Master Course Entrance Examination Problem Booklet

Information Physics and Computing

Tuesday, August 23, 2016 10:00~13:00

Answer three out of Problems 1-5.

Note:

(1) Do not open this booklet until the starting signal is given.

(2) You should notify the examiner if there are missing or incorrect pages in your booklet. No questions relating to the contents of the problems are acceptable in principle.

(3) Three answer sheets will be given. Use one sheet per a problem. You may use the back of the sheet if necessary.

(4) Do not forget to fill the examinee's number and the problem number in the designated place at the top of each answer sheet. Do never put your name.

(5) Do not separate the draft papers from this booklet.

(6) Any answer sheet with marks or symbols unrelated to the answer will be invalid.

(7) In the case that a problem can be interpreted in several ways, you may answer the problem adding suitable definitions or conditions.

(8) Do not take the answer sheets and this booklet out of the examination room.

Examinee's number	

Fill this box with your examinee's number.

Problem numbers you selected			

Fill these boxes with the problem numbers you selected.

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Problem 1

Consider a causal discrete-time linear time-invariant system whose system function is given by

$$H(z) = \frac{1}{1 - a^2 z^{-2}} ,$$

where a is a real constant and $0 < |a| < 1$. Let $y[n]$ be the output signal when white noise $\epsilon[n]$ with zero mean and variance σ^2 is input to this system, where n is the discrete time. Answer the following questions.

(1) Obtain the impulse response of system $H(z)$. Derive the difference equation that $\epsilon[n]$ and $y[n]$ satisfy.

(2) Obtain the autocorrelation function $R_{yy}[m]$ of $y[n]$ and its power spectral density $S_{yy}(\exp(\mathrm{j}\,\omega))$.

(3) Let us predict $y[n]$ with $\hat{y}[n] = -\sum_{k=1}^{K} c_k y[n-k]$. Define the prediction error $e[n] := y[n] - \hat{y}[n]$.

   (a) Determine the coefficients $c_1, c_2, \ldots, c_K$ such that the mean squared error $E[|e[n]|^2]$ is minimized, where E denotes the expectation operation.

   (b) Discuss the relationship between the coefficients $c_1, c_2, \ldots, c_K$ that are obtained in Question (3)-(a) and $H(z)$.

   (c) Obtain the autocorrelation function of $e[n]$.

(4) Suppose that $y[n]$ is not observed at a certain time n. Discuss a method for estimating $y[n]$ from the observed $y[n-1], y[n-2], y[n+1]$, and $y[n+2]$. Also, compare the estimation error obtained by the method and the prediction error obtained in Question (3) with $K = 4$.

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Problem 2

The operation of an operational amplifier circuit with resistors connected can be analyzed by using a block diagram as shown in Fig. 1, where $G(s)$, $V_I(s)$, $V_O(s)$, and $V_F(s)$ are the open loop gain, the input voltage, the output voltage of the operational amplifier, and the input voltage to $G(s)$, respectively.

When Fig. 1 is made correspondence with an actual operational amplifier circuit, p and q are either + (non-inverting input terminal) or – (inverting input terminal), α is the input gain from $V_I(s)$ to $V_F(s)$ with $V_O(s)$ grounded, and β is the feedback gain from $V_O(s)$ to $V_F(s)$ with $V_I(s)$ grounded.

As to the specification of the operational amplifier in this problem, $G(s)=A/(1+Ts)$ ($A>0$, $T>0$), the input impedance is infinite, and the output impedance is zero. Answer the following questions.

(1) Set p and q to – and +, respectively, in Fig. 1. Find the transfer function from $V_I(s)$ to $V_O(s)$.

(2) Consider the operation of a non-inverting amplifier as shown in Fig. 2 according to Fig. 1. In this case, the electric circuits as shown in Figs. 3 and 4 can be used for the calculation of α and β, respectively.

(a) Find the transfer function in Fig. 2.

(b) Show that the circuit in Fig. 2 is stable.

(c) Assume that A is infinite. Find the transfer function, $H_1(s)$, and the input impedance, Z_1.

(3) Consider the operation of an inverting amplifier as shown in Fig. 5 according to Fig. 1.

(a) Find the transfer function in Fig. 5.

