Pyow-Hack

Ordered Compositions in Lewis-Skyrms Signaling Games

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Abstract

This paper presents a Lewis-Skyrms signaling game that can exhibit a type of compositionality novel to the signaling game literature. The structure of the signaling game is motivated by an analogy to the alarm calls of putty-nosed monkeys (*Cercopithecus nictitans*). Putty-nosed monkeys display a compositional system of alarm calls with a semantics that is sensitive to the ordering of terms. This sensitivity to the ordering of terms has not been previously modeled with a Lewis-Skyrms signaling game literature. Signaling games are valued for showing how communicative systems can arise with minimal learning tools. Simulation results show that basic (Roth-Erev) reinforcement learning is sufficient for the acquisition of a compositional signaling system sensitive to the ordering of terms.

1 Introduction

Compositionality is exhibited when a full statement in a language is comprised of component parts that contribute to its meaning. E.g. "the apple is red" and "the traffic light is red" are two distinct statements that have "red" as a component term. There are a number of ways of precisifying informal notions of compositionality. It is thought to be a key feature of human language, differentiating it from other animal communication [11] [18]. Lewis initially developed signaling games as a way of showing how communicative conventions can arise without prior knowledge of a language [12]. Later, Skyrms showed how Lewis signaling games can be understood as evolutionary games accessible to agents with low-rationality learning dynamics [19, 20]. Recently, Sterelny and Planar have advocated the use of Lewis-Skyrms signaling games for modeling the early evolution of human language [15]. There are some signaling game models that have been invoked in explanations of how and why humans acquired compositional language [10] [21]. However, the signaling game literature has yet to provide concrete models that exhibit important baseline types of compositionality, e.g. what Sterelney and Planar call linear syntax. This paper shows how such compositionality can obtain.

The pyow-hack signaling game model exhibits meaningful compositions sensitive to the ordering of terms. However, some of the game's structural features have not yet been explicated in the established literature. Consequently, the paper begins with describing two simpler signaling games along with some basic terms and diagrams for describing signaling games. Section 2.1 presents a trivially compositional game, which was initially developed by Barrett [7] to describe the evolution of kind language. Next, Section 2.2 outlines the progression of three properties of signaling games that will be combined to produce ordered compositions in the pyow-hack game: sender-compositionality, receiver-compositionality, and sender independent terms. It also shows that the model from Section 2.1 already meets the definition of sender-compositionality. Section 2.3 further explicates receiver-compositionality using a hierarchical signaling game model, which was originally developed in a pair of papers by Barrett, Cochran, and Skyrms [6] [5]. Then, Section 3 outlines the behavior of putty-nosed monkeys, which have an alarm call system with linear syntax, and then presents the pyow-hack game, in which a signaling system analogous to the monkey's can obtain. Lastly, Section 4 reviews the three properties of the pyow-hack game which allow for ordered compositions to obtain.

2 Ancient Artisans: Compositionality in Prior Signaling Game Models

This section precisifies three types of compositionality in order of increasing complexity. Simultaneously, it introduces diagrams of the signaling game models. These diagrams represent *both* the structure of a game *and* the reinforced dispositions of its players. Understanding how game structure relates to reinforcement of dispositions is essential to understanding compositionality in signaling games. This is not immediately apparent section 2.1's *trivially compositional* game. However, section 2.2, on *sender-compositionality* and its extensions, shows how one can erroneously consider distinct terms as identical when attending to only game structure. Finally, Section 2.3 explicates *receiver-compositionality* with direct reference to the reinforcement of dispositions.

This paper illustrates the signaling game models with a fictional story of prehistoric artisans, Devasena, Valli, and Narundi. Devasena is a logistics expert. She knows whether there is a better supply of wood or clay; when Devasena wears blue (**b_**) there is a better supply of wood, and when she wears red (**r_**) there is a better supply of clay. Valli is a market strategist. She knows whether pots or figurines are in greater demand; when Valli wears blue (**_b**) pots are in demand, and when she wears red (**_r**) figurines are in demand. Narundi manages acquisitions. She observes what colors Devasena and Valli are wearing and brings either wood or clay along with either tools for making pots or tools for making figurines. Thus when **rb** (Devasena wears red and Valli blue), Narundi brings clay and tools for making pots. Since the two term statements in this signaling system are merely conjunctive (i.e. the intersection of two properties), it is called *trivially compositional* [22]. There are no overbearing reasons for considering **rb** as a single statement with two component parts rather than understanding it as two independent statements, one made by Devasena and the other by Valli.

2.1 Learning Trivial Compositionality

The ancient artisans story is called a two-term two-sender one-receiver game in the established literature [7] [6] [5]. In this game there are four states of nature (needing wood pot, wood figurine, clay pot, or clay figurine supplies), four correspondingly appropriate actions (bringing the corresponding supplies), and two terms (**b** and **r**). How could Devasena, Valli, and Narundi acquire their signaling system without any established communicative conventions, without knowing whether Devasena or Valli is more attune to market demand or material supplies? The prior literature has shown that such signaling systems can arise through basic Roth-Erev reinforcement learning.

The reinforcement procedure is as follows. Suppose that Devasena and Valli each have four urns associating one and only one with the four states of nature: supply and demand for wood pots, wood figurines, clay pots, and clay figurines. In each urn, they place one red stone and one blue stone. Narundi has four urns associating each with one of the four signals she can receive: **bb**, **br**, **rb**, and **rr**. Narundi places four tokens in each of her urns corresponding to the four actions she can perform: bring wood pot, wood figurine, clay pot, or clay figurine supplies. Each day Devasena and Valli observe the state of nature and then draw a stone at random with equal probability from their corresponding urns to determine what color to wear. Narundi then observes Devasena and Valli's colors and likewise draws randomly from her corresponding urn to determine what action to perform. If the action matches the state of nature, the day is a success. Consequently, each player returns what was drawn to the urn from which it was drawn along with an additional stone or token of the same type; thus, it is more likely that, when the same state of nature occurs in the future, the same signals and then action will occur. If Narundi's action does not match the state of nature, then the day is a failure. Consequently, stones and tokens are returned to the urns that they were drawn from leaving the probabilities of signals and actions unchanged.

