Application of Calculus in Modelling the Growth of L.Plantarum Bacteria

Group Name: Runtime Terror

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1 Introduction

This paper discuss about how calculus more specifically differential calculus can be used to analyse and interpret results from experimental data collected about Lactobacillus plantrum bacteria. Before we discuss about the maths behind the modelling of the experimental data, let's try to know more about the Lactobacillis plantrum bacteria.

Lactobacillus is the largest genus(category) within the group of lactic acid bacteria. What then is lactic acid bacteria? Lactic acid bacteria are the main bacterial species involved in beverage spoilage. It is also presented through all stages of wine-making. They can be isolated on many surfaces and environments including vine leaves, grapes, winery equipment, and barrels.

Now that we know what lactobacillus is, lets look at what plantarum is and then join the two together. Plantarum in latin means 'The species of plant' which is self explanatory. Therefore, Lactobacillus plantarum is a versatile bacterium found in a variety of ecology niches, ranging from vegetable and plants fermentations to the human gastrointetinal tract. L. plantarum cells are rods with rounded ends, straight, generally 0.9–1.2 m wide and 3–8 m long, occuring singly in pairs or in shot chains.

Figure 1: Lactobacillus plantarum

2 Beneficial Influences Of Lactobacillus plantarum On Human Health and Disease

L. plantarum is a potent, high quality probiotic that can aid the human host in several ways:

- **Fights off pollutants**
  L. plantarum is scientifically proven to combat cadmium (Cd)—an almost-invisible, deadly substance that derives from the Earth's crust and is slowly making its way into more human food.L. plantarum shields the intestinal wall from Cd, which enables the body to alleviate oxidative stress. The multi-beneficial L. plantarum also protects the human host from aluminum and copper toxicity.

- **Boosts iron absorption**
  When the human host absorbs iron, the body works to bind oxygen to hemoglobin, enabling oxygen to transport from tissues to lungs—a process that is necessary for human survival. Consuming probiotic-infused fruit drink improves iron absorption 50 percent.

- **Improves heart issues** There are lists of fermented foods with different strands and strains of probiotics and, naturally, probiotic-rich foods can help mend the gut which in turn can enhance cognitive function, digestion, as well as heart problems.

3 Technical Terms

- **Asymptote:** An asymptote is a line that a curve approaches, as it heads towards infinity

- **Lag Time:** Frequently, when a microbial population is transferred into a new habitat, noticeable cell division only commences after what is known as a "lag time". This lag time can be determined by finding the tangent line x-axis intercept to the growth curve at its inflection point to the line.

- **Gompertz Function:** It is a sigmoid function which describes growth as being slowest at the start and end of a given time period. The right-hand or future value asymptote of the function is approached much more gradually by the curve than the left-hand or lower valued asymptote.

- **Sum of Squares Error SSE or SOS:** It is the difference between the observed value and the predicted value. It measures the unexplained variability by a regression or model. By unexplained variation, it means output of a function that is beyond the expected result of a model. We usually use it when we want to minimize the error. The smaller the error, the better the estimation power of the regression.

4 Application of Calculus to measure Bacteria Growth

The Gompertz model is used in modelling the experimental data. Gompertz function or curve is a well known and widely used in many aspects of biology. It has been frequently used to describe the growth of animals and plants, as well as the number or volume of bacteria and cancer cells. It is very useful because it defines specific parameters that characterize the S-shaped curve. In addition, it uses relative growth, which is the logarithm of the given population compared to the initial population. Lag time and asymptote are the two very important parameters found.

The typical growth curve for bacteria is S-shaped as shown in the figure below. N/N_0 is the ration of number of bacteria, N, to the initial population, No, and t is the time. The curve shows an initial lag, followed by exponential growth and then leveling off.

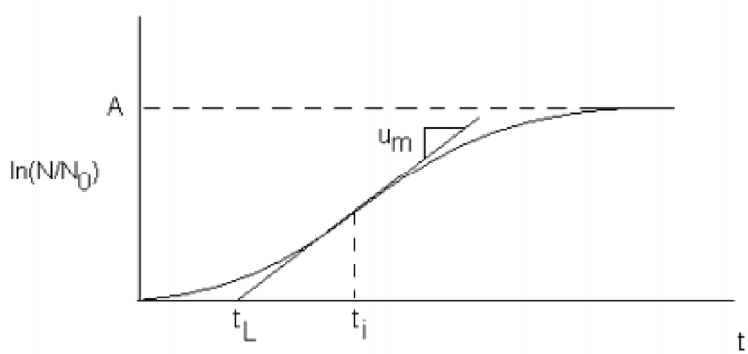


Figure 2: Gompertz Function

There are several parameters that are useful for characterizing the growth curve. One is the asymptote A, which represents the ultimate value of lnN/N_0. Another is the lag time t_L. The lag time can be found as follows:

