 0.8kg ball moving horizontally strikes a wall with a velocity of 10m/s (i). If it re bounces with a velocity of 6m/s (– i), what is the impulse due to the collision? **Ans**. $\vec{I}$ = – 12.8 N S i

Exercise

1. A ball of mass 200g mass moves towards a wall with a speed of 10m/s. If it bounces back with 80% of its original speed along the same line, what is the impulse? **Ans**. I = 3.6 NS
2. An unbalanced force of 15N is applied for 2 sec to bring an object moving with a velocity V to rest. What is the magnitude change in momentum? **Ans**. ΔP= 30 kg m/s
3. A force of 200N acts on a body for 0.1sec. Find the magnitude of change in momentum. **Ans**. ΔP=20kg m/s
4. What constant force must be applied on a 4kg mass in order to change its speed from 6m/s to 12m/s in 3sec? **Ans**. F= 8N.
5. What constant force on an object would increase its momentum by 125kg m/s in 5sec? **Ans**. F=25N
6. What physical quantity does
   a) a slope of momentum versus time graph represent?
   b) the area under resultant force versus time graph represent?

Questions from national exams

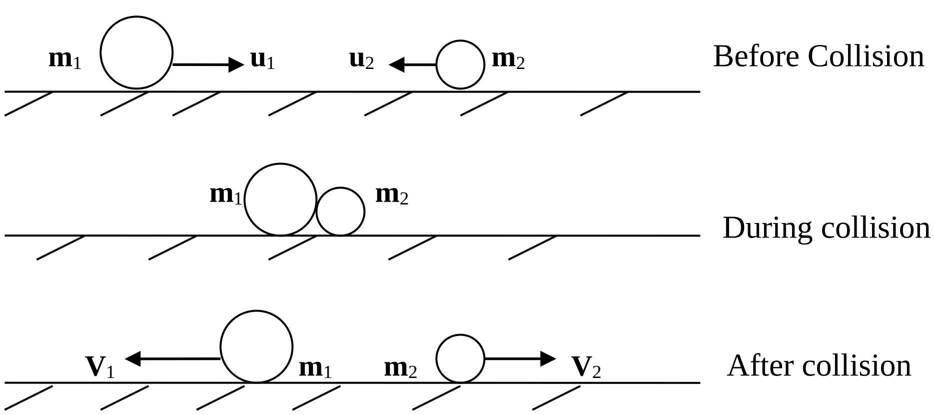
1. If we represent the dimension of mass, length and time by M, L and T, respectively, then the dimensions of impulse are
   A) [ML/T] B) [LT/M] C) [ML^2/T^2] D) [ML/T^2]
2. A 0.5kg steel ball is dropped onto a steel plate with a speed of 12m/s. If it re bounds with a speed of 10m/s and is in contact for 0.02sec, then what is the magnitude of the average force exerted on the ball? **Ans**. F_{av}= 550N
   A) 75N B) 100N C) 25N D) 50N
3. A force **F** acts on a ball initially at rest on a smooth surface for a time **t**. The variation of **F** with **t** is shown in the figure below. The momentum of the ball, in NS, after 4sec is
   A) 10 C) 30
   B) 20 D) 40



4. A man standing on a frictionless surface continuously fired 120 bullets horizontally in 1 minute using a machine gun. If each bullet has a mass of 10 *g* and leaves the muzzle of the gun with a speed of 800 m/s, what is the average force exerted by the gun on the man?
   A) 8N B) 16 N C) 960 N D) 16000 N

Classification of collisions

1. Elastic collision:- is a collision in which both kinetic energy and linear momentum are conserved.
   Consider two billiard balls $\mathbf{m_1}$ and $\mathbf{m_2}$ moving toward each other with velocities $\mathbf{u_1}$ and $\mathbf{u_2}$ respectively before collision.
   Their velocities after collision will be $\mathbf{V_1}$ and $\mathbf{V_2}$ respectively.



