Київський національний університет імені Тарас Шевченка

Факультет комп’ютерних наук та кібернетики

Лабораторна робота №1
з курсу
«Управління динамічними системами»
на тему:

«Аналітичне розв’язування диференціальних рівнянь за допомогою комп’ютерних пакетів програм»

Виконав:
Студент групи ІПС-21
факультету комп’ютерних
наук та кібернетики
Ольховатий І.В.

Київ 2022

Зміст

Умови задач згідно варіанту……………………………………………….3

Представлення аналітичного розв’язку в зошиті ……………...…………4

Код програм..……………………………………………………………….11

Знімки екрану з відповідними результатами роботи………………………16

Варіант №1

Задачі:

№53

$$(x^2-1)y' + 2xy^2 = 0$$

$$M_1(\sqrt{2};1), M_2(-\sqrt{2};1), M_3\left(\sqrt{\frac{e^2+1}{e^2}};-1\right), M_4(-\sqrt{1+e};-1);$$

№109

$$xy' - y = (x+y) * \ln\frac{(x+y)}{x}$$

$$M_1(1;e-1), M_2(-1;1-e), M_3(-2;2-2e^{-2}), M_4(1;e^{-1}-1);$$

№137

$$(2x+1)y' = 4x + 2y$$

$$M_1(1;3ln3+1), M_2\left(-\frac{1}{2};1\right), M_3(1;-1), M_4(-1;-1);$$

№549

$$y'' - 2y' + 2y = e^x + xcosx$$

$$M_{1,1}(1;e), M_{1,2}(2;e^2), M_{2,1}(-1;e^{-1}), M_{2,2}(2;e^2),$$
$$M_{3,1}(-2;e^{-2}), M_{3,2}(-1;e^{-1}) M_{4,1}(1;e), M_{4,2}(-1;e^{-1});$$

№855

$$x' = Ax$$

$$A = \begin{matrix} 2 & -1 & -1 \\ 1 & 0 & -1 \\ 3 & -1 & -2 \end{matrix}$$

$$M_1\left(1;\begin{matrix}1\\1\\1\end{matrix}\right), M_2\left(1;\begin{matrix}-e\\0\\-e\end{matrix}\right), M_3\left(-1;\begin{matrix}1+e\\1+e\\1+2e\end{matrix}\right), M_4\left(-1;\begin{matrix}\frac{1-e^3}{e}\\-e\\\frac{1-2e^3}{e}\end{matrix}\right);$$

№53

$(x^2-1)y' + 2xy^2 = 0$

Це рівняння зі змінними, що розділяються.

$$(x^2-1)\frac{dy}{dx} + 2xy^2 = 0$$

Помножимо обидві частини на dx та поділимо на $(x^2-1)y^2$. Маємо:

$$\frac{dy}{y^2} + 2\frac{x\,dx}{(x^2-1)} = 0$$

Проінтегруємо ліву та праву частини:

$$\int\frac{dy}{y^2} + \int 2\frac{x\,dx}{(x^2-1)} = C$$

$$\int\frac{x\,dx}{(x^2-1)} = \left|\begin{matrix} u = x^2-1 \\ du = 2x\,dx\end{matrix}\right| = \frac{1}{2}\int\frac{du}{u} = \frac{1}{2}\ln|x^2-1| + C_1$$

$$\int\frac{dy}{y^2} = -\frac{1}{y} + C_2$$

$-\frac{1}{y} + \ln|x^2-1| = C$ – вигляд загального розв'язку

Задача Коші:

1) $y(\sqrt{2}) = 1$ $\quad -1 + \ln|2-1| = C_1 \Rightarrow C_1 = -1$

$$y = \frac{1}{1+\ln|x^2-1|}$$

2) $y(-\sqrt{2}) = 1$ $\quad -1 + \ln|2-1| = C_2 \Rightarrow C_2\ -1$

$$y = \frac{1}{1+\ln|x^2-1|}$$

3) $y\left(\sqrt{\frac{e^2+1}{e^2}}\right) = -1$ $\quad 1 + \ln\left|1+\frac{1}{e^2}\right| = C_3 \Rightarrow C_3 = 1$

$$y = \frac{1}{1+\ln|x^2-1|}$$

4) $y(-\sqrt{1+e}) = -1$

$1 + \ln|1+e-1| = C_4 \Rightarrow C_4 = 2$

$$y = \frac{1}{\ln|x^2-1|-2}$$

№109

$$xy' - y = (x+y)\ln\frac{x+y}{x}$$

$$x\frac{dy}{dx} - y = (x+y)\ln\frac{x+y}{x}$$

$$x\,dy - y\,dx = (x+y)\ln\left(\frac{x+y}{x}\right)dx$$

$$x\,dy - \left((x+y)\ln\frac{x+y}{x} + y\right)dx = 0$$

Маємо однорідне рівняння, зробимо заміну:

