カルノーサイクル

$$\eta = \frac{Q_{in} + Q_{out}}{Q_{in}}$$

$$= 1 + \frac{Q_{out}}{Q_{in}}$$

$$= 1 - \frac{\cancel{R} T_0 \ln\left(\frac{V_3}{V_4}\right)}{\cancel{R} T_i \ln\left(\frac{V_2}{V_1}\right)}$$

$$\begin{cases} T_2 V_2^{k-1} = T_3 V_3^{k-1} \\ T_4 V_4^{k-1} = T_1 V_1^{k-1} \end{cases}$$

$\to T_1 = T_2 = T_0$

$T_3 = T_4 = T_i$

$$T_2 V_2^{k-1} = T_3 V_3^{k-1}$$

$$\frac{T_4 V_4^{k-1} = T_1 V_1^{k-1}}{T_3 V_3^{k-1} = T_2 V_2^{k-1}}$$

$$\to \cancel{\frac{T_i}{T_i}} \cdot \left(\frac{V_4}{V_3}\right)^{k-1} = \cancel{\frac{T_0}{T_0}} \cdot \left(\frac{V_1}{V_2}\right)^{k-1}$$

$$\to \frac{V_4}{V_3} = \frac{V_1}{V_2}$$

$$\Rightarrow \frac{V_3}{V_4} = \frac{V_2}{V_1}$$

$$\eta = 1 + \frac{T_0}{T_i} \cdot \cancel{\frac{\ln\left(\frac{V_2}{V_1}\right)}{\ln\left(\frac{V_2}{V_1}\right)}}$$

$$= 1 - \frac{T_0}{T_i}$$

Oct/8/2019 (Tue)

エネルギー変換工学

①

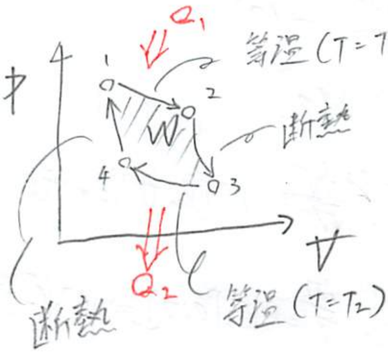
[温熱源. $T = T_1$]

$\downarrow Q_1$

[カルノー熱機関] $\to W = Q_1 - Q_2$

$\downarrow Q_2$

[低温熱源. $T = T_2$]



$$W = \int_1^2 PdV + \int_2^3 PdV + \underbrace{\int_3^4 PdV}_{= -\int_4^3 PdV} + \underbrace{\int_4^1 PdV}_{= -\int_1^4 PdV}$$

$$= \oint PdV \quad —(1),(2)$$

周積分

熱力学の第一法則より

$$dQ = \underbrace{dU}_{= mC_v dT\ (\text{理想気体のとき})} + PdV$$

過程 $1 \to 2$: $dQ = Pd\forall$

$\Rightarrow Q_1 = \int_1^2 Pd\forall$ —(4)

過程 $3 \to 4$: $dQ = Pd\forall \Rightarrow Q_2 = \int_3^4 Pd\forall$ (負)

$\left(= \int_4^3 Pd\forall \right)$ (正) —(5)

状態方程式より

$1 \to 2 : P\forall = mRT_1 \Rightarrow P = \frac{mRT_1}{\forall}$ —(6)

$3 \to 4 : P\forall = mRT_2 \Rightarrow P = \frac{mRT_2}{\forall}$ —(7)

式(6)を式(4)に, 式(7)を式(5)に代入して,

$1 \to 2 : Q_1 = \int_1^2 \frac{mRT_1}{\forall} \cdot d\forall = mRT_1 \int_1^2 \frac{d\forall}{\forall} = mRT_1 [\ln \forall]_1^2 = mRT_1 \ln\left(\frac{V_2}{V_1}\right)$

$3 \to 4 : Q_2 = \int_4^3 \frac{mRT_2}{\forall} \cdot d\forall = mRT_2 \int_4^3 \frac{d\forall}{\forall} = mRT_2 [\ln \forall]_4^3 = mRT_2 \ln\left(\frac{V_3}{V_4}\right)$

仕事Wは,

$W = Q_1 - Q_2 = mRT_1 \ln\left(\frac{V_2}{V_1}\right) - mRT_2 \ln\left(\frac{\forall_3}{\forall_4}\right)$ —(11)

熱効率は

$\eta = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}$ $\left(= 1 - \frac{T_2}{T_1} \right)$ ← カルノーサイクルに限る.

