第1問

　図1のように，半径 $a, 2a, 3a, 4a$ の極めて薄い導体球殻 A, B, C, D が同心的に置かれコンデンサを形成しており，球殻 D は接地されている．球殻 B, C の間は誘電率 $2\varepsilon_0$ の誘電体で満たされており，またその他の領域は真空（誘電率 ε_0）である．ただし，この誘電体は完全な絶縁体ではなく，極めて小さな導電率を有している．スイッチ SW は開いており，球殻 A, B, C, D の電荷は 0 である．いま，スイッチ SW を閉じて球殻 AD 間に電圧 V_0 を印加すると，その直後に球殻 A, B, C, D の電荷はそれぞれ $Q_0, 0, 0, -Q_0$ となった．この時刻を $t=0$ とする．その後 $t>0$ で，誘電体内部の電界により誘電体に漏れ電流が流れ始め，十分時間が経過すると誘電体内部の電界は 0 となった．以下の問に答えよ．

(1) $t=0$ における，球の中心から距離 r の電界強度を ε_0, r, Q_0 で表せ．

(2) Q_0 と V_0 の関係を求め，$t=0$ におけるコンデンサの静電エネルギー U_0 を ε_0, a, V_0 で表せ．

(3) 十分時間が経過したあとのコンデンサの静電エネルギーを U_∞ とおく．$t=0$ から十分時間が経過するまでの，静電エネルギーの変化量 $U_\infty - U_0$ を ε_0, a, V_0 で表せ．

(4) $t=0$ から十分時間が経過するまでの間に，図1中の電源がした仕事 W_s と，誘電体内部で発生したジュール熱 W_J を求め，それぞれ ε_0, a, V_0 で表せ．

(5) (4)のあと，図2のように導体球殻 A を中心から $a/2$ だけ移動させた．十分時間が経過すると，漏れ電流により再び誘電体内部の電界は 0 となった．この誘電体内部の電界が 0 になったあとの，コンデンサ内部の電気力線の概略を描け．

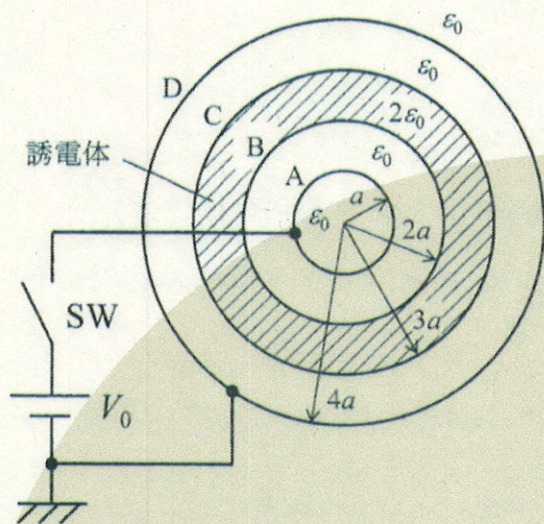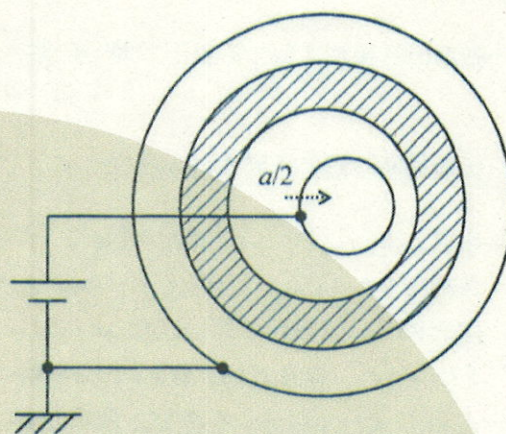(6) (5)のとき，導体球殻 A に働く静電気力の向きを，その理由とともに述べよ．

図1

図2

Problem 1

A concentric spherical capacitor is formed by infinitely-thin spherical conducting shells A, B, C, and D having radii a, $2a$, $3a$, and $4a$, respectively, as shown in Fig. 1. The shell D is grounded. The space between the shells B and C is filled with a dielectric substance of permittivity $2\varepsilon_0$. Other spaces are in vacuum of permittivity ε_0. The dielectric substance is not an ideal insulator, but has quite a small conductivity. The switch SW is open and the charges on the shells A, B, C, and D are zero. Now, the switch SW is closed to apply a voltage V_0 between the shells A and D. Immediately after the switch is closed, the charges on the shells A, B, C, and D become Q_0, 0, 0, and $-Q_0$, respectively. This time is defined as $t = 0$. Then, at $t > 0$, a leakage current flows in the dielectric substance due to the electric field in the dielectric substance. After a sufficiently long time, the electric field in the dielectric substance becomes zero. Answer the following questions.

(1) Express the electric field strength at a distance r from the center of the spherical shells at $t = 0$ in terms of ε_0, r, and Q_0.

(2) Find the relation between Q_0 and V_0. Then, express the electrostatic energy U_0 of the capacitor at $t = 0$ in terms of ε_0, a, and V_0.

(3) After a sufficiently long time, the electrostatic energy of the capacitor becomes U_∞. Express the change in the electrostatic energy from $t = 0$, i.e. $U_\infty - U_0$, in terms of ε_0, a, and V_0.

(4) Find the energy W_s supplied from the power source in Fig. 1 and the Joule heating energy W_j generated in the dielectric substance after the sufficiently long time from $t = 0$. Express W_s and W_j in terms of ε_0, a, and V_0.

(5) After (4), the shell A is moved from the center by $a/2$, as shown in Fig. 2. After a sufficiently long time, the electric field in the dielectric substance becomes zero again due to the leakage current in the dielectric substance. Sketch the electric lines of force inside the capacitor after the electric field in the dielectric substance becomes zero.

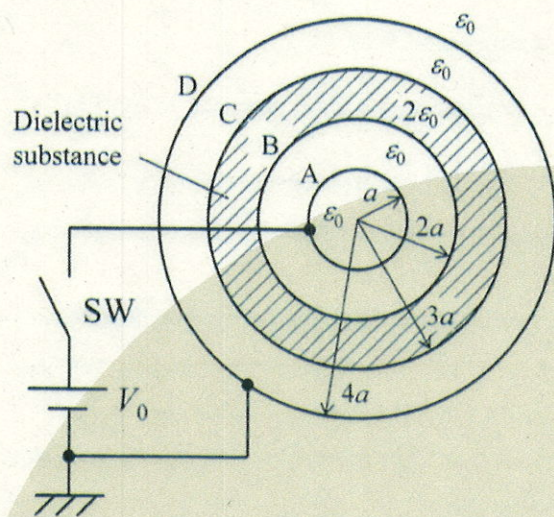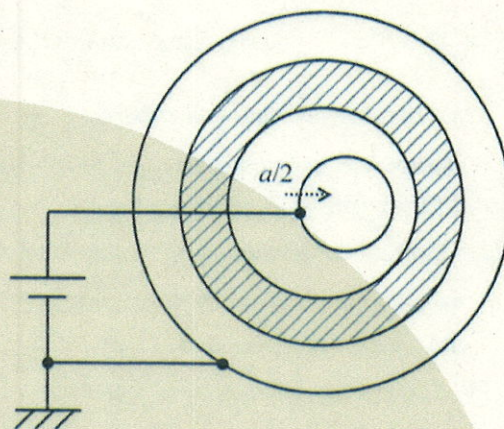(6) In (5), what is the direction of electrostatic force exerted on the shell A? Explain the reason.

Fig. 1

Fig. 2