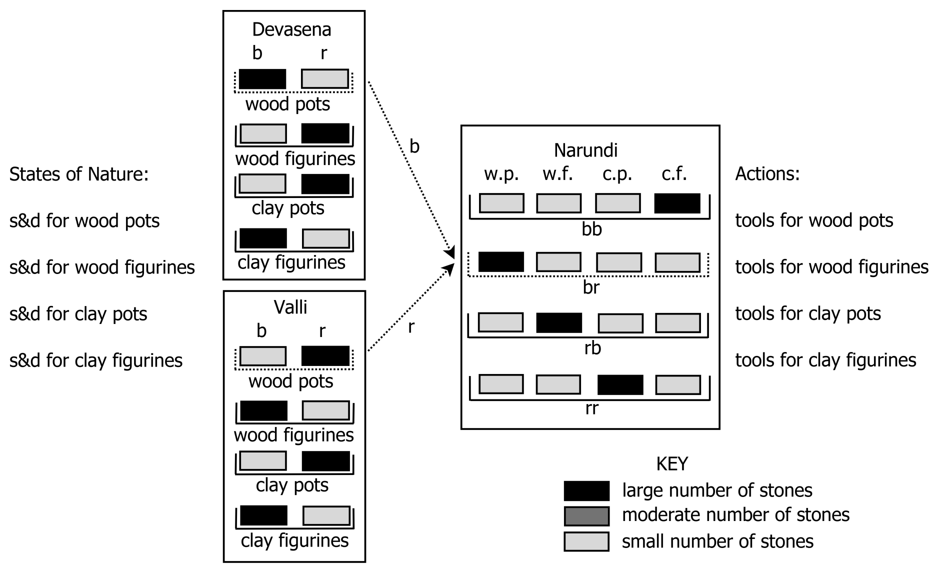

Figure 1: A Signaling System for the two-term two-sender one-receiver signaling game. To indicate the quantities of different types of stones, each urn is depicted with multiple boxes, one box for each type of stone/token in the urn. The most likely stones/tokens to be drawn from a urn are indicated the darker shaded in boxes.

The entirety of what occurs on a single day is called a single play of the game. Here's an example of a successful play:

1. Nature chooses a state at random, with each state having equal probability of being selected. Suppose the state of supply and demand for clay pots is chosen.
2. Observing supply and demand for clay pots. Devasena draws at random with equal probability from her corresponding urn. Suppose she draws a red stone and consequently wears red.
3. Observing supply and demand for clay pots. Valli draws at random with equal probability from her corresponding urn. Suppose she draws a blue stone and consequently wears blue.
4. Narundi then draws at random with equal probability from her corresponding urn, in this case the urn associated with **rb**. Suppose she draws a clay pot token and consequently brings clay and pot making tools.
5. Since this action corresponds with the state of nature, this counts as a success.
6. Given the success, Devasena replaces the stone that was drawn into the urn it was drawn from along with an additional stone of the same type. In this case, Devasena returns the red stone to the to the urn associated with supply and demand for clay pots and adds an additional red stone to the urn.
7. Given the success, Valli replaces the stone that was drawn into the urn it was drawn from along with an additional stone of the same type. In this case, Valli returns the blue stone to the to the urn associated with supply and demand for clay pots and adds an additional blue stone to the urn.
8. Given the success, Narundi replaces the token that was drawn into the urn it was drawn from along with an additional token of the same type. In this case, Narundi returns the clay pot token to the to the urn associated

with **rb** and adds an additional clay pot token to the urn.

9. In the event that Narundi does not bring material and tools corresponding to the state of nature, this counts as a failure.

10. When a failure occurs, each artisan returns what was drawn to the urn it was drawn from and nothing additional added to the urns.

This concludes a single play of the game.

A set of n repeated plays is called a n-run. The only way to know the outcome of a run is to actually perform all of the plays in a run. A strategy profile describes a sender or receiver's dispositions for all states of nature or signals that she can observe. Figure 1 shows both the structure of the game and a set of strategy profiles that could, in principle, be reached through the reinforcement procedure just described. When a set of strategy profiles describes dispositions such that for a state of nature the correct action is performed, it is called a signaling system. The set of strategy profiles in figure 1 describe a signaling system in this sense. A run is successful if it converges to a signaling system. An easy and close approximate measure of a run's success is commonly performed by checking whether the run's cumulative success rate is above an appropriate cutoff. The cumulative success rate is defined as $\frac{\#\text{ of successful plays}}{\#\text{ of plays}}$. In the ancient artisans game there are four equally probable states of nature with unique corresponding actions. So if a strategy profile tends to lead to successful plays for all but one of the states of nature, we should expect a run with that strategy profile to have a cumulative success rate near 0.75. Runs that tend to have successful plays for all states of nature have a cumulative success rate that converges towards 1. Consequently, in the ancient artisans game, it is reasonable to use a cutoff of 0.8 and measure only those runs that have a cumulative success rate greater than 0.8 as successful. Using this measure, after 10^6 plays per run under simple reinforcement learning, the run success rate in the ancient artisans game is approximately 73% [7].

2.2 Sender-compositionality and its Extensions

The definition of trivial compositionality has mostly been used in critique. It does not define a type of compositionality that can easily be extended to yield a more sophisticated type of compositionality. Sender-compositionality is a type of compositionality that is useful for building up more sophisticated types of compositionality; it is also exhibited in the ancient artisans story. Call a set of strategy profiles in a signaling game *sender-compositional* if there is a term that is transmitted as a component of at least two distinct statements. E.g. Devasena wearing red ($\mathbf{r}\_$) is a component of statements **rb** and **rr**.