$y = ux$

$dy = u\,dx + x\,du$

$$xu\,dx + x^2du - \left((x+ux)\ln\frac{x+ux}{x} + ux\right)dx = 0 \,|:x$$

$$u\,dx + x\,du - (\ln(u+1) + u\ln(u+1) + u)\,dx = 0$$

$$x\,du - \ln(u+1)\cdot(u+1)\,dx = 0$$

Маємо рівняння зі змінними, що розділяються:

$$\frac{du}{dx} - \frac{(u+1)\ln(u+1)}{x} = 0$$

$$\frac{du}{(u+1)\ln(u+1)dx} = \frac{dx}{x}$$ Інтегруємо:

$$\ln(\ln(u+1)) = \ln x + C$$

$$u = e^{xe^c} - 1 = e^{cx} - 1$$

$$y = ux = xe^{cx} - x$$ – загальний розв'язок

Задачі Коші:

1) $y(1) = e-1$ $\quad e-1 = 1\cdot e^{c_1} - 1 \Rightarrow c_1 = 1$, $y = xe^x - x$

2) $y(-1) = 1-e$ $\quad 1-e = -1\cdot e^{-c_2} - 1 \Rightarrow c_2 = -1$, $y = xe^{-x} - x$

3) $y(-2) = 2-2e^{-2}$ $\quad 2-2e^{-2} = -2e^{-2c_3} + 2 \Rightarrow c_3 = -1$, $y = xe^{-x} - x$

4) $y(1) = \frac{1}{e} - 1$ $\quad = c_4 = -1$, $y = xe^{-x} - x$.

№137

$(2x+1)y' = 4x+2y$

Це лінійне диференціальне рівняння I порядку, тому можемо застосувати формулу Коші:

$$(2x+1)\cdot\frac{dy}{dx} = 4x+2y$$

$$\frac{dy}{dx} - \left(\frac{2}{2x+1}\right)y = \frac{4x}{2x+1}$$

$$y(x) = e^{\int\frac{2dx}{2x+1}}\cdot\left(\int\frac{4x}{2x+1}\cdot e^{-\ln(2x+1)}dx\right) = e^{\ln(2x+1)}\cdot\left(\int\frac{4x}{(2x+1)^2}dx\right) =$$

$$= (2x+1)\cdot\left(\frac{1}{2x+1}+\ln(2x+1)+C\right) = (2x+1)(\ln(2x+1)+C)+1$$

$y = (2x+1)(\ln(2x+1)+C)$ – загальний розв'язок

Задача Коші:

1) $y(1) = 3\ln 3+1$

$$(2\cdot 1+1)(\ln(2\cdot 1+1+c_1))+1 = 3\ln 3+1$$

$$\underline{c_1=0}, \quad y = (2x+1)\ln|2x+1|+1$$

2) $y(-1) = 1$

$$(2\cdot(-1)+1)\cdot(\ln(2\cdot(-1)+1)+c_2)+1 = 1$$

$$\underline{c_2=0}$$

$$y = (2x+1)\ln(2x+1)+1$$

3) $y(1) = -1$

$$(2\cdot(1)+1)\cdot(\ln|2\cdot 1+1)+c_3)+1 = -1$$

$$3\ln 3+c_3 = -2$$

$$\underline{c_3 = -2-3\ln 3} \quad y = (2x+1)(\ln(2x+1)+(-2-3\ln 3))+1.$$

4) $y(-1) = -1$

$$(2-(-1)+1)(\ln(-2+1)+c_4)+1 = -1$$

$$-1\cdot(1+c_4)+1 = 1$$

$$\underline{c_4 = -1} \quad y = (2x+1)(\ln(2x+1)-1)+1$$

N549.

$$y'' - 2y' + 2y = e^x + x\cos x$$

Спочатку знайдемо розв'язок загального р-ня:

$$y'' - 2y' + 2y = 0$$

Характеристичне р-не: $\lambda^2 - 2\lambda + 2 = 0$

$\lambda_1 = 1+i,\ \lambda_2 = 1-i$

$$y_{зо} = e^x \cdot (c_1\cos x + c_2\sin x)$$

$$y_{чн} = y_{чн_1} + y_{чн_2}$$

1) $y_{чн_1}$ – частк. розв. неодн. р-не:

