HW03

Textbook, Problem 3.42

3.42 Show that subtracting the Laplacian from an image gives a result that is proportional to the unsharp mask in Eq. (3-55). Use the definition for the Laplacian given in Eq. (3-53).

$$g_{\text{mask}}(x, y) = f(x, y) - \bar{f}(x, y) \qquad (3\text{-}55)$$

$$\nabla^2 f(x, y) = f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1) - 4f(x, y) \qquad (3\text{-}53)$$

Textbook, Problem 4.9

4.8* Show that $\Im\{e^{j2\pi t_0 t}\} = \delta(\mu - t_0)$, where t_0 is a constant. (*Hint:* Study Example 4.2.)

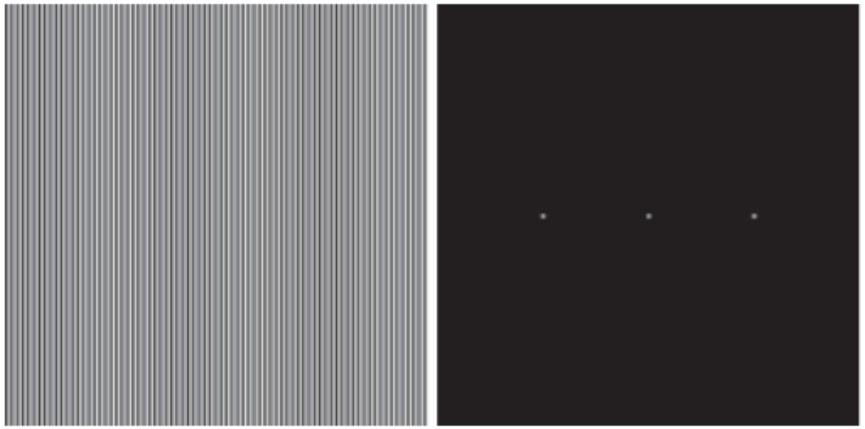
4.9 Show that the following expressions are true. (*Hint:* Make use of the solution to Problem 4.8):

(a)* $\Im\{\cos(2\pi\mu_0 t)\} = \frac{1}{2}[\delta(\mu - \mu_0) + \delta(\mu + \mu_0)]$

(b) $\Im\{\sin(2\pi\mu_0 t)\} = \frac{1}{2i}[\delta(\mu - \mu_0) - \delta(\mu + \mu_0)]$

Textbook, Problem 4.21

4.21 The image on the left in the figure below consists of alternating stripes of black/white, each stripe



being two pixels wide. The image on the right is the Fourier spectrum of the image on the left, showing the dc term and the frequency terms corresponding to the stripes. (Remember, the spectrum is symmetric so all components, other than the dc term, appear in two symmetric locations.)

(a)* Suppose that the stripes of an image of the same size are four pixels wide. Sketch what the spectrum of the image would look like, including only the dc term and the two highest-value frequency terms, which correspond to the two spikes in the spectrum above.

(b) Why are the components of the spectrum limited to the horizontal axis?

(c) What would the spectrum look like for an image of the same size but having stripes that are one pixel wide? Explain the reason for your answer.

(d) Are the dc terms in (a) and (c) the same, or are they different? Explain.

Textbook, Problem 4.57

4.57 Consider the hand X-ray images shown below. The image on the right was obtained by lowpass

(Original image courtesy of Dr. Thomas R. Gest, Division of Anatomical Sciences, University of Michigan Medical School.)

filtering the image on the left with a Gaussian lowpass filter, and then highpass filtering the result with a Gaussian highpass filter. The images are of size 420×344 pixels and $D_0 = 25$ was used for both filter transfer functions.

(a)* Explain why the center part of the finger ring in the figure on the right appears so bright and solid, considering that the dominant characteristic of the filtered image consists of edges of the fingers and wrist bones, with darker areas in between. In other words, would you not expect the highpass filter to render the constant area inside the ring as dark, since a highpass filter eliminates the dc term and reduces low frequencies?

(b) Do you think the result would have been different if the order of the filtering process had been reversed?