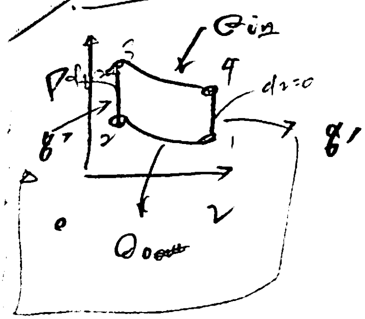
ターリングサイクル



$$\eta = \frac{Q_i + Q_o}{Q_{in}}$$

$$= 1 + \frac{Q_o}{Q_i}$$

$$Q_i = \int_3^4 dq = \int_3^4 (du + Pdv)$$

($du = 0$)

$$= \int_3^4 P\,dv$$

$$= \int_3^4 \frac{RT_i}{v}\,dv$$

$$= RT_i \ln\left(\frac{v_4}{v_3}\right)$$

$$Q_o = \int_1^2 dq = \int_1^2 \frac{RT_o}{v}\,dv$$

$$= RT_o \ln\left(\frac{v_2}{v_1}\right)$$

$$\eta = 1 + \frac{RT_o \ln\left(\frac{v_2}{v_1}\right)}{RT_i \ln\left(\frac{v_4}{v_3}\right)}$$

Tの話れまでにし、

$$\begin{cases} v_1 = v_4 \\ v_3 = v_2 \end{cases}$$

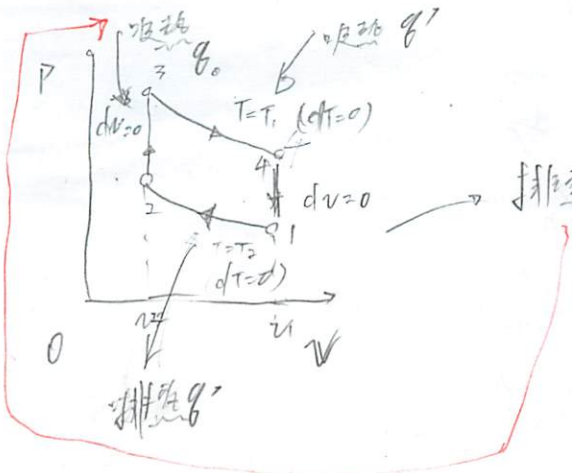
$$\eta = 1 + \frac{T_o}{T_i} \cdot \frac{\ln\left(\frac{v_3}{v_1}\right)}{\ln\left(\frac{v_1}{v_3}\right)}$$

$$= 1 - \frac{T_o}{T_i} \frac{\ln\left(\frac{v_1}{v_2}\right)}{\ln\left(\frac{v_1}{v_2}\right)}$$

$$= 1 - \frac{T_o}{T_i}$$

/5/2019 エネルギー変換工学 ⑪

スターリングサイクル



$$\begin{cases} \text{過程 } 2\to3 : q_0 = C_P(T_1-T_2) & -(1) \\ \text{過程 } 4\to1 : q_0 = C_v(T_1-T_2) & -(2) \end{cases}$$

吸熱の q_0 と排熱の q_0 は互いに等しく、キャンセルし合う

熱効率

$$\eta = 1 - \frac{q'}{q} \quad -(3)$$

熱力学の第一法則

$$dq = du + Pdv$$

du ← $C_v dT$（予備知識のため）

過程 1→2 と 3→4 は等温変化 $(dT=0)$ なので、

$$dq = \underbrace{C_v dT}_{0} + PdV \Rightarrow dq = PdV$$

過程 3→4 : $q = \int_3^4 p\,dv$

$= \int_3^4 \frac{RT}{v}\,dv$ ← $pv = TR$

$= RT_1 \int_3^4 \frac{dv}{v}$

~~$= RT_1 [\ln(\frac{v_3}{v_4})$~~

$= RT_1 \ln\left(\frac{v_4}{v_3}\right)$ —(4)

