Master Course Entrance Examination Problem Booklet

Information Physics and Computing

25th (Tuesday) August 2015 10:00~13:00

This booklet is the English version of the entrance examination problem booklet in Japanese for your assistance. Note that the most accurate expression of the problem is in the Japanese version.

Answer three problems out of the six problems.

Note:

(1) Do not open this booklet until the starting signal is given.

(2) You should notify the examiner if there are missing or incorrect pages in your booklet. No questions relating to the contents of the problems are acceptable in principle.

(3) Three answer sheets will be given. Use one sheet per a problem. You may use the back of the sheet if necessary.

(4) Do not forget to fill the examinee's number and the problem number in the designated place at the top of each answer sheet. Do never put your name.

(5) Do not separate the draft papers from this booklet.

(6) Any answer sheet with marks or symbols unrelated to the answer will be invalid.

(7) In the case that the problem can be interpreted in several ways, you may answer the problem adding suitable definitions or conditions to the original ones.

(8) Do not take the answer sheets and this booklet out of the examination room.

Examinee's number	No.

Fill this box with your examinee's number.

Problem numbers you selected			

Fill these boxes with numbers of three problems you selected.

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Problem 1

A causal IIR filter F_1 is expressed by the difference equation:

$$y[n]-\frac{1}{2}y[n-1]+\frac{13}{16}y[n-2]$$
$$=x[n]+\frac{1}{2}x[n-1]-\frac{3}{16}x[n-2]-\frac{1}{2}x[n-3]-\frac{13}{16}x[n-4]$$

where n is the discrete time, $y[n]$ is the output signal, $x[n]$ is the input signal whose values are 0 for durations of $n < 0$ and $n \geq N > 0$ for a certain N. Assume that all the IIR filters appearing in this problem yield the identically 0 output signal for the identically 0 input signal. Answer the following questions.

(1) Find the transfer function of F_1, $H_1(z)$, and its poles and zeros, and evaluate whether F_1 is BIBO stable or not. Calculate $\sum_{n=0}^{\infty} h_1[n]$ if it is converged, where $h_1[n]$ is the impulse response of F_1. Note that a system is called BIBO stable when the system yields a bounded output signal for any bounded input signal.

(2) Illustrate the poles and zeros of $H_1(z)$ by $\times$ and $\circ$, respectively, in the complex plane. On the basis of the pole/zero location, discuss which type of filter F_1 is categorized into the lowpass, highpass, bandpass, or bandstop filter, if you dare to categorize it.

(3) A causal IIR filter F_2 is expressed by the difference equation:

$$y[n]-\frac{1}{2}y[n-1]+\frac{13}{16}y[n-2]$$
$$=\frac{13}{16}x[n]+\frac{1}{2}x[n-1]+\frac{3}{16}x[n-2]-\frac{1}{2}x[n-3]-x[n-4]$$

(a) Let the transfer function of F_2 be denoted by $H_2(z)$. Supposing $H_2(z)=H_1(z)H_3(z)$, illustrate the poles and zeros of $H_3(z)$ by $\times$ and $\circ$, respectively, in the complex plane.

(b) Let the normalized angular frequency be denoted by ω $(0 \leq \omega \leq \pi)$. Find the amplitude response of $H_3(z)$, $|H_3(\exp(j\omega))|$. Show that the phase response of $H_3(z)$ satisfies $\arg H_3(\exp(j\omega)) \leq 0$.

(c) What kind of useful property in signal processing is provided in the systems having such a pole/zero location as $H_3(z)$ has?

(4) Let the causal IIR filter having an impulse response $h[n]$ be denoted by F and the anti-causal IIR filter having the impulse response $h^{\mathrm{R}}[n]=h[-n]$ be denoted by F^{R}. Let the series of operations of applying F followed by F^{R} be denoted by $\mathrm{F}^{\mathrm{R}} \circ \mathrm{F}$.

(a) Find the difference equation of $\mathrm{F}_1^{\mathrm{R}}$, its transfer function, $H_1^{\mathrm{R}}(z)$, and its poles and zeros, and evaluate whether $\mathrm{F}_1^{\mathrm{R}}$ is BIBO stable or not.