(b) Discuss arising problems when a conventional operational amplifier, in which A is significantly greater than 1 but finite, is connected to resistors as shown in Fig. 6.

(c) Assume that A is infinite in Fig. 5. Find the transfer function, $H_2(s)$, and the input impedance, Z_2.

(d) Consider that non-inverting amplifier $H_1(s)$ and inverting amplifier $H_2(s)$ are connected in series in two cases, i.e., $H_1(s)$ is chosen for the first stage and $H_2(s)$ is chosen for the first stage. Discuss appropriate uses for each of the two connections.

(4) Consider the operation of an operational amplifier circuit as shown in Fig. 7.

(a) Find the transfer function in Fig. 7.

(b) Show that $R_5 > R_7$ is a sufficient condition for stability of the circuit in Fig. 7.

(c) Assume that A is infinite as well as $R_5 > R_7$. Find the transfer function, $H_3(s)$, and the input impedance, Z_3.

(d) Discuss whether or not operational amplifier circuit $H_3(s)$ can be replaced with either a non-inverting or an inverting amplifier with infinite A.

(e) According to the discussion in Question (4)-(d), discuss an appropriate use of operational amplifier circuit $H_3(s)$.

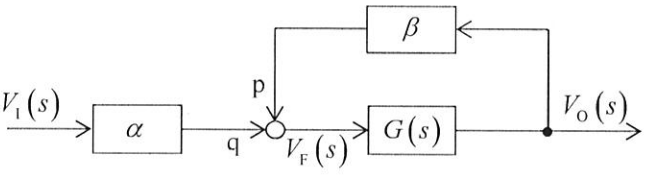

Fig. 1

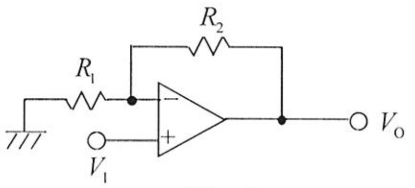

Fig. 2

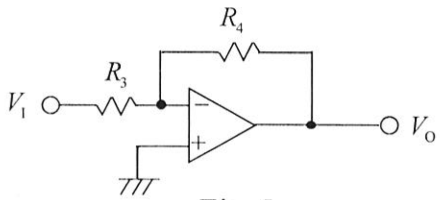

Fig. 5

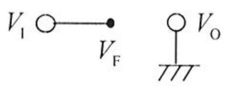

Fig. 3

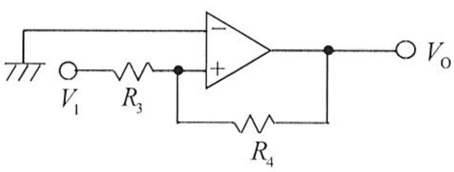

Fig. 6

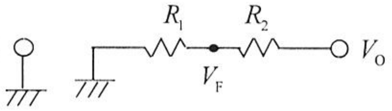

Fig. 4

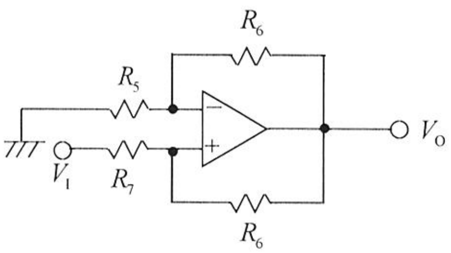

Fig. 7

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Problem 3

Consider a plant P given by a state space representation;

$$P: \quad \begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t}x(t) &= Ax(t) + Bu(t), \\ y(t) &= Cx(t), \end{aligned}$$

where $x(t)$, $u(t)$, and $y(t)$ are the state vector, the input, and the output, respectively. Answer the following questions.

(1) Assume that

$$A = A_{\mathrm{O}} := \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}, \quad B = B_{\mathrm{O}} := \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad C = C_{\mathrm{O}} := \begin{bmatrix} -1 & 1 \end{bmatrix}. \tag{1}$$

(a) Explain the controllability and the observability of P in this case.

(b) Find $F = F_{\mathrm{O}}$ of a state feedback $u(t) = Fx(t)$ which assigns the poles of the closed loop system to -1 and -2.

(2) Consider a system G given by a state space representation;

$$G: \quad \begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t}\hat{x}(t) &= A\hat{x}(t) + Bv(t) - Le(t), \\ e(t) &:= z(t) - C\hat{x}(t), \end{aligned} \tag{2}$$

where $\hat{x}(t)$ is the state vector, and $v(t)$ and $z(t)$ are certain signals.

