

Survey Paper

Process Fault Detection Based on Modeling and Estimation Methods—A Survey

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A review on detection and diagnosis illustrate that process faults can be detected when based on the estimation of unmeasurable process parameters and state variables.

Key Words—Fault detection; supervision; reliability; safety; process models; parameter estimation; state estimation; d.c. motor; centrifugal pump; leak detection; pipeline.

Abstract—The supervision of technical processes is the subject of increased development because of the increasing demands on reliability and safety. The use of process computers and microcomputers permits the application of methods which result in an earlier detection of process faults than is possible by conventional limit and trend checks. With the aid of process models, estimation and decision methods it is possible to also monitor nonmeasurable variables like process states, process parameters and characteristic quantities. This contribution presents a brief summary of some basic fault detection methods. This is followed by a description of suitable parameter estimation methods for continuous-time models. Then two examples are considered, the fault detection of an electrical driven centrifugal pump by parameter monitoring and the leak detection for pipelines by a special correlation method.

1. Introduction

BECAUSE of the increasing demands on reliability and safety of technical plants and their elements methods for improving the supervision and monitoring as part of the overall control of processes are getting an increasing interest. This holds as well for advanced processes with highest demands on reliability and safety, e.g. aeronautics and nuclear power stations, as for many other large and also small processes. An essential prerequisite for the further development of automatic supervision is an *early process fault detection*. Whereas previously methods of fault detection in technical processes only permitted recognition when limit values of measurable output signals had already transgressed, an attempt is now made to detect the faults earlier and to locate them better by the use of the measurable signals. This is possible for a range of process faults by the application of *mathematical process models* and *signal models* using process computers and microcomputers. Methods can be used for *predicting signals* and for *estimating nonmeasurable process state variables, process parameters* and other quantities characteristic to the process. Process monitoring by use of these and decision methods seems to develop to a new area within automatic control.

In the next section the elementary functions of process supervision are considered. Then fault detection methods by monitoring measurable signals and nonmeasurable quantities,

such as state variables, process parameters and others, are briefly reviewed. Because parameter estimation methods for continuous-time models are required, some basic methods are discussed. The methods and applications for fault detection are then described for a d.c. driven centrifugal pump and for pipelines.

2. Elementary functions of process supervision

A *fault* is to be understood as a nonpermitted deviation of a characteristic property which leads to the inability to fulfil the intended purpose.

Figure 1 shows a block diagram for *process supervision*. If a process fault appears it has to be detected as early as possible. This can be done by checking if particular measurable or unmeasurable estimated variables are within a certain tolerance of the normal value. If this check is not passed, this leads to a fault message. The functions up to this point are usually called *monitoring* or, as indicated in the first block of Fig. 1, as *fault detection*. If necessary, this is followed by a *fault diagnosis*: the fault is located and the cause of it is established. The next step is a *fault evaluation*, that means an assessment is made of how the fault will affect the process. The faults can be divided into different hazard classes according to an incident/sequence analysis or a fault tree analysis. After the effect of the fault is known, a *decision* on the action to be taken can be made. If the fault is evaluated to be tolerable, the operation may continue and if it is conditionally tolerable a *change of operation* has to be performed. However, if the fault is intolerable, the *operation* must be *stopped* immediately and the *fault* must be *eliminated*.

Figure 1 indicates that a looped signal flow exists in the supervision of processes in a similar way as in a closed loop control system. It is therefore possible to also refer to a supervision loop. This is only closed on the appearance of a fault and displays very different dynamic characteristics depending on the error. The time delay originates mainly in the blocks 'change operation' (e.g. transfer to another operating condition), or 'stop operation' and 'fault elimination' and in the process itself. In this contribution the primary tasks of fault detection and fault diagnosis are considered.

3. Fault detection methods

Previous supervision of technical processes was restricted to checking directly measurable variables for upward or downward transgression of fixed limits or trends. This could be automated by using simple limit-value monitors. Various faults in the process could then be detected, but only after the measurable output values had been effected considerably. The use of digital process computers and microprocessors enables the use of further methods which can detect faults in the process earlier and which can locate them better. The problem is to orientate process faults with the aid of the measurable input and output variables $U(t)$ and $Y(t)$ (Fig. 2). Mathematical models of the process and its signals.

$$Y = f\{U, N, \theta, X\} \quad (1)$$

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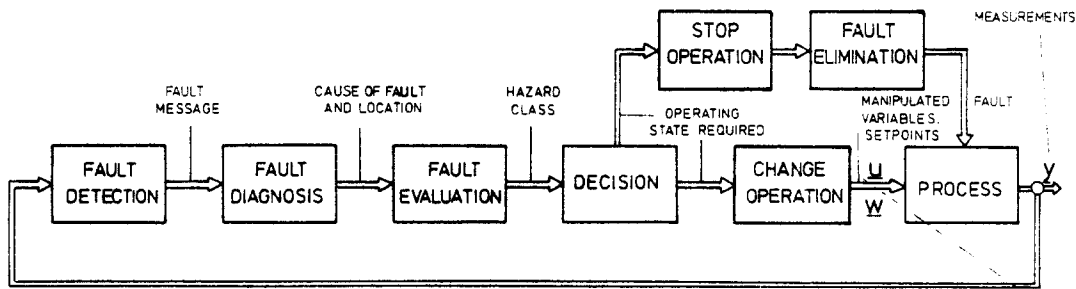


FIG. 1. Supervision loop (on appearance of a fault).

can be used to this end, where N generally represents nonmeasurable disturbance signals from the process and its manipulating and measuring equipment, θ nonmeasurable process parameters, and X partially measurable and partially nonmeasurable internal state variables (signals). In the process models considered the process parameters are constants or slowly time-variable coefficients and the state variables are time-dependent. The methods for fault detection can be divided as being mainly based on the following quantities:

- (1) Measurable signals U, Y .
- (2) Nonmeasurable state variables X .
- (3) Nonmeasurable process parameters θ .
- (4) Nonmeasurable characteristic quantities $\eta = g\{U, Y, \theta\}$.

It is typical for more sophisticated monitoring methods to use nonmeasurable quantities which can be obtained by process models and estimation methods. Many literature references are available on the subject of process supervision and fault detection for class (1), and they are so widely scattered that it is hardly possible to present a complete summary of them. Much less is known about class (2) and only a very limited number of references can be given for classes (3) and (4).

In this section the principles of these methods are briefly described. Recent surveys on failure detection methods in general are given by Himmelblau (1978), Pau (1981), and on class (2) by Willsky (1976). Some examples for all classes are given in Isermann (1981a).

3.1. Measurable signals

Measurable input signals $U(t)$ and output signals $Y(t)$ can be directly used to monitor changes in the process. The various methods are briefly described for a measurable output signal $Y(t)$, the most frequent case.

Limit and trend checking. In the case of the well-known and very commonly used limit check of a signal $Y(t)$ a signal is released as soon as an adjustable maximum value Y_{max} is exceeded or a minimum value Y_{min} fallen below (e.g. the water level in a steam generator drum). The normal state is

$$Y_{min} < Y(t) < Y_{max} \tag{2}$$

This is referred to as an *absolute value check*. The limits are usually set such that a large enough distance to the appearance of damage is retained on the one hand, unnecessary fault alarms being avoided on the other. The limit check can also be applied on the *trend* $\dot{Y}(t)$ of the signal $Y(t)$. If the limit values are set small

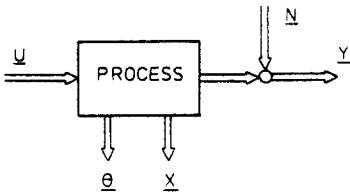


FIG. 2. Representation of a process with measurable input variables U , measurable output variables Y and nondirectly measurable disturbance variables N , process parameters θ and state variables X .

enough the fault alarm can take place earlier than in the last case since the trend permits a certain prediction of the signal progression (applications for, e.g. bearing turbine oil pressures or vibrations). The normal state is

$$\dot{Y}_{min} < \dot{Y}(t) < \dot{Y}_{max} \tag{3}$$

Also a combination of absolute value and trend checking is possible (Isermann, 1981a).

Prediction of signals. If only limit checking is applied, the limits usually are set on the safe side to allow sufficient time for counteractions. However, this can lead to unnecessary false alarms if the variable returns to the normal state without external action. This disadvantage can be avoided, if the affected signals $Y(t)$ can be predicted. This also allows to predict the time of exceeding a threshold.

In order to do this, mathematical models of deterministic signals, or stochastic signals, or a deterministic process and a stochastic signal model have to be used. The (nonmeasurable) parameters of these signal models can be obtained by applying recursive parameter estimation methods, e.g. by using a multistep prediction described by de Keyser and van Cauwenberghe (1981). An absolute value check can then be employed on the predicted signal $\hat{Y}(k)$. An application to a steam generator with deterministic signals is shown by Baur (1977).

Analysis of signals. Output signals $Y(t)$ often consist of lower frequency components $Y_{LF}(t)$ with large magnitudes which mainly determine the nominal values of the signal and higher frequency components $Y_{HF}(t)$ with small amplitudes, which give additional information on the inner state of the process. Then attempts can be made to identify high frequency signal models and to pinpoint process errors from changes in the corresponding signal model parameters. There are many examples in connection with nonparametric signal models, e.g. autocorrelation functions, spectral densities or other methods for vibration analysis. The noise analysis of transmission systems, internal combustion engines and turboengines are well-known examples. Other applications are reported by Zwingelstein and Upadhyaya (1979), and Saedtler (1979) on signal analysis from vibration sensors, pressor sensors and neutron flux measurements for nuclear reactors (Williams and Sher, 1979).

3.2. Nonmeasurable state variables

If process faults are indicated by internal, nonmeasurable process state variables, attempts can be made to reconstruct or estimate these *state variables* from the measurable signals by using a known *process model*.

Static case. A static process model gained from theoretical modeling is sufficient, if the relationship between the measurable input signals U and the output signals Y can be considered as static. The state variable estimates \hat{X} required are then a function of d.c. values \bar{U} and \bar{Y}

$$\hat{X} = f\{\bar{U}, \bar{Y}\} \tag{4}$$

Included in this case are, e.g. variables in the monitoring of material stresses (e.g. drill breakages from measurements of pressure, temperature, advancement) or the overload and tilt protection of cranes.

Dynamic case. In general, dynamic relationships do exist

$$X(t) = f\{U, Y, t\}. \quad (5)$$

Then it is expedient to change to state representation, which is for linearization about one operating point.

$$\dot{x}(t) = Ax(t) + Bu(t) \quad (6)$$

$$y(t) = Cx(t) \quad (7)$$

where $y = \Delta Y$, $u = \Delta U$, $x = \Delta X$ are changes of Y , U and X . The representation must be selected such that the state variables of interest $x_i(t)$ are elements of the state vector $x(t)$. To reconstruct these states from measurable input and output signals a state variable observer (deterministic case) or state variable filter (stochastic case.)

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + H[y(t) - C\hat{x}(t)] \quad (8)$$

can be used, whereby the feedback matrix H must be selected or designed properly, as is well known.

A comprehensive survey of methods for the detection of *abrupt faults* which appear in the state variables and output variables of dynamic systems is given by Willsky (1976). Certain changes in the process, in its noise or in the actuators can be modelled by $v(t)$ in

$$\dot{x}(t) = Ax(t) + Bu(t) + Fv(t) + v(t) \quad (9)$$

and changes in the sensors by $\mu(t)$ in

$$y(t) = Cx(t) + n(t) + \mu(t) \quad (10)$$

where $v(t)$ is process noise and $n(t)$ measurement noise with known statistics. If $v(t)$ and $\mu(t)$ are assumed to be delta impulses or step functions, abrupt changes of the states and the outputs can be modeled. These changes can then be detected by the use of Kalman-Bucy filters where the residuals

$$\tilde{y}(t) = y(t) - C\hat{x}(t) \quad (11)$$

are generated. *Fault decisions* can then be made by special testing methods, e.g.

- 'Fault sensitive filters', where the feedback matrix H is chosen so that particular fault modes manifest themselves as residuals in a fixed direction or in a fixed plane (Beard, 1971; Jones, 1973), a deterministic approach.
- A *Whiteness* and a *chi-squared test* of the residuals of the normal Kalman-filter (Mehra and Peschon, 1971).
- A finite bank of Kalman-filters with a standard multiple hypothesis testing that the systems most likely respond to one of the assumed models with hypothesized faults included (Montgomery and Caglayan, 1974; Montgomery and Price, 1974).
- A *generalized likelihood ratio test* which results in a correlation of the observed residuals with the precomputed filter responses due to certain faults (fault signatures) (Willsky and Jones, 1974).

A recent description of the last two statistic methods is given by Willsky (1980). A block diagram of these state variable techniques is shown in Fig. 3.