Franke [10] expresses dissatisfaction with the type of compositionality exhibited in the ancient artisans game. Some of this dissatisfaction comes from the fact that it can only generate statements that are composite with respect to the senders' dispositions. The receiver acquires its dispositions by reinforcing actions as if each composite statement is unitary; Narundi's reinforcement of an action picked from the **br** urn has no direct effect on the contents of the **bb** urn or the **rr** urn. Call a game state *receiver-compositional* if there is a term that is a component of at least two distinct statements (sender-compositionality) *and* reinforcing an action for one of the statements has a direct effect on the receiver's dispositions towards the other statement(s) containing the given term. E.g. the ancient artisans game is sender-compositional since $\mathbf{r}\_$ is a component of statements **rb** and **rr**; it is not receiver-compositional because the receiver's dispositions towards the statement **rb** are defined by the contents of the **rb** urn, her dispositions towards **rr** are defined by the contents of the **rr** urn, and when a play results in reinforcement of dispositions towards one of those statement (i.e. adding a stone to the urn that was drawn from) there is no change in the contents of the other urn. The model described in section 2.3 will exhibit receiver-compositionality by having a single urn that has an effect on the receiver's dispositions towards two statements that contain the same term (the statements **r_** and **rb**); when a stone is added to the shared urn, it directly effects the receiver's dispositions towards both statements rather than only effecting dispositions towards the statement that was transmitted on the successful play.

Receiver-compositionality is intuitively desirable. Suppose someone, who already knows what "square" and "circle" mean, learns the appropriate response to hearing "bring the red circle". If she simultaneously fails to learn the appropriate response to hearing "bring the red square", then it seems doubtful that she has learned the meaning of "red" as a component part of the statement "bring the red circle". Conversely, if learning the appropriate response to "bring the red circle" causes a person to be disposed to act appropriately in response to "bring the red square", then this seems more reflective of "red" being treated as a component term. This is what receiver-compositionality allows.

It is worth attending to the reinforcment of dispositions when claiming that a term is a component of two distinct statements. If this is ignored, one might be tempted to claim that **r** is a component term of both **br** and **rb** in the ancient artisans story. However, there are strong reasons to reject this claim. One reason is that it is straightforwardly apparent that the meaning of **r** in **br** is entirely different than its meaning in **rb**. Another reason for rejecting the claim is as follows. Consider a signaling game identical to the ancient artisans game but with one alteration; Valli wears green and yellow instead of blue and red. This game is isomorphic to the original and in simulations will lead to the same 73% run success rate. It is difficult to defend the claim that statements **br** and **rb** share a component term when there exists an isomorphic signaling system containing statements **by** and **rg** which clearly do not share a component term.

So why not avoid confusion and describe Devasena and Valli as using different pairs of colors from the start?

In maintaining the superficial similarity between Devasena and Valli's terms, there is an immediately available extension of signaling game dynamics that creates a substantive identity between the terms used by distinct senders.[1] This extension is sender independence. An individual term X in a signaling game is *sender independent* if a receiver cannot condition her actions based on which sender transmitted X. In the pyow-hack game there will be two basic senders that can transmit a statement with a single P or H. Since the receiver will not be able to conditionalize on which basic sender transmitted the term, her dispositions towards a single P from basic sender A will be the same as her dispositions towards a single P from basic sender B; that is, the receiver will draw from (and on success reinforce) the same urns irrespective of which sender transmitted the single term. The power of sender independent terms will be more apparent after receiver-compositionality has been more thoroughly elaborated.

2.3 Receiver-compositionality in a Hierarchical Game

Receiver-compositionality can be illustrated with an extension of the ancient artisans story. As trade networks expand, the three artisans are joined by industrialists Nekhbet and Hestia. Nekhbet sees a broader market context than Devasena and Valli; she determines whether only the material (clay or wood), only the form (pot or figurine), or both the material and form of the product are relevant. If only the material is relevant, Nekhbet only allows Devasena to observe the state of nature to determine what color to wear, while Valli wears an uninformative color; if only the from is relevant, then Nekhbet only allows Valli to observe nature to determine blue or red, and Devasena is uninformative;

[1] It is also the case that all prior descriptions of the model have maintained this superficial similarity between the terms of distinct senders [7] [6] [5].

if both material and form are relevant she allows both Devasena and Valli to observe nature to determine what color to wear. When Narundi only sees a single term statement from Devasena and Valli ($\mathbf{b}\_$, $\mathbf{r}\_$, $\_\mathbf{b}$, or $\_\mathbf{r}$), then she draws at random with equal probability from either of the two corresponding urns; e.g. if Devasena wears red and Valli is uninformative, $\mathbf{r}\_$, then Narundi draws from either the **rb** or **rr** urn with equal probability. Hestia acquires supplies in bulk. She sees the signal from Devasena and Valli as well as Narundi's draw from the corresponding urn. If only Devasena wears an informative color, then Hestia only attends to the material of Narundi's draw and brings tools for both pots and figurines; e.g. if Narundi draws a wood pot token, Hestia brings wood and tools for both pots and figurines. If only Valli wears an informative color, then Hestia only attends to the form of Narundi's draw and brings the corresponding tools along with both wood and clay. If both Devasena and Valli are informative, then Hestia brings material and tools corresponding to Narundi's draw.