$q = RT_1 \ln\left(\frac{v_4}{v_3}\right)$ $v_4 = v_1$, $v_3 = v_2$

過程 1→2 : $q' = \int_1^2 \frac{RT}{v}\,dv$

~~$= RT_2 \int$~~

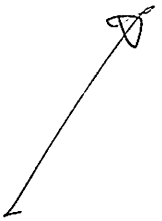
$= RT_2 \ln\left(\frac{v_2}{v_1}\right)$ —(5) (<0)

↳ 正の値に直す。

~~$q' =$~~ $q = RT_2 \ln\left(\frac{v_1}{v_2}\right)$

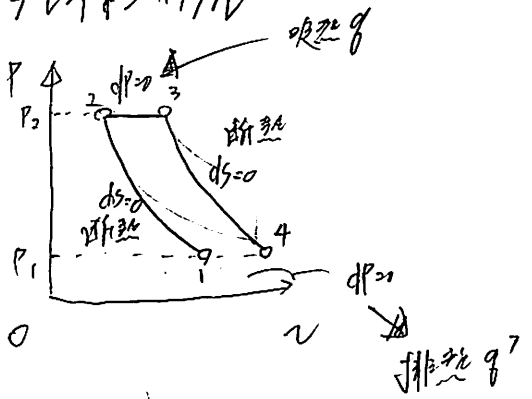
求める熱効率は

$\eta = 1 - \frac{RT_2 \ln\left(\frac{v_1}{v_2}\right)}{RT_1 \ln\left(\frac{v_4}{v_3}\right)} = 1 - \frac{T_2}{T_1}$ —(6)



同じ

ブレイトンサイクル



熱効率 η

$$\eta = 1 - \frac{q'}{q} \quad -(9)$$

過程 2→3 : $q = C_p (T_3 - T_2)$ −(10)

過程 4→1 : $q' = C_p (T_4 - T_1)$ −(11)

式(9)に代入して

$$\eta = 1 - \frac{C_p (T_4 - T_1)}{C_p (T_3 - T_2)}$$

断熱変化のポアソンの式より

$$Pv^k = (一定) \xrightarrow{\text{状態方程式 } PV = RT} P\left(\frac{RT}{P}\right)^k = (一定)$$

$$\therefore P^{1-k} T^k = (一定)$$

過程 1→2 : $P_1^{1-k} T_1^k = P_2^{1-k} T_2^k$

$$\Rightarrow T_2^k = \left(\frac{P_1}{P_2}\right)^{1-k} \cdot T_1^k$$

$$\Rightarrow T_2^k = \left(\frac{P_2}{P_1}\right)^{k-1} \cdot T_1^k \quad \therefore T_2 = \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} \cdot T_1 \quad -(13)$$

(δ = 圧力比)

過程 3→4 : $P_3^{1-k} \cdot T_3^k = P_4^{1-k} \cdot T_4^k$

$$\therefore T_4 = \left(\frac{P_2}{P_1}\right)^{(1-k)/k} \cdot T_3 \quad -(14)$$

$$\eta = 1 - \frac{\delta^{(1-k)/k} T_3 - T_1}{T_3 - \delta^{(1-k)/k} \cdot T_1}$$

$$= 1 - \delta^{-(1-\frac{1}{k})}$$

$$= 1 - \left(\frac{P_1}{P_2}\right)^{1-\frac{1}{k}} \quad -(16)$$

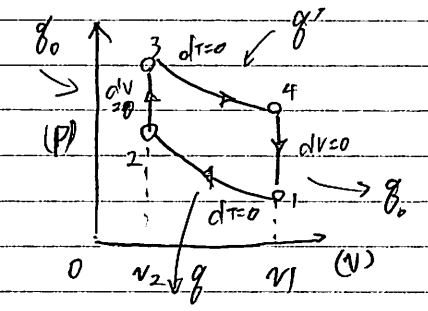
式(9)を代入して

$$\eta = 1 - \frac{C_p(T_4 - T_1)}{C_p(T_3 - T_2)}$$

Date

No.1 / 5 / 20 / 9　エネルギー変換と平

[スターリングサイクル]



$du = C_V \Delta T$ より,

$Q_{23} = \int_2^3 C_V dT$

$= C_V (T_3 - T_2)$

$4 \to 1$: $Q_{41} = \int_4^1 dq$

$dq = du + pdv$

$Q_{41} = \int_4^1 du + pdv$ ($dv = 0$)

$Q_{41} = \int_4^1 du = C_V (T_1 - T_4)$

$\therefore Q_{41} < 0$, $Q_{23} > 0$ より.

