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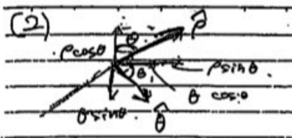
Date.

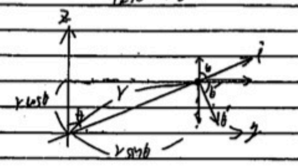
(2)



図にかける

$\rho \sin\theta + \theta\cos\theta = y$

$\rho\cos\theta - \theta\sin\theta = z$



$\rho\sin\theta\cos\theta + \theta\cos^2\theta = y\cos\theta$

$-)\ \rho\sin\theta\cos\theta - \theta\sin^2\theta = z\sin\theta$

$\underline{\theta = y\cos\theta - z\sin\theta}$ $\quad B_\theta = B_y\cos\theta - B_z\sin\theta$

$\rho\sin^2\theta + \theta\sin\theta\cos\theta = y\sin\theta$

$+)\ \rho\cos^2\theta - \theta\sin\theta\cos\theta = z\cos\theta$

$\underline{\rho = y\sin\theta + z\cos\theta}$ $\quad B_r = B_y\sin\theta + B_z\cos\theta$

$\rho = y\sin\theta + z\cos\theta$.

$= -\alpha\dfrac{3yz}{r^5}\sin\theta + \alpha\dfrac{2z^2 - y^2}{r^5}\cos\theta$.

$\rho = \sqrt{y^2 + z^2} = r$

$y = r\sin\theta,\ z = r\cos\theta$ を (1) に代入

$B_y = \alpha\dfrac{3yz}{r^5}$, $\quad B_z = \alpha\dfrac{2z^2 - y^2}{r^5}$

$= \alpha\dfrac{3\sin\theta\cos\theta}{r^3}$ $\quad B_z = \alpha\dfrac{2r^2\cos^2\theta - r^2\sin^2\theta}{r^5} = \alpha\dfrac{2\cos^2\theta - \sin^2\theta}{r^3}$

No.
Date.

$B_r = B_y\sin\theta + B_z\cos\theta$.

$= \dfrac{\alpha}{r^3}(3\sin^2\theta\cos\theta + 2\cos^3\theta - \sin^2\theta\cos\theta)$

$= \dfrac{\alpha}{r^3}\{\cos\theta(2\sin^2\theta + 2\cos^2\theta)\}$

$= \dfrac{\alpha}{r^3}(2\cos\theta) = \dfrac{2\alpha\cos\theta}{r^3}$.

$B_\theta = B_y\cos\theta - B_z\sin\theta$.

$= \dfrac{\alpha}{r^3}(3\sin\theta\cos^2\theta - 2\sin\theta\cos^2\theta + \sin^3\theta)$

$= \dfrac{\alpha}{r^3}(\sin\theta\cos^2\theta + \sin^3\theta)$

$= \dfrac{\alpha\sin\theta}{r^3}$.

(3) 磁力線の方程式は、極座標を用いて

$\dfrac{dr}{\frac{B_r}{\mu_0}} = \dfrac{r\,d\theta}{\frac{B_\theta}{\mu_0}}$ $\quad$ 磁力線 $\dfrac{dx}{B_x} = \dfrac{dy}{B_y} = \dfrac{dz}{B_z}$

(2) を代入して

$\dfrac{dr}{2\cos\theta} = \dfrac{r\,d\theta}{\sin\theta}$ $\quad$ $\boxed{\text{電力線の方程式}\quad \dfrac{dx}{E_x} = \dfrac{dy}{E_y} = \dfrac{dz}{E_z}}$

両辺 (微分方程式) $\dfrac{1}{2}\dfrac{1}{r}dr = \dfrac{\cos\theta}{\sin\theta}d\theta$.