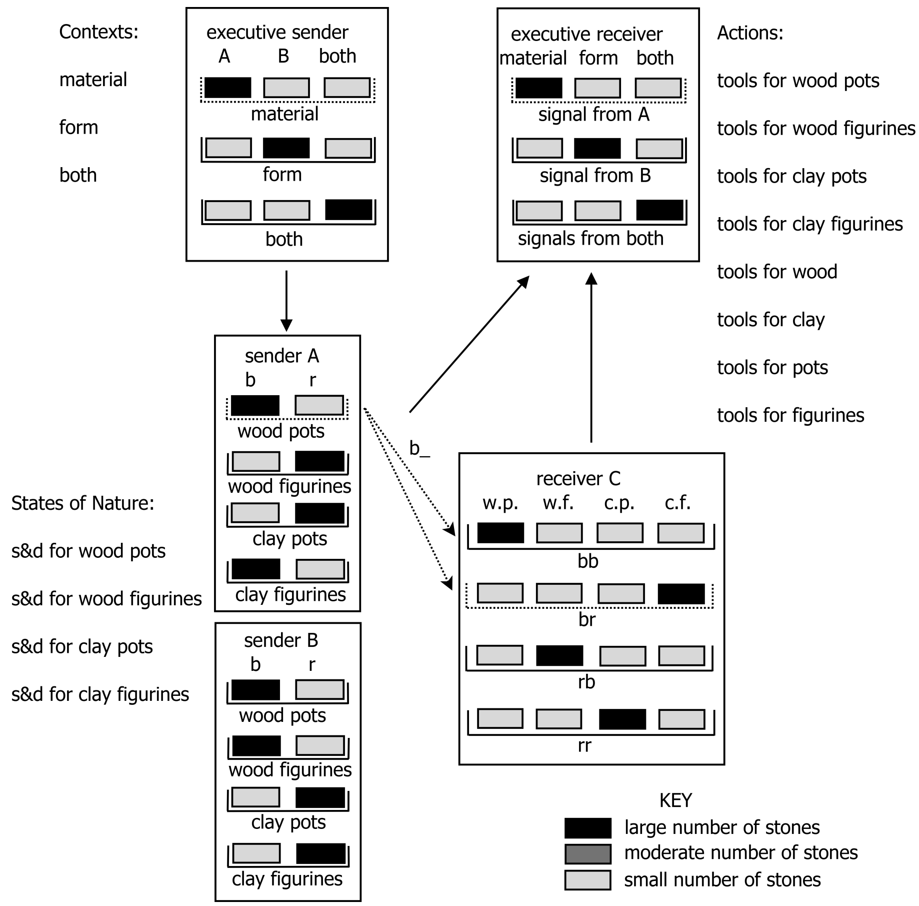

Figure 2: A Partial Pooling Equilibrium for the Hierarchical Signaling Game

Barrett, Cochran and Skyrms [6][5] show how the extended ancient artisans signaling system can be acquired through reinforcement learning using a hierarchical extension of the two-term two-sender one-receiver signaling game that exhibits receiver-compositionality. The hierarchical signaling game has two basic senders (e.g. Devasena and Valli), one executive sender (e.g. Nekhbet), one basic receiver (e.g. Narundi) and one executive receiver (e.g. Hestia). A state of nature features two binary properties (e.g. material and form) and a context (e.g. only material, only form, or both are relevant). In the game the executive sender sees the context and the basic senders see the properties. Correspondingly, the executive sender has three urns and the basic senders have four urns each, as they did in the previous game. The executive sender determines which of basic senders' signals gets transmitted. She begins the game with three stones in each urn, a sender A stone, sender B stone, and a both stone. The basic receiver sees what signal was transmitted and has four urns as she did in the previous game. The executive receiver sees both what was transmitted (a single term from sender A, a single term from sender B, or two terms), and correspondingly has three urns. The executive receiver sees whether sender A, B, or both transmitted a signal and determines whether the basic receiver's draw is interpreted as material, form, or both. Thus, the executive receiver begins the game with three urns, A, B, and Both, and each urn containing a material, form or both stone.

Acquiring a signaling system is considerably more difficult in the hierarchical game. To illustrate this, the procedure for a single play of the game is described for a run, Figure 2, in which several plays have already occurred:

1. Nature chooses a state and context at random with equal probability. Suppose the state of supply and demand for wood pots is chosen, but the context in which only material is relevant is chosen.
2. The executive sender observes the context and chooses a stone at random with equal probability from her material urn. Suppose the executive sender chooses a sender A stone. In the Figure 2 diagram, this is the executive senders most likely choice since sender A stones are the most prevalent in the material urn.
3. Since the sender A ball was chosen, only sender A will transmit a signal.
4. Sender A observes the state of supply and demand for wood pots. So, she draws at random with equal probability from her wood pot urn. Suppose

she draws a **b** ball. Again, this is the most likely choice in the example equilibrium.

5. Given her draw, sender A transmits $\mathbf{b}_{-}$.
6. Receiver C sees the $\mathbf{b}_{-}$ and it is determined at random with equal probability whether she will draw from the **bb** urn or **br** urn. When only sender A transmits a signal and that signal is $\mathbf{b}_{-}$, the probability either the **bb** or **br** urn being determined remains 0.5 for the entirety of the game on every run of the game. Suppose it is determined that receiver C will draw from the **br** urn.
7. Receiver C draws at random with equal probability from the **br** urn. Suppose receiver C draws a clay figurine token. In the Figure 2 diagram, this is the ball that she is most likely to draw in the example equilibrium.
8. Receiver C's draw is now interpreted by the executive receiver.
9. The executive receiver sees that only sender A transmitted a signal and draws from her sender A urn. Suppose the executive receiver draws a material stone. In the example run, this is her most likely draw.
10. Given that the executive receiver drew a material stone, she interprets receiver C's draw, a clay figurine token, as needing clay and tools for both pots and figurines; correspondingly, she brings clay and tools for both pots and figurines.
11. Since this action does not correspond with the state of nature, this counts as a failure.
12. Given the failure, each player returns the stone that she drew and no additional stones are added to the urns that were drawn from.

This concludes a play of the game.

One might wonder how the dispositions depicted in Figure 2 could be acquired if they lead to failure in action. This is possible since, when nature is supply and demand for clay figurines and the context is both, then the most likely signal to be transmitted is **br** and the corresponding most likely action is bringing clay and tools for pots, a success. Thus, in this case, the clay pot tokens will be reinforced in the **br** urn. Likewise, when nature is supply and demand for clay figurines and the context is only the form is relevant, then there is about a 50% chance that receiver C will draw from the **br** urn (given the most likely signal is **_r**) and consequently reinforce clay figurine tokens in the **br** urn if the play is a success, which is likely.