$$\vec{P}_i = \vec{P}_f$$

$$m_1\vec{u}_1 + m_2\vec{u}_2 = m_1\vec{V}_1 + m_2\vec{V}_2 \;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\quad (1)$$

$$m_1\vec{u}_1 - m_1\vec{V}_1 = m_2\vec{V}_2 - m_2\vec{u}_2$$

$$m_1(\vec{u}_1 - \vec{V}_1) = m_2(\vec{V}_2 - \vec{u}_2) \;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\quad (2)$$

$$KE_i = KE_f$$

$$\tfrac{1}{2} m_1u_1^2 + \tfrac{1}{2} m_2u_2^2 = \tfrac{1}{2} m_1V_1^2 + \tfrac{1}{2} m_2 V_2^2 \;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\quad (3)$$

$$m_1(u_1^2 - V_1^2) = m_2(V_2^2 - u_2^2)$$

$$m_1(u_1 - V_1)(u_1 + V_1) = m_2(V_2 - u_2)(V_2 + u_1) \;.\;.\;.\;.\;.\;.\;.\;.\quad (4)$$

Dividing equation (4) by equation (1) gives $\vec{u}_1 + \vec{V}_1 = \vec{u}_2 + \vec{V}_2$

<u>Example</u>

1. An object of mass 3kg traveling at 10m/s i collide head-on with an object of mass 6kg traveling at 2m/s ($-i$). If the second object moves at 6m/s (i) after collision, what will be velocity of the first object?

   Ans. $\vec{V}_1$ = 6m/s ($-i$)

2. A ball of mass 0.5kg moving at 20m/s east collides with another ball of mass 0.3kg moving at 10m/s west. After collision, velocity of the first ball is 2.5m/s west. Determine velocity of the second ball after collision.

   Ans. $\vec{V}_2$ =27.5m/s east.

3. Two identical balls collide head-on. The initial velocity of one is 4m/s east while that of the other is 2m/s west. If the collision is elastic, what is the final velocity of each ball? **Ans**. $\vec{V}_1$ = 2m/s west, $\vec{V}_2$ = 4m/s east.

<u>Exercise</u>

1. A 1kg ball moving at 5m/s i collides with a 2kg ball moving at 4ms ($-i$). If the collision is elastic, what are the velocities of the balls directly after collision? **Ans**. $\vec{V}_1$ =7m/s ($-i$), $\vec{V}_2$ = 2m/s i
2. A block with mass m_A=5kg and $\vec{u}_A$ = 2m/s i collides elastically with a block of mass m_B = 3kg that is moving with $\vec{u}_B = -$ 2m/s i. After collision block B moves with $\vec{V}_B$ =2m/s i. What is velocity of m_A after collision?

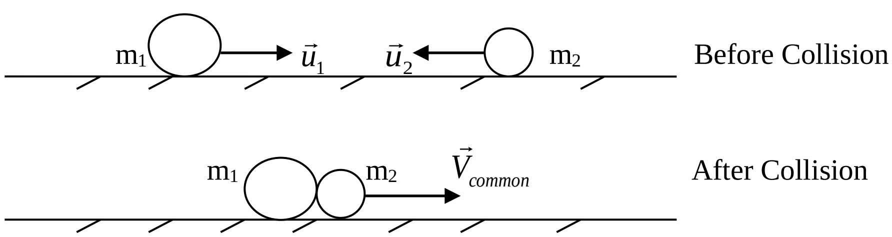
   Ans. $\vec{V}_A = -$ 0.4 m/s i

3. A 3kg billiard ball initially moving with a speed of 8m/s makes an elastic head-on collision with a 5kg block initially at rest. What is the speed of the block immediately after collision?