式(12)に式(8), (9)を代入して,

$\eta = 1 - \frac{Q_2}{Q_1} = 1 - \frac{mRT_2 \ln\left(\frac{V_3}{V_4}\right)}{mRT_1 \ln\left(\frac{V_2}{V_1}\right)}$

$= 1 - \frac{T_2}{T_1} \cdot \frac{\ln\left(\frac{V_3}{V_4}\right)}{\ln\left(\frac{V_2}{V_1}\right)}$ —(13)

/8/2019 (Tue) ②

エネルギー変換工学

過程 $2\to3$ と $4\to1$ は

断熱変化なので、

ポアソンの公式：$\begin{cases} TV^{k-1} = (一定) \\ PV^{k} = (一定) \end{cases}$

$2\to3 : \underbrace{(T_1, V_2) \to (T_2, V_3)}$

$\hookrightarrow T_1 V_2^{k-1} = T_2 V_3^{k-1}$ — (14)

$4\to1 : \underbrace{(T_2, V_4) \to (T_1, V_1)}$

$\hookrightarrow T_2 V_4^{k-1} = T_1 V_1^{k-1}$ — (15) $\left(T_1 V_1^{k-1} = T_2 V_4^{k-1} \right)$

式(14)を(15)で割れば、

$$\frac{\cancel{T_1} V_2^{k-1}}{\cancel{T_1} V_1^{k-1}} = \frac{\cancel{T_2} V_3^{k-1}}{\cancel{T_2} V_4^{k-1}}$$

$$\left(\frac{V_2}{V_1}\right)^{k-1} = \left(\frac{V_3}{V_4}\right)^{k-1}$$

$$\therefore \frac{V_3}{V_4} = \frac{V_2}{V_1} \quad (16)$$

(13)に、(16)を代入して、

$$\eta = 1 - \frac{T_2}{T_1} \cdot \frac{\cancel{\ln\left(\frac{V_2}{V_1}\right)}}{\cancel{\ln\left(\frac{V_2}{V_1}\right)}} =$$

$$= 1 - \frac{T_2}{T_1} \cdot 1 = \underline{1 - \frac{T_2}{T_1}}$$ //

$dS = \frac{dQ}{T} \Rightarrow dQ = TdS$ より,

$1 \to 2: Q_1 = \int_1^2 dQ = \int_1^2 \underbrace{T}_{T_1\text{で(一定)}} dS$

$$= T_1 \int_1^2 dS = T_1(S_2 - S_1) \quad —(20)$$

$\left(\int_1^2 dS = \int_1^2 1\, dS = [S]_1^2 = S_2 - S_1 \right)$

$3 \to 4: Q_2 = \int_3^4 dQ = \int_3^4 \underbrace{T}_{T_2\text{で(一定)}} dS$

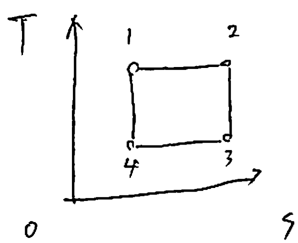
$$= T_2 \int_3^4 dS = T_2 \cdot (\underset{S_1}{\underset{\shortparallel}{S_4}} - \underset{S_2}{\underset{\shortparallel}{S_3}})$$

(∵ 断熱変化)

$$= -T_2(S_2 - S_1) < 0 \quad —(22)$$

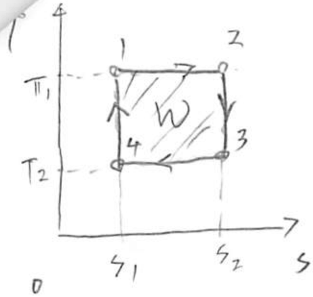
仕事は, 式(20),(22)より,

$$W = Q_1 - Q_2 = T_1(S_2 - S_1) - T_2(S_2 - S_1)$$
$$= (S_2 - S_1)(T_1 - T_2) \quad —(24)$$



断熱変化: S = (一定)
等温変化: T = (一定)

5/2019/(Tue) エネルギー変換工学 ③



第4回

Oct/8/2019 (火) エネルギー変換工学

[カルノーサイクル]

P-V線図の面積 W_{PV} と、T-S線図の面積 W_{TS} が等しいことを示せ.