The type of game state depicted in Figure 2 is called a partial pooling equilibrium. A set of strategy profiles is a partial pooling equilibrium if the players perform better than chance, but worse than optimal, and their strategy profiles are a equilibrium in the sense that, for any given urn, changing which type of ball is most populous will not improve the success rate. The partial pooling equilibrium depicted in Figure 2 is worth highlighting because it is the first example of a signaling game exhibiting non-trivial compositionality; i.e. **br** indicates supply and demand for clay figurines, but this is not merely an intersection of what is indicated by **b_** (which sometimes indicates wood) and **_r** (which sometimes indicates pots). In the hierarchical extension of the ancient artisans game, all optimal signaling systems are trivially compositional. It is not until the pyow-hack game that optimal signaling systems exhibit non-trivial compositionality.

2.3.1 Run Success Rates and Learning Dynamics

Under basic Roth-Erev reinforcement learning, Barrett, Cochran and Skyrms' hierarchical signaling game model exhibits run success rates around 20%.[2] This run success rate can be increased to around 97% when the reinforcement dynamics are supplemented with punishment via costly signals [6][5]. Since the focus of this paper is on how a particular type of compositionality can obtain, the details of stronger learning dynamics are omitted. However, it will be noted that the same type of reinforcement supplemented with punishment via costly signals produces similar gains in the pyow-hack signaling game model.

3 Sender Independent Terms in the Pyow-Hack Game

3.1 A Brief Description of Putty-nosed Monkey Behavior

Cercopithecus nictitans martini, putty-nosed monkeys, are a West African species. They typically live in groups of 13-22 individuals comprised of one adult male with several females and dependent juveniles [1]. Their common predators are crowned eagles and leopards. Group leaders give different alarm calls that correlate fairly robustly with the presence of leopards and eagles. They also have a call associated with group movement [1][2][4][16][17].

Putty-nosed monkey alarm calls are comprised of two basic calls: a hack (H) and a pyow (P). These basic calls are strung together in sequences of varying length. A sequence of repeated hacks, perhaps HHHHHHHH, is associated with aerial predators (eagles) and invokes the behavior of looking up. A sequence of repeated pyows, perhaps PPPPP, is associated with ground predators (leop-

[2]Based on 1000 simulations of 10^8 plays per run and using a cutoff of 0.98 cumulative success rate for counting a run as a success.

ards). Behaviorally, the pyow call sequences are associated with moving towards the caller since ground predators rely on stealth and the monkeys can collectively scare off the predator [4]. Sequences of pyows followed by hacks, perhaps PPPPHH, are associated with group movement. Sequences of hacks followed by pyows, perhaps HHHHPPP, occur when a nearby eagle moves away from the group. Longer call sequences seem to correlate with more urgent contexts when signaling for predators and increased distance traveled when signaling group movement; behaviorally, this correlates respectively with faster reaction times and potentially moving longer distances [3][4][16].

Schlenker et al. [16] give a detailed overview of putty-nosed monkey alarm calls, and reasons for interpreting the calls as semantically compositional. Additionally, they propose some possible referential or imperative semantics for the alarm calls.[3] In developing a compositional semantics, Schlenker et al. make a particularly insightful observation about the relation between calls associated with ground predators and calls associated with group movement. Though over a shorter distance, the monkeys move towards the caller when the call for a ground predator is issued (so they can collectively mob the predator). This provides some reason for interpreting a "pyow" as contributing similar meaning to the ground predator call as a "pyow" contributes to a group movement call.

The pyow-hack signaling game will simplify things by only allowing six different statements in the game: P, PP, PH, H, HH, and HP. On this simplification, putty-nosed monkey behavior translates to the following call system. When a leopard is nearby, the group leader issues a P call, to which group members are disposed to move towards the group leader. When a leopard is very nearby, the group leader issues a PP call, to which group members are disposed to quickly

[3] That is, the semantics is referential if PPPP means there's a leopard nearby and is imperative if PPPP is a command to move towards the caller. Schlenker et al. are incredibly thoughtful in their discussion of these semantics even if they do not think there is any strong reason to prefer one semantic over the other.

move towards the group leader. When moving, the group leader issues a PH call, to which group members are disposed to move an extended distance towards the group leader. When an eagle is nearby, the group leader issues a H call, to which group members are disposed to look up. When an eagle is very nearby, the group leader issues a HH call, to which group members are disposed to quickly look up. When a nearby eagle is leaving, the group leader issues a HP call, to which group members are disposed to look up and then elsewhere.

3.1.1 Modeling an Agent with Multiple Senders or Receivers

The hierarchical ancient architects game was described as being played by five distinct agents. However, the reinforcement dynamics of drawing stones from an urn is intended to represent an organism's internal mechanisms for learning through reinforcement conditioning. Consequently, one might think of different senders in a game as representing different functional components of a single organism. In the ancient architects story, this might look like Devasena's draws determining whether to wear a blue or a red top while Valli's determining whether to wear blue or red pants; perhaps the executive sender merely reflects some mechanism in the prefrontal cortex. In modeling monkeys, different basic senders might be responsible for sending the first or second term in a sequence. Certainly there is evidence that different areas of the human brain realize different functional roles in language production and comprehension [9][13]. But, it is also possible that neurons within a particular brain area could be arranged to realize different functional roles.

3.2 The Pyow-Hack Game

The pyow-hack signaling game abstracts and simplifies away from several of the details of putty nose monkeys' environment and behavior. Most noticeably, it only allows for call sequences of at most two signals. Like the Barrett, Cochran and Skyrms's model [6] [5], it is a hierarchical signaling game consisting of an executive sender, two basic senders, an executive receiver, and a basic receiver.

In the pyow-hack game there are six states of nature: a leopard is nearby, a leopard is very near (urgent), an eagle is nearby, an eagle is very near (urgent), a nearby eagle is moving away, and the group is moving. There are six corresponding appropriate actions: move towards caller, quickly move towards caller, look up, quickly look up, look up and elsewhere, and move an extended distance towards caller.