   A) 4m/s B) 6m/s C)8m/s D) 10m/s

ii) <u>Inelastic collision</u> :- is a collision in which **kinetic energy is not conserved, while linear momentum is conserved.**

When two objects collide and stick together after collision, the collision is known as **inelastic collision**.
After collision, the objects move with common velocity V_{com}.



$$\vec{P}_i = \vec{P}_f$$

$$m_1\vec{u}_1 + m_2\vec{u}_2 = (m_1+m_2)\vec{V}_{com}$$

$K.E_i > K.E_f$. K***inetic energy is not conserved.***

One of the applications of conservation of momentum is in describing the motion of a rocket. When it is fired, exhaust gases shoot downward at high speed and the rocket moves upward to balance the momentum of the gas. If the rocket is far enough from the sun or any other planet, the rocket's increase in speed is given by

$$v - u = v_e \ln\frac{M_i}{M_f}$$

where v is the final speed of the rocket relative to the earth
u is the initial speed of the rocket relative to the earth
v_e is the speed of the exhaust gases
M_i is initial mass of the rocket
M_f is final mass of the rocket.

Example

1. How do we distinguish between elastic and inelastic collisions?
2. A 2kg ball moving at 12m/s east collides head-on with a 2kg ball moving at 24m/s west. If they stick together after collision, calculate:
   a) common velocity. **Ans**. $\vec{V}_{com}$ = 6m/s west.
   b) kinetic energy before collision. **Ans**. $K.E_i$ =720J
   c) kinetic energy after collision. **Ans**. $K.E_f$ = 72J
   d) the decrease in kinetic energy. **Ans**. $K.E_i - K.E_f$ = 648J
   e) the percentage loss. **Ans**. percentage loss= 90%
3. A 2.5kg rifle fires a 7g bullet with a speed of 600m/s. What is the recoil velocity of the rifle?
   Ans. $V_{rec} = -1.68$m/s
4. Prove that when a moving body makes an inelastic collision with a second body of equal mass, initially at rest, one half of the initial kinetic energy is lost.

Exercise

1. A 5000kg goods wagon moving at 10m/s collides with a loaded car of mass 15000kg at rest. If the cars couple together, find
   a) their common velocity. **Ans**. V_c = 2.5m/s
   b) the decrease in K.E. **Ans**. $K.E_{dec}$ = 187500J
   c) the percentage decrease in K.E. **Ans**. percentage decrease in K.E=75%
2. A 3kg rifle fires a 5g bullet with a velocity of 300m/s. Find
   a) the recoil velocity of the rifle. **Ans**. $V_{rec} = -0.5$m/s.
   b) K.E of the rifle and bullet after the bullet is fired. **Ans**. $K.E_{rifle}$= 0.375J, $K.E_{bullet}$ = 225J
3. A 3kg block moving with a speed of 8m/s collides with a 1kg block initially at rest. If the two blocks stick together, what is the kinetic energy of the system after collision? **Ans**. $K.E_{after}$ = 72J

Questions from national exams

1. A 4kg block moving with a speed of 8m/s to the right collides head-on with a 2kg block moving with a speed of 10m/s to the left. If the collision is inelastic how much kinetic energy is lost in the collision?
   A) 216J B) 128J C) 100J D) 12J
2. Two heavily loaded trucks collide head on and became entangled as a result of collision. One of them which was at rest has a mass of 2500kg. The other truck having a mass of 2000kg was moving with a velocity of 100km/h before collision. What is the velocity, in m/s, of the entangled mass after collision?
   A) 20.34 B) 18.23 C) 15.34 D) 12.34
3. A cart moving at a speed V collides with an identical stationary cart on an air truck and the two stick together after collision. What is their velocity after collision?
   A) – 0.5V B) zero C) 0.5V D) V
4. A car (mass m_1) is moving at velocity V, when it meshes into an unmoving car (mass m_2), locking bumpers. Both cars move together at the same velocity. The common velocity will be given by
   A) $\frac{m_1V}{m_2}$ B) $\frac{m_2V}{m_1}$ C) $\frac{m_1V}{m_1+m_2}$ D) $\frac{(m_1+m_2)V}{m_1}$
5. A stationary mass explodes into two parts of masses 0.4kg and 4kg. If the larger mass has kinetic energy of 100J, what is kinetic energy of the smaller mass?
   A) $4\sqrt{50}$ J
   B) The change in kinetic energy is the same for both particles
   C) 1000J
   D) 100J

6. A rocket moving in free space has a speed of 3×10^3m/s relative to the earth. Its engines are turned on, and fuel is ejected in a direction opposite the rocket's motion at a speed of 5×10^3m/s relative to the earth. What is the speed of the rocket relative to the earth once the rocket's mass is reduced to half its mass before ignition?