P-V線図において、

$$W = \int_1^2 PdV + \int_2^3 PdV + \int_3^4 PdV + \int_4^1 PdV = \oint PdV$$

(囲む部分)

熱力学第一法則より、

$$dQ = dU + PdV$$

T-S線図から、状態 1→2, 3→4 においては、$\Delta T = 0$ なので、

$$dq = dU + PdV$$

$(C_v dT = 0)$ より、

$$\begin{cases} dq_{12} = PdV \\ dq_{34} = PdV \end{cases}$$

→ これらを積分して、

$$\begin{cases} Q_{12} = \int_1^2 dq_{12} = \int_1^2 PdV \\ Q_{34} = \int_3^4 dq_{34} = \int_3^4 PdV \end{cases}$$

$$\eta = \frac{Q_{in} - Q_{out}}{Q_{in}} = \frac{Q_{12} - Q_{34}}{Q_{12}} = 1 - \frac{Q_{34}}{Q_{12}}$$

→ 温度Tの式にしたい!!

$$Q_{12} = \int_1^2 PdV$$

$$PV = T_1 R_m$$

$$\Rightarrow P = \frac{T_1 R_m}{V} \leftarrow \text{状態方程式}$$

$$Q_{12} = \int_1^2 \frac{T_1 R_m}{V} dV = T_1 R_m \ln\left(\frac{V_2}{V_1}\right)$$

同様に、

$$Q_{34} = \int_3^4 PdV$$

$$P = \frac{T_3 R_m}{V}$$

$$Q_{34} = \int_3^4 \frac{T_3 R_m}{V} dV = T_3 R_m \ln\left(\frac{V_4}{V_3}\right)$$

$$\eta = 1 - \frac{T_3 R_m \ln\left(\frac{V_4}{V_3}\right)}{T_1 R_m \ln\left(\frac{V_2}{V_1}\right)}$$

→ Vを消去...

$$2\to3:\ T_2 V_2^{\kappa-1} = T_3 V_3^{\kappa-1} \qquad T_1 = T_2$$

$$4\to1:\ T_4 V_4^{\kappa-1} = T_1 V_1^{\kappa-1} \qquad T_3 = T_4$$

$$\begin{cases} T_1 V_2^{\kappa-1} = T_3 V_3^{\kappa-1} \\ T_3 V_4^{\kappa-1} = T_1 V_1^{\kappa-1} \end{cases}$$

$$\cancel{T_3}\left(\frac{V_3}{V_2}\right)^{\kappa-1} = \cancel{T_3}\left(\frac{V_4}{V_1}\right)^{\kappa-1}$$

$$\frac{V_3}{V_2} = \frac{V_4}{V_1}$$

$$\eta = 1 - \frac{T_3}{T_1}$$

Date

仕事 $W = \dfrac{Q_{in} - Q_{out}}{\cancel{Q_{in}}}$

$= \dfrac{Q_{12} - Q_{34}}{\cancel{Q_{12}}}$

$1 \to 2 : Q_{12} = \int_1^2 dQ$

エントロピー $\Delta S = \dfrac{\Delta Q}{T}$

$Q_{12} = \int_1^2 T \cdot \Delta S$

$= T_1 \int_1^2 \Delta S$

$Q_{12} = T_1 (S_2 - S_1)$

$3 \to 4 : Q_{34} = \int_3^4 dQ$

$= \int_3^4 T \cdot \Delta S$

$Q_{34} = T_3 (S_4 - S_3)$

符号マイナス なる!

$W = Q_{12} - Q_{34}$
$Q_{12} = T_1 (S_2 - S_1)$
$Q_{34} = T_3 (S_4 - S_3)$

$W = T_1 (S_2 - S_1) - T_3 (S_4 - S_3)$

$S_3 = S_2$
$S_4 = S_1$
$T_3 = T_2$

$\Rightarrow W = T_1 (S_2 - S_1) - (-T_2 (S_1 - S_2))$

$\Rightarrow W = (T_1 - T_2)(S_2 - S_1)$

[illegible]