(a) Assume $v(t) = u(t) = F\hat{x}(t)$ and $z(t) = y(t)$. In this case, G can be regarded as an observer of P. Explain the pole distribution of the augmented system composed of P and G.

(b) Find $L = L_{\mathrm{O}}$ which assigns the poles of the augmented system to -1, -2, -3, and -4 in the case of $A = A_{\mathrm{O}}$, $B = B_{\mathrm{O}}$, $C = C_{\mathrm{O}}$, and $F = F_{\mathrm{O}}$ in Question (1).

(3) On system G in Question (2), assume $v(t) = F\hat{x}(t) + w(t)$, where $w(t)$ is a new exogenous input.

(a) Let us express the transfer characteristic from the signal $\begin{bmatrix} w^\top(t) & e^\top(t) \end{bmatrix}^\top$ to the signal $\begin{bmatrix} v^\top(t) & z^\top(t) \end{bmatrix}^\top$ as

$$\begin{bmatrix} v(s) \\ z(s) \end{bmatrix} = \begin{bmatrix} D(s) & U(s) \\ N(s) & V(s) \end{bmatrix} \begin{bmatrix} w(s) \\ e(s) \end{bmatrix}, \tag{3}$$

where $v(s)$, $z(s)$, $w(s)$, and $e(s)$ denote the Laplace transforms of $v(t)$, $z(t)$, $w(t)$, and $e(t)$, respectively. Represent $D(s)$, $N(s)$, $U(s)$, and $V(s)$ by using A, B, C, F, and L.

(b) Find the transfer functions $D(s)$, $N(s)$, $U(s)$, and $V(s)$ in the case of $A = A_\mathrm{O}$, $B = B_\mathrm{O}$, $C = C_\mathrm{O}$, and $F = F_\mathrm{O}$ in Question (1) and $L = L_\mathrm{O}$ in Question (2).

(4) In Question (3), assume signal $w(t)$ as $w(s) = Q(s)e(s)$, where $Q(s)$ is a certain transfer function. We regard this system G as a system named K, where $z(t)$ and $v(t)$ are the input and the output, respectively. In other words, $v(s) = K(s)z(s)$, where $K(s)$ is the transfer function of K.

(a) Represent $K(s)$ by using $D(s)$, $N(s)$, $U(s)$, $V(s)$, and $Q(s)$.

(b) Assume $u(t) = v(t)$ and $z(t) = y(t)$. In this case, the whole closed loop system can be regarded as a feedback system composed of P and K. In the case of $A = A_\mathrm{O}$, $B = B_\mathrm{O}$, $C = C_\mathrm{O}$, and $F = F_\mathrm{O}$ in Question (1) and $L = L_\mathrm{O}$ in Question (2), rewrite a function

$$S(s) := \frac{1}{1 - P(s)K(s)} \tag{4}$$

by using $Q(s)$, where $P(s)$ is the transfer function of P. Also explain the condition on $Q(s)$ such that the closed loop system is stable.

(c) Discuss roles of $Q(s)$ in the control system design.

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Problem 4

Answer the following questions on computer systems.

(1) Answer the following questions on the truth table of Table 1.

(a) Show the Boolean equations of the output X and Y in the minimal sum-of-products form.

(b) Sketch a schematic of combinational logic corresponding to the truth table by using 2-input 1-output AND, OR, and XOR gates in MIL symbols, where the number of gates is as small as possible.

(c) Four logic circuits, each of which is obtained in Question (1)-(b), are connected as shown in Fig. 1. When 1101 is given to $A_4,\ldots,A_1$, 0111 is to $B_4,\ldots,B_1$, and 1 is to C_1, find Y_4 and $X_4,\ldots,X_1$ with the reason.