The executive sender as well as the basic senders can observe the state of nature. The executive sender determines whether just one or both of the basic senders will transmit a signal. This corresponds with the executive sender having six urns, one for each state of nature. These urns contain two types of balls, single transmission balls and dual transmission balls. As in the previous games, all of the players' urns start with one ball of each type. The basic senders each have six urns corresponding to the states of nature. The basic senders have two types of balls, P balls and H balls. On plays in which the executive sender draws a single transmission ball, it is determined at random with equal probability whether sender A or sender B transmits a signal.[4]

The basic receiver has four urns: PP, PH, HP, and HH. When as single P is transmitted, it is determined at random with equal probability whether the basic sender draws from the PP or PH urn. When a single H is transmitted, it is determined at random with equal probability whether the basic sender draws from the HP or HH urn. As in the previous hierarchical game, the basic receiver draws balls that can be given multiple interpretations by the executive. The

[4]This is part of how the pyow hack game avoids the oddity discussed at the end of section 2.1.1. In this game, executives are only sensitive to signal length. As will be seen shortly, the executive receiver cannot form dispositions relative to which basic sender transmitted a signal. She can only form dispositions relative to signal length.

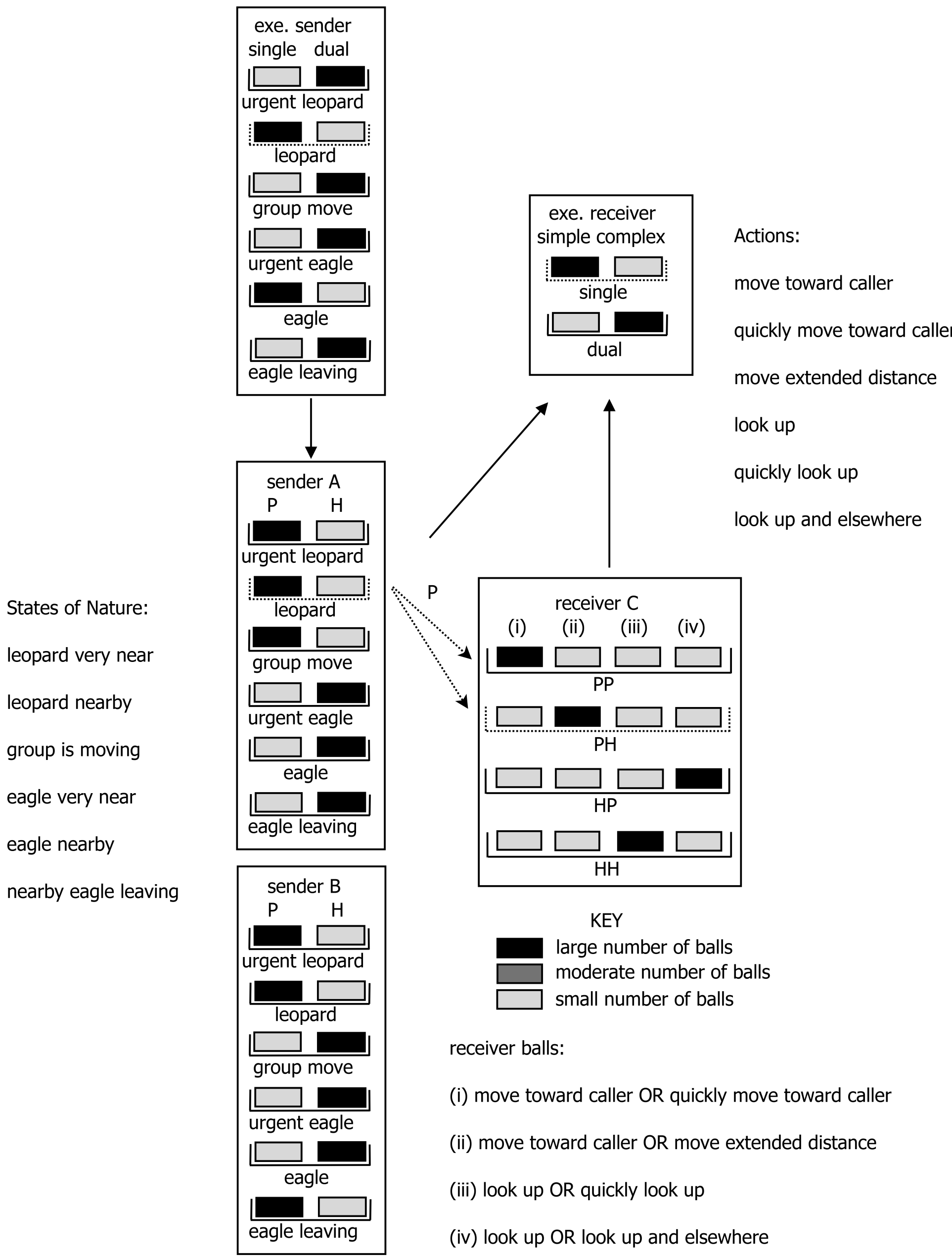

Figure 3: A Signaling System for the Pyow Hack Game

basic receiver's urns contain four types of balls labeled: (i) 'quickly move towards caller', (ii) 'move an extend distance towards caller', (iii) 'quickly look up', and (iv) 'look up and elsewhere'. These labels are the complex interpretations that the executive can give to the balls. Type (i) and (ii) balls can be given the simple interpretation "move towards the caller". Type (iii) and (iv) balls can be given the simple interpretation "look up". Thus, the executive receiver has two urns, a single transmission urn and a dual transmission urn. Each of these urns has two types of balls, simple interpretation balls and complex interpretation balls.