$\dfrac{1}{2}\log r = \log|\sin\theta| + C$

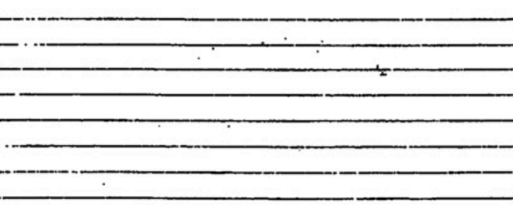
$r = C\sin^2\theta$

No.
Date.

$r = C\sin^2\theta$.

$C = 1$ と取ると.

θ	0	$\pi/6$	$\pi/3$	$\frac{\pi}{2}$
r	0	$\frac{1}{4}$	$\frac{3}{4}$	1



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電気磁気学

[1]

(a) $V_1 = P_{11}Q_1 + P_{12}Q_2$.

$V_2 = P_{21}Q_1 + P_{22}Q_2$

(i) $S_1 = Q\,[C]$, $S_2 = 0\,[C]$ とすると、

$V_1 = P_{11}Q$.

$V_2 = P_{21}Q$.

$V_1 = \dfrac{Q}{4\pi\varepsilon_0 a} \Rightarrow P_{11} = \dfrac{1}{4\pi\varepsilon_0 a}$

$V_2 = -\displaystyle\int_\infty^{a+c}\dfrac{Q}{4\pi\varepsilon_0 x^2}dx = \dfrac{Q}{4\pi\varepsilon_0(a+c)} \approx \dfrac{Q}{4\pi\varepsilon_0 c}$

$c \gg a$

$P_{12} = \dfrac{1}{4\pi\varepsilon_0 c}$

(ii) $S_1 = 0\,(C)$, $S_2 = Q\,[C]$ とすると、

$V_1 = -\displaystyle\int_\infty^{b+c}\dfrac{Q}{4\pi\varepsilon_0 x^2}dx = \dfrac{Q}{4\pi\varepsilon_0(b+c)} \approx \dfrac{Q}{4\pi\varepsilon_0 c}$

$V_2 = \dfrac{Q}{4\pi\varepsilon_0 b} \Rightarrow P_{22} = \dfrac{1}{4\pi\varepsilon_0 b}$ $\quad P_{21} = \dfrac{1}{4\pi\varepsilon_0 c}$

(b) $Q_1 = Q'$

$Q_2 = Q - Q'$ $\quad \underline{V_1 = V_2 = V}$

方程式の連立
$$\begin{cases} V_1 = \dfrac{1}{4\pi\varepsilon_0}\left(\dfrac{Q'}{a} + \dfrac{Q - Q'}{c}\right) \\ V_2 = \dfrac{1}{4\pi\varepsilon_0}\left(\dfrac{Q'}{c} + \dfrac{Q - Q'}{b}\right) \end{cases}$$

$$\frac{Q'}{a}+\frac{Q-Q'}{c}=\frac{Q'}{c}+\frac{Q-Q'}{b}$$

$$\frac{Q'}{a}+\frac{Q}{c}-\frac{Q'}{c}=\frac{Q'}{c}+\frac{Q}{b}-\frac{Q'}{b}$$

$$\left(\frac{1}{a}+\frac{1}{b}-\frac{2}{c}\right)Q'=\left(\frac{1}{b}-\frac{1}{c}\right)Q$$

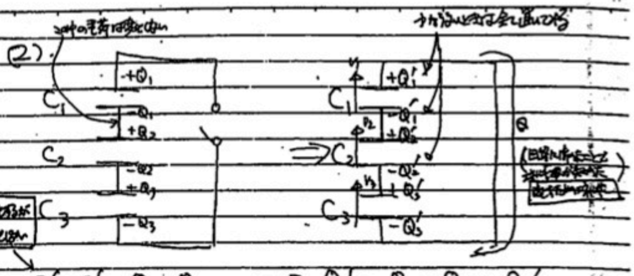
$$Q'=\frac{\frac{1}{b}-\frac{1}{c}}{\frac{1}{a}+\frac{1}{b}-\frac{2}{c}}Q$$

