II

(1) 1次遅れ系より伝達関数は $\frac{B}{s+A}$ (実際は $\frac{K}{Ts+1}$)

ステップ応答なので $y(s) = \frac{B}{s(s+A)} = \frac{a}{s} + \frac{b}{s+T} = \frac{(a+b)s + aT}{s(s+T)}$

$\therefore\ y(t) = \frac{B}{A}(1 - e^{-Tt})$

図より $t \to \infty$ で 1 より $A = B$

また $y(0)$ で傾きで、 $y'(t) = B\, e^{-At} = A\, e^{-At}$ より

$y'(0) = A = 20$

$\therefore$ 伝達関数は $\frac{20}{s+20}$ //

(2) $C(s) = \frac{20}{s+20}$

(a) $\frac{y}{r} = \frac{\frac{20K}{s+20} \cdot \frac{10}{s(s+10)}}{1 + \frac{20K}{s+20} \cdot \frac{10}{s(s+10)}} = \frac{200K}{s(s+20)(s+10) + 200K} = \frac{200K}{s^3 + 30s^2 + 200s + 200K}$

ラウスの安定判別法より

$$\begin{array}{c|ccc} s^3 & 1 & 200 & 0 \\ s^2 & 30 & 200K & 0 \\ s^1 & b_{n-1} & 0 & \\ s^0 & 200K & & \end{array}$$

$b_{n-1} = \frac{6000 - 200K}{30} > 0$

$K < 30$

$\therefore\ 0 < K < 30$ //

(b) $E = R(1 - \frac{Y}{R})$

$E(s) = R(s)\{1 - G_c(s)\} = R(s)\{1 - \frac{K(s)G(s)}{1 + K(s)G(s)}\}$

$\therefore\ E(s) = R(s) \cdot \frac{1}{1 + K(s)G(s)}$ //

$E = \frac{s^3 + 30s^2 + 200s}{s^3 + 30s^2 + 200s + 200K}$

$\therefore\ e(\infty) = \lim_{s \to 0} sE = \lim_{s \to 0} s \cdot \frac{1}{s^2} \cdot \frac{s^3 + 30s^2 + 200s}{s^3 + 30s^2 + 200s + 200K} = \frac{1}{K} \leqq \frac{1}{20}$

$\therefore\ K \geqq 20$ より $20 \leqq K \leqq 30$ //

(c) $E = -\frac{Y}{D} \cdot D$

$= -\frac{G}{1 + KG} \cdot D = -\frac{10(s+20)}{s^3 + 30s^2 + 200s + 200K} \cdot D$

$\therefore\ e(\infty) = \lim_{s \to 0} sE = \lim_{s \to 0} s \cdot \frac{1}{s} \cdot -\frac{10(s+20)}{s^3 + 30s^2 + 200s + 200K} = -\frac{1}{K} = -\frac{1}{20}$ //