**問題１８　量子力学　解答**

I (1) $\boxed{h\nu}$　(2) $\boxed{h\nu/c}$

(3) $\boxed{h\nu = h\nu' + K}$　(4) $\boxed{\dfrac{h\nu}{c} = \dfrac{h\nu'}{c}\cos\phi + p\cos\theta}$　(5) $\boxed{0 = \dfrac{h\nu'}{c}\sin\phi - p\sin\theta}$

(6) 与式を $p^2$ について解くと，

$$p^2 = \frac{1}{c^2}\left[(K+mc^2)^2 - m^2c^4\right] = \frac{K^2}{c^2} + 2Km \qquad \text{(d)}$$

従って答は，$\boxed{\dfrac{1}{c^2}\left[(K+mc^2)^2 - m^2c^4\right]}$ または，$\boxed{\dfrac{K^2}{c^2} + 2Km}$

(7) (b) より　$p^2\cos^2\theta = \left(\dfrac{h\nu}{c}\right)^2 + \left(\dfrac{h\nu'}{c}\right)^2\cos^2\phi - \dfrac{2h^2\nu\nu'}{c^2}\cos\phi$

(c) より　$p^2\sin^2\theta = \left(\dfrac{h\nu'}{c}\right)^2\sin^2\phi$

2 つの式を足し合わせると，

$$p^2 = \left(\frac{h\nu}{c}\right)^2 + \left(\frac{h\nu'}{c}\right)^2 - \frac{2h^2\nu\nu'}{c^2}\cos\phi \qquad \text{(e)}$$

従って答は，$\boxed{\left(\dfrac{h\nu}{c}\right)^2 + \left(\dfrac{h\nu'}{c}\right)^2 - \dfrac{2h^2\nu\nu'}{c^2}\cos\phi}$

(8) (a) を $K$ について解いて，$K = h(\nu - \nu')$ を (d) に代入すると，

$$p^2 = \left(\frac{h\nu}{c}\right)^2 + \left(\frac{h\nu'}{c}\right)^2 - \frac{2h^2\nu\nu'}{c^2} + 2hm(\nu - \nu') \qquad \text{(d)'}$$

(e) と (d)' の差をとってまとめると，

$$\frac{c}{\nu'} - \frac{c}{\nu} = \frac{h}{mc}(1-\cos\phi) \qquad \text{(f)}$$

従って答は，$\boxed{\dfrac{h}{mc}(1-\cos\phi)}$

II

(1) $$\frac{\partial \rho}{\partial t}=\psi^*\frac{\partial \psi}{\partial t}+\psi\frac{\partial \psi^*}{\partial t}$$

(2) (1) の解にシュレディンガー方程式を代入する.

$$\begin{aligned}
\frac{\partial \rho}{\partial t}&=\psi^*\frac{\partial \psi}{\partial t}+\psi\frac{\partial \psi^*}{\partial t}\\
&=\psi^*\left[\frac{-\hbar}{2im}\frac{\partial^2\psi}{\partial x^2}+\frac{1}{i\hbar}V\psi\right]+\psi\left[\frac{\hbar}{2im}\frac{\partial^2\psi^*}{\partial x^2}-\frac{1}{i\hbar}V\psi^*\right]\\
&=\frac{-\hbar}{2im}\left(\psi^*\frac{\partial^2\psi}{\partial x^2}-\psi\frac{\partial^2\psi^*}{\partial x^2}\right)\\
&=\frac{-\hbar}{2im}\frac{\partial}{\partial x}\left(\psi^*\frac{\partial\psi}{\partial x}-\psi\frac{\partial\psi^*}{\partial x}\right)\\
&=-\frac{\partial}{\partial x}\left[\frac{-i\hbar}{2m}\left(\psi^*\frac{\partial\psi}{\partial x}-\psi\frac{\partial\psi^*}{\partial x}\right)\right]\\
&=-\frac{\partial}{\partial x}j(x,t)
\end{aligned}$$

(3) $$\begin{aligned}
j&=\frac{-i\hbar}{2m}\left(\psi^*\frac{\partial\psi}{\partial x}-\psi\frac{\partial\psi^*}{\partial x}\right)\\
&=\frac{-i\hbar}{2m}\Big[\left(A_1^*e^{-ikx}-+A_2^*e^{ikx}\right)\left(ikA_1e^{ikx}-ikA_2e^{-ikx}\right)\\
&\qquad\qquad -\left(A_1e^{ikx}+A_2e^{-ikx}\right)\left(-ikA_1^*e^{-ikx}+ikA_2^*e^{ikx}\right)\Big]\\
&=\frac{\hbar k}{m}|A_1|^2-\frac{\hbar k}{m}|A_2|^2
\end{aligned}$$