A) 7×10^3m/s B) 5×10^3m/s C) 4×10^3m/s D) 2×10^3m/s

Collisions in two dimension

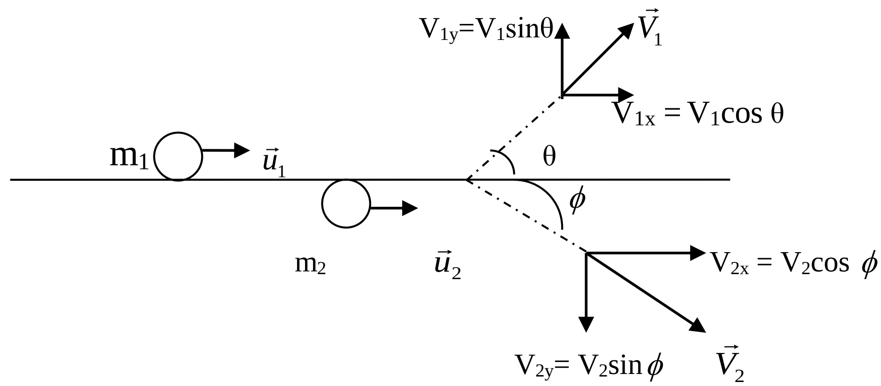
In two dimensional collisions, we have two component equations for the conservation of momentum.

$$m_1u_{1x} + m_2u_{2x} = m_1V_{1x} + m_2V_{2x}$$

$$m_1u_{1y} + m_2u_{2y} = m_1V_{1y} + m_2V_{2y}$$

Conservation of kinetic energy gives

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$



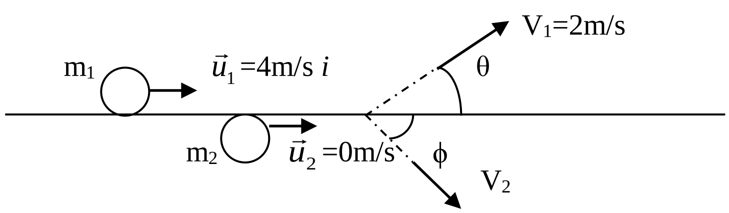
Example

1. A 6kg ball moving at 3m/s i collides elastically with a 4kg ball initially at rest. After collision the first ball moves at 2m/s 37^0 above x-axis. What is velocity of the second ball after collision?

   Ans. $\vec{V}_2$ = 2.74m/sec 40.6^0 below x-axis.

2. A 5kg ball moving at 4m/s i collides elastically with a 4kg ball initially at rest as shown in the figure below. Find

   a) V_2. **Ans**. $V_2 = \sqrt{15}$ m/s
   b) θ. **Ans**. θ=49.5^0
   c) ϕ. **Ans**. 29.4^0



3. A 5kg sphere moving at (8i – 6j) m/s collides with a 2kg sphere and both spheres come to rest after collision. Find velocity of the second ball before collision. **Ans**. $\vec{u}_2$ = (–20 i+15j) m/s
4. A 1kg object explodes into three pieces, m_1=400g, m_2=200g and m_3. m_1 and m_2 are moving at (15i+10j)m/s and (20i–10j) m/s respectively. What is velocity of the third piece? **Ans**. $\vec{V}_3$ = (–25i–5j) m/s

