演算法 Homework 20191121
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Mini HW #6

繳交作業

截止時間 無截止日期　　**總分** 1　　**繳交** 線上輸入或者檔案上傳　　**檔案類型** pdf
接受繳交時間 11月21日 14:20 - 11月28日 14:20 7 天

Given a free tree (a tree without any designated root).

Our goal is to choose a vertex to be the root of the tree, making the height of the tree as minimal as possible.

Consider all the simple paths on the tree. Let the length of the longest simple paths on the tree be x, and a set S containing all the vertices on all of the longest simple paths.

Please simply explain the correctness of the following statement.

(1) (20%) The height of the tree with arbitrary root $\geq \lfloor \frac{x}{2} \rfloor$.

(2) (40%) The vertex v we choose to minimize the height must satisfy : $v \in S$

(3) (40%) The middle vertex of one of the longest simple paths is always an answer to the problem.

(1)
Let the length of path between 2 adjacent vertices is 1.
x = The length of the longest simple path ≥ The length of any simple paths.
At first, the tree of the simple path has one root of tree which do not has any verticle with more than 2 branches except for the root. That is the tree looks like line. Then, the height of the tree can be indicated like below.

x/2 ≤ the height ≤ x

So, the correctness of (1) holds. The root of the tree is minimal and the minimal height is x/2, when the middle of the height of the tree has a vertex.

(2)
a) The simple path always has v when the vertex v is in the simple path.
b) According to (1), the root of the tree which makes the height of tree minimal is in the longest simple path
c) According to the definition of simple path, the root of the tree v must be in the simple path.

From a), b) and c) above, the statement of (2) holds

(3)

According to (1), “the root of the tree is minimal and the minimal height is x/2, when the middle of the height of the tree has a vertex”. So The middle of the longest path always makes the minimal tree.

Here, there are counterpart. Let the length of the paths between 2 vertices is different each like (a,b)=1, (b,c)=10, (c,d)=10, (d,e)=10, (e,f)=10, (f,g)=20. In this case the statement is not correct because the middle of the vertex is d and the height is 40 but the minimal height would be 31. The reason why the statement does not always hold.

The correct statement is “the nearest vertex to the middle of the longest path is always the answer to the problem”.