$$=\frac{\frac{c-b}{bc}}{\frac{bc+ac-2ab}{abc}}Q$$

$$=\frac{ac-ab}{bc+ac-2ab}Q$$

$$=\frac{a(c-b)}{b(c-a)+a(c-b)}Q$$

(2)



$$Q_1'-Q_2'=Q_1-Q_2 \longrightarrow Q_1'=Q_1-Q_2+Q_2'$$

$$-Q_1'+Q_2'=-Q_1+Q_2 \longrightarrow Q_1'=Q_1-Q_2+Q_2'$$

$$-Q_2'+Q_3'=-Q_2+Q_3 \longrightarrow Q_3'=-Q_2+Q_3+Q_2'$$

$$\boxed{\frac{Q_1'}{C_1}+\frac{Q_2'}{C_2}+\frac{Q_3'}{C_3}=0}$$ 閉路→電位差が0

$V_1 \quad V_2 \quad V_3$

$$\frac{Q_1-Q_2+Q_2'}{C_1}+\frac{Q_2'}{C_2}+\frac{-Q_2+Q_3+Q_2'}{C_3}=0$$

$$C_2C_3(Q_1-Q_2+Q_2')+C_1C_3Q_2'+C_1C_2(-Q_2+Q_3+Q_2')=0$$

$$(C_1C_2+C_2C_3+C_3C_1)Q_2'+C_2C_3Q_1-(C_1C_2+C_2C_3)Q_2$$
$$+C_1C_2Q_3=0$$

$$Q_2'=\frac{-C_2C_3Q_1+(C_1C_2+C_2C_3)Q_2-C_1C_2Q_3}{C_1C_2+C_2C_3+C_3C_1}$$

流れる電荷は

$$Q_2-Q_2'=\frac{C_3C_1Q_2+C_2C_3Q_1+C_1C_2Q_3}{C_1C_2+C_2C_3+C_3C_1}$$

Q_2の変化⇒Q_1とQ_3の変化→回路に流れる電荷

(b) $W_1=\frac{1}{2}\left(\frac{Q_1^2}{C_1}+\frac{Q_2^2}{C_2}+\frac{Q_3^2}{C_3}\right)$

$W_2=\frac{1}{2}\left(\frac{Q_1'^2}{C_1}+\frac{Q_2'^2}{C_2}+\frac{Q_3'^2}{C_3}\right)$

より残っているエネルギーは

導線の抵抗によりジュール熱が消費されている

$$W_1-W_2=\frac{1}{2}\left(\frac{Q_1^2-Q_1'^2}{C_1}+\frac{Q_2^2-Q_2'^2}{C_2}+\frac{Q_3^2-Q_3'^2}{C_3}\right)$$

[2] $(r>a)$

$$\oint_C B\,dl=\mu_0 I$$

$$2\pi r B=\mu_0 I$$

$$B=\frac{\mu_0 I}{2\pi r} \qquad (\pi a^2-\pi b^2)J=I$$

$(a>r>b)$

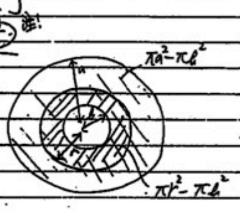
$$J=\frac{I}{\pi(a^2-b^2)}$$

$$2\pi r B=\mu_0(\pi r^2-\pi b^2)J$$

$$B=\frac{\mu_0 I}{2\pi r}\frac{r^2-b^2}{a^2-b^2}$$

$(b>r>0)$

$$B=0$$

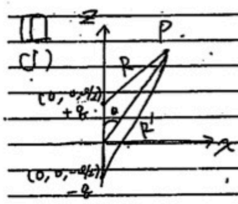


H13

電気磁気学

[1]

(1)



- $R^2=r^2+\left(\frac{\ell}{2}\right)^2-2\cdot r\cdot\frac{\ell}{2}\cos\theta$. 余弦定理

$R=\sqrt{r^2+\left(\frac{\ell}{2}\right)^2-r\ell\cos\theta}=r\sqrt{1+\left(\frac{\ell}{2r}\right)^2-\frac{\ell}{r}\cos\theta}$