Of course, these methods require a relatively exact knowledge of the process parameters (A, B, C) and the influencing signals.

3.3. Nonmeasurable process parameters

Process model parameters are understood as constants or time-dependent coefficients in the process which appears in the mathematical description of the relationship between the input and output signals, the *process model*. A distinction is made between static process models, e.g. in the form of a polynomial equation

$$Y(U) = \beta_0 + \beta_1 U + \beta_2 U^2 + \dots \quad (12)$$

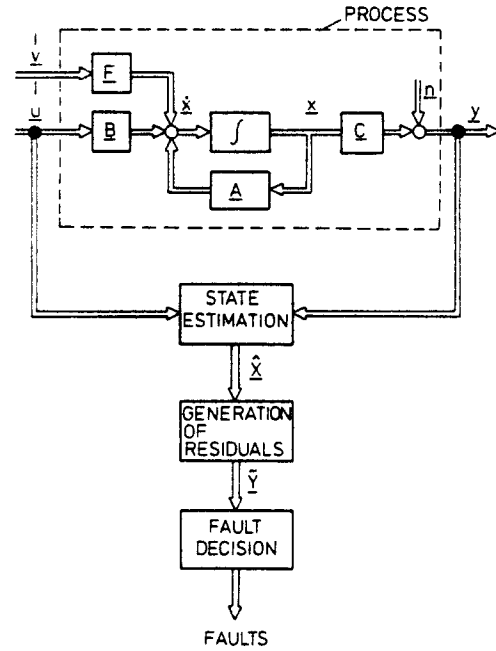


FIG. 3. Fault detection based on state variable estimation.

and dynamic process models which, for processes with lumped parameters, are usually differential equations

$$y(t) + a_1 \dot{y}(t) + a_2 \ddot{y}(t) + \dots + a_n y^{(n)}(t) = b_0 u(t) + b_1 \dot{u}(t) + b_2 \ddot{u}(t) + \dots + b_m u^{(m)}(t) \quad (13)$$

in the simplest case linearized about one operating point. The process model parameters $\theta^T = [\beta_0 \beta_1 \beta_2 \dots]$ or $\theta^T = [a_1 \dots a_n : b_1 \dots b_m]$ are mostly more or less intricate relationships of several physical process coefficients, e.g. length, mass, speed, drag coefficient, viscosity, resistances, capacities. Faults which make themselves noticeable in these physical process constants are therefore also expressed in the process model parameters.

If the physical process coefficients which indicate process faults are not directly measurable, an attempt can be made to determine their changes via the changes in the process model parameters θ . The following procedure is available:

- Establishment of the process equation for the measurable input and output variables

$$Y(t) = f\{U(t), \theta\} \quad (14)$$

mostly by theoretical modeling.

- Determination of the relationship between the model parameters θ_i and the physical process coefficients p_j : $\theta = f(p)$.
- Estimation of the model parameters θ_i as a result of measurements of the signals $Y(t)$ and $U(t)$.
- Calculation of the process coefficients:

$$p = f^{-1}(\theta) \quad (15)$$

and determination of their changes Δp_j .

- Possible process faults can be pinpointed (if need be by pattern recognition) by the use of a catalogue of faults in which the relationship between process faults and changes in the coefficients Δp_j has been established.

Hence, the basis of this class of methods is the combination of theoretical modeling and parameter estimation of continuous-time models. A block diagram is given in Fig. 4. A necessary requirement of this procedure is, however, the existence of the inverse relationship equation (15). Therefore, it may be restricted to well-defined processes, see Section 5.

The process parameter techniques try to monitor the process directly, based on physical laws whereas the state variable

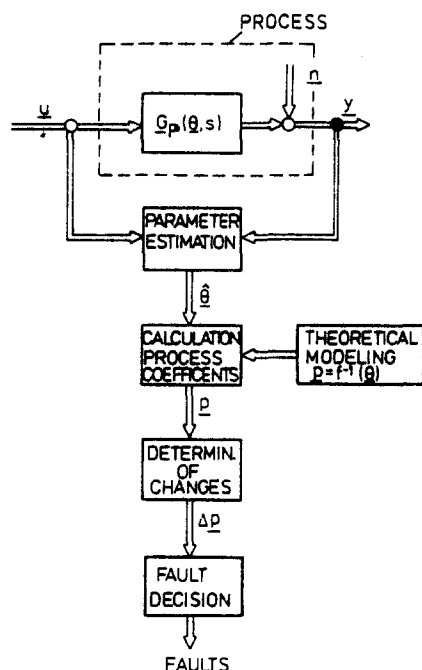


FIG. 4. Fault detection based on parameter estimation and theoretical modeling.

techniques must assume the process parameters as known and try to monitor the signals. Of course, both techniques complement one another.

3.4. Nonmeasurable characteristic quantities

The current checking of characteristic quantities can give important information on the inner state when supervising larger plants or sections.

Examples of characteristic quantities are:

- Efficiency (e.g. all types of engines and machines, steam generators, heat exchangers, furnaces, vehicles).
- Fuel consumption per production unit or time (e.g. cement burning, milling, drying).
- Oil consumption per production unit or time (e.g. internal combustion engines, compressors).
- Tool usage per production unit or time (e.g. machine tools).
- Wear per production unit or time (e.g. tools, motors, grinding devices).

The characteristic quantities must be determined from measurable variables:

$$\eta = g\{U, Y\}. \quad (16)$$

Mostly, static relationships are sufficient. Changes in the quantities can point to faults, e.g. contamination, deposits, wear, friction, icing, leaks. To this end, absolute value or trend checks should be carried out on the characteristic quantities.

3.5. A general structure of process fault detection methods

Looking at the various tasks of process fault detection methods which make use of nonmeasurable quantities and therefore are based on process models one recognizes several similarities. Hence, it is straightforward to present these methods in a generalized structure (Fig. 5). Based on the *a priori* knowledge and experience of the real process we will use:

- a model of the *normal* process;
- a model of the *observed* process;
- models of the *faulty* process.

As the methods rely on the determination of changes in comparison to the *normal status*, the model of the normal process must be known and tracked with high precision. This also includes the definition of what is 'normal', e.g. parameter values including their 'normal' (allowable) tolerances. The normal model can, for

example, be the model obtained just before a fault is alarmed, i.e. the previous model. The models of the faulty process show the effects of the faults on the analysed quantities. These effects are called *fault signatures*.

Dependent on the faults to be detected one has to use:

- state estimation methods;
- parameter estimation methods;
- calculation of characteristic quantities.

A comparison of these nonmeasurable quantities $\hat{x}, \hat{\theta}, \hat{\eta}$ as part of the observed model with the corresponding quantities of the normal model (i.e. previous model) results in *changes*

$$\Delta \hat{x}, \Delta \hat{\theta} \text{ or } \Delta \hat{\eta}$$

or in *error signals* or *residuals*, e.g.

$$\tilde{y} = y - C\hat{x} \text{ or } e = y - \Psi\hat{\theta}$$

if the effect of several estimates on particular signals is considered (Ψ : see next section). These changes or residuals form *comparison quantities* and are one basis for the next step, the *fault decision*. The other basis is the *fault signatures* which show the effect of faults on these quantities, for example, by:

- changes (bias) in definite directions;
- changes in opposite directions;
- changes due to a certain pattern;
- increase of variances.

Therefore, the changes can be examined with respect to the indication of possible faults. To compare the changes with the fault signatures just binary decisions combined with exceeding predetermined thresholds can be made, or more sophisticated methods like correlations of the changes with the failure signatures may be used. This is obviously a field for the application of *statistical decision theory* and *pattern recognition*. The result of the fault decision is the *fault type* and the *time of its occurrence*.

The next task consists of the *fault diagnosis* with the goals to determine the *fault location*, the *fault size* and the *cause of the fault*. Again the observed and normal process model may be used to perform this. Based on this information the fault evaluation can start, etc., see Fig. 1.

In the following two examples process fault detection methods are considered which try to monitor changes of process parameters and process state variables. As parameter estimation for continuous-time models is required, suitable methods are considered next.

4. Parameter estimation for continuous-time models

Fault detection, based on process parameters which are mostly not directly measurable, requires on-line parameter estimation methods. As the goal is not only to detect but to diagnose process faults, the process models should express as close as possible the physical laws which govern the process behaviour. Therefore, the process models have to be developed first by *theoretical modeling*, that means by stating the balance equations for mass, energy and momentum, the physical-chemical state equations and the phenomenological laws for any irreversible phenomena. The models will then appear in the continuous-time domain, in the form of ordinary or partial differential equations. Their parameters θ_i are expressed in dependence on process coefficients p_j like storage or resistance quantities, whose changes may tell about process faults. Hence, the parameters θ_i of *continuous-time models* have to be estimated.

Also, in the case of fault detection methods based on state variable techniques, parametric models are required. If their parameters are not known exactly enough, parameter estimation methods also have to be applied.

A comprehensive survey on parameter estimation methods for continuous-time models was given quite recently by Young (1981). For fault detection the single parameters must be estimated very accurately and one of the questions is, for which parameters and for how many parameters (model order) this will be possible in a noisy environment.

Having in mind these requirements, the next sections will briefly discuss some parameter estimation methods for linear,

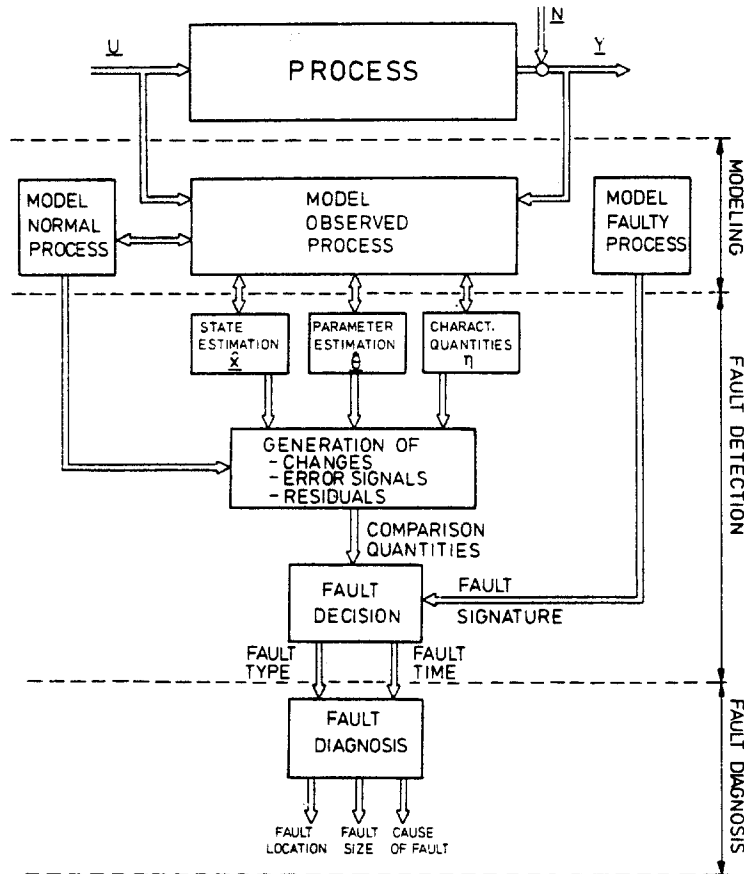


FIG. 5. Generalized structure of fault detection methods based on process models and nonmeasurable quantities.

continuous-time models, based on sampled signals, obtained by a process or microcomputer.

4.1. *Least-squares parameter estimation.* It is assumed that a stable process with lumped parameters is time-invariant and linearizable so that it can be described by a linear differential equation,

$$y_u^n(t) + a_{n-1}y_u^{n-1}(t) + \dots + a_1y_u^1(t) + a_0y_u^0(t) = b_nu^n(t) + b_{n-1}u^{n-1}(t) + \dots + b_1u^1(t) + b_0u^0(t) \quad (17)$$

where the superscript notations means the time derivative operation, that means $y^j(t) = d^jy(t)/dt^j$, and $y(t)$ and $u(t)$ are the deviations

$$y(t) = Y(t) - Y_{00}; \quad u(t) = U(t) - U_{00}. \quad (18)$$

The measured output $y(t)$ is assumed to be contaminated by a stationary stochastic noise $n(t)$, see Fig. 4,

$$y(t) = y_u(t) + n(t). \quad (19)$$

Substituting for $y_u^j(t)$ in terms of its measurements one obtains

$$y^n(t) = \psi^T(t)\theta + e(t) \quad (20)$$

where $e(t)$ is the equation error.