$$y'' - 2y' + 2y = e^x$$

$y_{чн_1} = ae^x$ $\qquad ae^x - 2ae^x + 2ae^x = e^x$

$y'_{чн_1} = y'' = ae^x$ $\qquad a = 1$

$$\underline{y_{чн_1} = e^x}$$

2) $y_{чн_2}$ = –частк. розв. неодн. р-не:

$$y'' - 2y' + 2y = x\cos x.$$

$y_{чн_2} = (ax+b)\cos x + (cx+d)\sin x$

$y'_{чн_2} = \cos x\,(a + cx + d) - \sin x\,(ax + b - c)$

$y''_{чн_2} = \cos x\,(-ax - b + 2c) - \sin x\,(2a + cx + d)$

$\cos(-ax - b + 2c) - \sin x\,(2a + cx + d) - 2\cos x\,(a + cx + d) + 2\sin x\,(ax + b - c) +$

$+\ 2\cos x\,(ax + b) + 2\sin x\,(cx + d) = x\cos x$

$\sin x\,(2ax + cx - 2a + 2b - 2c + d) + \cos x\,(ax - 2cx - 2a + b + 2c - 2d)$

$= x\cos x$

$$\begin{cases} a - 2c = 1 \\ 2a + c = 0 \\ -2a + b + 2c - 2d = 0 \\ 2a + 2b - 2c + d = 0 \end{cases}$$

$a = \frac{1}{5}$ $\quad c = -\frac{2}{5}$

$d = \frac{-14}{25}$ $\quad c = \frac{2}{25}$

$$y_{чн} = e^x - \frac{2x}{5}\sin x + \frac{x}{5}\cos x - \frac{14}{25}\sin x + \frac{2}{25}\cos x.$$

– частк. розв'язок.

$y_{зно} = y_{зо} + y_{чн}$

$y = e^x(c_1\cos x + c_2\sin x) + e^x \cdot \frac{2x}{5}\sin x + \frac{x}{5}\cos x - \frac{14}{25}\sin x + \frac{2}{25}\cos x.$

– загальний розв'язок.

Задача Коші:

1) $y(1) = e$, $y(2) = e^2$

$$\begin{cases} e(c_1\cos 1 + c_2\sin 1) + e + \sin 1\left(\frac{2}{5} - \frac{14}{25}\right) + \cos 1 \cdot \left(\frac{1}{5} + \frac{2}{25}\right) = e \\ e^2(c_1\cos 2 + c_2\sin 2) + e^2 + \frac{2}{5}\cdot 2\sin 2 + \frac{1}{5}\cos 2 - \frac{14}{25}\sin 2 + \frac{2}{25}\cos 2 = e^2 \end{cases}$$

$$c_1 = -\frac{12\cos 2\sin 1 - 7\cos 1 \cdot e \cdot \sin 2 + 6\sin 1 \cdot \sin 2 + 4e\sin 1 \cdot \sin 2}{25e^2(\cos 2\sin 1 - \cos 1\sin 2)}$$

$$c_2 = \frac{12\cos 1\cos 2 - 7\cos 1\cos 2 \cdot e + 4\cos 2 \cdot e \cdot \sin 1 + 6\cos 1\sin 2}{25e^2(\cos 2\sin 1 - \cos 1\sin 2)}$$

2) $y(-1) = \frac{1}{e}$, $y(2) = e^2$.

$$\begin{cases} e^{-1}\cdot(c_1\cos -1 + c_2\cdot\sin -1) + e^{-1} - \sin(-1) - \left(\frac{1}{5}\right)\cos -1 - \frac{14}{25}\sin -1 + \frac{2}{25}\cos -1 = e^{-1} \\ e^2\cdot(c_1\cos 2 + c_2\sin 2) + e^2 + \frac{2}{5}2\cdot\sin 2 + \frac{1}{5}2\cos 2 - \frac{14}{25}\sin 2 + \frac{2}{25}\cos 2 = e^2 \end{cases}$$

$$c_1 = -\frac{12\cos 2\sin(-1) + 3e^3\cos -1\sin 2 - 34\sin -1\sin 2 + 4e^3\sin -1\sin 2.}{25e^2(\cos 2\sin -1 - \cos -1\sin 2)}$$

$$c_2 = -\frac{12\cos -1\cos 2 - 3e^3\cos(-1)\cos 2 - 4e^3\cos 2\sin -1 + 34\cos -1\sin 2}{25e^2(\cos 2\sin 1 - \cos -1\sin 2)}$$