Simple reinforcement learning (Roth-Erev) is the dynamic that was presented in Section 2.1. When a play is successful, drawn balls are returned to their urns and one additional ball of the type drawn is added to the urn that was drawn from for each player. On failures, balls are returned to the urns they were drawn from. Here's an example play for the equilibrium depicted in Figure 3:

1. Nature chooses a state and context at random with equal probability. Suppose the state of a nearby leopard is chosen.
2. The executive sender observes the state and chooses a ball at random with equal probability from her nearby leopard urn. Suppose the executive sender chooses a single transmission ball; this is the executive sender's most likely choice in the example equilibrium.
3. Since the single transmission ball was drawn, either sender A or sender B is chosen at random with equal probability to transmit a signal. Suppose sender A is chosen.
4. Sender A observes the state of a nearby leopard. So, she draws at random with equal probability from her leopard urn. Suppose she draws a P ball. Again, this is the most likely choice in the example equilibrium.
5. Given her draw, sender A transmits 'P'.

6. Receiver C sees the 'P' and it is determined at random with equal probability whether she will draw from the PP urn or PH urn. Suppose it is determined that receiver C will draw from the PH urn.
7. Receiver C draws at random with equal probability from the PH urn. Suppose receiver C draws a (ii) ball. This is the ball that she is most likely to draw in the example equilibrium.
8. Receiver C's draw is now interpreted by the executive receiver.
9. The executive receiver sees that only single signal was transmitted and draws from her single urn. Suppose the executive receiver draws a simple ball. In the example equilibrium, this is her most likely draw.
10. Given that the executive receiver drew a simple ball, she interprets receiver C's draw, (ii), as needing to move towards the caller. So this action is performed.
11. Since this is the correct action for the given state, this counts as a success.
12. Given the success, each player returns the ball that she drew along with an additional ball of the type that was drawn.
13. When a failure occurs, drawn balls are returned to the urns that they were drawn from.

This concludes a play of the game.

Simulating 1000 runs of the pyow-hack game, with 10^7 plays per run, produced the run success rate of 19.4%. This was calculated by measuring each run's cumulative success rate, the number of successful plays divided by the total number of plays. This was calculated by counting a run as successful if it had a cumulative success rate above 0.92. This was an appropriate cutoff for determining whether a run was successful as $0.92 > 5.5/6$. That is, a cumulative success rate greater than 0.92 is indicative of plays being successful for each of the six states of nature.

3.2.1 A Brief Note on Run Success Rates and Learning Dynamics

Under basic Roth-Erev reinforcement learning, the pyow-hack game has a run success rate of around 19%. This increases to around 58% when using costly signals analogous to Barrett, Cochran and Skyrms' reinforcement with punishment via costly signals [6][5]. An even stronger learning dynamics, described by Barrett and Gabriel[8], can give a run success rate of 94.3% (of 1000 runs, 10^7 plays per run, and 10 iterations of [+2, -9] reinforcement with iterated punishment). However, even on the weakest learning dynamics, it remains the case that when an optimal signaling system obtains, the compositionality is novel in that it is sensitive to the ordering of terms.

4 Discussion

4.1 Review of Technical Terms

Two types of compositionality are emphasized in this paper: (i) a game state is *sender-compositional* if there is a term that is transmitted as a component of at least two distinct statements; (ii) a game state is *receiver-compositional* if there is a term that is a component of at least two distinct statements and reinforcing an action for one of the statements has a direct effect on the receivers dispositions towards the other statement(s) containing the given term. Receiver-compositionality allows a term to contribute similar dispositions to multiple statements that have the given term as a component part. Additionally, motivating some of the differences between the pyow-hack hierarchical game and the Barrett, Cochran, and Skyrms' [6][5] hierarchical game, a term in a stable game state is *sender independent* just in case transmission of the term typically results in the same action regardless of which sender transmitted it. The signaling game literature discusses a third type of compositionality defined by Schlenker et al. [17] and introduced to the signaling game literature by Steinert-Threlkeld [22][5]: (iii) a game state is *trivially compositional* just in case complex expressions are always interpreted by intersection (generalized conjunction) of the meanings of the parts of the expression. It can be checked that optimal signaling systems for the Barrett, Cochran, and Skyrms hierarchical game are trivially compositional.[6]

4.2 Order Sensitive Compositionality

Signaling systems for the pyow-hack game are not trivially compositional. If they were, since the terms are sender independent, HP would be associated with the same dispositions, as PH. But this cannot occur in a signaling system since the game only allows for six possible statements and requires distinct actions to be performed for each of the six states of nature. For a given game state, if transmitting HP and transmitting PH typically results in the same action being performed in response to either statement, then at most only five of the six states of nature can be mapped to the correct action by the senders' and receiver's strategy profiles. This is a quick method of demonstrating that compositionality

[5]Although, it is not obvious that this is an appropriate definition for the sort of compositionality that Steinert-Threlkeld is concerned with. Steinert-Threlkeld constructs a game in which an artificial neural network is supposed to learn the function words "most" and "least" across different dimensions of properties that these function words can be applied to. In his analysis, Steinert-Threlkeld does not show (though it may be true) that the network's learning the correct output for "most blue" contributes to its learning the correct output for "most green". But, this is the case for the human language that Steinert-Threlkeld is attempting to model. When a human learns to use "most" appropriately for some small domain of properties, she then is able to use the term appropriately for novel properties. This is because the content of the term is not specific to particular properties. Furthermore, there is no obvious impediment to Steinert-Threlkeld showing that his model exhibits the desired behavior. To do this, Steinert-Threlkeld should show that first training the network on one or two dimensions of properties allows it to learn "most" and "least" for a second or third dimension at a faster rate than it would with no pre-training. This is exactly the sort of generalization that artificial neural networks are valued for.