Now measurements of the input and output signals are made and all required derivatives are determined at discrete times $t = kT_0$, $k = 0, 1, 2, \dots, N$ with T_0 the sampling time. Then $N + 1$ equations

$$y^n(k) = \psi^T(k)\theta + e(k) \quad (21)$$

result where $e(k)$ can be interpreted as an equation error, resulting in a vector equation

$$y^n = \Psi\theta + e. \quad (22)$$

The data matrix consists of $N + 1$ rows of the data vector $\psi^T(k)$. With the cost function

$$V = \sum_{k=0}^N e^2(k) = e^T e \quad \text{and} \quad dV/d\theta = 0 \quad (23)$$

the least squares estimate of the parameter vector becomes the well-known nonrecursive estimation equation

$$\hat{\theta} = [\Psi^T\Psi]^{-1}\Psi^Ty^n. \quad (24)$$

However, these parameters are biased for any noise $n(t)$. Therefore, the least-squares method should not be used, if the noise-to-signal ratio is not small.

4.2. *Determination of the time derivatives.* The parameter estimation of least-squares requires the time derivatives of the input signal $u(t)$ and the (noisy) output signal $y(t)$ up to the m th and the $(n - 1)$ th degree, respectively. There are mainly following possibilities to calculate these values from sampled measurements $u(k)$ and $y(k)$.

In the case of *numerical differentiation* the simplest way is to replace the derivatives by the corresponding (backward) differences. To reduce somewhat the influence of the noise interpolation formulas there is another way. For example, interpolation by splines (third order polynomials) or Newton interpolation can be used. However, the remaining noise influence restricts the application to second and third order processes.

The use of *state variable filtering* according to both the input signal $u(t)$ and the output signal $y(t)$, simultaneously provides the

time derivatives and filters the noise without differentiation (Young, 1981).

Since only the time derivatives of the signals are required there is some freedom in the choice of the filter parameters (Young and Jakeman, 1980).

4.3. Instrumental variables parameter estimation. To overcome the bias problem estimation of the instrumental variable (IV) concept can be used (Young, 1970, 1981). Instrumental variables are introduced which are only insignificantly correlated with the noise-free process output $y_n(t)$. A suitable way to generate the instrumental variables is to use an auxiliary model of the process which generates the instrumental variables.

This method yields consistent parameter estimates. To start the procedure, the least-squares methods can be used. It is also well suited to be programmed in a recursive form. For further details and improvements see Young (1981).

A major advantage of the IV method is that no strong assumptions and knowledge on the noise are required. However, in closed loop configurations biased estimates are obtained, because the input signal is correlated with the noise.

If on-line real-time parameter estimation is required both the least squares and the instrumental variables method can be written in recursive form (Isermann, 1981b).

4.4. Parameter estimation via discrete-time models. As the parameter estimation methods for discrete-time systems are fairly well developed one can first try to estimate the parameters of the discrete-time model and then to calculate the parameters of the continuous-time model by suitable transformation relationships. Mainly, two approaches are used, which are described, e.g. by Sinha and Lastman (1982), Strmčnik and Bremsák (1979) and Hung, Liu and Chou (1980).

However, these methods require extensive computation and are, in part, not straightforward, so that they are, at least in the present status, not feasible for on-line real-time applications.

In conclusion, it can be stated that contrary to discrete-time models the parameter estimation methods for continuous-time models have not obtained the same status. There is still a need for robust parameter estimation methods with less computational effort which provide consistent and efficient parameter estimates under many noise conditions and also in closed loop and for time-variant processes.

5. Fault detection for an electromotor driven centrifugal pump

The early detection of process faults is of course especially attractive for engines. Therefore, as a first example, a centrifugal pump with a water circulation system, driven by a speed-controlled direct current motor is considered (Fig. 6). The goal is to detect changes (faults) in the d.c. motor, the pump and the circulation system based on theoretically derived process models and parameter estimation. It is shown how the process coefficients can be determined. Some experiments demonstrate results obtained by using a microcomputer connected to the sensors of the engine set (Geiger, 1982).

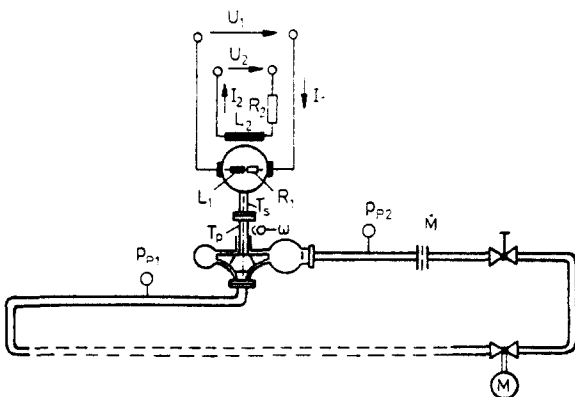


FIG. 6. Scheme of a speed controlled d.c. motor and centrifugal pump. p pressure, \dot{M} mass flow, ω angular velocity, T torque, U voltage, I current, R resistance, L inductance.

5.1. Mathematical process models. The dynamic models of the d.c. motor, the centrifugal pump and the pipe system are gained by stating the balance equations for energy and momentum and by using special physical relationships. In order not to obtain too many parameters, appropriate simplifications have to be made.

The derivation of the models and the symbols used are given in Appendix A. Some symbols are also indicated in Fig. 6.

Based on the equations (A6), (A17), (A25) and (A31) the block diagram of Fig. 7 results.

It shows that some process coefficients are lumped together, e.g. the friction coefficients of the motor c_{FM1} and the pump $c_{FP\omega}$ and the torque coefficient g_{ω} of the pump.

The resulting four basic equations will be used for parameter estimation in the following form:

(a) Armature circuit

$$\frac{dI_1(t)}{dt} = a_{11}\Delta I_1(t) + a_{12}\Delta\omega(t) + b_1\Delta U_1(t). \quad (25)$$

(b) Mechanics of motor and pump.

$$\frac{d\omega(t)}{dt} = a_{21}\Delta I_1(t) + a_{22}\Delta\omega(t) + a_{23}\Delta\dot{M}(t). \quad (26)$$

(c) Pipe system.

$$\frac{d\dot{M}(t)}{dt} = a_{33}\Delta\dot{M}(t) + d_3\Delta Y(t). \quad (27)$$

(d) Pump (specific energy Y).

$$\Delta Y(t) = h_{\omega}(t) + h_M\Delta\dot{M}(t). \quad (28)$$

The parameters are

$$\left. \begin{aligned} a_{11} &= -\frac{R_1}{L_1} & b_1 &= \frac{1}{L_1} \\ a_{12} &= -\frac{\Psi}{L_1} \\ a_{21} &= \frac{\Psi}{\theta} = \frac{\Psi}{\theta_M + \theta_P} \\ a_{22} &= -\frac{c_{F1} + g_{\omega}}{\theta} \\ a_{23} &= -\frac{g_M}{\theta} \\ a_{33} &= -\frac{a_R}{a_{ac}} & d_3 &= \frac{1}{a_{ac}} \end{aligned} \right\} \quad (29)$$

A state variable representation

$$\begin{aligned} \dot{x}(t) &= Ax(t) + bu(t) \\ y(t) &= Cx(t) \end{aligned} \quad (30)$$

can now be given with following definitions

$$\left. \begin{aligned} x(t) &= \begin{bmatrix} \Delta I_1(t) \\ \Delta\omega(t) \\ \Delta\dot{M}(t) \end{bmatrix}; & A &= \begin{bmatrix} a_{11} & a_{12} & 0 \\ a_{21} & a_{22} & a_{23} \\ 0 & a_{32} & a_{33} \end{bmatrix} \\ y(t) &= \begin{bmatrix} \Delta I_1(t) \\ \Delta\omega(t) \\ \Delta\dot{M}(t) \\ \Delta Y(t) \end{bmatrix}; & C &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & h_{\omega} & h_M \end{bmatrix} \\ b &= \begin{bmatrix} b_1 \\ 0 \\ 0 \end{bmatrix} & u(t) &= \Delta U_1(t) \\ a_{32} &= \frac{h_{\omega}}{a_{ac}}; & a_{33} &= a_{33} + \frac{h_{\omega}}{a_{ac}} \end{aligned} \right\} \quad (31)$$

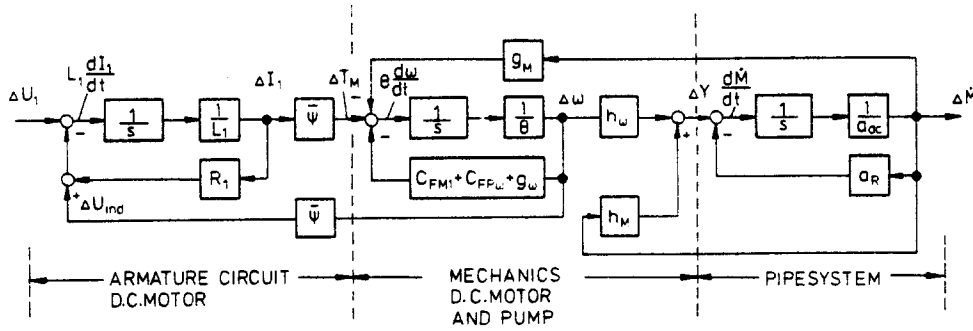


FIG. 7. Block diagram of the linearized d.c. motor-pump-pipe system.

5.2. *Process parameter monitoring.* The parameters of (25)–(28) can be estimated by bringing them into the form of (21) and by applying, for example, the least-squares method. One of the questions then is, how the various process coefficients which are required for fault detection can be calculated. Therefore several cases of operation and measurements are discussed.

(a) *D.c. motor and pump, closed valve.* In this case $\dot{M}(t) = 0$ is valid, so that only (25) and (26) are to be used.

(i) *Measured signals:* $\Delta U_1, \Delta I_1, \Delta \omega$

Both equations are written due to (21)

$$\begin{aligned} y_1(t) &= \psi_1^T(t) \hat{\theta}_1 \\ y_2(t) &= \psi_2^T(t) \hat{\theta}_2 \end{aligned} \quad (32)$$

where

$$\left. \begin{aligned} y_1(t) &= dI_1(t)/dt & y_2(t) &= d\omega(t)/dt \\ \psi_1^T(t) &= [\Delta I_1(t) \Delta \omega(t) \Delta U_1(t)] \\ \theta_1^T &= [\hat{a}_{11} \quad \hat{a}_{12} \quad \hat{b}_1] \\ \psi_2^T(t) &= [\Delta I_1(t) \Delta \omega(t)] \\ \theta_2^T &= [\hat{a}_{21} \quad \hat{a}_{22}] \end{aligned} \right\} \quad (33)$$

Using (29), the following five process coefficients can be calculated based on the five parameter estimates $\hat{\theta}_1$ and $\hat{\theta}_2$:

$$\begin{aligned} \hat{L}_1 &= \frac{1}{\hat{b}_1} \\ \hat{R}_1 &= -\hat{a}_{11} \hat{L}_1 = -\hat{a}_{11} / \hat{b}_1 \\ \hat{\Psi} &= -\hat{a}_{12} \hat{L}_1 = -\hat{a}_{12} / \hat{b}_1 \\ \hat{\theta} &= \hat{\Psi} / \hat{a}_{21} = -\hat{a}_{12} / \hat{b}_1 \hat{a}_{21} \\ \hat{c}_{F1} &= c_{FM1} + c_{FP\omega} = \hat{a}_{22} \hat{\theta} = \hat{a}_{22} \hat{a}_{12} / \hat{b}_1 \hat{a}_{21} \end{aligned} \quad (34)$$

Hence, all process coefficients which describe the linearized dynamic behaviour can be calculated. However, the friction coefficients of the motor c_{FM1} and the pump $c_{FP\omega}$ and the moments of inertia θ_m and θ_p are lumped together so that only their sum can be gained.

If not the dynamic behaviour, but only the static behaviour could be identified, L_1 and θ could be not obtained, see (A6) and (A25) for $d/dt = 0$. This shows that by identifying the dynamics more parameters can be estimated and therefore more process coefficients can be monitored.

A disadvantage of the linearized dynamic relationships is that the coefficients c_{FM0} and c_{FP0} for the adhesive friction do not appear. However, from (A3) and (A10) it follows with the assumption that the friction torque only depends linearly on the speed

$$\begin{aligned} T_{FM}(t) &= c_{FM0} + c_{FM1} \omega(t) \\ T_{FP}(t) &= c_{FP0} + c_{FP1} \omega(t) \end{aligned}$$

$$\underbrace{(\theta_p + \theta_m)}_0 \frac{d\omega(t)}{dt} = \Psi I_1(t) - c_{F0} - c_{F1} \omega(t) \quad (35)$$

$$c_{F0} = c_{FM0} + c_{FP0}$$

$$c_{F1} = c_{FM1} + c_{FP1}$$

Then the absolute values $\omega(t)$ and $I_1(t)$, and not their deviations, are used and the estimation of c_{F0} also becomes possible (Geiger, 1982).