- Obtain the time of the inflection point ti
- Find the slope of Um of the tangent line at the inflection point. This represents the maximum growth rate.
- The lag time t_L is obtained by finding the x-axis intercept of this tangent line

We will be using the Gompertz model to represent the growth curve. The model is given by the equation:

$$ln\frac{N}{N_0} = ae^{-e^{(b-ct)}}$$

Where a, b and c are constants, which are obtained by fitting experimental data. We will use this table below which provides experimental data for the growth rate of L plantarum, at 41.5^o C. This data is recorded from experiments performed under specific conditions in a laboratory. Our goal now is to find the asymptote. Here are the steps we will be taking to get the asymptote:

- First, we fit the Gompertz equation to the experimental data in order to obtain a, b and c.
- Then, we show how the parameters A and t_L relate to the constants
- Finally, we obtain numerical values of A and t_L from the above step

t(hr)	$ln(N/N_0)$
2	0.2
3	0.5
4	1.1
5	1.8
6	2.5
8	4.0
12	6.4
13	6.6

Mathematical Description and Solution Approach

1. Finding the constant in the Gompertz model that best fit the given data:

$$f(t) = ae^{-e^{(b-ct)}}$$

2. Determining the asymptote by finding the limit as t approaches the infinity. As an An asymptote is a line that a curve approaches, as it heads towards infinity we use the

$$lim_{t\to\infty} f(t)$$

to get that value of line

$$Asymptote = lim_{t\to\infty} f(t) = lim_{t\to\infty} ae^{-e^{(b-ct)}}$$

3. Obtaining lag time with the first and second derivatives: Here we will use the chain rule and product rule of differentiation to find the first and second derivatives of f(t). Let:

$$g(x) = ae^x \qquad h(x) = e^x \qquad i(x) = b - cx \tag{1}$$

$$Then, f(x) = g(h(i(x))) \tag{2}$$

$$Hence, f'(t) = g'(h(i(t)) * h'(i(t)) * i'(t)$$

$$f'(t) = ae^{-e^{(b-ct)}} * -e^{(b-ct)} * -c$$

$$f'(t) = ace^{(b-ct)-e^{(b-ct)}}$$

$$f''(t) = ac^2 e^{(b-ct)-e^{(b-ct)}} (e^{(b-ct)} - 1)$$

To find the point of inflection, set the second derivative equal to zero. The x-axis intercept of the tangent line gives the lag time.

4. After finding the constants with the Gompertz model, Google sheets is used to input the given data and guess the three constants. Also, the square in the error of the guesses is calculated by using the solver application to manipulate the constants until the sum of the squared errors (sos) is as small as possible.

   The table below tabulates is the results of this procedure. The first two columns give the experimental data; the third column gives the value of lnN/N_0 as determined by the Gompertz model with constants a, b, and c given at the bottom of the table. The final column gives the squared difference. The "sos" gives the sum of the square errors.

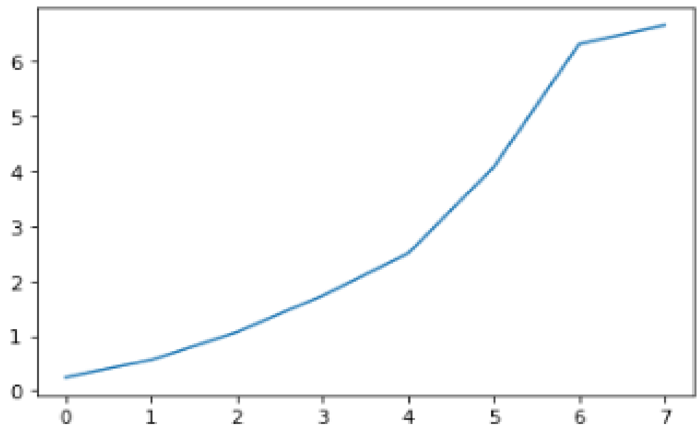
   A graph of the curve with the constants given in the table is also shown, along with the given experimental data.

