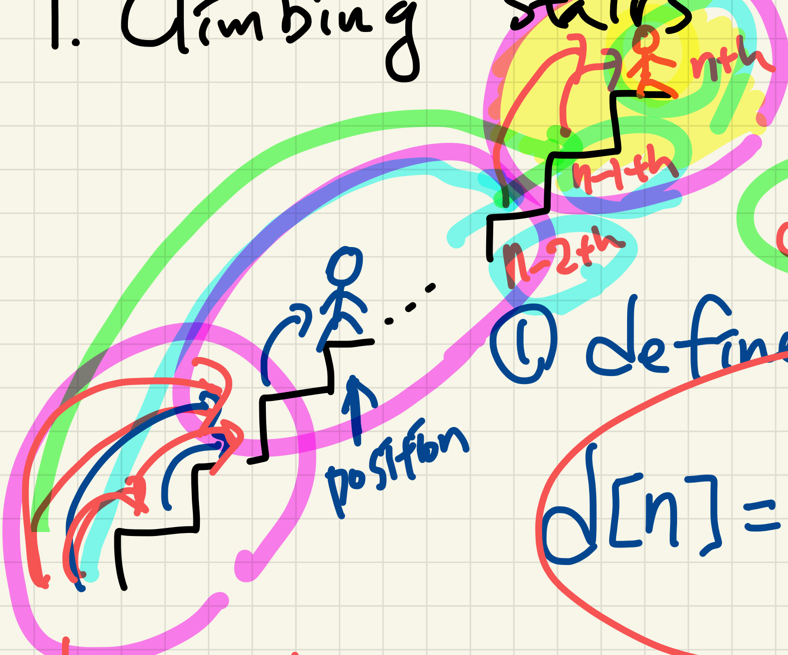
1. Climbing stairs



$d[1] = 1$
$d[2] = 2$

① define subproblem

$d[n]$ = # of ways to climb n stairs.

② guessing. end

③ Recurrence Relation

$$d[n] = \underline{d[n-1]} + \underline{d[n-2]}$$

2. Make one

① define the subproblem.

$$x \xrightarrow{/3 \quad /2 \quad -1} \cdots \to 1$$

min # of operation.

$d[n]$ = min # of operations to make 1.

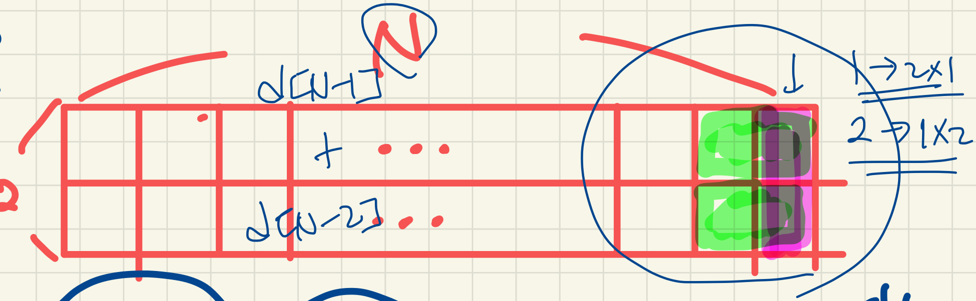
②

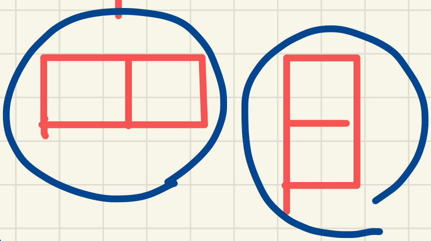
$$n \to \cdots \to x \to 1$$

③

$$\boxed{\min\left(\begin{array}{l} d[x/3], \quad \%3==0 \\ d[x/2], \quad \%2==0 \\ d[x-1] \end{array} \right) + 1}$$

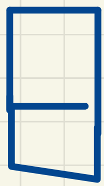
3.





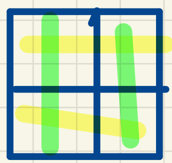
how many possible ways to fill up 2xN tiles

N=1



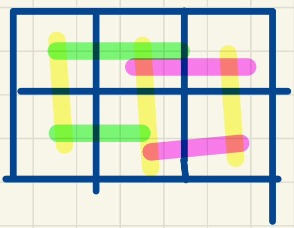
1 way

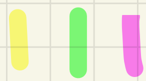
N=2



2 ways

N=3





3 ways ...

① define subproblem (N)

$\boxed{d[N] = \# \text{ of ways } 2 \times N}$

, d[1]=1 , d[2]=2 , d[3]=3

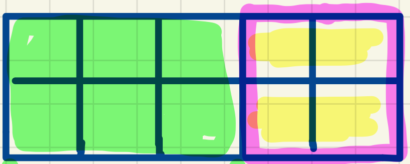
② guess to find Recurrence relation

③ $d[N] = \underline{d[N-1]} + \underline{d[N-2]}$

4. $N = 3$

$$d[N] = d[N-1] + d[N-2] \times 2$$

$d[3] = d[2]$ How it ends



$d[N-2]$

$d[N-2]$

$N-2$

5. 1, 2, 3

find the num of ways
to rep n as the sum of 1, 2, 3.

$N = 1 \rightarrow 1$ 1

$N = 2 \rightarrow 1+1, 2$ 2

$N = 3 \rightarrow 1+1+1, 1+2, 2+1, 3$ 4

① $d[N]$ = the num of ways to rep n as the sum of 1, 2, 3.