- $R'^2=r^2+\left(\frac{\ell}{2}\right)^2-2\cdot r\cdot\frac{\ell}{2}\cos(\pi-\theta)$

$R'=\sqrt{r^2+\left(\frac{\ell}{2}\right)^2+r\ell\cos\theta}=r\sqrt{1+\left(\frac{\ell}{2r}\right)^2+\frac{\ell}{r}\cos\theta}$

$R\fallingdotseq r\sqrt{1-\frac{\ell}{r}\cos\theta} \qquad R'\fallingdotseq r\sqrt{1+\frac{\ell}{r}\cos\theta}$

$=r\left(1-\frac{\ell}{r}\cos\theta\right)^{\frac{1}{2}} \qquad =r\left(1+\frac{\ell}{2r}\cos\theta\right)$

$\fallingdotseq r\left(1-\frac{\ell}{2r}\cos\theta\right) \qquad \boxed{(1+x)^a\fallingdotseq 1+ax}$

$$V=\frac{q}{4\pi\varepsilon_0}\left(\frac{1}{R}-\frac{1}{R'}\right)=\frac{q}{4\pi\varepsilon_0 r}\left(\frac{1}{1-\frac{\ell}{2r}\cos\theta}-\frac{1}{1+\frac{\ell}{2r}\cos\theta}\right)$$

$\left(1-\frac{\ell}{2r}\cos\theta\right)^{-1}\fallingdotseq 1+\frac{\ell}{2r}\cos\theta$

$$\fallingdotseq\frac{q}{4\pi\varepsilon_0 r}\left(1+\frac{\ell}{2r}\cos\theta-1+\frac{\ell}{2r}\cos\theta\right)$$

$$=\frac{q}{4\pi\varepsilon_0 r}\left(\frac{\ell}{r}\cos\theta\right)=\frac{q\ell\cos\theta}{4\pi\varepsilon_0 r^2}=\frac{p\cos\theta}{4\pi\varepsilon_0 r^2}$$

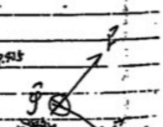
電位から電場を出す

(2) $E=-\nabla V \quad (E=-\mathrm{grad}\,V)$

球座標

$$=-\left(\hat{r}\frac{\partial V}{\partial r}+\hat{\theta}\frac{1}{r}\frac{\partial V}{\partial\theta}+\hat{\varphi}\frac{1}{r\sin\theta}\frac{\partial V}{\partial\varphi}\right)$$

$$=-\left\{\hat{r}\left(-\frac{p\cos\theta}{2\pi\varepsilon_0 r^3}\right)+\hat{\theta}\frac{1}{r}\left(-\frac{p\sin\theta}{4\pi\varepsilon_0 r^2}\right)\right\}$$



$$=\hat{r}\frac{p\cos\theta}{2\pi\varepsilon_0 r^3}+\hat{\theta}\frac{p\sin\theta}{4\pi\varepsilon_0 r^3}$$

$\hat{r}$成分 $\qquad \hat{\theta}$成分

$$\frac{Q'}{a} + \frac{Q - Q'}{c} = \frac{Q'}{c} + \frac{Q - Q'}{b}$$

$$\frac{Q'}{a} + \frac{Q}{c} - \frac{Q'}{c} = \frac{Q'}{c} + \frac{Q}{b} - \frac{Q'}{b}$$

$$\left(\frac{1}{a} + \frac{1}{b} - \frac{2}{c}\right) Q' = \left(\frac{1}{b} - \frac{1}{c}\right) Q$$

