



ESCUELA POLITÉCNICA NACIONAL



FACULTAD DE INGENIERÍA DE SISTEMAS

INGENIERÍA *EN COMPUTACION*

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PERÍODO ACADÉMICO: 2025-A

ASIGNATURA: ICCD412 Métodos Numéricos

GRUPO: GR2

TIPO DE INSTRUMENTO: *Tarea N°10*

FECHA DE ENTREGA LÍMITE: **[1/06/2025]**

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## TEMA

*Splines cúbicos*

## OBJETIVOS

- *Aplicar el método en un ejercicio y aprender a dominarlo de mejor manera.*
- *Desarrollar habilidades matemáticas y encontrar las soluciones a los ejercicios planteados de manera correcta.*

## DESARROLLO

1. Dados los puntos  $x = [-2, -1, 1, 3]$ ,  $y = [3, 1, 2, -1]$ 
  - a) Determine el spline cúbico con frontera natural
  - b) Determine el spline cúbico con frontera condicionada
$$B_0 = 1$$
$$B_n = -1$$
2. Dados los puntos  $(0,1); (1,5); (2,3)$ , determine el spline cúbico.
3. Dados los puntos  $(-1,1); (1,3); (0.5,4.8)$ , determine el spline cúbico sabiendo que  $f'(x_0) = 1$ ,  $f'(x_n) = 2$ .

# Método de Splines

$$x = [-2, -1, 1, 3]$$

$$y = [3, 1, 2, -1]$$

$$x_0, x_1 \quad x_1, x_2 \quad x_2, x_3$$

$$S_0(x) = a_0 + b_0(x+2) + c_0(x+2)^2 + d_0(x+2)^3 = 3$$

$$S_1(x) = a_1 + b_1(x+1) + c_1(x+1)^2 + d_1(x+1)^3 = 1$$

$$S_2(x) = a_2 + b_2(x-1) + c_2(x-1)^2 + d_2(x-1)^3 = 2$$

- Evalúa  $S_0(x_0)$  y  $S_0(x_1)$   $x_0, x_1$

$$S_0(x_0) = a_0 = 3 \quad \textcircled{a}$$

$$S_0(x_1) = a_0 + b_0 + c_0 + d_0 = 3 \quad \textcircled{b}$$

- Evalúa  $S_1(x_0)$  y  $S_1(x_1)$   $x_0, x_1$

$$S_1(x_0) = a_1 = 1 \quad \textcircled{c}$$

$$S_1(x_1) = a_1 + 2b_1 + 4c_1 + 8d_1 = 1 \quad \textcircled{d}$$

- Evalúa  $S_2(x_0)$  y  $S_2(x_1)$   $x_0, x_1$

$$S_2(x_0) = a_2 = 2 \quad \textcircled{e}$$

$$S_2(x_1) = a_2 + b_2 + c_2 + d_2 = 2 \quad \textcircled{f}$$

- Verifican continuidad

- Continuidad Natural

$$S_0'(x_1) = S_1'(x_1)$$

$$b_0 + 2c_0(-1+2) + 3d_0(-1+2)^2 = b_1 + 2c_1(-1+1) + 2d_1(-1+1)^2$$

$$b_0 + 2c_0 + 3d_0 = b_1 \quad \textcircled{g}$$

$$S_1'(x_2) = S_2'(x_2)$$

$$b_1 = b_2 + 2c_2(2-1) + 2d_2(2-1)^2$$

$$b_1 + 4c_1 + 12d_1 = b_2 \quad \textcircled{h}$$

$$S_0''(x_1) = S_1''(x_1)$$

$$2c_0 + 6d_0(-1+2) = 2c_1 + 6d_1(-1+1)$$

$$c_0 + 3d_0 = c_1 \quad (9)$$

$$S_1''(x_2) = S_2''(x_2)$$

$$2c_1 + 6d_1(-1+1) = 2c_2 + 6d_2(-1+1)$$

$$c_1 + 6d_1 = c_2 \quad (10)$$

- Condiciones de frontera

$$S_0''(x_0) = S_{n-1}''(x_n) = 0 \quad n = \# \text{ de splits}$$

$$S_0''(x_0) = S_2''(x_3)$$

$$2c_0 + 6d_0(-2+2) = 2c_2 + 6d_2(3-4)$$

$$2c_0 = 2c_2 + 12d_2$$

$$c_0 = c_2 + 6d_2$$

$$\begin{array}{cc} \downarrow & \downarrow \\ c_0 = 0 & c_2 + 6d_2 = 0 \\ (11) & (12) \end{array}$$