(b) Find the amplitude response of $\mathrm{F}_1^{\mathrm{R}}$, $|H_1^{\mathrm{R}}(\exp(\mathrm{j}\omega))|$, and the phase response of $\mathrm{F}_1^{\mathrm{R}}$, $\arg H_1^{\mathrm{R}}(\exp(\mathrm{j}\omega))$, by using the amplitude response of F_1, $|H_1(\exp(\mathrm{j}\omega))|$ and the phase response of F_1, $\arg H_1(\exp(\mathrm{j}\omega))$, respectively.

(c) Let the transfer function of $\mathrm{F}_1^{\mathrm{R}} \circ \mathrm{F}_1$ be denoted by $H_1^{\mathrm{RL}}(z)$. Find $\arg H_1^{\mathrm{RL}}(\exp(\mathrm{j}\omega))$. What kind of useful property in signal processing is provided in $\mathrm{F}_1^{\mathrm{R}} \circ \mathrm{F}_1$?

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Problem 2

Figures 1, 2, 3, and 4 show operational amplifier circuits. All the operational amplifiers can be assumed to be ideal. Answer the following questions.

(1) In the circuit shown in Fig. 1, switches φ_1 and φ_2 turn ON and OFF repeatedly, where a clock signal is provided to the switches such that φ_2 is OFF when φ_1 is ON and φ_2 is ON when φ_1 is OFF. Obtain the input-output relation from input voltages V_{in1} and V_{in2} to output voltage V_{out} for each of the periods that φ_1 is ON and φ_2 is ON. Also, find the average output voltage when the duty ratio of the clock signal is 50%, which means that the ON and OFF periods of each switch are equal.

(2) In the circuit shown in Fig. 1, the switching frequency for switches φ_1 and φ_2 is f. Since the output signal contains the frequency components relevant to the switching, let us consider removing these frequency components by a lowpass filter circuit shown in Fig. 2, where we assume the frequency of the input signal is much lower than f. Specify the relationship between f and circuit parameters R_1, R_2, R_3, and C_4 such that the voltage gain at f is 20 dB smaller than the voltage gain at the cutoff frequency of the lowpass filter circuit in Fig. 2.

(3) Find output voltage V_{out} by using input voltage V_{in} (> 0) for each of circuits shown in Figs. 3 and 4. Assume that the diode has the following property: the relationship between the forward electric current I_A and voltage V_A is approximated by $I_A = I_S \exp(V_A/U_T)$, where $-I_S$ and U_T denote the reverse saturation current and the thermal voltage, respectively.

(4) It is possible to design a circuit such that output voltage V_o is in proportion to the product of input voltages V_1 (> 0) and V_2 (> 0), V_1V_2, by using the operational amplifier circuits shown in Figs. 1, 2, 3, and 4 with setting appropriate capacitance and resistance ratios. Illustrate a circuit diagram to realize it. Find also the relationship between the output voltage and the input voltages for the circuit. Note that the frequency components relevant to the switching are assumed to be ignored by using the lowpass filter circuit in Fig. 2.