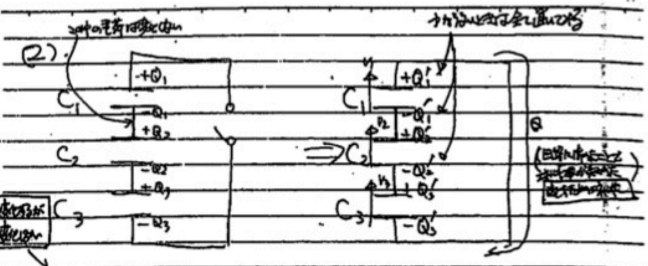
$$Q' = \frac{\frac{1}{b} - \frac{1}{c}}{\frac{1}{a} + \frac{1}{b} - \frac{2}{c}} Q$$

$$= \frac{\frac{c-b}{bc}}{\frac{bc + ac - 2ab}{abc}} Q$$

$$= \frac{ac - ab}{bc + ac - 2ab} Q$$

$$= \frac{a(c-b)}{b(c-a) + a(c-b)} Q$$

(2)



導体の電荷の和は変化しない

$$Q_1' - Q_2' = Q_1 - Q_2 \longrightarrow Q_1' = Q_1 - Q_2 + Q_2'$$

$$-Q_1' + Q_2' = -Q_1 + Q_2 \longrightarrow Q_1' = Q_1 - Q_2 + Q_2'$$

$$-Q_2' + Q_3' = -Q_2 + Q_3 \longrightarrow Q_3' = -Q_2 + Q_3 + Q_2'$$

$$\boxed{\frac{Q_1'}{C_1} + \frac{Q_2'}{C_2} + \frac{Q_3'}{C_3} = 0}$$ 回路→電位差が0

$V_1 \quad V_2 \quad V_3$

$$\frac{Q_1 - Q_2 + Q_2'}{C_1} + \frac{Q_2'}{C_2} + \frac{-Q_2 + Q_3 + Q_2'}{C_3} = 0$$

$$C_2C_3(Q_1 - Q_2 + Q_2') + C_1C_3Q_2' + C_1C_2(-Q_2 + Q_3 + Q_2') = 0$$

$$(C_1C_2 + C_2C_3 + C_3C_1)Q_2' + C_2C_3Q_1 - (C_1C_2 + C_2C_3)Q_2 + C_1C_2Q_3 = 0$$

$$Q_2' = \frac{-C_2C_3Q_1 + (C_1C_2 + C_2C_3)Q_2 - C_1C_2Q_3}{C_1C_2 + C_2C_3 + C_3C_1}$$

流れる電荷は

$$Q_2 - Q_2' = \frac{C_3C_1Q_2 + C_2C_3Q_1 + C_1C_2Q_3}{C_1C_2 + C_2C_3 + C_3C_1}$$

~~Q_2の変化→Q_1とQ_3の変化→回路に流れる電荷~~

(b)

$$W_1 = \frac{1}{2}\left(\frac{Q_1^2}{C_1} + \frac{Q_2^2}{C_2} + \frac{Q_3^2}{C_3}\right)$$

$$W_2 = \frac{1}{2}\left(\frac{Q_1'^2}{C_1} + \frac{Q_2'^2}{C_2} + \frac{Q_3'^2}{C_3}\right)$$

より残っているエネルギーは

電荷(電流)が流れたことによるジュール熱が消費されている

$$W_1 - W_2 = \frac{1}{2}\left(\frac{Q_1^2 - Q_1'^2}{C_1} + \frac{Q_2^2 - Q_2'^2}{C_2} + \frac{Q_3^2 - Q_3'^2}{C_3}\right)$$

[2] $(r > a)$

$$\oint_C \mathbb{B} \cdot d\ell = \mu_0 I$$

$$2\pi r \mathbb{B} = \mu_0 I$$

$$\mathbb{B} = \frac{\mu_0 I}{2\pi r}$$

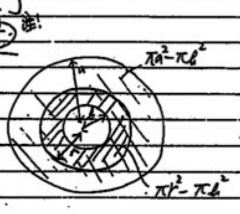
$$(\pi a^2 - \pi b^2) J = I$$

$(a > r > b)$

$$J = \frac{I}{\pi(a^2 - b^2)}$$

$$2\pi r \mathbb{B} = \mu_0(\pi r^2 - \pi b^2) J$$

$$\mathbb{B} = \frac{\mu_0 I}{2\pi r} \frac{r^2 - b^2}{a^2 - b^2}$$



$(b > r > 0)$

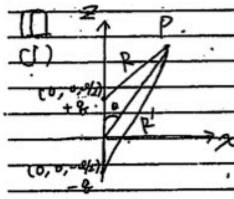
$$\mathbb{B} = 0$$

H13

電気磁気学

[1]