- Con 2 entradas adicionales ( $B_0, B_n$ )

$$B_0 = S_0'(x_0)$$

$$B_n = S_{n-1}'(x_n)$$

$$B_0 = b_0 \quad (13)$$

$$B_n = b_2 + 4c_2 + 12d_2 \quad (14)$$

$$a_0 = 3$$

$$a_0 + b_0 + c_0 + d_0 = 3$$

$$a_1 = 1$$

$$a_1 + 2b_1 + 4c_1 + 8d_1 = 1$$

$$a_2 = 2$$

$$a_2 + b_2 + c_2 + d_2 = 2$$

$$b_0 + 2c_0 + 3d_0 = b_1$$

$$b_1 = 4c_1 + 12d_1 + b_2$$

$$c_1 = c_0 + 3d_0$$

$$c_2 = c_1 + 6d_1$$

$$c_0 = 0$$

$$c_2 + 6d_2 = 0$$

$$b_0 = b_0 = 1$$

$$b_1 = b_2 + 4c_2 + 12d_2 = -1$$

Fronteira Natural y condicionada

$$a_0 = \frac{95}{23}$$

$$a_1 = -\frac{79}{23}$$

$$b_0 = -\frac{164}{23}$$

$$b_1 = \frac{121}{23}$$

$$c_0 = 1$$

$$c_1 = -\frac{113}{23}$$

$$d_0 = 2$$

$$d_1 = +\frac{1}{23}$$



## Metodos Números

Tarea N° 40

Nombre: Kevin Garcia

2) con  $x = [0, 1, 2]$   $y = [1, 0, 3]$   $[0, 1]$   $[1, 2]$

$$S_0(x) = a_0 + b_0(x-0) + c_0(x-0)^2 + d_0(x-0)^3 = 1$$

$$S_1(x) = a_1 + b_1(x-1) + c_1(x-1)^2 + d_1(x-1)^3 = 0$$

- Evaluamos  $(0, 1)$

$$S_0(x_0) = a_0$$

$$S_0(x_1) = a_0 + b_0 + c_0 + d_0$$

- Evaluamos  $(1, 2)$

$$S_1(x_0) = a_1$$

$$S_1(x_1) = a_1 + b_1 + c_1 + d_1$$

• Verificar continuidad

$$S'_0(x_1) = S'_1(x_1)$$

$$b_0 + 2c_0(0-1) + 3d_0(0-1)^2 = b_1 + 2c_1(1-1) + 3d_1(1-1)^2$$

• Final

$$S(x) = \begin{cases} -\frac{3}{2}x^3 + \frac{11}{2}x + 1 & x \in [0, 1) \\ \frac{3}{2}(x-1)^3 - \frac{9}{2}(x-1)^2 + (x-1) + 0 & x \in [1, 2] \end{cases}$$

3) Dadas  $(-1, 4)$   $(1, 3)$   $(0.5, 4.8)$  con  $f'(x_0) = 4$ ,  $f'(x_n) = 2$

$[-1, 0.5] \cup [0.5, 1]$

$$S_0 = a_0 + b_0(x+1) + c_0(x+1)^2 + d_0(x+1)^3 = 4$$

$$S_1 = a_1 + b_1(x-0.5) + c_1(x-0.5)^2 + d_1(x-0.5)^3 = 3$$

- Evaluamos en  $[-1, 0.5] \cup [0.5, 1]$

$$S_0 = 0.0$$

$$S_0(x) = a_0 + 4.5b_1 + 2.5c_1 + 1.5d_1$$

$$S_1(x_0) = a_1$$

$$S_1(x_1) = a_1 + 0.5b_1 + 0.5c_1 + 0.5d_1$$

Finalmente

$$S_0(x) = 4 + 4(x+1) + 6.3833(x+1)^2 - 3.5741(x+1)^3$$

$$S_1(x) = 4.8 - 3.875(x-0.5) - 9.7(x-0.5)^2 + 20.9(x-0.5)^3$$