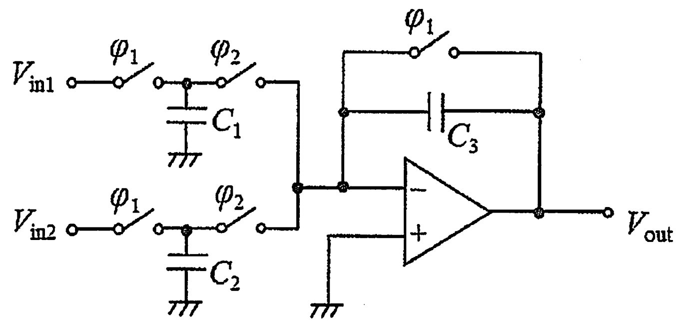

Fig. 1

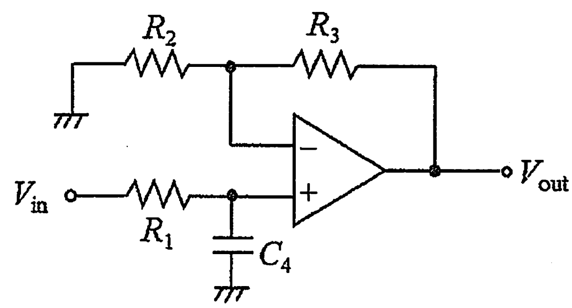

Fig. 2

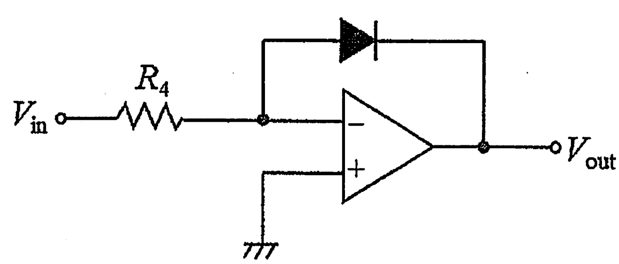

Fig. 3

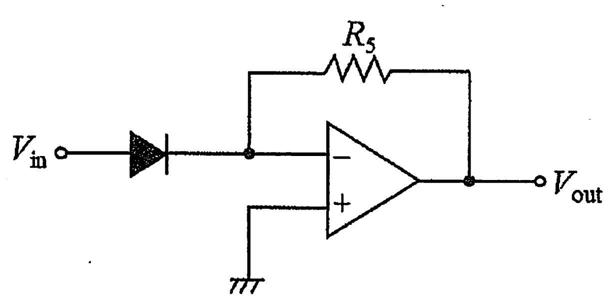

Fig. 4

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Problem 3

Consider the closed loop system composed of a plant:

$$P(s) = \frac{1 + T_3 s}{(1 + T_1 s)(1 + T_2 s)}$$

and a controller $C(s) = K$ (K: constant gain) as shown in Fig. 1, where $T_1 > |T_3| > T_2 > 0$. Let $S(s)$ denote the transfer function from r to e. Answer the following questions.

(1) (a) Illustrate the Bode gain plot of $P(s)$ by the straight-line approximation as similar to an example in Fig. 2. In particular, specify the break angular frequencies, the slopes, and the approximated gains at the break angular frequencies.

(b) Illustrate the unit step response of $P(s)$ in the case of $T_3 > 0$ and that of $P(s)$ in the case of $T_3 < 0$ as similar to an example in Fig. 3. In particular, specify the slopes at $t = 0$ and the steady-state values at $t = \infty$.

(2) Derive the condition on K for stability of the closed loop system.

(3) Assume $T_3 > 0$.

(a) Obtain the formula of $|S(\mathrm{j}\omega)|^2$. By using this formula, find the sign of $\frac{\mathrm{d}}{\mathrm{d}K}|S(\mathrm{j}\omega)|$, where $|\omega| < \infty$, in the region of $K \geq 0$ such that the closed loop system is stable.

(b) Prove the following inequality:

$$-\frac{\pi}{2} \leq \arg P(\mathrm{j}\omega) \leq 0, \ \forall \omega \geq 0$$

(c) Explain the result on the sign in Question (3)–(a) by using the property of $P(s)$ shown in Question (3)–(b).

(d) Discuss meaning of the result on the sign in Question (3)–(a) from the viewpoint of control engineering.

(4) Assume that $T_1 = 1$, $T_2 = \frac{1}{20}$, $T_3 = -\frac{1}{7}$, $\omega_1 = 1$, and $\omega_2 = 20$.

(a) Find the signs of $\dfrac{\mathrm{d}}{\mathrm{d}K}|S(\mathrm{j}\omega_1)|$ and $\dfrac{\mathrm{d}}{\mathrm{d}K}|S(\mathrm{j}\omega_2)|$ in the region of $K \geq 0$ such that the closed loop system is stable.

(b) Discuss meaning of the result of Question (4)–(a) from the viewpoint of control engineering in comparison with the discussion in Question (3)–(d).