② $N=3$

$N-1$

$$\underbrace{\bigcirc + \bigcirc + \cdots}_{N-2} + \bigcirc = N$$

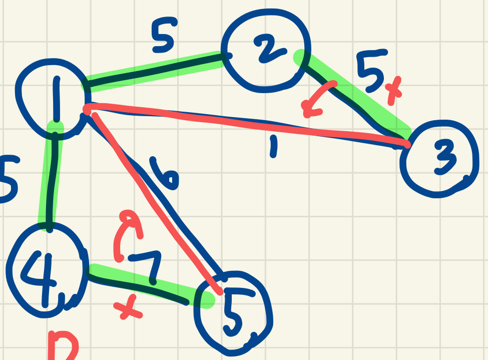
2

1,

3,

$d[N] = d[N-1] + d[N-2] + d[N-3]$

$D = 5$ or 7



5+5+1+6 = 17

2 days

↳ Enhancer 1 day ?

N	M	D
5	6	5
1	2	5
1	4	5
4	5	7
2	3	5
1	5	6
1	3	1

active: (1 2 5), (1 4 5), (4 5 7), (2 3 5)

inactive: (1 5 6), (1 3 1)

① Kruskal's algorithm to get the MST and days (w/o Enhancer)

$\{(1,3,1), (1,2,5), (1,4,5), ~~(2,3,5)~~, (1,5,6), (4,5,7)\}$ cycle.

i−1   i

② 2nd Kruskal's algorithm to check if there's an active edge that could replace the heaviest inactive edge in prev MST.

③ the edge' weight ≤ D and should be active.

6.

$1 \le N \le 100$ # of digits

"pretty number" # cannot start with 0

ex 45656 ✓

$N=1 \rightarrow 9$ (1, 2, 3, 4, ..., 9)

$N=2 \rightarrow 17$ (10, 12, 23, 34, 45, 56 ... 21, 32, 43, 54, 65)

*N

① $\underbrace{X X \ldots X\,y}_{N-1}\,X$ L

$y \rightarrow L-1$ or $L+1$

$(1 \le L \le 8)$

d[N][L] = # of pretty numbers where the last digit is L

N \ L	0	1	2	3	4	...	9
1	0	1	1	1	1	1 ...	1
2	1	1	2	2	2	...	
3				4			

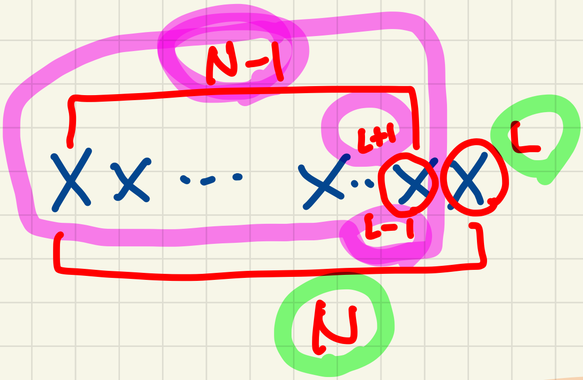
N-1 L

L+1 32 43

L-1 12 23

_ 2/4 _ 3 _

d[3][3] ⇐ d[2][2] + d[2][4]



$$d[N][L] = d[N-1][L-1] + d[N-1][L+1]$$

$$(1 \le L \le 8)$$

$L=0$ $\underline{L=9}$

7. Non-decreasing number (can start '0')

$N=1 \rightarrow 10 \quad (0, 1, 2, \cdots, 9)$

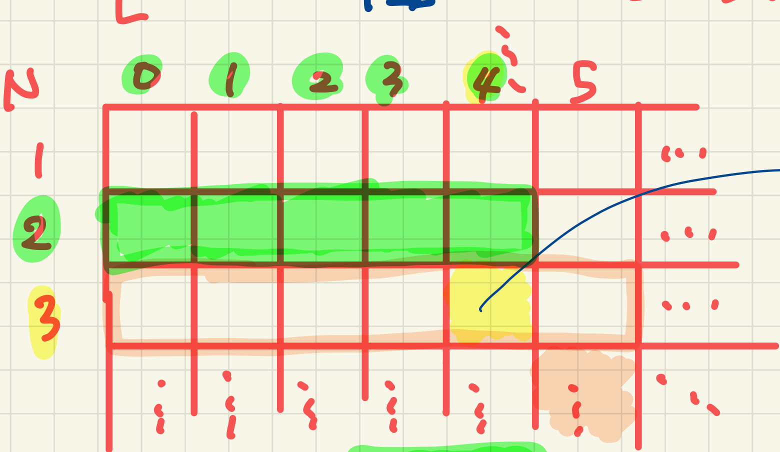
$N=2 \rightarrow 00, 01, 02, 12, 22, \cdots\cdots \quad 55$

$\boxed{10+9+8+\cdots+1=55}$

$9\boxed{x} \rightarrow 10 \quad 0 \le x \le 9 \rightarrow 10$

$8\boxed{x} \rightarrow 9 \quad 0 \le x \le 8 \rightarrow 9$

$7\boxed{x} \rightarrow 8 \quad 0 \le x \le 7 \rightarrow 8$



$d[3][4] = d[2][4] + d[2][3] + d[2][2] + d[2][1] + d[2][0]$

sum

$$d[N][L] = \sum d[N-1][i] \quad \text{where } 0 \le i \le L$$

Base

$$d[1][x] = 1 \quad \text{for all } x \quad 0 \le x \le 9.$$

8. Longest Increasing Subsequence.

(LIS)

[10, 9, 2, 5, 3, 7, 101, 18]

LIS ⇒ "4"