Exercise

1. A 4kg ball moving at (3i–2j) m/s collides elastically with a 3kg ball at rest. After collision the second ball moves with (2i+j)m/s. What is velocity of the first ball after collision? **Ans**. $\vec{V}_1$ = (1.5i– 2.75j) m/s
2. A 5kg sphere moving at (3i+7j) m/s collides with a 4kg sphere moving at $\vec{u}_2$. After collision both spheres move together at (i+4j)m/s. What is velocity of the 4kg sphere before collision? **Ans**. $\vec{u}_2$ = (–1.5i+0.25j)m/s
3. A mass m explodes into three pieces, m_1= 1.25kg, m_2= 3.25kg and m_3. If the masses are moving at (30i –10j) m/s, (–17i+10j) m/s and (35.5i + 40j) m/s respectively. What is mass of the third piece?

   Ans. m_3= 0.5kg

4. A proton moving with 3.5×10^5m/s i collides with a second proton initially at rest. After collision the first proton moves at an angle of 37^0 above the x-axis and the second proton moves at an angle ϕ below the x-axis. Find V_1 and V_2. $\vec{V}_1=2.8\times10^5$m/s at 37^0 above x-axis, $\vec{V}_2=2.1\times10^5$m/s at 53^0 below x-axis.

5. A clay is accidentally dropped and breaks into three pieces m_1, m_2 and m_3=600g which moves at 2m/s 53^0 N of W, 3m/s N of E and 4m/s South respectively. Find m_1 and m_2. **Ans**. m_1= 0.96kg, m_2= 0.48kg

6. A 4kg mass moving at 3m/s i collides with a 10kg mass moving at 4m/s j. If they stick together after collision, determine their common velocity. **Ans**. $\vec{V}_c = (\frac{6}{7}i + \frac{20}{7}j)$ m/s

7. A body explodes into three pieces, m_A=2kg, m_B=3kg and m_c=1kg, then move at $\vec{V}_A$ =3m/s i, $\vec{V}_B = -$ 1m/s j. Determine V_C. **Ans**. $\vec{V}_C = (-6i + 3j)$ m/s

8. Two identical objects with the same initial speed collide and stick together. If the combined objects move with half the initial speed of either object, What was the angle between the initial velocities?
   A) 30^0 B) 60^0 C) 90^0 D) 120^0

9. A 200kg motor bicycle moving east with a speed of 30m/s collides with a 250kg motor bicycle moving south with a speed of 32m/s. Assuming perfectly inelastic collision, what is the magnitude and direction of the final momentum?
   A) 6,000 NS 53^0 east of south
   B) 8,000 NS 37^0 east of south
   C) 10,000 NS 53^0 south of east.
   D) 10,000 NS 37^0 south of east

10. Two particles with masses 2*m* and 3*m* are moving toward each other along the *x* axis with the same initial speed *v*. Particle 2*m* is traveling to the left, and particle 3*m* is traveling to the right. They undergo an elastic glancing collision such that particle 2*m* is moving in the negative *y* direction after the collision. What are the *x* component of the final velocity of the particle 3*m* and the kinetic energy of the particle 2*m*, respectively?
    A) 0.33 v and 0.7 mv^2
    B) 0.33 v and 1.40 mv^2
    C) 0.67 v and 0.70 mv^2
    D) 0.67 v and 1.4 mv^2

11. A hard ball moving on a horizontal frictionless surface with a speed of 5 m/s in the positive *x* direction strikes a stationary hard ball of the same mass. After the collision, the first ball moves with a speed of 4 m/s along a direction that makes an angle of 37.0^0 with its initial direction of motion. What is the struck ball's speed and the smallest angle between its direction of motion and the positive *x* axis after the collision?
    A) 3 m/s and 53.0^0 B) 3 m/s and 37.0^0 C) 4 m/s and 53.0^0 D) 4 m/s and 37.0^0

Center Of Mass

Center of mass:- is the point at which the whole mass of a body may be considered to be concentrated. Suppose two masses m_1 located at (x_1, y_1) and m_2 at (x_2, y_2). The center of mass (x_c, y_c) of the system is given by

$$x_c = \frac{m_1x_1 + m_2x_2}{m_1 + m_2} \quad \text{and} \quad y_c = \frac{m_1y_1 + m_2y_2}{m_1 + m_2}$$