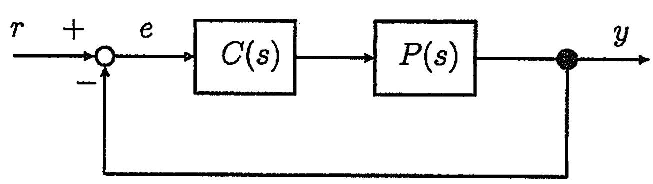

Fig. 1 Closed loop system

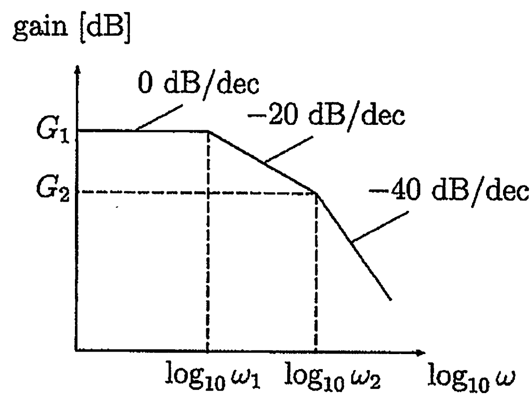

Fig. 2 An example of Bode gain plot

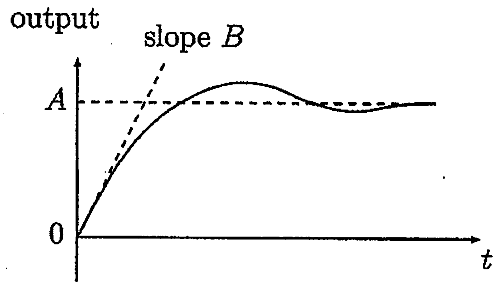

Fig. 3 An example of unit step response

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Problem 4

Answer the following questions on the 8-bit CPU with the instruction set architecture in Fig. 3. Note that the prefix "0x" indicates that the following number is hexadecimal.

(1) Convert the assembly instructions in Fig. 1 into the corresponding machine code instructions and show them in hexadecimal binary format.

```
SET    0x08
COPY   r1, r0
STORE  [r1], r0
```

Fig. 1

(2) The binary list "0x08 0xE8 0x04 0xC8" is executed as machine code instructions. Show the value of register r1 after executing it.

(3) Consider the instruction execution time in Fig. 2. This CPU takes 1 cycle per instruction if no cache miss occurs, and it has separate instruction and data caches. The cache miss penalties are 20 cycles for both instructions and data, and the clock frequency is 20 Hz. The cache hit rates of the instructions in Fig. 2 were 95% for instructions and 75% for data. Answer the following questions.

```
SET    12
COPY   r1, r0
SET    0
STORE  [r1], r0
DEC    r1
JNZ    -3
SET    1
```

Fig. 2

(a) Show the total number of instructions executed.
(b) Show the total number of data memory accesses.
(c) Show the total cycles of the instruction cache miss penalty.
(d) Show the total execution time of these instructions in seconds.

(4) Let us calculate the sum of 10 unsigned 8-bit data stored at the 10-byte memory area starting with address 0x10. The calculated sum is stored at address 0x08. The overflow can be ignored. Answer the following questions.

(a) Show assembly instructions to store 0 as the initial value to address 0x08.
(b) Show assembly instructions to store 0x10, the address of the data, to register r2 and store 10, the number of the data to be added, to register r3.
(c) Show assembly instructions to load the value at address 0x08 to register r0, to add the value at the address pointed by register r2 to it, and to store the result value to address 0x08.

(d) Show assembly instructions to achieve the original purpose by referring to the results of Questions (4)-(a), -(b), and -(c).

(e) Show optimization techniques to reduce the number of instructions of the result of Question (4)-(d).