(ii) *Measured signals:* $U_1, \Delta \omega$

It is now assumed that the armature current $I_1(t)$ cannot be measured, but only the input $U_1(t)$ and the output $\omega(t)$. Equation (A6) and (A25) then lead to the model

$$\frac{d^2\omega(t)}{dt^2} + \alpha_1 \frac{d\omega(t)}{dt} + \alpha_0 \omega(t) = \beta_0 \Delta U_1(t) \quad (36)$$

with

$$\left. \begin{aligned} \alpha_1 &= \frac{R_1}{L_1} + \frac{c_{F1}}{\theta} & \alpha_0 &= \frac{1}{\theta L_1} (c_{F1} R_1 + \Psi^2) \\ \beta_0 &= \frac{\Psi}{L_1} & (c_{F1} &= c_{FM1} + c_{FP\omega}) \end{aligned} \right\} \quad (37)$$

As there are five unknown process coefficients and only three estimated parameters, the process coefficients cannot be calculated uniquely. Two coefficients have to be known. If θ and L_1 are known, R_1 , Ψ and c_{F1} can be determined. This example shows that it is important to measure as many variables as possible. If the static behaviour can be identified, only one parameter

$$\left(\frac{\alpha_0}{\beta_0} \right) = \left[\frac{c_{F1} R_1}{\Psi} + \Psi \right]$$

could be estimated. Then two coefficients have to be assumed as known, to determine one coefficient. Hence, also in this case more coefficients are obtained by the use of dynamic models.

If too little or no coefficients can be assumed as known, the single process coefficients cannot be calculated. However, then the changes of the parameter estimates can be monitored. Table 1 shows the sign of these parameter estimates in dependence on the changes (faults) of the process coefficients. In this case only one pattern is unique, that for $\Delta \Psi$. Then also the magnitudes of the changes may be taken into account, to detect which of the process coefficients has changed, using their nominal values and sensitivity functions.

(b) *D.c. motor, pump and pipe system (valve opened).* Now all four equations (25)–(28) have to be used.

(i) *Measured signals:* $\Delta U_1, \Delta I_1, \Delta \omega, \Delta Y(t), \Delta \dot{M}(t)$

The four equations are now

$$y_j(t) = \psi_j^T(t) \hat{\theta}_j \quad j = 1, 2, 3, 4 \quad (38)$$

TABLE 1. PATTERN FOR THE SIGN OF THE PARAMETER CHANGES

	$\Delta \hat{x}_1$	$\Delta \hat{z}_0$	$\Delta \hat{\beta}_0$
$+\Delta R_1$	+	+	0
$+\Delta \Psi$	0	+	+
$+\Delta L_1$	-	-	-
$+\Delta \theta$	-	-	-
$+\Delta c_{F1}$	+	+	0

where

$$\left. \begin{aligned} y_1(t) &= dI_1(t)/dt & y_2(t) &= d\omega(t)/dt \\ y_3(t) &= d\dot{M}(t)/dt & y_4(t) &= \Delta Y(t) \end{aligned} \right\} \quad (39)$$

$$\begin{aligned} \psi_1^T(t) &= [\Delta I_1(t) \quad \Delta \omega(t) \quad \Delta U_1(t)] \\ \theta_1^T &= [a_{11} \quad a_{12} \quad b_1] \\ \psi_2^T(t) &= [\Delta I_1(t) \quad \Delta \omega(t) \quad \Delta \dot{M}(t)] \\ \theta_2^T &= [a_{21} \quad a_{22} \quad a_{23}] \\ \psi_3^T(t) &= [\Delta \dot{M}(t) \quad \Delta Y(t)] \\ \theta_3^T &= [a'_{33} \quad d_3] \\ \psi_4^T(t) &= [\Delta \omega(t) \quad \Delta \dot{M}(t)] \\ \theta_4^T &= [h_\omega \quad h_M] \end{aligned} \quad (40)$$

By using (29) ten process coefficients can be calculated from the ten parameter estimates $\hat{\theta}_j$.

$$\begin{aligned} \hat{L}_1 &= \frac{1}{\hat{b}_1} \\ \hat{R}_1 &= -\hat{a}_{11}\hat{L}_1 = -\hat{a}_{11}/\hat{b}_1 \\ \hat{\Psi} &= -\hat{a}_{12}\hat{L}_1 = -\hat{a}_{12}/\hat{b}_1 \\ \hat{\theta} &= \hat{\Psi}/\hat{a}_{21} = -\hat{a}_{12}/\hat{b}_1\hat{a}_{21} \\ \hat{c}_{F1} + \hat{g}_\omega &= -\hat{a}_{22}\hat{\theta} = \hat{a}_{22}\hat{a}_{12}/\hat{b}_1\hat{a}_{21} \\ \hat{g}_M &= -\hat{a}_{23}\hat{\theta} = \hat{a}_{23}\hat{a}_{12}/\hat{b}_1\hat{a}_{21} \\ \hat{a}_{ac} &= 1/\hat{d}_3 \\ \hat{a}_R &= -\hat{a}'_{33}\hat{a}_{ac} = -\hat{a}'_{33}/\hat{d}_3 \\ \left. \begin{aligned} \hat{h}_\omega \\ \hat{h}_M \end{aligned} \right\} &\text{directly available from } \theta_4. \end{aligned} \quad (41)$$

Also in this case all process coefficients are obtainable and can be monitored. In addition to the last case, c_{F1} and g_ω are lumped together.

The use of dynamic models instead of static models enables one to estimate L_1 , θ and a_{ac} .

(ii) Some signals not measurable

The problems which arise, if $I_1(t)$ cannot be measured, have been already discussed, see (36). Therefore it will be assumed that at least $U_1(t)$, $I_1(t)$ and $\omega(t)$ are measured.

$Y(t)$ not measurable, but $\dot{M}(t)$.

Introduction of (28) into (27) leads to

$$\frac{d\dot{M}(t)}{dt} = (a'_{33} + d_3h_M)\Delta \dot{M}(t) + d_3h_\omega\Delta \omega(t). \quad (42)$$

Hence, as a'_{33} , d_3 , h_M and h_ω are not obtained separately, the process coefficients a_R , a_{ac} , h_M , h_ω and c_{F1} cannot be determined

uniquely. However, all remaining six coefficients can still be obtained.

$\dot{M}(t)$ not measurable, but $Y(t)$.

Replacing $\Delta \dot{M}(t)$ in (26) and (27) by use of (28) leads to

$$\frac{d\omega(t)}{dt} = a_{21}\Delta I_1(t) - \left(a_{22} - a_{23}\frac{h_\omega}{h_M} \right) \Delta \omega(t) + \frac{a_{23}}{h_M} \Delta Y(t) \quad (43)$$

$$\frac{dY(t)}{dt} = (a'_{33} + d_3h_M)\Delta Y(t) + h_\omega \frac{d\omega(t)}{dt} - h_\omega a'_{33} \Delta \omega(t) \quad (44)$$

Now d_3 , h_M and a_{23} are not obtainable separately and therefore the process coefficients a_R , a_{ac} , h_M , g_M and c_{F1} cannot be determined uniquely, but only the remaining five coefficients.

$\dot{M}(t)$ and $Y(t)$ not measurable.

Inserting (28) and (27) into (26) yields

$$\frac{d^2\omega(t)}{dt^2} = \alpha_1 \frac{d\omega(t)}{dt} + \alpha_0 + \beta_1 \frac{dI_1(t)}{dt} + \beta_0 \Delta I_1(t) \quad (45)$$

with

$$\left. \begin{aligned} \alpha_1 &= -(a'_{33} + d_3h_M - a_{22}) \\ \alpha_2 &= a_{22}(d_3h_M - a'_{33}) - a_{23}d_3h_\omega \\ \beta_1 &= a_{21} \\ \beta_0 &= a'_{33}d_3h_Ma_{21}. \end{aligned} \right\} \quad (46)$$

Based on (25), three process coefficients can be estimated: L_1 , R_1 and Ψ . As β_1 enables us to estimate θ , three parameter estimates remain to determine six coefficients, which is not possible uniquely. Therefore three process coefficients are assumed to be known. Hence, in this case, only four process coefficients can be calculated uniquely.

If the sensors do have dynamics which cannot be neglected in comparison to the process, they must also be included in the process model.

5.3. Experimental results. Experiments were made with a centrifugal pump driven by a speed controlled d.c. motor. The technical data are

- (a) D.c. motor:
 - maximum power $P_{\max} = 4 \text{ kW}$
 - maximum rotation speed $N_{\max} = 3000 \text{ rev min}^{-1}$.
- (b) Centrifugal pump, one stage:
 - maximum total head $H_{\max} = 39 \text{ m}$
 - for $N_{\max} = 3000 \text{ rev min}^{-1}$.
- (c) Pipe system: length: $L \sim 10 \text{ m}$
diameter: $d_i = 50 \text{ mm}$.

The d.c. motor is controlled by an a.c./d.c. converter with cascade control of the speed and the armature current as auxiliary control variable. The manipulated variable is the armature current U_1 . A microcomputer DEC-LSI 11/23 was connected on-line to the process. For the experiments the reference value $W(t)$ of the speed control has been changed stepwise with a magnitude of 2% of N_{\max} , i.e. 60 rev min^{-1} every 60 s. The measured signals were sampled with sampling time $T_0 = 2 \text{ ms}$ over a period of 2 s, so that 1000 samplings were obtained.

These measurements were stored in the core-memory. After 2 s measurements the parameters were estimated off-line, using the recursive least-squares method with state variable filters for the determination of the time derivatives. The available computation time for the parameters was 58 s. Hence, one set of parameter estimates was obtained every minute. As the noise is negligibly small, the parameter estimates can be assumed to be unbiased.

The first experiments were performed with closed valve. As

described before (d.c. motor and pump, closed valve) then two equations can be applied for the parameter estimation. In order to also obtain the adhesive friction coefficient, (35) has been used

$$\theta \frac{d\omega(t)}{dt} = \Psi I_1(t) - c_{F0} - c_{F1}\omega(t)$$

together with (A1) for the armature circuit

$$L_1 \frac{dI_1(t)}{dt} = U_1(t) - R_1 I_1(t) - \Psi \omega(t).$$

Therefore the deviations of the signals have to be replaced in (33) by their absolute values. The process coefficients are obtained by (34) with $\hat{c}_{F0} = -\hat{a}_{23}\hat{\theta}$ in addition.

In Figs 8–11 results of the parameter monitoring are presented. Figure 8 shows the step responses after a speed setpoint change. The resulting process coefficients after a start of the cold engine (Fig. 9), indicate that the armature resistance increases during the first 10 min, the flux linkage decreases during 20 min and the friction torque coefficient decreases during the first hour. Hence, small changes of the process coefficients can be detected.

Figures 10 and 11 show the reaction on artificial changes (faults). A significant change of the armature resistance estimate is detectable after a 7% change (Fig. 10). The effect of tightening and loosening the screws of the pump packing box cap is clearly seen in Fig. 11. More details are given in Geiger (1982). Results of more experiments including multiple hypothesis testing for the fault decision are described in Geiger (1984).

The known literature shows only very few applications of this way of process parameter fault detection. Results with a hydraulic drive of a machine tool are given in Hohmann (1977) and with jet engines in Baskiotis, Raymond and Rault (1979) and Baskiotis *et al.* (1981). The last reference also includes parameter monitoring of the human glucose control system. Results for a d.c. motor are published by Filbert and Metzger (1982).

5.4. Conclusion for parameter monitoring. Fault detection and diagnosis based on parameter monitoring require precise theoretical models and parameter estimation methods.

The use of dynamic models instead of static models allows to monitor more process coefficients, the more the higher the order.

A unique calculation of the process coefficients and a parameter estimation with high precision is only possible for low order elements between measured variables. Therefore the measured variables should be selected such that the process is divided in first order elements or, in other words, all state variables should be measurable.

Easy to implement parameter estimation methods for continuous-time models to be used on-line, real-time and in closed loop need to be developed.

6. Leak detection for pipelines

The detection and localization of leaks in liquid and gas pipelines is important because of safety, environment and economy. The applied methods which work during normal operation are mainly based on balancing the input and output flows. However, leaks smaller than about 2% of the total flow for liquids and about 10% for gases cannot be detected by these methods due to noise effects and inherent dynamics. Opposite to engines and other industrial plants pipelines are, in general, not well instrumented. The only available sensors are mostly flow-rate and pressure at both ends of a pipeline section (Fig. 12). The question now is if, by the use of process models and estimation techniques, it will be possible to detect and localize small leaks rapidly.

