演算法 Homework 20191219 山下夏輝(Yamashita Natsuki) R08922160

## Mini HW #9

課題の提出

**期限** 期日なし **点数** 1 **提出しています** テキスト入力ボックスまたはファイルのアップロード **ファイル タイプ** pdf **使用可能** 12月19日 日 0.00 ~ 12月26日 日 14.20 8 日

Both <u>Hamiltonian path</u> <u>w</u> and <u>Traveling salesman problem</u> <u>w</u> are NP-complete.

Please provide an explanation or proof: how to reduce Hamiltonian path problem to traveling salesman problem?

(You may find many solutions on the Internet, but please write down the answer in your own words)

## **Hamiltonian Path to Hamiltonian Cycle**

The difference between Hamiltonian path and cycle is only whether the start vertex is adjacent to the destination vertex or not. To check them only takes liner time. Then Hamiltonian cycle can be verified in polynomial time and be NP-complete the time as well as Hamiltonian Path.

## **Traveling Salesman is NP**

About time complexity, Hamiltonian Cycle is NP-complete and verified in polynomial time. Then to find the simple cycle in G' also only takes polynomial time. To check whether the total weight of simple cycle in G' is smaller than k or not can be done in liner time. So it is proved that the travel salesman is verified in polynomial time.

## **Reduce Hamiltonian Cycle to Traveling Salesman**

For the graph of Hamiltonian cycle problem, a unweighted graph G(V, E) which there is a Hamiltonian cycle is given. There is a simple cycle through every vertex. The cycle passes through all of the vertex and through them exactly once.

For the graph of traveling salesman problem, a weighted graph G'(V', E') and integer k' is given. There is a simple cycle through every vertex and the cycle passes through all of the vertex and through them exactly once as well as the Hamiltonian cycle problem. In addition to it, the goal of this reduction problem is to find whether there is any cycle with the smaller total weights than k.

For reduction, G' is duplicated from G and all E' weights in G' are weighted 0. In addition to it, G' is added edges which weighted 1 between all of the pair of vertexes which E of G does not have any edge, which G' become a complete graph.

Here, the graph G has a Hamiltonian cycle. Then, the graph G' also has a Hamiltonian cycle. So the simple cycle in G' can be find. Also, the judgement whether the total weight of simple cycle in G' is smaller than k or not.