The instruction set architecture

○ Register set (8 bit)

register r0	register r1	register r2	register r3
pc register	flag register		

○ Instruction format

i-format:

3 bit	5 bit
opcode	operand

r-format:

3 bit	2 bit	2 bit	
opcode	dst	src	0

○ Assembly instruction list

```
SET imm (i-format)            example:  SET   0x10
    format:   opcode = 0, operand = imm (immediate value)
    behavior: r0 ← imm (unsigned 8-bit extension) ; pc ← pc + 1
JNZ imm (i-format)            example:  JNZ   -4
    format:   opcode = 1, operand = imm (offset)
    behavior: if flag == 0 then pc ← pc + 1 + imm (signed 8-bit extension)
                          else pc ← pc + 1
LOAD dst, [src] (r-format)    example:  LOAD  r0, [r1]
    format:   opcode = 2, dst = dest. reg., src = memory address reg.
    behavior: dst ← MEM[src]; pc ← pc + 1
STORE [dst], src (r-format)   example:  STORE [r0], r1
    format:   opcode = 3, dst = memory address reg., src = source reg.
    behavior: MEM[dst] ← src; pc ← pc + 1
INC dst (r-format)            example:  INC   r1
    format:   opcode = 4, dst = destination register, src = unused
    behavior: dst ← dst + 1; pc ← pc + 1
DEC dst (r-format)            example:  DEC   r1
    format:   opcode = 5, dst = destination register, src = unused
    behavior: dst ← dst - 1; if dst == 0 then flag ← 0 else flag ← 1;
              pc ← pc + 1
ADD dst, src (r-format)       example:  ADD   r0, r1
    format:   opcode = 6, dst = destination reg., src = source reg.
    behavior: dst ← dst + src (unsigned); pc ← pc + 1
COPY dst, src (r-format)      example:  COPY  r0, r1
    format:   opcode = 7, dst = destination reg., src = source reg.
    behavior: dst ← src; pc ← pc + 1
```

Note that ← means assignment. MEM[reg] is the memory access of an address reg. Immediate and offset values are hexadecimal if they are starting with 0x, otherwise decimal. The values of dst and src are r0 = 0, r1 = 1, r2 = 2, and r3 = 3.

Fig. 3

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Problem 5

Consider the compass-like biped robot shown in Fig. 1. This robot has masses only at the hip and the tips of the legs. The leg with a mass of tip m_1 is called Leg 1 and the other with a mass of tip m_2 is called Leg 2. The mass of hip is M. These are supposed to be mass points. The leg touching on the inclined ground is called support leg and the other separated from the ground is called swing leg. Let b be the length of both legs, γ ($\gamma > 0$) be the angle of the ground, and g be the gravity acceleration. Here, assume that the tip of the support leg does not slip and bounce and there is no friction in the rotation motion of the swing leg. Suppose that angles of the legs are defined as shown in Fig. 1 and the arrow directions in the figure indicate positive. In this problem, the motion is restricted to the two dimensional space in Fig. 1. However, assume the swing leg does not touch the ground and the robot continues the motion when Leg 1 and Leg 2 intersect each other without being in touch. Answer the following questions.

(1) As shown in Fig. 1, set the axis origin O to the tip of the support leg, the x axis to the downward direction of the ground, and the y axis to be perpendicular to the x axis. On the basis of this coordinate system, obtain the position of the hip and the positions of the both leg tips. Also obtain the potential energy of this system with reference to the axis origin.

(2) Obtain the kinetic energy of this system in the form of $\frac{1}{2}\boldsymbol{\omega}^{\mathrm{T}}V\boldsymbol{\omega}$ by using angular velocity vector $\boldsymbol{\omega} = \left[\frac{d\theta}{dt}, \frac{d\varphi}{dt}\right]^{\mathrm{T}}$. $\boldsymbol{x}^{\mathrm{T}}$ denotes the transpose of $\boldsymbol{x}$.

As shown in Fig. 2, Leg 2 leaves the ground at the moment that Leg 1 lands on the ground. In this way, the swing and the support legs switch and the next biped walking starts. Assume that in this transition Leg 1 lands without slipping and bouncing and the impact with the ground is perfectly inelastic. Let τ $(0 < \tau < \frac{\pi}{2})$ be the angle between both legs at the landing. In Questions (3) and (4), suppose that $m_1 = m_2 = m$.