Table 1

Input			Output	
A	B	C	X	Y
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

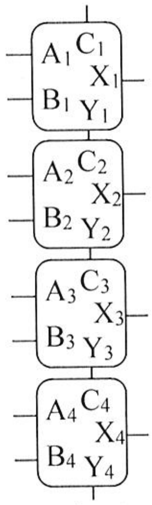

Fig. 1

(2) The logic circuit shown in Fig. 2 is constructed by connecting the circuit shown in Fig. 1 with input and output registers. The registers are constructed with flip-flops. The output of this logic circuit must be fixed within one clock cycle. The propagation delay of each circuit from A and B to X and Y is 30 ps, and that from C to X and Y is 20 ps. The flip-flop setup and hold times are 35 ps and 10 ps, respectively. Answer the following questions.

(a) Show the maximum operating frequency of this circuit in GHz when there is no clock skew.

(b) Show the allowable maximum clock skew when this circuit is operated in 5 GHz.

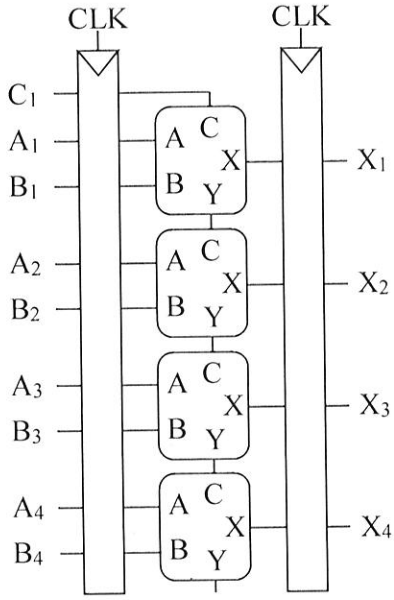

Fig. 2

(3) Answer the following questions on the 32 bit MIPS program in Fig. 3. Fig. 4 shows a subset of the MIPS instruction set, where MIPS uses byte addressing, $0 is a register that always returns 0, $sp is a register that stores the address of the stack, and $ra is a register that stores a return address.

(a) Show the return value in $v0 when the procedure "func" is called with 3 in $a0.

(b) Draw the contents of the stack when the procedure "func" is called with 5 in $a0 and returned.

(c) Show the return value in $v0 by using n when the procedure "func" is called with n in $a0. Also, show the number of bytes consumed in the stack.

```
func:
        addi    $sp, $sp, -12
        sw      $a0, 4($sp)
        sw      $ra, 0($sp)
        addi    $t0, $0, 2
        slt     $t0, $a0, $t0
        beq     $t0, $0, else
        addi    $v0, $0, 1
        addi    $sp, $sp, 12
        jr      $ra
else:
        addi    $a0, $a0, -1
        jal     func
        add     $v0, $v0, $v0
        sw      $v0, 8($sp)
        addi    $a0, $a0, -1
        jal     func
        lw      $t0, 8($sp)
        add     $v0, $v0, $t0
        lw      $ra, 0($sp)
        lw      $a0, 4($sp)
        addi    $sp, $sp, 12
        jr      $ra
```

Fig. 3

```
MIPS Instruction Set
(a name starting with $ is a register)
addi    $a, $b, c
  add $b and c then store the result to $a
add     $a, $b, $c
  add $b and $c then store the result to $a
sw      $a, offset($b)
  store $a to the memory at $b+offset
lw      $a, offset($b)
  load the memory at $b+offset to $a
slt     $a, $b, $c
  if $b < $c then store 1 to $a,
  otherwise store 0 to $a
beq     $a, $b, c
  if $a = $b then jump to the label c,
  otherwise go to the next instruction
jal     a
  store the address of the next instruction
  to $ra, and jump to the label a
jr      $a
  jump to the address $a
j       a
  jump to the label a
```

Fig. 4

(4) Rewrite the program in Fig. 3 so that it runs faster and consumes less stack. Explain the contents of the program and the reason why it runs faster.

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Problem 5

As shown in Fig.1, a string A_1 with a mass point M_1 of mass m fixed in the middle of the string is stretched with a tension T and attached perpendicularly to two parallel walls separated by a distance L. Here, assume that gravity force, the size of the mass point, the diameter of the string, and the mass except the mass point are negligible. The motion of the mass point is restricted on the x axis. Answer the following questions.

(1) The force necessary to keep mass point M_1 with a small displacement Δx ($|\Delta x| << L$) from its equilibrium position can be described in the form of $F = k\Delta x$. Obtain the equivalent spring constant k. In the following questions, the movement of mass point M_1 can be approximated by that of a mass point of mass m attached to a spring with the equivalent spring constant k.

