Модульна контрольна робота
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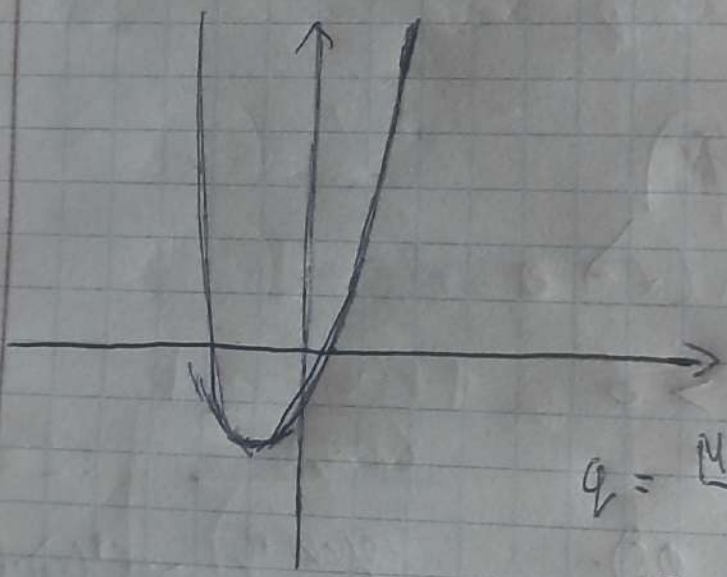
Варіант 19

№2

$x^2 + 5\sin x - 1 = 0$ $\quad \varepsilon = 0{,}01$



$f' = 2x + 5\cos(x)$

$f'' = 2 - 5\sin(x)$ зростає на $[-1, 0{,}2]$

$f' \neq 0$ $[a, b]$

$q = \frac{M_2 |x_0 - x^*|}{2m_1} < 1$

$n_0(\varepsilon) = \left[\log_2\left(\frac{\ln|x_0 - x^*|/\varepsilon}{\ln(1/q)}\right) + 1\right] + 1$

$m_1 \approx 6{,}3$

$M_2 \approx 6{,}2$

№ 3 $-x_1 - x_2 = -4$

$x_1 - 3x_2 - 2x_3 = -2$

$x_2 + 2x_3 = 0$

$\text{cond}(A) = \|A^{-1}\| \|A\|$

$$A^{-1} = \begin{pmatrix} -\frac{2}{3} & \frac{1}{3} & \frac{1}{3} \\ -\frac{1}{3} & -\frac{1}{3} & -\frac{1}{3} \\ \frac{1}{6} & \frac{1}{6} & \frac{2}{3} \end{pmatrix}$$

$$\|A\| = \sqrt{(-1)^2 + (-1)^2 + 0^2 + 1^2 - (-3)^2 + (-2)^2 + 0^2 + 1^2 + 2^2} = \sqrt{21}$$

$$\|A^{-1}\| = \frac{\sqrt{21}}{3}$$

$$\text{cond}(A) = \frac{\sqrt{21}}{3} \cdot \sqrt{21} = 7$$

№ 4.

$$A = \begin{pmatrix} 0 & 1 & 2 \\ 1 & 1 & 1 \\ 2 & 1 & 1 \end{pmatrix}$$

A - симметрична $\varepsilon = 10^{-2}$

$k = 0$ $a_{13}^{(k)} = 2 > \varepsilon$

$p^{(0)} = \frac{1}{2} \operatorname{arctg} \frac{2a_{13}^{(0)}}{a_{11}^{(0)} - a_{33}^{(0)}} = \frac{1}{2} \operatorname{arctg} \frac{4}{0-1} = \frac{1}{2} \operatorname{arctg}$

$\approx 0,66$ $\sin p \approx 0,61$

$\cos p = 0,79$

$$U = \begin{pmatrix} 0,79 & 0 & -0,61 \\ 0 & 1 & 0 \\ +0,61 & 0 & 0,79 \end{pmatrix}$$

$$A^{(1)} = (H^{(0)})^T A^{(0)} H^{(0)} = \begin{pmatrix} 2,6565 & 2,17 & 0,8411 \\ 2,17 & 1 & 0,89 \\ 0,8411 & 0,89 & -1,5563 \end{pmatrix}$$

$k = 1$

$a_{12}^{(1)} = 2,17 > \varepsilon$ ✓

№5 $f(x) = \ln x$ $a, b \in [1; 2]$ $n = 4$

$f'(x) = e^x$

$f''(x) = e^x$

$f'''(x) = e^x$

$\dots\dots\dots$

$$\frac{M_{n+1}}{(n+1)} \frac{(b-a)^{n+1}}{2^{2n+1}} = \frac{e^2}{5} \cdot \frac{1}{2^9} \approx 0,0028$$

($M_{n+1} = e^2$, $(b-a)^{n+1} = 1^{n+1} = 1$, $n+1 = 5$)

№2 $f(x, y, z) = xz - y^2$ $x = 2,3 \pm 0,02$

$y = 1,5 \pm 0,02$; $z = 3,5 \pm 0,02$

$$\Delta(f^*) \leq (z^*)\Delta(x^*) + (2y)\Delta(y^*) + (x^*)\Delta(z^*)$$

$$= |3,5|_{\cdot 0,02} + |2 \cdot 1,5|_{\cdot 0,02} + |2,3|_{\cdot 0,02} = 0,176$$

$f(x^*, y^*, z^*) = 2,3 \cdot 3,5 - 1,5^2 = 5,8$

$\delta(f^*) = \frac{0,176}{5,8} \approx 0,172$