In general, for n particles lying in the *x-y* plane

$$x_c = \frac{m_1x_1 + m_2x_2 + ... + m_nx_n}{m_1 + m_2 + ... + m_n} \quad \text{or} \quad x_c = \frac{\sum m_ix_i}{\sum m_i}, \quad y_c = \frac{m_1y_1 + m_2y_2 + ... + m_ny_n}{m_1 + m_2 + ... + m} \quad \text{or} \quad y_c = \frac{\sum m_iy_i}{\sum m_i}$$

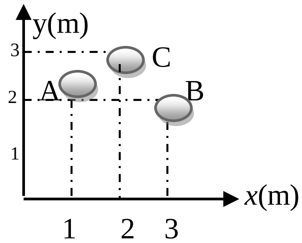
Example

1. Three masses, m_1=2kg, m_2=3kg and m_3=5kg, are placed at $\vec{r}_1 = (4i+2j-3k)$ m, $\vec{r}_2 = (-3i+2j-2k)$ m and $\vec{r}_3 = (2i-2j+2k)$ respectively. Find the center of mass. **Ans**. $\vec{r}_{c.m} = (0.9i - 0.2k)$ m.
2. Find x_c of the masses from point A. The rod is mass less. **Ans**. x_c= 3.4m

3. A system consists of 4kg at (0,5)m, 7kg at (3,8)m and 5kg at (–3,–6)m. Find the center of mass. **Ans**. $\vec{r}_{c.m}$ =(0.375i+2.875j)m

Exercise

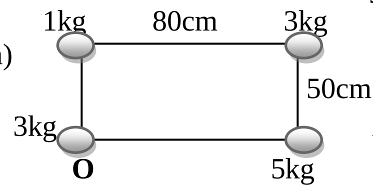
1. Where does the center of mass of the system lie?



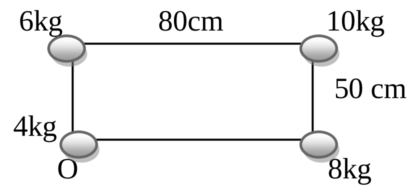
Position	Mass
Block A (1,2)m	M
Block B (3,2)m	M
Block C (2,3)m	3M

Ans. x_c= 2m and y_c= 2.5m

2. Find the center of mass of a system with respect to corner O.

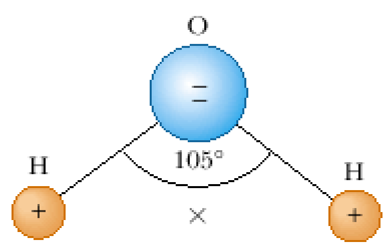
a) 

Ans. $\vec{r}_{c.m}$ =(0.58i+0.23j)m

b) 

Ans. $\vec{r}_{c.m}$ =(0.51i+0.29 j)m

3. Find the center of mass for water molecule from the center of oxygen atom.
   m_{oxygen}=16 $m_{hydrogen}$, (Distance from oxygen to hydrogen atom= 0.097nm)



Ans. $\vec{r}_{cm} = (0, -6.6\times10^{-12})$ m

Motion of center of mass

The velocity of center of mass is

$$\vec{V}_{c.m} = \frac{m_1\vec{V}_1 + m_2\vec{V}_2 + ... + m_n\vec{V}_n}{m_1 + m_2 + ... + m_n}$$

That is $V_{x\,c.m} = \dfrac{m_1V_{x1} + m_2V_{2x} + ... + m_nV_{nx}}{m_1 + m_2 + m_3}$ and $V_{x\,c.m} = \dfrac{m_1V_{y1} + m_2V_{y2} + ... + m_nV_{ny}}{m_1 + m_2 + m_3}$