3) $y(-2) = e^{-2}$, $y(-1) = e^{-1}$

$$c_1 = \frac{e(3\cos -1\sin -2 - 8e\cos -2\sin -1 + 4\sin -2\sin -1 + 6e\sin -2\sin -1}{25(\cos -1\sin -2 - \cos -2\sin -1)}$$

$$c_2 = \frac{-3e\cos -2\cdot\cos -1 + 8e^2\cos -2\cdot\cos -1 - 6e^2\cos -1\sin 2 - 4e\cos -2\sin -1}{25(\cos -1\sin -2 - \cos -2\sin -1)}$$

4) $y(1) = e$, $y(-1) = e^{-1}$

$$c_1 = \frac{-7\cos 1\sin -1 + 3e^2\cos -1\sin 1 - 24\sin -1\sin 1 + 4e^2\sin -1\sin 1}{25e(\cos 1\sin -1 - \cos -1\sin 1)}$$

$$c_2 = -\frac{-7\cos -1\cos 1 - 3e^2\cos -1\cos 1 - 4e^2\cos 1\sin -1 + 24\cos -1\sin 1}{25e(\cos 1\sin -1 - \cos -1\sin 1)}$$

№855

$\dot{x} = Ax \qquad A = \begin{pmatrix} 2 & -1 & -1 \\ 1 & 0 & -1 \\ 3 & -1 & -2 \end{pmatrix}$

Розвʼязок р-ня буде мати вигляд:

$x = c_1 \cdot e^{\lambda_1 t} \cdot a_1 + c_2 e^{\lambda_2 t} \cdot a_2 + c_3 e^{\lambda_3 t} \cdot a_3$, де λ_i – власні числа, a_i – власні вектори.

$\det|A - \lambda E| = 0.$

$\begin{vmatrix} 2-\lambda & -1 & -1 \\ 1 & -\lambda & -1 \\ 3 & -1 & -2-\lambda \end{vmatrix} = -\lambda(2-\lambda)(-2-\lambda) + 3 + 1 - (2-\lambda) + (-2-\lambda) =$

$= (-2\lambda + \lambda^2)(-2-\lambda) + 4 - 3\lambda - 2 + \lambda - 2 - \lambda =$

$= -\lambda^3 + \lambda. \qquad \lambda_1 = 0, \ \lambda_2 = 1, \ \lambda_3 = -1.$

1) $\lambda_1 = 0 \quad \left(\begin{array}{ccc|c} 2 & -1 & -1 & 0 \\ 1 & 0 & -1 & 0 \\ 3 & -1 & 2 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 2 & -1 & -1 & 0 \\ 3 & -1 & -2 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & -1 & 1 & 0 \end{array}\right) \sim$

$\sim \left(\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & -1 & 1 & 0 \end{array}\right) \qquad \alpha_1 = \alpha_3, \ \alpha_2 = \alpha_3 \qquad a_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

2) $\lambda_2 = 1: \quad \left(\begin{array}{ccc|c} 1 & -1 & -1 & 0 \\ 1 & -1 & -1 & 0 \\ 3 & -1 & -3 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & -1 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \end{array}\right) \Rightarrow a_2 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}.$ $\quad \beta_1 = \beta_2 + \beta_3; \ \beta_2 = 0$

3) $\lambda_3 = -1: \quad \left(\begin{array}{ccc|c} 3 & -1 & -1 & 0 \\ 1 & 1 & -1 & 0 \\ 3 & -1 & -1 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 3 & -1 & -1 & 0 \\ 3 & -1 & -1 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & -4 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \Rightarrow$

$\Rightarrow \gamma_3 = \gamma_1 + \gamma_2 \quad \gamma_3 = 2\gamma_2 \Rightarrow a_3 \begin{pmatrix} \gamma_1 \\ \gamma_2 \\ \gamma_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}.$

$x = c_1 \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + c_2 e^{t} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + c_3 e^{-t} \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$ – загальний розвʼязок

Задача Коші:

1) $x(1) = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \quad c_1 \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + c_2 \begin{pmatrix} e \\ 0 \\ e \end{pmatrix} + c_3 \begin{pmatrix} \frac{1}{e} \\ \frac{1}{e} \\ \frac{2}{e} \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

$c_1 = 1, \ c_2 = 0, \ c_3 = 0$

2) $x(1) = \begin{pmatrix} -e \\ 0 \\ -e \end{pmatrix} \quad c_1 \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + c_2 \begin{pmatrix} e \\ 0 \\ e \end{pmatrix} + c_3 \begin{pmatrix} \frac{1}{e} \\ \frac{1}{e} \\ \frac{2}{e} \end{pmatrix} = \begin{pmatrix} -e \\ 0 \\ -e \end{pmatrix}$