(3) The angular momentum of the system about the landing point (tip of Leg 1) is conserved before and after the landing of Leg 1. Also the angular momentum of Leg 2 about the hip is conserved. Using these conditions, the relationship between the angular velocity vector just before the landing, $\boldsymbol{\omega}^{-}$, and the angular velocity vector

just after the landing, $\boldsymbol{\omega}^+$, is described as $\boldsymbol{\omega}^+ = K\boldsymbol{\omega}^-$. Obtain matrix K.

(4) Consider the system continues the stable biped walking with the same angle between both legs at the landing, τ.

(a) Obtain the difference in potential energy of this system, ΔU, between the potential energy just after the landing of the swing leg in the $(n-1)$th step and the potential energy just before the landing of the swing leg in the nth step.

(b) Show the condition to keep the stable motion by using the angular velocity vector just before the landing, $\boldsymbol{\omega}^-$, the matrix expressing the state just before the landing, V^-, matrix K of Question (3), and the difference of the potential energy of Question (4)-(a), ΔU. Here, $\frac{1}{2}\boldsymbol{\omega}^{-\mathrm{T}} V^- \boldsymbol{\omega}^-$ is the kinetic energy just before the landing.

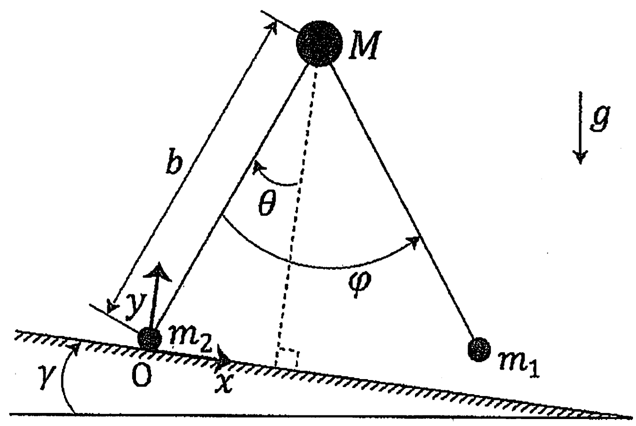

Fig. 1

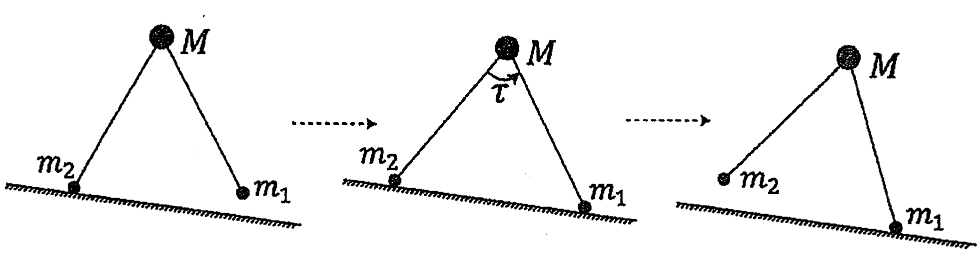

Fig. 2

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Problem 6

As shown in Fig. 1, consider a parallel plate capacitor C_1 which consists of two conducting plates P_1 and P_2 and two dielectrics L_1 and L_2. The charges Q and $-Q$ are put on P_1 and P_2, respectively. The cross sections of P_1, P_2, L_1, and L_2 parallel to the yz-plane are squares of side length b, and the thicknesses of L_1 and L_2 are $x_0 (\ll b)$. The permittivities of L_1 and L_2 are ϵ_1 and ϵ_2, respectively. In this problem, assume that the effect at the edges of P_1 and P_2 can be neglected so that the electric field by the charges Q and $-Q$ is uniform between P_1 and P_2. Assume also that the electric flux density $\boldsymbol{D}$ and the electric field $\boldsymbol{E}$ satisfy $\boldsymbol{D} = \epsilon \boldsymbol{E}$ in a dielectric of permittivity ϵ. Answer the following questions.

(1) Find the electric field and the electric flux density in dielectrics L_1 and L_2. Also, obtain the capacitance of C_1.