   The link to the google sheet file can be found here
   The link to the Github repository can be found here

	t (hr)	ln(N/No)	Estimated ln(N/No)	Squared error
0	2	0.2	0.241710	0.00173969
1	3	0.5	0.562655	0.00392559
2	4	1.1	1.066725	0.00110722
3	5	1.8	1.731326	0.00471616
4	6	2.5	2.498122	3.52798e-06

Expanding based on the definition provided in the technical terms section, we use the SSE or sum of squared errors to check the accuracy of our model because the smaller the SSE value gets the more accurate our model,f(t), becomes.

$$f(t) = 7.8324.e^{-e^{(1.8031-0.27828t)}}$$



The next step in analyzing the growth cure is to find the parameters of the asymptote A and the lag time t_L. A can be found by determining the limit of the given function as t approaches the infinity.

$$A = lim_{t\rightarrow\infty} 7.8324e^{-e^{(1.8031-0.27828t)}}$$

$$A = lim_{t\rightarrow\infty} 7.8324e^{-e^{(1.8031-0.27828(\infty)}}$$

$$A = lim_{t\rightarrow\infty} 7.8324e^{-e^{-(\infty)}}$$

$$A = lim_{t\rightarrow\infty} 7.8324e^{0}$$

$$=> A = a = 7.8324$$

It is also necessary to take the first and second derivatives of the equation in order to find the required parameters. Thus, setting the second derivative to zero and solving for t will get the point of inflection.

$$0 = ac^2 e^{(b-ct)-e^{(b-ct)}}(e^{(b-ct)} - 1)$$

$$ln(1) = ln(ac^2 e^{(b-ct)-e^{(b-ct)}} e^{(b-ct)}) - ln(ac^2 e^{(b-ct)-e^{(b-ct)}})$$

$$ln(1) = ln(ac^2) + ln(e^{(b-ct)-e^{(b-ct)}}) + ln(e^{(b-ct)}) - ln(ac^2) - ln(e^{(b-ct)-e^{(b-ct)}})$$

$$ln(1) = ln(e^{(b-ct)})$$

$$ln(1) = b - ct$$

$$=> t_i = \frac{b}{c} = \frac{1.8031}{0.27828} = 6.479$$

Then, the slope of the tangent line of the point of inflection, which represents the maximum growth rate is found by using the first derivative:

$$Maxgrowthrate : f'(t_i) = 0.80108$$

Finally, the lag time, t_L is obtained by finding the x-axis intercept of the tangent line. The slope-intercept form of a line, y = mx + b is used to solve for b at the point (6.4286, 2.8404).

$$2.8404 = 0.801846.4286 + b$$

$$=> b = -2.3146$$

$$0 = 0.80814t_L - 2.3146$$

$$=> t_L = 2.8867$$

5 Insights and Conclusion

5.1 Interpretations

- **Interpretation of the asymptote**
  As observed, the growth of Lactobacillus plantarum bacteria is successfully modeled using the Gompertz curve at 41.5 degrees Celsius. The asymptote, which represents the maximum relative population of the bacteria, was found to be 7.8324. What this means is that if there are n number of Lactobacillus plantarum bacteria in a sample, then they can multiply to reach a maximum population size of 7.8324.

- **Interpretation of the the derivative of growth function**
  The maximum rate of growth of Lactobacillus plantarum bacteria was found to be 0.80184. This means that approximately eight-tenths of the relative population increased in an hour during the fastest period of growth.

- **Interpretation of the lag time**
  The lag time which is the time required for the bacteria to become accustomed to the new environment is found to be 2.8867 hours. During this time, there is little or no growth of bacterial population.

5.2 Inaccuracies in the result obtained

While the model used (Gompertz model) was a very realistic model of the growth of Lactobacillus plantarum, it has limitation as well especially in terms of result inaccuracies. The constants a, b and c that were found did give some error when compared to the values from the experimental data as they were found with trial and error. And the sum of the squares of each of the errors of the records in the experimental data is approximately 0.26. Hence, a model that is able to reduce the sum of the squared error even further would greatly improve further studies of this problem.

5.3 Importance of the findings

The results found from the Gompertz model have direct implications for biology and medicine because they provide the fastest rate of growth of the bacteria. In addition, the maximum relative population and showed the time required before the bacteria reaches its maximum growth rate.These results are vital to the development of products using Lactobacillus plantarum bacteria. Industries or firms involved in using the bacteria can use these results to maximise their efficiency in using Lactobacillus plantarum bacteria. For instance:

The lag time helps an industry that uses Lactobacillus plantarum bacteria in their process to setup a clear and defined processing system, specifically in-terms of setting up a timing schedule when utilising the bacteria because the lag time is the time required for the bacteria to become accustomed to the new environment.

The maximum relative population of the bacteria, was found to be 7.8324. An industry using the bacteria can determine what amount of bacteria it needs to have at the beginning of the process where the bacterium would be used because it is possible to estimate the final size of the bacterium from the starting amount as follows:

$$ln(\frac{N}{N_0}) = 7.8324$$

$$N = N_0 * e^{7.8324}$$

6 Group Member Roles

- Natnael: Research on bacteria growth model and editing the LaTEx document
- Mahalinoro: Research on bacteria growth model and editing the LaTex document
- Chris: Gathering information and compiling everything by editing the LaTex document
- Zubery: Research on the technology side and the calculation on the bacteria growth dataset.

7 Bibliography

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