$c_1 = 0, \ c_2 = -1, \ c_3 = 0.$

3) $x(-1) = \begin{pmatrix} 1+e \\ 1+e \\ 1+2e \end{pmatrix} \Rightarrow c_1 \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + c_2 \begin{pmatrix} \frac{1}{e} \\ 0 \\ \frac{1}{e} \end{pmatrix} + c_3 \begin{pmatrix} e \\ e \\ 2e \end{pmatrix} = \begin{pmatrix} 1+e \\ 1+e \\ 1+2e \end{pmatrix}$

$c_1 = 1, \; c_2 = 0, \; c_3 = 1$

4) $x(-1) = \begin{pmatrix} \frac{1-e^2}{e} \\ -e \\ \frac{1-2e^2}{e} \end{pmatrix} \qquad c_1 \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + c_2 \begin{pmatrix} \frac{1}{e} \\ 0 \\ \frac{1}{e} \end{pmatrix} + c_3 \begin{pmatrix} e \\ e \\ 2e \end{pmatrix} = \begin{pmatrix} \frac{1-e^2}{e} \\ -e \\ \frac{-1-2e^2}{e} \end{pmatrix}$

$c_1 = 0, \; c_2 = 1, \; c_3 = -1$

№53

```
#general solution
y = function('y')(x)
de = (x^2-1)*diff(y,x)+2*x*y^2
solution = desolve(de, y)
solution.show()

#Couchi problem solution
y=function('y')(x)
de = (x^2-1)*diff(y,x)+2*x*y^2
solution=desolve(de,y,ics=[sqrt(2),1])
solution.show()
solution1=desolve(de,y,ics=[-sqrt(2),1])
solution1.show()
solution2=desolve(de,y,ics=[sqrt((e^2+1)/e^2),-1])
solution2.show()
solution3=desolve(de,y,ics=[-sqrt(1+e),-1])
solution3.show()

#direction fields
x = var('x')
y = var('y')
f(x,y)=(x^2-1)*diff(y,x)+2*x*y^2
p=plot_slope_field(f,(x,-10,10),(y,-10,10), headaxislength=3,
headlength=3,axes_labels=['$x$','$y(x)$'])

#plot of Couchi problem solution
p+=desolve_rk4(f,y,ics=[sqrt(2),1],ivar=x,output='plot',
end_points=[-10,10],thickness=6,rgbcolor=hue(1))
p1=desolve_rk4(f,y,ics=[-sqrt(2),1],ivar=x,output='plot',
end_points=[-10,10],thickness=3,rgbcolor=hue(0.2))
p2=desolve_rk4(f,y,ics=[sqrt((gp(e)^2+1)/gp(e)^2),-1],ivar=x,output='plot',
end_points=[-10,10],thickness=2,rgbcolor=hue(1))
p3=desolve_rk4(f,y,ics=[-sqrt(1+gp(e)),-1],ivar=x,output='plot',
end_points=[-10,10],thickness=2,rgbcolor=hue(0.3))
show(p+p1+p2+p3,xmin=-10,xmax=10,ymin=-10,ymax=10)
```

№109

```
#general solution
y = function('y')(x)
de = (x)*diff(y,x)-y-(x+y)*log((x+y)/x)
solution = desolve(de, y)
solution.show()

#Couchi problem solution
y=function('y')(x)
de = (x)*diff(y,x)-y-(x+y)*log((x+y)/x)
solution=desolve(de,y,ics=[1,e-1])
solution.show()
solution1=desolve(de,y,ics=[-1,1-e])
solution1.show()
solution2=desolve(de,y,ics=[-2,2-2*e^(-2)])
solution2.show()
solution3=desolve(de,y,ics=[1,1/e-1])
solution3.show()

#direction fields
x = var('x')
y = var('y')
f(x,y)=x*diff(y,x)-y-(x+y)*log((x+y)/x)
p=plot_slope_field(f,(x,-10,10),(y,-10,10), headaxislength=5,
headlength=5,axes_labels=['$x$','$y(x)$'])

#plot of Couchi problem solution
p+=desolve_rk4(f,y,ics=[1,gp(e)-1],ivar=x,output='plot',
end_points=[-10,10],thickness=2,rgbcolor=hue(1))
p1=desolve_rk4(f,y,ics=[-1,1-gp(e)],ivar=x,output='plot',
end_points=[-10,10],thickness=2,rgbcolor=hue(0.2))
p2=desolve_rk4(f,y,ics=[-2,2-2*gp(e)^(-2)],ivar=x,output='plot',
end_points=[-10,10],thickness=2,rgbcolor=hue(0.5))
p3=desolve_rk4(f,y,ics=[1,1/gp(e)-1],ivar=x,output='plot',
end_points=[-10,10],thickness=2,rgbcolor=hue(0.3))
show(p+p1+p2+p3,xmin=-10,xmax=10,ymin=-10,ymax=10)
```