$-Q_{41} = Q_{23}$

$\Rightarrow C_V (T_4 - T_1) = C_V (T_3 - T_2)$

$T_4 - T_1 = T_3 - T_2$ //

効率を求める.

$2 \to 3$: $Q_{23} = \int_2^3 dq$.

熱力学第一法則 $dq = du + pdv$

$Q_{23} = \int_2^3 (du + pdv) = \int_2^3 du$ ($dv = 0$)

$3 \to 4$: $Q_{34} = \int_3^4 dq = \int_3^4 du + pdv$ ($dv = 0$)

$= \int_3^4 pdv = \int_3^4 \frac{T_3 R}{V} dV$

($PV = TR$ より, $P = \frac{TR}{V}$)

$= T_3 R \int_3^4 \frac{1}{V} dV$

$= T_3 R \ln\left(\frac{V_4}{V_3}\right)$

$1 \to 2$: $Q_{12} = \int_1^2 dq = \int_1^2 du + pdv$

$= \int_1^2 \frac{T_1 R}{V} dV$

$= T_1 R \ln\left(\frac{V_2}{V_1}\right)$

* スターリングサイクルにおいて,
熱の出入りは $1 \to 2$ の放熱, $3 \to 4$ の吸熱だけ
($2 \to 3$, $4 \to 1$ は土台)

$\eta = \frac{Q_{in} - Q_{out}}{Q_{in}} = 1 - \frac{Q_{out}}{Q_{in}}$

$= 1 - \frac{|Q_{12}|}{|Q_{34}|} = 1 - \frac{T_1 R \ln\left(\frac{V_2}{V_1}\right)}{T_3 R \ln\left(\frac{V_4}{V_3}\right)}$

$T_3 = T_4 = T_H$, $T_2 = T_1 = T_L$ より

$\eta = 1 - \frac{T_L R \ln\left(\frac{V_2}{V_1}\right)}{T_H R \ln\left(\frac{V_4}{V_3}\right)}$

V を消したい!!

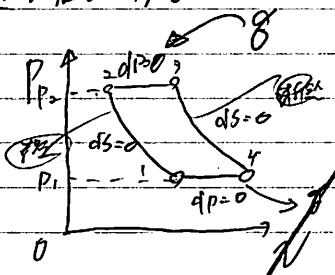
$V_4 = V_1$, $V_3 = V_2$ より,

$\eta = 1 - \frac{T_L \ln\left(\frac{V_2}{V_1}\right)}{T_H \ln\left(\frac{V_1}{V_2}\right)}$ ← 負の位

$= 1 - \frac{T_L \ln\left(\frac{V_2}{V_1}\right)}{T_H \ln\left(\frac{V_2}{V_1}\right)} = 1 - \frac{T_L}{T_H}$ //

Date

ブレイトンサイクル



$\eta = 1 - \frac{q'}{q}$

$dq = du + pdv$

$2 \to 3$: $Q_{2\to3} = \int_2^3 dq = \int_2^3 (du + pdv)$

$(du = C_p \Delta T)$

$\int_2^3 (C_p \Delta T + pdv)$

$= C_p(T_3 - T_2) + P_2(V_3 - V_2)$

$S = \frac{Q}{T}$

$Q_{4\to1} = \int_4^1 dq = \int_4^1 du + pdv$

$= \int_4^1 C_p dT + \int_4^1 pdv$

$= C_p(T_1 - T_4) + p_4(V_1 - V_4)$

$3 \to 4$

Q. 断熱 $(dq = 0)$

$T_3 V_3^{k-1} = T_4 V_4^{k-1}$ $(\frac{PV}{T} = (-k))$

$P_3 V_3^k = P_4 V_4^k$

$V_4 = \left(\frac{P_3}{P_4}\right)^{1/k} \cdot V_3$

$1 \to 2$

$P_1 V_1^k = P_2 V_2^k$

$V_2 = \left(\frac{P_1}{P_2}\right)^{1/k} \cdot V_1$

$Q_{2\to3} = C_p(T_3 - T_2) + P_2\left(V_3 - \left(\frac{P_1}{P_2}\right)^{1/k} V_1\right)$