(2) Let $x_1(t)$ be the displacement of mass point M_1 from its equilibrium position at time t. Mass point M_1 is subject to damping force proportional to its velocity (damping coefficient $c > 0$). Describe the general solution of $x_1(t)$, and explain the movement of mass point M_1. Here, damping force acts only on mass point M_1.

(3) In the following questions, the damping force is negligible. As shown in Fig.2, another string A_2 having the same characteristics as A_1 is stretched with a tension T and attached perpendicularly to the walls with a distance a from A_1. Two mass points M_1 and M_2 are connected with a coil spring (natural length a, spring constant S). Let $x_1(t)$ and $x_2(t)$ be the displacements of mass points M_1 and M_2 at time t, respectively, from each equilibrium position.

(a) Obtain $x_1(t)$ and $x_2(t)$ under the following initial condition at time $t = 0$:

$$x_1 = X_0\ (0 < |X_0| < a), \quad x_2 = 0, \quad \frac{dx_1}{dt} = 0, \quad \frac{dx_2}{dt} = 0.$$

(b) Assume that the spring constant S of the coil spring is significantly small compared with the equivalent spring constant k of strings A_1 and A_2. Explain by using equations about a "beat phenomenon" which is observed under this condition. Then, obtain the relationship between the beat frequency and the eigenfrequency of string A_1 alone as shown in Fig.1.

(c) Under the condition described in Question (3)-(b), sketch the time-courses of $x_1(t)$ and $x_2(t)$.

(4) As shown in Fig.3, another string A_3 having the same characteristics as A_1 is added to the system described in Fig.2. Let $x_1(t)$, $x_2(t)$ and $x_3(t)$ be the displacements of mass points M_1, M_2 and M_3 at time t, respectively, from each equilibrium position. Obtain the eigenmodes of the system, and describe the movements of the mass points in each eigenmode by using equations and figures.

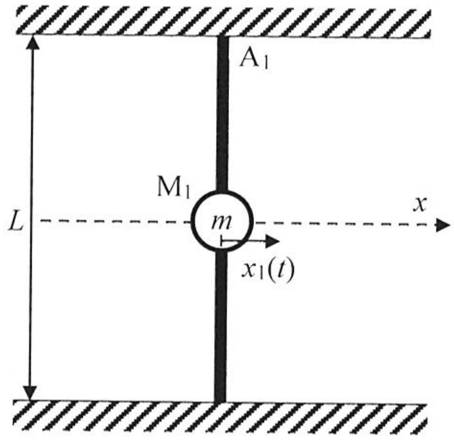

Fig.1

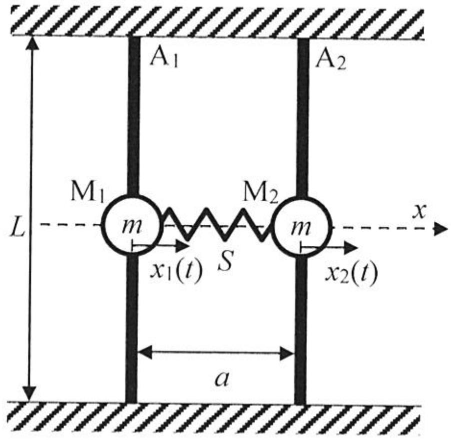

Fig.2

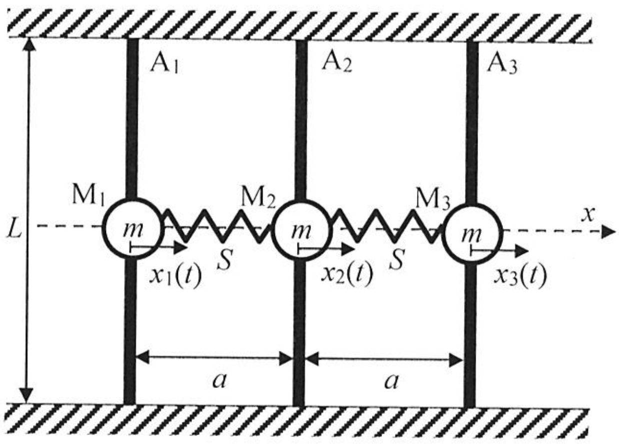

Fig.3

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