№137

```
#general solution
y = function('y')(x)
de = (2*x+1)*diff(y,x)-4*x-2*y
solution = desolve(de, y)
solution.show()

#Couchi problem solution
y=function('y')(x)
de = (2*x+1)*diff(y,x)-4*x-2*y
solution=desolve(de,y,ics=[1,3*log(3)+1])
solution.show()
solution1=desolve(de,y,ics=[-1,1])
solution1.show()
solution2=desolve(de,y,ics=[-1,-1])
solution2.show()
solution3=desolve(de,y,ics=[1,-1])
solution3.show()

#direction fields
x = var('x')
y = var('y')
f(x,y)=(2*x+1)*diff(y,x)-4*x-2*y
p=plot_slope_field(f,(x,-10,10),(y,-10,10), headaxislength=5,
headlength=5,axes_labels=['$x$','$y(x)$'])

#plot of Couchi problem solution
p+=desolve_rk4(f,y,ics=[1,3*log(3)+1],ivar=x,output='plot',
end_points=[-10,10],thickness=2,rgbcolor=hue(1))
p1=desolve_rk4(f,y,ics=[-1,1],ivar=x,output='plot',
end_points=[-10,10],thickness=2,rgbcolor=hue(0.2))
p2=desolve_rk4(f,y,ics=[-1,-1],ivar=x,output='plot',
end_points=[-10,10],thickness=2,rgbcolor=hue(0.5))
p3=desolve_rk4(f,y,ics=[1,-1],ivar=x,output='plot',
end_points=[-10,10],thickness=1,rgbcolor=hue(0.3))
show(p+p1+p2+p3,xmin=-10,xmax=10,ymin=-10,ymax=10)
```

№549

```
#general solution
y = function('y')(x)
de = diff(diff(y,x),x) - 2 * diff(y,x) + 2*y - e^x - x*cos(x)
solution = desolve(de, y)
solution.show()

#Couchi problem solution
y=function('y')(x)
de =diff(diff(y,x),x) - 2 * diff(y,x) + 2*y - e^x - x*cos(x)
solution=desolve(de,y,ics=[1,e,2,e^2])
solution.show()
solution1=desolve(de,y,ics=[-1,1/e,2,e^2])
solution1.show()
solution2=desolve(de,y,ics=[-2,e^(-2),-1,e^(-1)])
solution2.show()
solution3=desolve(de,y,ics=[1,e,-1,e^(-1)])
solution3.show()

#direction fields
x = var('x')
y = var('y')
f(x,y)= diff(diff(y,x),x) - 2 * diff(y,x) + 2*y - e^x - x*cos(x)
p=plot_slope_field(f,(x,-10,10),(y,-10,10), headaxislength=5,
headlength=5,axes_labels=['$x$','$y(x)$'])

#plot of Couchi problem solution
p+=desolve_rk4(f,y,ics=[1,gp(e),2,gp(e)^2],ivar=x,output='plot',
end_points=[-10,10],thickness=2,rgbcolor=hue(1))
p1=desolve_rk4(f,y,ics=[-1,1/gp(e),2,gp(e)^2],ivar=x,output='plot',
end_points=[-10,10],thickness=2,rgbcolor=hue(0.2))
p2=desolve_rk4(f,y,ics=[-2,gp(e)^(-2),-1,gp(e)^(-1)],ivar=x,output='plot',
end_points=[-10,10],thickness=2,rgbcolor=hue(0.5))
p3=desolve_rk4(f,y,ics=[1,gp(e),-1,gp(e)^(-1)],ivar=x,output='plot',
end_points=[-10,10],thickness=2,rgbcolor=hue(0.3))
show(p+p1+p2+p3,xmin=-10,xmax=10,ymin=-10,ymax=10)
```

