No.

Date

平成16年 電磁気・電磁波（1のみ）

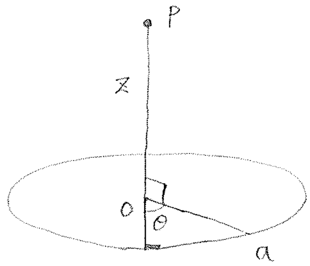
1

(1) ガウスの定理より

$$2E_\infty \cdot \pi r^2 = \frac{\pi r^2 \sigma}{\varepsilon_0}$$

$$\therefore\ E_\infty = \frac{\sigma}{2\varepsilon_0}$$

(2)



$$dV = \frac{dQ}{4\pi\sqrt{z^2+r^2}\cdot\varepsilon_0}\quad , dQ = dr \cdot r d\theta \cdot \sigma$$

$$\therefore\ dV = \frac{\sigma r dr\, d\theta}{4\pi\sqrt{z^2+r^2}\cdot\varepsilon_0}$$

$$V = \frac{\sigma\ r\, dr}{4\pi\varepsilon_0\sqrt{z^2+r^2}}\int_0^{2\pi} d\theta = \frac{\sigma}{2\varepsilon_0}\int_0^a \frac{r}{\sqrt{z^2+r^2}}\, dr = \frac{\sigma}{2\varepsilon_0}\left[\sqrt{z^2+r^2}\right]_0^a$$

$$= \frac{\sigma}{2\varepsilon_0}\left(\sqrt{z^2+a^2} - z\right)$$

ここで $E_a = -\mathrm{grad}\, V$ より

$$E_a = -\frac{\sigma}{2\varepsilon_0}\left\{\frac{1}{2}(z^2+a^2)^{-\frac{1}{2}}\cdot 2z - 1\right\}$$

$$= \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{z^2+a^2}}\right)$$

(3) $E_\infty = \dfrac{\sigma}{2\varepsilon_0}$, $E_a = \dfrac{\sigma}{2\varepsilon_0}\left(1 - \dfrac{z}{\sqrt{z^2+a^2}}\right)$ より

$$2\cdot\frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{z^2+a^2}}\right)$$

$\therefore\ \dfrac{z}{\sqrt{z^2+a^2}} = \dfrac{1}{2}$ となるので

$$2z = \sqrt{z^2+a^2}$$

$$4z^2 = z^2 + a^2$$

$$z = \frac{a}{\sqrt{3}}$$