[6]Though it should be noted that the hierarchical game can exhibit non-trivial compositionality in suboptimal partial pooling equilibria [5].

exhibited in the pyow-hack game is different from the compositionality exhibited in the Barrett, Cochran, and Skyrms hierarchical game. However, it does not show how compositionality with sensitivity to term ordering obtains in the pyow-hack game.[7]

Sensitivity to term ordering is allowed by the combination of both sender independent terms and receiver-compositionality. To see how this sensitivity is allowed, consider the signaling system diagrammed in Figure 3. It is easy to see that PH and HP are associated with different dispositions, actions. PH is typically transmitted when the state of nature is group movement and typically results in the action of moving an extended distance towards the caller. HP corresponds with the nearby eagle leaving state of nature and the look up & elsewhere action. However, this does not necessarily entail that the compositionality is sensitive to term ordering because of the worry described at the end of Section 2.2. Recall that this worry raises the concern that the P in PH is not the same term as the P in HP, perhaps one is an A-tone P and the other is a B-tone P making them functionally distinct terms. To establish that the P in the PH statement is the same term as the P in the HP statement, it must be shown that there is a connection between the dispositions associated with the P term in PH statements and the P term in HP statements. If one adopts a dispositional account of term meaning for the pyow-hack signaling game, then this amounts to a requirement that the P in PH and the P in HP have the same meaning.[8]

A clear dispositional connection between the P in PH statements and the second P in PP statements is as follows. Note that this is still following the signaling system depicted in Figure 3. The P in PH statements is connected

[7]This sensitivity to term order further highlights the differences between the pyow-hack game and the Barrett, Chochran, and Skyrms hierarchical game, which can exhibit non-trivial compositionality in suboptimal pooling equilibria, but cannot exhibit sensitivity to term order.

[8]Depending on how a dispositional account of meaning is understood, this might just translate to a requirement that the P in PH and P in HP have strongly related meanings.

to the P in solitary P statements by receiver-compositionality. In both the corresponding group movement and leopard states successful action reinforces the prevalence of (ii)-balls in receiver C's PH urn.[9] Since terms are sender independent solitary P statements transmitted by sender A result in the same action as solitary P statements transmitted by sender B. That is, the reinforcement of P-balls in sender A's leopard urn is connected to the reinforcement of P-balls in sender B's leopard urn because when nearby leopard states occur the receiver probabilistically chooses from one of the same two urns regardless of whether the solitary P is transmitted by sender A or B. The P in solitary P statements is connected to the second P in PP statements by receiver-compositionality. In both the corresponding leopard and urgent leopard states, successful action reinforces the prevalence of (i)-balls in receiver C's PP urn. But this seems to be sufficient for establishing a dispositional connection between a P in PH statements and a P in HP statements. This is because the second P in a PP statement is transmitted by the same functional component, sender B, as the P in a HP statement; both P's being transmitted by sender B eliminates the worry raised at the end of section 2.2 that the two P's might in some way be different terms.

More abstractly, the pyow-hack game is able to realize a type of compositionality analogous to the compositionality exhibited by putty-nosed monkeys because of a combination of receiver-compositionality and sender independent terms. Receiver compositionality allows for a dispositional connection between a basic sender's term, say P, when used in a composite statement and when used in isolation. Sender independent terms allow for a dispositional connection between an isolated term transmitted by sender A and that same term transmitted by sender B. Thus there is an indirect connection between the dispositions

[9]Note that (ii)-balls are only reinforced in the PH urn 25% of the time that leopard states occur, since sender A transmits with 0.5 probability and receiver C then draws from the PH urn with 0.5 probability.

associated with a term contributed by sender A to a composite statement and that same term contributed by sender B to a composite statement.

4.3 Final Remarks

Developing a Lewis-Skyrms signaling game that exhibits a compositionality analogous to the compositionality exhibited in putty-nosed monkey alarm calls generates a variety of interesting results. It shows how signaling games can be more directly connected to the behavior of a specific species. The pyow-hack signaling game motivates new terms for categorizing the types of compositionality that can be exhibited in signaling games, such as receiver-compositionality and sender independent terms. It highlights how a hierarchical signaling game can be used to develop a novel type of compositionality. In particular, Barrett, Chochran, and Skyrms' introduction of executive senders to signaling games makes the implementation of sender independent terms particularly easy.

This paper leaves as an open question the extent to which the pyow-hack game might be used to explain putty-nosed monkey behavior. If taken as providing a how-possibly explanation of putty-nosed monkey's acquisition of a compositional alarm call system, one might wonder whether the various models generate sufficiently high run success rates. This seems to deserve an affirmative answer. Having only basic reinforcement available to guide learning is a particularly hostile environment. Success rates from models with moderate partial payoffs show that small amounts of noise do not radically alter results. Furthermore, the pyow-hack signaling game abstracts away from a variety of real world details that might aid learning. In the real world, infant monkeys are born into an already established signaling system. This means that the established signaling system was likely selected over several generations allowing social groups with suboptimal signaling to die off. Additionally, in the real world visual cues can aid learning correct responses by reinforcement.

Some of the plausibility of taking the pyow-hack game to provide a how-possibly explanation of putty-nosed monkey alarm call behavior relies on it being plausible that the monkey's acquire their alarm call system through reinforcement learning. The non-exclusive alternative to this is that putty-nosed monkey's alarm call behavior is genetically hardwired. Thus, the pyow-hack game might motivate a number of research questions for comparative psychologists. For example, a comparative psychologist might consider attempting to habituate putty-nosed monkeys to grammatical structures that they don't exhibit in their alarm calls. This could even be attempted with sounds not native to their environment.[10] E.g. experimenters could test whether they could condition a group to expect food when they hear a ABC pattern and expect a predator when they hear an ABA pattern. Success in such habituation would support the plausibility of the monkeys acquiring their signaling system via reinforcement learning.

The general simplicity of Lewis-Skyrms signaling games is a particularly beneficial feature of the pyow-hack game. As just stated, this can translate into showing how compositionality can arise in a particularly hostile environment. The game's simplicity also has the benefit of allowing it to be paired with a variety of different dynamics. Future research might explore how the game behaves when paired with various replicator dynamics, which might more immediately resemble evolutionary dynamics. So, the pyow-hack game suggests multiple paths to a putty-nosed monkey type of compositionality obtaining.

[10] Such an experiment would not be dissimilar to the experiments in which Neiworth et al. have had success [14].

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