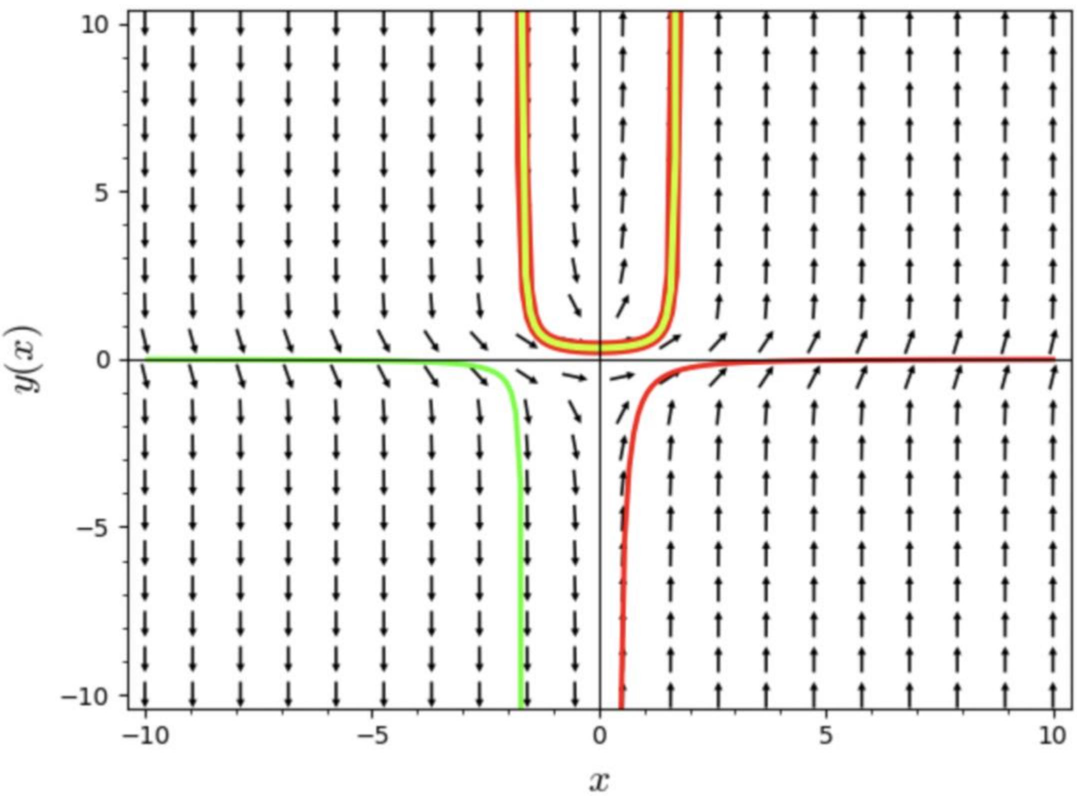
№855

```
#general solution
t = var('t')
x1 =function ('x1')(t)
x2 =function ('x2')(t)
x3 =function ('x3')(t)
de1 = diff(x1,t) == 2*x1-x2-x3
de2 = diff(x2,t) == x1-x3
de3 = diff(x3,t) == 3*x1-x2-2*x3
sol = desolve_system([de1,de2,de3],[x1,x2,x3],ivar=t)
solx1,solx2,solx3 = sol[0].rhs(),sol[1].rhs(),sol[2].rhs()
solx = matrix ([[solx1],[solx2],[solx3]])
show(solx)

#Couchi problem solution
sol = desolve_system([de1,de2,de3],[x1,x2,x3],ics=[0,-e,0,-e],ivar=t)
solx1,solx2,solx3 = sol[0].rhs(),sol[1].rhs(),sol[2].rhs()
solx=matrix ([[solx1],[solx2],[solx3]])
show(solx)
#plot of Couchi problem solution
p1 = plot((solx1),(0,0.2),figsize=5,rgbcolor= hue(0.1))
sol = desolve_system([de1,de2,de3],[x1,x2,x3],ics=[0,1+e,1+e,1+2*e],ivar=t)
solx1,solx2,solx3 = sol[0].rhs(),sol[1].rhs(),sol[2].rhs()
solx=matrix ([[solx1],[solx2],[solx3]])
show(solx)
#plot of Couchi problem solution
p2 = plot((solx1),(0,0.2),figsize=5,rgbcolor= hue(0.5))
sol = desolve_system([de1,de2,de3],[x1,x2,x3],ics=[0,(1-e^2)/e,-e,(1-
2*e^2)/e],ivar=t)
solx1,solx2,solx3 = sol[0].rhs(),sol[1].rhs(),sol[2].rhs()
solx=matrix ([[solx1],[solx2],[solx3]])
show(solx)
#plot of Couchi problem solution
p3 = plot((solx1),(0,0.2),figsize=5,rgbcolor= hue(0.3))
sol = desolve_system([de1,de2,de3],[x1,x2,x3],ics=[0,1/e,0,1/e],ivar=t)
solx1,solx2,solx3 = sol[0].rhs(),sol[1].rhs(),sol[2].rhs()
solx=matrix ([[solx1],[solx2],[solx3]])
show(solx)
#plot of Couchi problem solution
p4 = plot((solx1),(0,0.2),figsize=5,rgbcolor= hue(0.9))
show(p1+p2+p3+p4)
```