The following cases have to be distinguished with respect to leak detection methods (Siebert, 1981):

- (i) Medium: liquid – gas – multiple phase.
- (ii) Operation: standstill – stationary – small changes – unstationary.
- (iii) Size of leakage: large – medium – small (pipe burst).
- (iv) Development of leakage: abrupt (cracking of welding seam); slow (hole corrosion) or already existing.
- (v) Leak monitoring: continuously – on request.

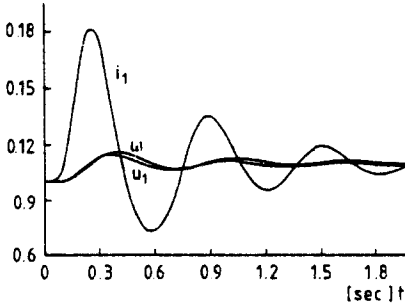


FIG. 8. Step responses for a change of the speed setpoint. $u_1 = U_1/\bar{U}_1$ armature voltage $\bar{U}_1 = 60$ V, $i_1 = I_1/\bar{I}_1$ armature current $\bar{I}_1 = 0.5$ A, $\omega = \omega_1/\bar{\omega}_1$ angular velocity $\bar{\omega} = 62.83$ s⁻¹ ($\cong 600$ rev min⁻¹).

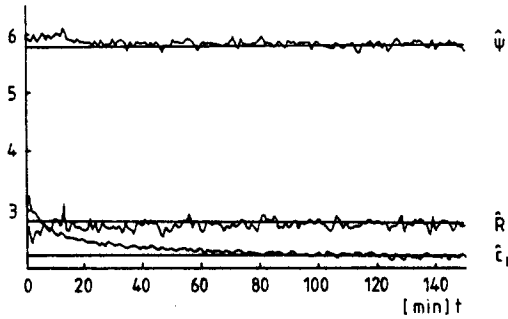


FIG. 9. Process coefficient estimates after start of the cold engine. R_1 armature resistance, Ψ flux linkage, c_{F0} friction coefficient.

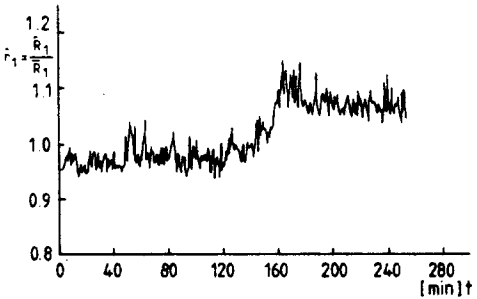


FIG. 10. Change of armature circuit resistance. $\Delta R_1 \sim 0.2 \Omega$, $\bar{R}_1 \sim 3 \Omega$, $r_1 = R_1/\bar{R}_1$.

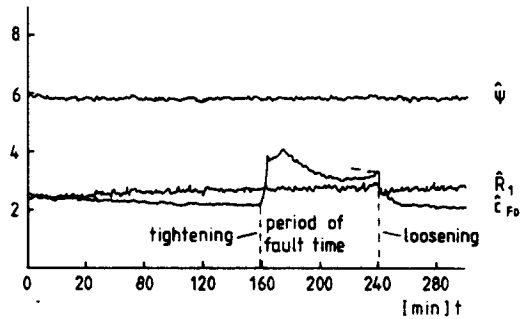


FIG. 11. Change of pump packing box friction by tightening and loosening of the cap screws.

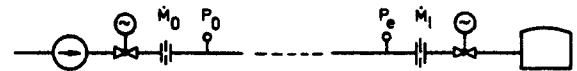


FIG. 12. Usual instrumentation of a pipeline p pressure, \dot{M} mass flow.

In the following, methods for leak detection will be discussed for liquids and gases, stationary operation with small changes of the variables, small leaks which appear abrupt, and continuous monitoring.

6.1. *Mathematical process models.* The dynamic models of a pipeline and the symbols used are described in Appendix B. Based on the mass and momentum balance equations, the quadratic friction law, the isothermic gas state equation and various simplifying assumptions a nonlinear hyperbolic partial differential equation system is obtained, (B9). After dividing the pipeline in $l/2$ sections and discretization of the differential equations, a set of ordinary differential equations results which forms a nonlinear state space model, see (B15).

For small changes and the flow \dot{M} in the positive z -direction, the momentum balance equation of (B12) can be linearized

$$\frac{\partial \dot{M}_j}{\partial t} = g_2(\Delta p_{j+1} - \Delta p_{j-1}) + 2g_{3(j-1)}\Delta \dot{M}_j \quad (47)$$

where all coefficients are taken for the steady-state values \bar{p}_j and \bar{M}_j . Further on the linearized valve equations are introduced in the form

$$\left. \begin{aligned} \Delta p_0 &= c'_{v0}\Delta \dot{M}_0 + \Delta p_{ia} \\ \Delta p_m &= c'_{vm}\Delta \dot{M}_m + \Delta p_{ex} \end{aligned} \right\} \quad (49)$$

Then a linear state representation

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) \end{aligned} \quad (49)$$

results, with

$$\left. \begin{aligned} x^T(t) &= [\Delta \dot{M}_0 \Delta \dot{M}_2 \dots \Delta \dot{M}_l \Delta p_1 \Delta p_3 \dots \Delta p_{l-1}] \\ u^T &= [\Delta p_{ia} \Delta p_{ex}] \\ y^T &= [\Delta \dot{M}_0 \Delta \dot{M}_l] \end{aligned} \right\} \quad (50)$$

However, for most of the gas pipelines the nonlinear (B15) must be used. It is because of their big storage capacities and the time-dependent consumption that they rarely come to a steady state.

6.2. *Methods for leak detection.* It is assumed that a small leak flow $d\dot{M}_L$ occurs at section $j = \xi$. The effect of the leak can be modeled by introducing this leak flow into the mass balance of this section, see (B18), leading to (B20). This changes the linearized state equation (49) to

$$\dot{x}(t) = Ax(t) + Lv(t) + Bu(t) \quad (51)$$

with the leak flow vector

$$v^T(t) = [0 \ 0 \dots \dot{M}_{L\xi} \dots 0 \ 0 \dots 0] \quad (52)$$

and the leak influence matrix

$$L = \begin{bmatrix} 0 & \dots & 0 & 0 & \dots & 0 \\ \vdots & & \vdots & \vdots & & \vdots \\ 0 & \dots & 0 & 0 & \dots & 0 \\ 0 & g_{11} & \dots & 0 & 0 & \dots & 0 \\ \vdots & & q_{1\xi} & \vdots & \vdots & & \vdots \\ 0 & \dots & g_{1(m-1)} & 0 & \dots & 0 \end{bmatrix} \quad (53)$$

Hence a leak flow appears as a disturbance of the state variables $x(t)$ (Fig. 13).

The leak monitoring task now consists in the detection of an appearing leakage, its localisation ξ_L along the pipeline and the estimation of its size \dot{M}_L . In most cases only measurements $\dot{M}_0(t)$, $p_0(t)$ and $\dot{M}_l(t)$, $p_l(t)$ at the inlet and the exit of the pipeline are available.

The results of simulations for a gasoline and an ethylene-gas pipeline assuming different locations of a suddenly appearing

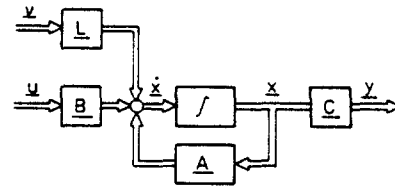


FIG. 13. A leak appears as disturbance $v(t)$ (without noise terms).

leak of 5 % of the flow-rate are shown in Siebert (1981). If the leak location is about in the middle of the pipeline the flows $\dot{M}_0(t)$ and $\dot{M}_l(t)$ change with about the same settling times and reach their new steady-states after about 4 min for the gasoline pipeline and 2 h for the gas pipeline. If the leak is closer to one end, the time responses and the magnitudes of the flow changes become rather different.

Now various approaches for leak monitoring are shown, followed by experimental results with a rather simple method.

(a) *Fault sensitive filters.* For the detection of the leak a state variable filter

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + H[y(t) - C\hat{x}(t)] \quad (54)$$

can be designed to estimate the states and to calculate the residuals

$$\tilde{y}(t) = y(t) - C\hat{x}(t) \quad (55)$$

thereby assuming that all parameters of the pipeline system are known.

In the first adjustment phase, the filter gain H can be large so that a fast adjustment occurs in order to make the filter less sensitive to high frequency residual changes. For leak detection the gain H is lowered. Then if a leak $\dot{M}_{L\xi}$ occurs suddenly the residuals $\tilde{y}(t)$ show a variation $+\Delta \dot{M}_0(t)$ and $-\Delta \dot{M}_l(t)$, i.e. the residuals change in predetermined directions what can be used for the fault detection.

One drawback of this method is that the filter tries to match the leak disturbed process after a while so that the information on the leak vanishes with time.

(b) *Fault model filters.* Another method consists in an estimation of the leak flow vector $v(t)$ by modeling the fault influence in the filter (Fig. 14). The influence of a suddenly (stepwise) appearing leak is modeled by

$$\dot{v}(t) = v(t) \quad (56)$$

so that the filter is able to reconstruct a remaining leak vector

$$\hat{v}(t) = H_v \int_{t_0}^t \tilde{y}(t') dt' \quad (57)$$

If the filter converges in the right way the estimated leak vector contains as well as the size, the section number ξ where the leak occurred. To extract this information under noisy conditions a 'bank of filters' could be used, assuming different locations ξ or the leak vector estimates could be correlated with a 'fault signature', for example the flow profile after a leak. This relies on the state variable fault detection methods discussed in Section 3.2.

A simulation with multiple model hypothesis probability testing for an oil pipeline of 30 km length, using parallel Kalman filters, is described by Digernes (1980). A leak of 1 % of the total flow was detected after about 160 s.

Another reference on a fault-sensitive state variable detector is Candy and Rozsa (1980). Here a simulation study on the detection of diverted or stolen nuclear material (corresponding to a leak) in a plutonium concentrator by using statistical decision methods for the residuals of an extended Kalman filter is shown. Relatively large computation time and storage was required.

In order to estimate the leak location for a pipeline of 100 km length with an accuracy of about ± 1 km at least 50 sections have

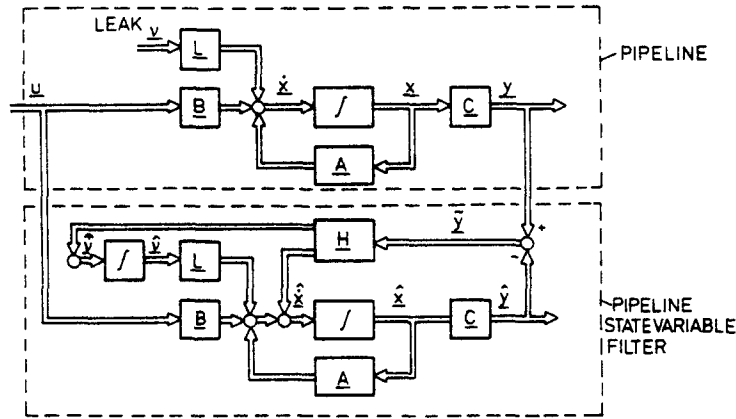


FIG. 14. Pipeline state variable filter including a leak influence model.

to be modeled so that a high system order results, which causes computational problems for standard process computers. Additionally, several process parameters are not known precisely enough, i.e. the friction coefficients, and also several temperature effects, so that the models have to be updated by parameter estimation methods (Billmann, 1983).

Therefore, a simpler method was developed first for liquid pipelines, taking into account several practical requirements and special cases of pipeline operation.

(c) *A cross-correlation method for leak detection of stationary liquid pipelines.* It is assumed that a liquid pipeline operates in a stationary steady-state and that only the flow-rates at the inlet $\dot{M}_0(k)$ and the exit $\dot{M}_l(k)$ can be measured.

The simplest well-known method of leakage monitoring is then by stating a static balance equation

$$\dot{M}_L(k) = \dot{M}_0(k) - \dot{M}_l(k) \quad (58)$$

and by triggering an alarm if the leak flow \dot{M}_L exceeds a certain limit. This pure balancing method is, however, not suitable for the detection of smaller leaks, because of the noise signals, drifting measurements and the dynamic changes of both flows.

