

演算法 Homework 20190919  
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## Mini HW #1

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截止時間 星期四, 14:20 總分 1 繳交 線上輸入或者檔案上傳  
檔案類型 pdf 接受繳交時間 9月19日 14:20 - 9月26日 14:20 7天

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Let  $f(n) = g(n) - h(n)$ .

Given  $g(n) = \Theta(F(n))$  and  $h(n) = o(F(n))$ .

Prove or disprove  $f(n) = \Omega(F(n))$  using the definitions of  $\Theta, o, \Omega$  given in the textbook.

Note that the definition of  $o$  is different from  $O$ . As for the definition of  $o$ , please refer to section 3.1 of the textbook.

Proof

The definitions of  $\Theta, o$  and  $\Omega$  is below.

$\Theta(g(n)) = \{f(n) : \text{there exist positive constant } c \text{ and } n_0 \text{ such that } 0 \leq f(n) \leq cg(n) \text{ for all } n \geq n_0\}$

$o(g(n)) = \{f(n) : \text{for any positive constant } c > 0, \text{ there exists a constant } n_0 > 0 \text{ such that } 0 \leq f(n) < cg(n) \text{ for all } n \geq n_0\}$

$\Omega(g(n)) = \{f(n) : \text{for any positive constant } c \text{ and } n_0 \text{ such that } 0 \leq cg(n) \leq f(n) \text{ for all } n \geq n_0\}$

At first, a function  $g(n)$  belongs to the set  $\Theta(F(n))$  if there exist positive constants  $c_1$  and  $c_2$ .  $g(n)$  is in the range between  $c_1g(n)$  and  $c_2g(n)$ .

Secondly, if there exist positive constants  $c_3$ , the value of function  $h(n)$  is below  $c_3h(n)$  and also  $c_3h(n)$  can not be equal  $h(n)$ .

Thirdly, if there exist positive constants  $c_4$ , the value of function  $f(n)$  is always above  $c_4f(n)$  and also  $c_4f(n)$  includes the value of  $f(n)$ .

In addition to them, according to the definitions, every formulas given is positive functions.

As a result, this equation,  $f(n) = \Omega(F(n))$ , holds in the given condition.