$Q_{4\to1} = C_p(T_4 - T_1) + P_4\left(\left(\frac{P_3}{P_4}\right)^{1/k} V_3 - V_1\right)$

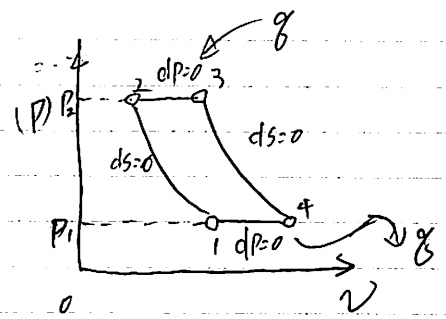
$S = \frac{Q}{T}$

~~$Q = \Delta S \cdot \Delta T$~~

エネルギー変換工学 Nov/5/2019

[ブレイトンサイクル]

$\eta = 1 - \frac{Q_{out}}{Q_{in}}$



$2 \to 3: Q_{23} = \int_2^3 dq = \int_2^3 du + \int p\,dv$

$= \int_2^3 C_V \Delta T$

$Q_{23} = C_V(T_3 - T_2)$ (>0)

$4 \to 1: Q_{41} = \int_4^1 dq = \int_4^1 du$

$= \int_4^1 C_V dT$

$Q_{41} = C_V(T_1 - T_4)$ (<0)

$\eta = 1 - \frac{Q_{out}}{Q_{in}} = 1 - \frac{(-Q_{41})}{Q_{23}}$

$= 1 - \frac{C_V(T_4 - T_1)}{C_V(T_3 - T_2)}$

$\eta = 1 - \frac{T_4 - T_1}{T_3 - T_2}$

圧力比の式をつかう(つくる)

(別ルート) ポアソン (断熱)

$1 \to 2: T_1 V_1^{k-1} = T_2 V_2^{k-1}$ $\left(\frac{PV}{T} = (一定)\right)$

$\Rightarrow T_1 = \left(\frac{V_2}{V_1}\right)^{k-1} T_2$

$3 \to 4: T_3 V_3^{k-1} = T_4 V_4^{k-1}$

$\Rightarrow T_3 = \left(\frac{V_4}{V_3}\right)^{k-1} T_4$

$= \left(\frac{\frac{T_4}{P_4}}{\frac{T_3}{P_3}}\right)^{k-1} \cdot T_4$

$T_3 = \left(\frac{P_3 T_4}{P_4 T_3}\right)^{k-1} \times T_4$

$= \left(\frac{P_3}{P_4}\right)^{k-1} \times \frac{T_4^k}{T_3^{k-1}}$

$\Rightarrow T_3^k = \left(\frac{P_3}{P_4}\right)^{k-1} \times T_4^k$

$T_3 = \left(\frac{P_3}{P_4}\right)^{(k-1)/k} \times T_4$

同様に,

$T_1 = \left(\frac{V_2}{V_1}\right)^{k-1} T_2$

$\Rightarrow T_1 = \left(\frac{P_1}{P_2}\right)^{(k-1)/k} \cdot T_2 \Rightarrow T_2 = \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} \cdot T_1$

$P_3 = P_2$, $P_4 = P_1$ より,

$T_3 = \left(\frac{P_2}{P_1}\right)^{(k-1)/k} \times T_4$

$\eta = 1 - \frac{T_4 - T_1}{\left(\frac{P_2}{P_1}\right)^{(k-1)/k} T_4 - \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} T_1}$

$= 1 - \frac{(T_4 - T_1)}{\left(\frac{P_2}{P_1}\right)^{(k-1)/k} (T_4 - T_1)}$

$= 1 - \left(\frac{P_1}{P_2}\right)^{(k-1)/k}$

$= 1 - \left(\frac{P_1}{P_2}\right)^{1 - \frac{1}{k}}$

$\frac{P_1}{P_2} = \delta$ 圧縮比

Date