An improvement of this method is obtained by determining the low frequency components of the flows by discrete-time low-pass filtering

$$\dot{M}_j^*(k) = \kappa_M \dot{M}_j^*(k-1) + (1 - \kappa_M) \dot{M}_j^*(k) \quad (j = 0, 1). \quad (59)$$

$\dot{M}_0^*(k)$ and $\dot{M}_l^*(k)$ are then reference values, which may be different because of calibration errors of the sensors, and change slowly due to temperature and viscosity changes also in the case of constant pumping. A leak is then obtained by

$$\begin{aligned} \Delta \dot{M}_0(k) &= \dot{M}_0(k) - \dot{M}_0^*(k) \\ \Delta \dot{M}_l(k) &= \dot{M}_l(k) - \dot{M}_l^*(k) \\ \dot{M}_L(k) &= \Delta \dot{M}_0(k) - \Delta \dot{M}_l(k) \end{aligned} \quad (60)$$

or by further low-pass filtering

$$\dot{M}_L''(k) = \kappa_L \dot{M}_L''(k-1) + (1 - \kappa_L) \dot{M}_L''(k) \quad (61)$$

where $\kappa_L < \kappa_M$ is required in order to be able to detect suddenly-appearing leaks. If $\dot{M}_L''(k)$ exceeds a certain threshold

$$\dot{M}_L''(k) > \dot{M}_{Lx}$$

a leak alarm is given.

By the described low-pass filtering, noise effects and slow drift effects can be partly eliminated, but changes of the flows according to the inherent fluid dynamics cause the adjustment of relatively large thresholds \dot{M}_{Lx} in order to avoid too frequent false alarms.

This is one of the reasons that it was proposed to cross-correlate the differences

$$\Phi_{MM}(\tau) = \frac{1}{N} \sum_{k=1}^N \Delta \dot{M}_0(k - \tau) \Delta \dot{M}_l(k) \quad (62)$$

(Isermann and Siebert, 1976, 1977). This cross-correlation function reacts sensitively even to small leaks, reduces noise effects and models inherent dynamic relationships between the flow changes. For further noise reduction the correlation function is averaged with respect to

$$\Phi_Z = \frac{1}{2P+1} \sum_{\tau=-P}^P \Phi_{MM}(\tau). \quad (63)$$

After a leak has occurred the fault signature consists in changes $+\Delta \dot{M}_0$ and $-\Delta \dot{M}_l$ so that the products become negative and Φ_Z decreases. An alarm is given, if

$$\Phi_Z < \Phi_{Zc}. \quad (64)$$

The cross-correlation function is calculated recursively with fading memory

$$\Phi_{MM}(\tau, k) = \lambda \Phi_{MM}(\tau, k-1) + (1 - \lambda) [\Delta \dot{M}_0(k - \tau) \Delta \dot{M}_l(k)] \quad (65)$$

where $0.9 < \lambda < 1$. Larger values of λ result in improved smoothing and thus reduce the noise, but lead in turn to a delayed alarm.

Assuming stationary operating conditions, the location of the leak can be estimated by calculation of the point of intersection of the pressure curves. Their gradient is given for a turbulent flow by

$$\frac{\partial p}{\partial z} = c_p = \frac{\dot{M}^2}{p_0 - p_l}. \quad (66)$$

Here, too, it is advantageous to use the deviations from a reference value. Therefore, the pressure gradients are estimated by recursive averaging with fading memory

$$c_j(k) = \kappa_c c_j(k-1) + (1 - \kappa_c) \frac{\dot{M}_j^2(k)}{p_0(k) - p_l(k)} \quad (j = 0, l). \quad (67)$$

Then reference mass flows are determined

$$\bar{\dot{M}}_j^{*2}(k) = c_j(k) [p_0(k) - p_l(k)] \quad (j = 0, l). \quad (68)$$

As soon as a leakage alarm is triggered, c_0 and c_l are no longer calculated but fixed and the differences

$$\Delta_{M,j}^2(k) = \dot{M}_j^2(k) - \bar{\dot{M}}_j^{*2}(k) \quad (j = 0, l) \quad (69)$$

are determined and summed up. Assuming small leaks the leak location is given by

$$\hat{z}_L = L \left(1 - \frac{\sum \Delta_{M0}^2(k)}{\sum \Delta_{Ml}^2(k)} \right) \quad (70)$$

see Siebert and Isermann (1977). In a similar way the leak flow is estimated.

6.3. Experimental results with pipelines. Figure 15 shows the course of the pipeline considering the height above sea level as well as the location of pumps. The two main pumps are driven by two 400 kW asynchronous machines, which may be operated individually or together. At full power about $330 \text{ m}^3 \text{ h}^{-1}$ are delivered at an initial pressure of 69 bar. This pressure is measured after the pumps, but before the entrance valve.

The line has a diameter of 273 mm and a wall thickness of 8 mm. Intermediate depots are located at 21.1, 27.3, 35.8, 43.9 and 46.7 km.

The volume flows are measured by means of measuring orifices and Barton cells, pressures with Barton cells (accuracy about 0.1 %). The volume flow $V(l)$ at the end of the line is transmitted by a telemetric device, i.e. deviations from the operating point—about 1/10 of the total measuring range ($0\text{--}400 \text{ m}^3 \text{ h}^{-1}$)—are encoded into an 8-bit-word. This corresponds to a resolution of $0.16 \text{ m}^3 \text{ h}^{-1}$ or 0.05 % with respect to $330 \text{ m}^3 \text{ h}^{-1}$.

Since the pressure at the end of the line was almost constantly equal to atmospheric pressure, recording and processing of the measurement variable $p(l)$ was dispensed with. Thus only both volumetric flows $V(0)$ and $V(l)$, as well as the pressure at the beginning of the pipeline, were used for leakage monitoring. For further details on the pipeline and the equipment see Siebert and Klaiber (1980).

An INTEL MDS 800 microcomputer development system was used to carry out the on-line experiments with the pipeline described above. An 8-bit microcomputer with the 8080 A central processor was used. The system is extended to 48 k of RAM.

An extensive program package—16 kbytes of program memory—for leakage monitoring was implemented on the microcomputer system in the ASM 80 assembler language.

A series of experiments for leakage detection were carried out on the pipeline described above, where leaks could be generated artificially at the branches to the intermediate depots.

The following values were assumed for the constants:

$$\kappa_c = 0.9975$$

$$\lambda = 0.99$$

$$\Phi_{Lr} = -0.5$$

$$P = 20.$$

Figure 16 shows one of the experiments carried out. The values of the signals $p(0)$, $V(0)$ and $V(l)$ collected by the microcomputer were recorded, below them the sum of cross-correlation functions Φ_L for the indication of a leakage is shown. The leak was generated at $t = t_L$. The leakage location and leakage flow calculation after detection of the leak is also shown.

In the experiment shown—leakage location at 35.8 km and mean leakage rate of 0.19 % which is about 0.21 sec^{-1} —the trigger level was exceeded 98 s after occurrence of the leak and the alarm was triggered. The leak location was estimated with an

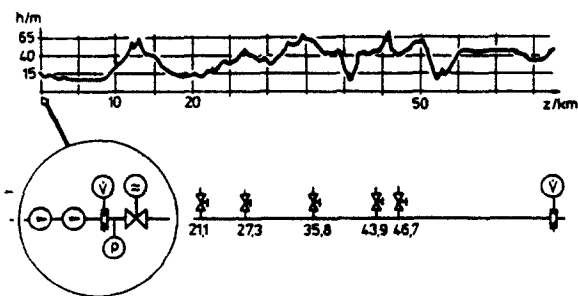


FIG. 15. Schematic diagram of the pipeline with topographic profile.

error of about $\pm 0.7\%$ or $\pm 500 \text{ m}$ at time 90 s after the alarm.

Since the characteristic variable Φ_L in all cases significantly exceeded its standard deviation during regular operation without a leak, it may be assumed that even smaller leaks than those in the experiments carried out can be detected and localized.

The described cross-correlation method which is well suited for liquid pipelines, was modified for the use of gas pipelines by taking into account the quadratic pressure profile and dynamic models of the pressures and flows (Siebert and Isermann, 1980; Siebert, 1981). Signals of an ethylene pipeline ($L = 150 \text{ km}$; $d_i = 260 \text{ mm}$; $\dot{M} = 17 \text{ t h}^{-1}$) have been measured to verify the dynamic models. Simulation of leaks with these models show (Fig. 17) that the modified cross-correlation method is able to detect leaks of about 2 % after about 10 h and 5 % after 3.5 h. These results allow to show a typical relationship for fault detection methods between the alarm delay time and the size of the alarm under noisy conditions (Fig. 18). The methods on leak detection are presently further developed (Billmann, 1983; Billmann and Isermann, 1984).

6.4. Conclusions for leak detection. Leak detection methods require a precise knowledge of the normal state (precise normal model, or precise reference values).

The use of dynamic models instead of static models enables detection of smaller leaks.

If no precise parametric model from theoretical modeling is known or the computational effort becomes too large, nonparametric models may also be used (cross-correlation method instead of state-model methods).

The leak detection method has to be insensitive to process noise but sensitive to the appearance of a leak. Thus the normal model should only follow low frequency signal components, whereas the leak signature components should be detected in the high-frequency range. The methods are then better suited to detect abrupt leaks than slowly developing leaks.

The influence of drift effects of the sensors and the pipeline are to be suppressed by special methods.

Due to the remaining noise effects there is approximately a hyperbolic relationship between the alarm delay and the size of the leak.

Many of these statements hold quite generally for fault detection methods.

7. Concluding remarks

An attempt has been made, based on the present knowledge and experience, to review some fault detection methods. Emphasis has been given on methods for monitoring unmeasurable quantities like process parameters and process state variables. Fault detection methods for these classes make use of process models, parameter and state estimation methods and statistical decision methods. In designing fault detection methods one should take into account the following aspects.

Process models. In many cases rather accurate process models are required. Hence, as well as theoretical process modeling continuous-time process parameter estimation plays a basic role. The discussed fault detection methods are therefore easier to apply for well-defined processes, like electrical and mechanical processes than for others, such as thermal and chemical processes. Since the methods are based on deviations to a normal process, it is important to define the normal process and to track its normal changes by estimation methods. For small changes of the variables linearized models may be sufficient. In general, however, nonlinear models have to be applied. The use of dynamic instead of static models yields more information and allows detection of more and/or smaller faults.

Parameter and state estimation. If the parameters and states are required with high precision, as for parameter monitoring and fault model filtering, the application is restricted to rather low order model elements. This means that many signals should be measurable for higher order processes.

Faults. Faults may either be already existing or appear at an unknown time, expected or unexpected. Their speed of appearance can be very different, from very large (abrupt faults) to very slowly (drifting faults).

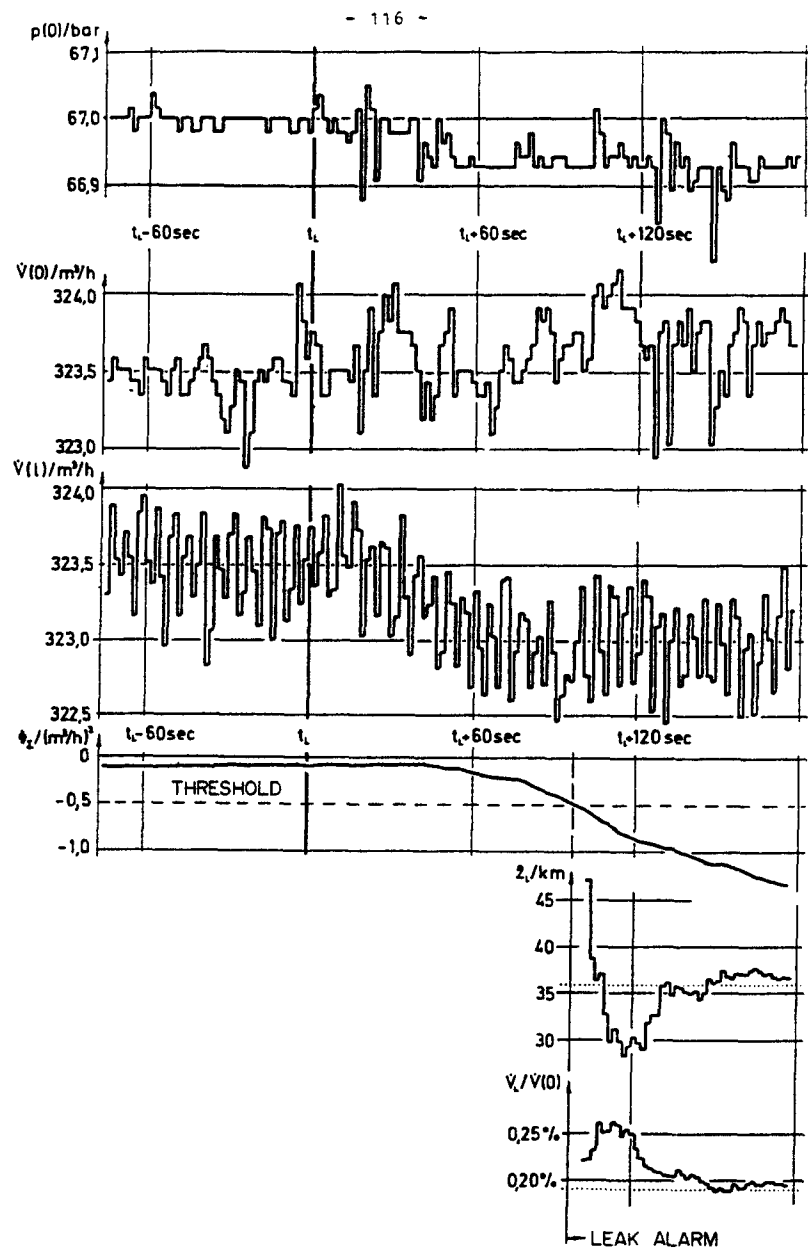


FIG. 16. Transients of $p(0)$, $V(l)$ and ϕ_{Σ} after occurrence of a leak at time $t = t_L$ at 35.8 km with a mean leakage rate of 0.19%. Detected leakage location and leakage flow.