(2) Let U_1 be the energy stored in C_1 in Question (1). Also, let U_2 be the energy stored in capacitor C_2 which is the same as C_1 except that the thicknesses of L_1 and L_2 are $x_0 + dx$ and $x_0 - dx$, respectively, as shown in Fig. 2. The charges Q and $-Q$ are put on P_1 and P_2 in C_2, respectively. Find $U_2 - U_1$. Also, obtain the force acting on the interface between L_1 and L_2 in C_1.

Now, as shown in Fig. 3, consider a parallel plate capacitor C_3 which consists of two conducting plates P_1 and P_2 and dielectric L. P_1 and P_2 are the same as those in Fig. 1, and the charges Q and $-Q$ are put on them, respectively. The cross section of L parallel to the yz-plane is a square of side length b, and the thickness of L is $2x_0$. L has a cylindrical hole of radius a centered at the z-axis. Assume that $a \ll 2x_0 \ll b$ and the electric field on the surface of L at $x = \pm x_0$ and $y = \pm \frac{b}{2}$ is uniform and parallel to the x-axis. Assume also that the electric field in L and that in the cylindrical hole are uniform with respect to the z-axis. The permittivities inside L and the cylindrical hole are ϵ and ϵ_0, respectively. Answer the following questions.

(3) Obtain the polarization charge on the cylindrical hole surface at $x = a\cos\theta, y = a\sin\theta$.

(4) Let $V_0(x, y)$ and $V_1(x, y)$ be the electric potential inside the cylindrical hole and L, respectively.

(a) Show that $V_k(x, y)$ satisfies the two-dimensional Laplace equation

$(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2})V_k(x, y) = 0$ for $k = 0, 1$.

(b) Write the boundary conditions for $V_0(x, y)$ and $V_1(x, y)$ at the cylindrical surface.

(c) The general solution to the two-dimensional Laplace equation with symmetry about the x-axis can be represented in polar coordinates as

$$V(r, \theta) = \sum_{n=1}^{\infty} \left(a_n r^n + \frac{b_n}{r^n} \right) \cos n\theta + c \log r + d,$$

where log is the natural logarithm and a_n, b_n, c, and d are constants. Using this, obtain V_0 and V_1.

(d) Discuss how to estimate the radius a of the cylindrical hole from measurements of V_1 on the line segment $\{(x, A, 0) \mid -x_0 \leq x \leq x_0\}$ for a constant A, where $a < A \ll \frac{b}{2}$.

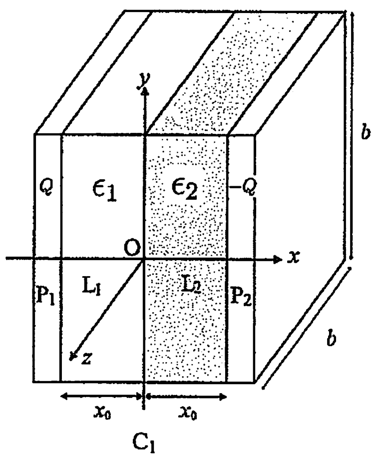

Fig. 1

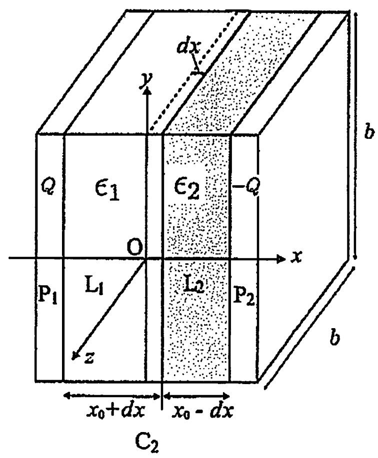

Fig. 2

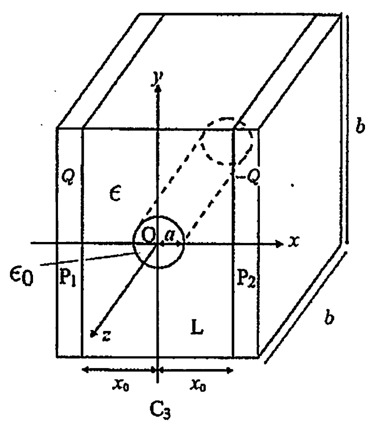

Fig. 3

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