(1)



$$\cdot\ R^2 = r^2 + \left(\frac{\ell}{2}\right)^2 - 2 \cdot r \cdot \frac{\ell}{2}\cos\theta$$ 余弦定理

$$R = \sqrt{r^2 + \left(\frac{\ell}{2}\right)^2 - r\ell\cos\theta} = r\sqrt{1 + \left(\frac{\ell}{2r}\right)^2 - \frac{\ell}{r}\cos\theta}$$

$$\cdot\ R'^2 = r^2 + \left(\frac{\ell}{2}\right)^2 - 2 \cdot r \cdot \frac{\ell}{2}\cos(\pi - \theta)$$

$$R' = \sqrt{r^2 + \left(\frac{\ell}{2}\right)^2 + r\ell\cos\theta} = r\sqrt{1 + \left(\frac{\ell}{2r}\right)^2 + \frac{\ell}{r}\cos\theta}$$

$$R \fallingdotseq r\sqrt{1 - \frac{\ell}{r}\cos\theta} \qquad R' \fallingdotseq r\sqrt{1 + \frac{\ell}{r}\cos\theta}$$

$$= r\left(1 - \frac{\ell}{r}\cos\theta\right)^{\frac{1}{2}} \qquad = r\left(1 + \frac{\ell}{2r}\cos\theta\right)$$

$$\fallingdotseq r\left(1 - \frac{\ell}{2r}\cos\theta\right) \qquad \boxed{(1+x)^a \fallingdotseq 1 + ax}$$

$$V = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{R} - \frac{1}{R'}\right) = \frac{q}{4\pi\varepsilon_0 r}\left(\frac{1}{1 - \frac{\ell}{2r}\cos\theta} - \frac{1}{1 + \frac{\ell}{2r}\cos\theta}\right)$$

$$\left(1 - \frac{\ell}{2r}\cos\theta\right)^{-1} \approx 1 + \frac{\ell}{2r}\cos\theta$$

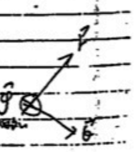
$$\fallingdotseq \frac{q}{4\pi\varepsilon_0 r}\left(1 + \frac{\ell}{2r}\cos\theta - 1 + \frac{\ell}{2r}\cos\theta\right)$$

$$= \frac{q}{4\pi\varepsilon_0 r}\left(\frac{\ell}{r}\cos\theta\right) = \frac{q\ell\cos\theta}{4\pi\varepsilon_0 r^2} = \frac{p\cos\theta}{4\pi\varepsilon_0 r^2}$$

(2) $\mathbb{E} = -\nabla V \quad (\mathbb{E} = -\mathrm{grad} V)$

$$= -\left(\hat{r}\frac{\partial V}{\partial r} + \hat{\theta}\frac{1}{r}\frac{\partial V}{\partial \theta} + \hat{\varphi}\frac{1}{r\sin\theta}\frac{\partial V}{\partial \varphi}\right)$$

$$= -\left\{\hat{r}\left(-\frac{p\cos\theta}{2\pi\varepsilon_0 r^3}\right) + \hat{\theta}\frac{1}{r}\left(-\frac{p\sin\theta}{4\pi\varepsilon_0 r^2}\right)\right\}$$



$$= \underbrace{\hat{r}\frac{p\cos\theta}{2\pi\varepsilon_0 r^3}}_{\hat{r}\text{成分}} + \underbrace{\hat{\theta}\frac{p\sin\theta}{4\pi\varepsilon_0 r^3}}_{\hat{\theta}\text{成分}}$$