Example

Thee balls, m_1= 2kg, m_2=3kg and m_3=5kg, connected by mass less springs are moving at $\vec{V}_1$=(I –2j+k) m/s, $\vec{V}_2$ = (2i+6k) m/s and $\vec{V}_3$ = (5i –3j –3k) m/s. Calculate $V_{c.m}$. **Ans**. $\vec{V}_{c.m}$ = (3.3i –1.9j +0.5k) m/s

Dynamics of circular motion

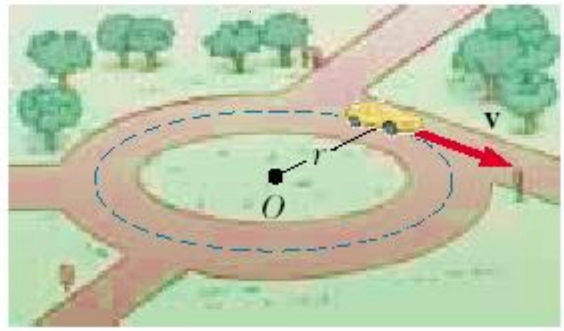
A car turning around a curved level road

For a car turning around a curved level road, the centripetal force that keeps the car in a circular path is sideways force of static friction.

$F_c = f_s$

$\frac{mV^2}{R} = f_s$ OR $V_{max} = \sqrt{\frac{f_s R}{m}}$

$\frac{mV^2}{R} = \mu_s mg$ OR $V_{max} = \sqrt{\mu_s gR}$



If speed of the car exceeds V_{max}, the frictional force that acts as a centripetal force is not sufficient to keep the car on the level road and the car will skid side way. In order to avoid the skidding, roads may be banked.

A car turning the corner of a banked curve

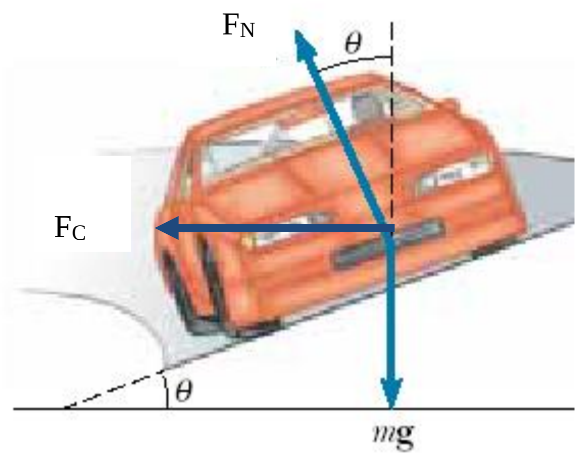
Suppose a car is moving round the corner of a banked curve, where θ is the banking angle of the road,

$F_c = F_N \sin\theta = \frac{mV^2}{R}$ (1)

$F_N \cos\theta = mg$ (2)

Dividing equation (1) by (2) gives

$\tan\theta = \frac{V^2}{Rg}$



Example

1. A 1400kg car is turning at the corner of a flat road of curvature 36m. If $\mu_k = 0.4$, calculate the maximum speed with which the car turns without skidding? **Ans**. $V_{max} = 12$m/s
2. A car is rounding a flat, unbanked curve of radius 50m. If $\mu_k = 0.2$, what is the maximum speed at which the car can round the curve without skidding? **Ans**. $V_{max} = 10$m/s
3. What should be the banking angle of the road in question 2 for the car not to depend on the force of static friction to go round the road without skidding? **Ans**. $\theta = \tan^{-1}(0.2) = 11.3^0$
4. What should be the coefficient of static friction at the pavement of a car moving with maximum speed of 12m/s in a circle of radius 48m on a flat road, to make turns successfully?
   A) 0.2 B) 0.3 C) 0.15 D) 0.4
5. A highway curve is banked (inclined) in such a way that a car travelling at a speed of 13.5 m/s can round the curve without skidding, in the absence of friction. If the banking angle is 30^0, what is the radius of the curve?
   A) 10.5 m B) 21.0 m C) 31.7 m D) 36.7 m