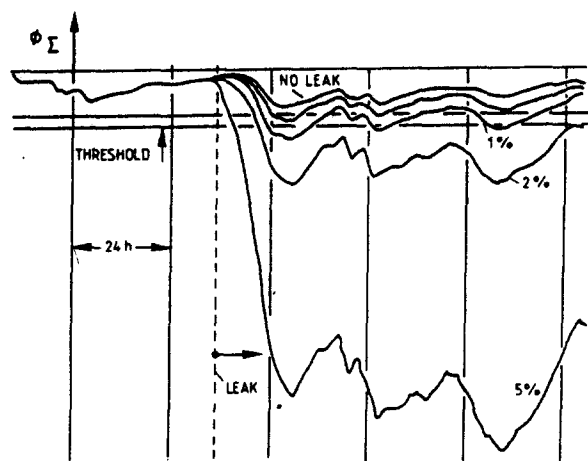


FIG. 17. Averaged cross-correlation function ϕ_{Σ} during normal operation (0%) and for simulated leaks of 1, 2 and 5% based on measured signals of an ethylene pipeline.

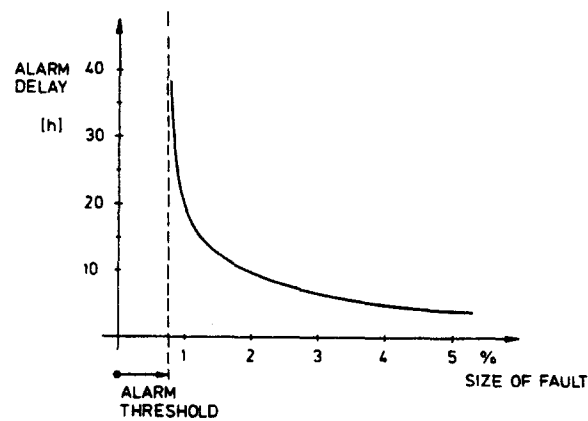


FIG. 18. Delay of the alarm in dependence on the size of a fault (leak in % of the total flow) for a gas pipeline.

Performance. Fault detection methods must be sensitive to the appearance of faults, but insensitive (robust) to other changes, like noise, modeling errors, operating points, normal signal variations, etc. Because these requirements often contradict each other, the following trade-offs do exist:

- (i) size of fault vs. detection time;
- (ii) speed of fault appearance vs. detection time;
- (iii) speed of fault appearance vs. process response time;
- (iv) size and speed of fault vs. speed of process parameter changes;
- (v) detection time vs. false alarm rate.

Methods which are, for example, designed for the detection of abrupt faults may be not suitable for slowly developing faults, and vice versa. Therefore several methods may be used in parallel, also to build up redundant fault detection systems.

Practical aspects. As the goal of fault detection methods is to improve the overall reliability of a process (or to provide safe back-up systems) and to run it more smoothly, they themselves must be very reliable. Otherwise, they produce too many false alarms and will be switched off, or they miss alarms.

In comparison to what is gained, the computational effort must be reasonable. Furthermore the methods should not be too complex, because they should be understandable and tunable by the process engineer. Otherwise, acceptance problems arise.

Testing. An important point for the application is the verification of the right functioning. This can be done by introducing artificial faults into well-conditioned (sound) processes or by connecting the detection system alternatively to a sound and to a faulty process.

In general, it is concluded that process fault detection and fault diagnosis methods may improve the overall reliability and safety of technical processes to a high degree. This is an area where several fields of engineering merge together. The available methods and experiences are in an early state of development. Therefore much remains to be done, especially with respect to applications.

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Appendix A. Derivation of the mathematical models of a d.c. motor, a centrifugal pump and a pipe system.

D.c. motor. The symbols used are:

U_1	armature voltage
I_1	armature current
R_1	armature resistance
L_1	armature inductance
Ψ	flux linkage
T_M	torque, generated by the motor
T_{FM}	friction torque of the motor
T_s	torque of the shaft
ω	angular velocity
θ_M	moment of inertia of the motor

It is assumed that the excitation is constant, so that a constant flux linkage Ψ is generated. Voltage equation for the armature circuit:

$$L_1 \frac{dI_1(t)}{dt} + R_1 I_1(t) + U_{ind}(t) = U_1(t) \quad (A1)$$

$$U_{ind}(t) = \Psi \omega(t) \quad (A2)$$

Balance equation for the angular momentum:

$$\theta_M \frac{d\omega(t)}{dt} = T_M(t) - T_{FM}(t) - T_s(t) \quad (A3)$$

$$T_M(t) = \Psi I_1(t) \quad (A4)$$

$$T_{FM}(t) = c_{FM0} + c_{FM1}\omega(t) \quad (\omega \geq 0). \quad (A5)$$

A quadratic term for the friction torque is neglected.

Now small deviations around the steady-state (\bar{U}_1 , \bar{I}_1 , $\bar{\Psi}$, $\bar{\omega}$, \bar{T}_s) are considered:

$$U_1(t) = \bar{U}_1 + \Delta U_1(t) \quad I_1(t) = \bar{I}_1 + \Delta I_1(t)$$

$$\omega(t) = \bar{\omega} + \Delta\omega(t); \quad T_s(t) = \bar{T}_s + \Delta T_s(t)$$

and from (A1)–(A5) results

$$L_1 \frac{d\Delta I_1(t)}{dt} + R_1 \Delta I_1(t) + \Psi \Delta\omega(t) = \Delta U_1(t) \quad (A6)$$

$$\theta_M \frac{d\Delta\omega(t)}{dt} + c_{FM1} \Delta\omega(t) = \Psi \Delta I_1(t) - \Delta T_s(t) \quad (A7)$$

with the steady-state equations

$$R_1 \bar{I}_1 + \Psi \bar{\omega} = \bar{U}_1 \quad (A8)$$

$$c_{FM0} + c_{FM1} \bar{\omega} = \Psi \bar{I}_1 - \bar{T}_s \quad (A9)$$

Centrifugal pump. The symbols are:

p_{P1}	pressure at pump inlet
p_{P2}	pressure at pump outlet
$Y = \frac{p_{P2} - p_{P1}}{\rho}$	specific energy
Y_{th}	theoretical specific energy
$H = Y/g$	head of the pump
ρ	density of the pumped medium
\dot{M}	mass flow-rate
T_P	torque generated by the pump
T_{FP}	friction torque of the pump
T_s	torque of the shaft
θ_P	moment of inertia of the pump
$P = T_s = \dot{M}Y(1/\eta)$	power at the shaft
η	overall efficiency coefficient

Balance equation for the angular momentum

$$\theta_P \frac{d\omega(t)}{dt} = T_s(t) - T_P(t) - T_{FP}(t). \quad (A10)$$

Based on the physical laws of centrifugal pumps the specific energy is given by Pfeleiderer (1955),

$$Y = h_1\omega + h_2\omega^2 + h_3\dot{M} + h_4\dot{M}^2 + h_5\dot{M}\omega + h_6\dot{M}^2\omega + h_7\dot{M}^2\omega^2 + h_8\dot{M}\omega^2 \quad (A11)$$

Typical characteristics of a centrifugal pump are shown in Fig. A1. For one variable kept constant the following quadratic equations result:

$$\omega = \text{const.}: \quad Y = h_{10} + h_{11}\dot{M} + h_{12}\dot{M}^2 \quad (A12)$$

$$\dot{M} = \text{const.}: \quad Y = h_{20} + h_{21}\omega + h_{22}\omega^2 \quad (A13)$$

where the coefficients follow from (A11). The torque of the pump becomes

$$T_P = \frac{\dot{M}}{\omega} \quad Y_{th} = \frac{\dot{M}}{\omega} [h'_2\omega^2 + h_3\dot{M}] \quad (A14)$$

The assumption of small deviations around the steady-state ($\bar{\omega}$, \bar{T}_s , \bar{T}_P , \bar{T}_{FP}) leads to

$$\omega(t) = \bar{\omega} + \Delta\omega(t); \quad T = \bar{T} + \Delta T(t) \quad (A15)$$

$$\theta_P \frac{d\Delta\omega(t)}{dt} = \Delta T_s(t) - \Delta T_P(t) - \Delta T_{FP}(t)$$

with the steady-state equations

$$\bar{T}_s - \bar{T}_P - \bar{T}_{FP} = 0. \quad (A16)$$

(A11) for the head is linearized around the steady-state

$$\Delta Y = \left(\frac{\partial Y}{\partial \omega} \right) \Delta\omega + \left(\frac{\partial Y}{\partial \dot{M}} \right) \Delta\dot{M} = h_\omega \Delta\omega + h_M \Delta\dot{M} \quad (A17)$$

where the coefficients h_ω and h_M follow from (A11) or from the measured characteristic curves of the pump.

The pump torque becomes, from (A14),

$$\Delta T_P = \left(\frac{\partial T_P}{\partial \omega} \right) \Delta\omega + \left(\frac{\partial T_P}{\partial \dot{M}} \right) \Delta\dot{M} = g_\omega \Delta\omega + g_M \Delta\dot{M} \quad (A18)$$

$$g_\omega = h'_2 \bar{\dot{M}} - h_3 \frac{\bar{\dot{M}}^2}{\bar{\omega}^2} \quad (A19)$$

$$g_M = h'_2 \bar{\omega} + 2h_3 \frac{\bar{\dot{M}}}{\bar{\omega}} \quad (A20)$$

The friction torque is assumed as

$$T_{FP}(t) = c_{FP0} + c_{FP1}\omega(t) + c_{FP2}\omega^2(t) \quad (A21)$$

and after linearization

$$\Delta T_{FP}(t) = c_{FP\omega} \Delta\omega(t) + c_{FPM} \Delta\dot{M}(t) \quad (A22)$$

with

$$c_{FP\omega} = \frac{\partial T_{FP}}{\partial \omega} = c_{FP1} + 2c_{FP2} \quad (A23)$$

c_{FPM} is assumed to be neglectable: $c_{FPM} = 0$.

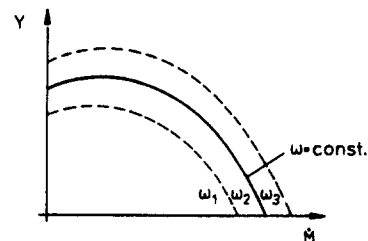


FIG. A1. Characteristics of a centrifugal pump.

Introduction of (A18) and (A22) leads to the linearized dynamic pump equation

$$\theta_P \frac{d\omega}{dt} + (g_\omega + c_{FP\omega})\Delta\omega(t) = \Delta T_s(t) - g_M \Delta \dot{M}(t). \quad (A24)$$

The equations for the mechanics of the motor (A7) and the pump (A24) can now be combined by eliminating $\Delta T_s(t)$:

$$\underbrace{(\theta_M + \theta_P)}_{\theta} \frac{d\omega(t)}{dt} + \underbrace{(c_{FM1} + c_{FP\omega} + g_\omega)}_{c_{F1}} \Delta\omega(t) = \Psi \Delta I_1(t) - g_M \Delta \dot{M}(t). \quad (A25)$$

This equation shows that various coefficients of the motor and the pump are added.

Pipe system. The pipe system is assumed to consist of length L , a valve and fluid resistances which can be lumped together. Figure A2 shows a scheme.

The symbols used are:

Δp_P	$p_{P2} - p_{P1}$ pressure difference of the pump
Δp_R	pressure drop of (lumped) fluid resistances
Δp_v	pressure drop of the valve
Δp_{ac}	pressure drop through acceleration
A_F	sectional area of the pipe
A_v	sectional area of the valve
w	velocity of the fluid
L	length of the pipes.

The balance equation for the momentum leads to

$$\Delta p_P(t) - \Delta p_v(t) - \Delta p_R(t) + \Delta p_{ac}(t) = 0 \quad (A26)$$

with

$$\Delta p_{ac}(t) = -\rho L \frac{dw}{dt} = -a'_{ac} \frac{d\dot{M}}{dt} \quad (A27)$$

$$\dot{M} = \rho A_F w \quad (A28)$$

$$a'_{ac} = L/A_F \quad (A28)$$

$$\Delta p_R(t) = c_R \dot{M}^2(t) \quad (A28)$$

$$\Delta p_v(t) = c_v \left(\frac{\dot{M}(t)}{A_v} \right)^2 \quad c_v = \frac{A_v^2}{k_v^2 \rho} \left(\frac{p_0}{\rho_0} \right) \quad (A29)$$

(k_v : k_v = value of the valve, $p_0 = 1$ bar; $\rho_0 = 1000$ kg m⁻³).

Introduction into (A26) leads to

$$a'_{ac} \frac{d\dot{M}(t)}{dt} + \left(\frac{c_v}{A_v^2} + c_R \right) \dot{M}^2(t) = \Delta p_P(t). \quad (A30)$$

Again small deviations of the signals are assumed

$$\dot{M}(t) = \dot{M} + \Delta \dot{M}(t) \quad \Delta p_P(t) = \overline{\Delta p_P} + \Delta \Delta p_P(t).$$

The term with $\Delta \dot{M}^2(t)$ is neglected (linearization) and with $\Delta p_P(t) = \rho \Delta Y(t)$ it follows that

$$a_{ac} \frac{d\dot{M}(t)}{dt} + a_R \Delta \dot{M}(t) = \Delta Y(t) \quad (A31)$$

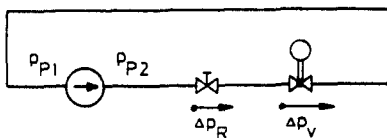


FIG. A2. Scheme of the pipe system with pump.

$$a_{ac} = a'_{ac}/\rho = L/A_F \rho \quad (A32)$$

$$a_R = a'_R/\rho = 2\dot{M}(c_v/A_v^2 + c_R)/\rho \quad (A33)$$

and the steady-state relation

$$\left(\frac{c_v}{A_v^2} + c_L \right) \dot{M}^2 = \overline{\Delta p_P}. \quad (A34)$$

Appendix B. Derivation of the mathematical models of a pipeline

A simplified scheme of a pipeline according to Fig. A3 is considered. The physical data are:

z	length coordinate
L	length of the pipeline
d_F	diameter of the pipe
$A_F = \pi d_F^2/4$	sectional area of the pipe
H	height of the pipeline
$p(z, t)$	fluid pressure
$\rho(z, t)$	fluid density
$T(z, t)$	fluid absolute temperature
$w(z, t)$	fluid velocity
M	fluid mass
$\dot{M}(z, t)$	fluid mass flow
R	gas constant
$c_F = \sqrt{(p/\rho)}$	speed of sound
λ	friction coefficient

For a pipe element of length dz the mass balance equation becomes

$$\frac{\partial \dot{M}}{\partial t} = A_F \rho w - A \left(w + \frac{\partial w}{\partial z} dz \right) \left(\rho + \frac{\partial \rho}{\partial z} dz \right) \quad (B1)$$

and with

$$\dot{M} = A_F \rho w \quad (B2)$$

and neglectation of small terms

$$\frac{\partial}{\partial z} (\rho w) + \frac{\partial \rho}{\partial t} = 0. \quad (B3)$$

The momentum balance is

$$\begin{aligned} \frac{\partial}{\partial t} (A_F \rho w dz) &= A_F \left(p + \frac{\rho w^2}{2} \right) - A_F \left(p + \frac{\partial p}{\partial z} dz + \frac{\rho w^2}{2} \right. \\ &\quad \left. + \frac{\partial (\rho w^2)}{\partial z} dz \right) - F - Y \end{aligned} \quad (B4)$$

or

$$\frac{\partial}{\partial t} (\rho w) + \frac{\partial}{\partial z} \left(p + \frac{\rho w^2}{2} \right) = -F - Y \quad (B5)$$

where the friction term, assuming turbulent flow, is

$$F = \frac{\partial p_F}{\partial z} = \frac{\lambda \rho}{2d_F} w|w| \quad (B6)$$

and the static pressure term

$$Y = \rho g \frac{dH}{dz}. \quad (B7)$$

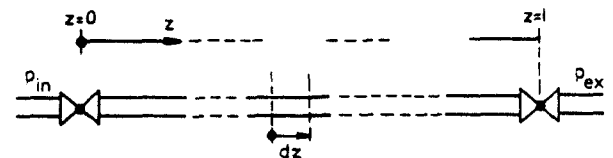


FIG. A3. Scheme of a pipeline ($H = \text{const.}$).

If an isothermic flow with temperature T_0 can be assumed the gas state equation is

$$p \frac{1}{\rho} = Z(p, T_0) R T_0 = c_F^2(p) \quad (\text{B8})$$

where $c_F(p)$ is the isothermic speed of sound. Then the two balance equations become, after the introduction of the variables $\dot{M}(z, t)$ and $p(z, t)$,

$$A_F \frac{\partial}{\partial t} \left(\frac{1}{c_F^2(p)} p \right) + \frac{\partial \dot{M}}{\partial z} = 0$$

$$\frac{1}{A_F} \frac{\partial \dot{M}}{\partial t} + \frac{\partial}{\partial z} \left(p + \frac{\dot{M}^2 c_F(p)}{2 A_F^2 p} \right) = \frac{1}{A_F} \frac{\partial \dot{M}}{\partial t} +$$

$$\left(1 - \frac{\dot{M}^2 c_F^2}{2 A_F^2 p^2} \right) \frac{\partial p}{\partial z} + \frac{\dot{M} c_F^2}{A_F^2 p} \frac{\partial \dot{M}}{\partial z} = -F - Y \quad (\text{B9})$$

$$F = \frac{\lambda}{2 d_F A_F^2 p} \dot{M} |\dot{M}|$$

$$Y = \frac{g p}{c_F^2(p)} \frac{dH}{dz}$$

This pipeline model can now be simplified by assuming:

- (i) the isothermic speed of sound is constant within a pipeline section j : $c_F(p) = c_{Fj}$;
- (ii) the fluid flow velocity w_F is small in comparison to the speed of sound c_F , so that

$$w_F^2 \ll c_F^2$$

$$\text{i.e. } \frac{\dot{M}^2 c_F^2}{A_F^2 p^2} = \frac{w_F^2}{c_F^2} \sim 0;$$

- (iii) for long pipelines the term

$$\frac{\dot{M} c_F^2}{A_F^2 p} \frac{\partial \dot{M}}{\partial z} \sim 0$$

can be neglected.

Then the simplified 'long pipeline model' results with the assumption $\partial H / \partial z = 0$:

$$k_1 \frac{\partial p}{\partial t} + \frac{\partial \dot{M}}{\partial z} = 0$$

$$k_2 \frac{\partial \dot{M}}{\partial t} + \frac{\partial p}{\partial z} = -k_3 \dot{M} \frac{|\dot{M}|}{p} \quad (\text{B10})$$

which is a hyperbolic partial differential equation system, with the coefficients

$$k_1 = \frac{A_F}{c_{Fj}^2}$$

$$k_2 = \frac{1}{A_F}$$

$$k_3 = \frac{\lambda}{2 d_F A_F^2} \frac{c_{Fj}^2}{p} \quad (\text{B11})$$

For the solution of the partial differential equation system the pipeline is subdivided (discretized) in sections j (Fig. A4) so that

$$\frac{\partial p_j}{\partial t} = g_{1j} (\dot{M}_{j+1} - \dot{M}_{j-1}) \quad j = 1, 3, \dots, l-1$$

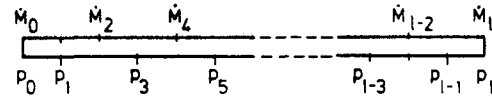


FIG. A4. Subdivision of the pipeline into sections.

$$\frac{\partial \dot{M}_j}{\partial t} = g_{2j} (p_{j+1} - p_{j-1}) + g_{3(j-1)} \dot{M}_j |\dot{M}_j|$$

$$j = 2, 4, \dots, l-2 \quad (\text{B12})$$

$$\frac{\partial \dot{M}_0}{\partial t} = g_{20} (p_1 - p_0) + g_{30} \dot{M}_0 |\dot{M}_0|$$

$$\frac{\partial \dot{M}_l}{\partial t} = g_{2l} (p_l - p_{l-1}) + g_{3l} \dot{M}_l |\dot{M}_l| \quad (l \text{ even})$$

with

$$g_{1j} = -\frac{1}{k_1 \Delta z} \quad g_{20} = -\frac{2}{k_2 \Delta z}$$

$$g_{2j} = -\frac{1}{k_2 \Delta z} \quad g_{2l} = -\frac{2}{k_2 \Delta z} \quad (\text{B13})$$

$$g_{3(j-1)} = -\frac{k_{3j}}{p_{j-1}} \quad g_{30} = -\frac{k_3}{p_0}$$

$$\Delta z = \frac{2L}{l} \quad g_{3l} = -\frac{k_3}{p_{l-1}}$$

The boundary conditions are given by the valve equations

$$\dot{M}_0 = k_{v0} \sqrt{\left(\frac{p_{00}}{p_{00}} \right) \rho_0} \sqrt{[(p_{in} - p_0)]} = c_{v0} \sqrt{(p_{in} - p_0)}$$

$$\dot{M}_l = k_{vl} \sqrt{\left(\frac{p_{00}}{p_{00}} \right) \rho_l} \sqrt{[(p_l - p_{ex})]} = c_{vm} \sqrt{(p_l - p_{ex})} \quad (\text{B14})$$

with k_v as the k_v values, see (A29). Equation (B11) can now be expressed in a nonlinear state variable representation

$$\dot{x}(t) = A(x)x(t) + B u_p(t)$$

$$y(t) = C x(t) \quad (\text{B15})$$

with

$$x^T = [\dot{M}_0 \dot{M}_2 \dots \dot{M}_l \quad p_1 p_3 \dots p_{l-1}]$$

$$u_p^T = [p_r p_l] \quad y^T = [\dot{M}_0 \dot{M}_l]$$

$$A = \begin{bmatrix} g_{30} |\dot{M}_0| & 0 & \dots & 0 & g_{20} & 0 & \dots & 0 \\ 0 & g_{31} |\dot{M}_2| & \dots & 0 & -g_{2j} & g_{2j} & 0 & \dots \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots \\ -g_{11} & g_{11} & \dots & 0 & 0 & \dots & 0 & \vdots \\ 0 & -g_{13} & g_{13} & 0 & 0 & \dots & 0 & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \\ 0 & \dots & g_{1(l-1)} & g_{1(l-1)} & 0 & \dots & 0 & \vdots \end{bmatrix}$$

$$B = \begin{bmatrix} -g_{20} & 0 \\ 0 & 0 \\ \vdots & \vdots \\ 0 & g_{2l} \\ 0 & 0 \\ \vdots & \vdots \\ 0 & 0 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 & \dots & 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & 1 & 0 & \dots & 0 \end{bmatrix} \quad (\text{B16})$$

The considered equations can also be used for *compressible liquids*. Then in (B9) the coefficient of $\dot{M}|\dot{M}|$

$$\frac{k_3}{\rho} = \frac{\lambda}{2d_F A_F^2} \frac{c_{Fj}^2}{p_j} = \frac{\lambda}{2d_F A_F^2} \rho \quad (B17)$$

and therefore $g_{3(j-1)}$ becomes a constant.

If a small *leak flow* $d\dot{M}_{L\zeta}$ occurs at section $j = \zeta$, see Fig. A5, this can be modeled by introducing it into the mass balance equation (B3)

$$\frac{\partial}{\partial z} (\rho w)_\zeta + \frac{\partial p_\zeta}{\partial t} + \frac{1}{A_F} \frac{\partial \dot{M}_{L\zeta}}{\partial z} = 0. \quad (B18)$$

Then (B9) changes to

$$k_1 \frac{\partial p_\zeta}{\partial t} + \frac{\partial \dot{M}}{\partial z} + \frac{\partial \dot{M}_{L\zeta}}{\partial z} = 0 \quad (B19)$$

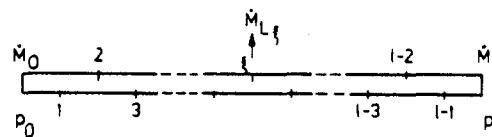


FIG. A5. Leak flow $\dot{M}_{L\zeta}$ at section ζ .

and (B11) to

$$\frac{\partial p_\zeta}{\partial t} = g_{1\zeta}(\dot{M}_{\zeta+1} - \dot{M}_{\zeta-1}) + g_{1\zeta}\dot{M}_{L\zeta}. \quad (B20)$$