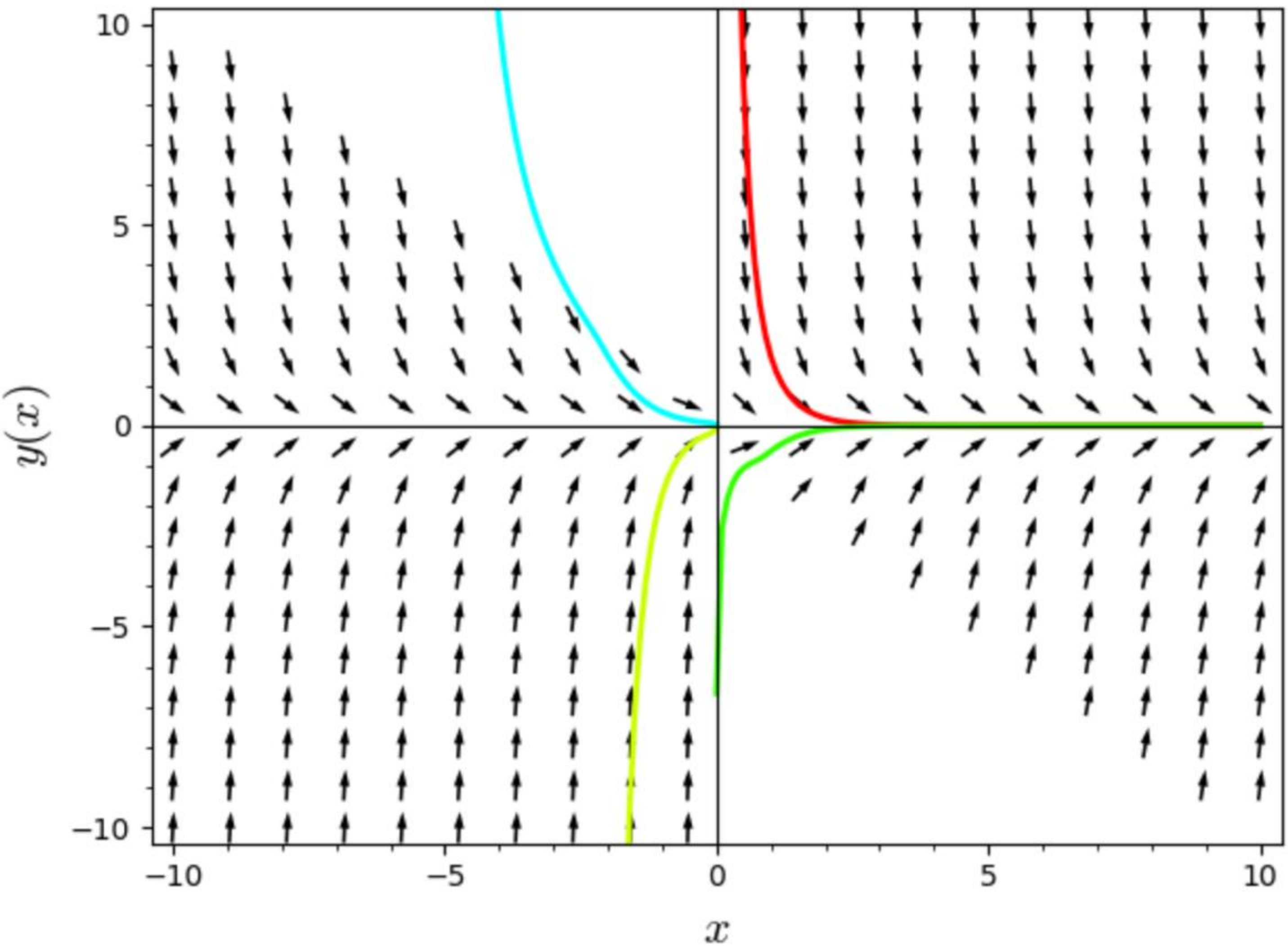
№53

$$\frac{1}{2\,y\,(x)} = C + \frac{1}{2}\log\left(x+1\right) + \frac{1}{2}\log\left(x-1\right)$$
$$\frac{1}{2\,y\,(x)} = \frac{1}{2}\log\left(x+1\right) + \frac{1}{2}\log\left(x-1\right) - \frac{1}{2}\log\left(\sqrt{2}+1\right) - \frac{1}{2}\log\left(\sqrt{2}-1\right) + \frac{1}{2}$$
$$\frac{1}{2\,y\,(x)} = \frac{1}{2}\log\left(x+1\right) + \frac{1}{2}\log\left(x-1\right) - \frac{1}{2}\log\left(-\sqrt{2}+1\right) - \frac{1}{2}\log\left(-\sqrt{2}-1\right) + \frac{1}{2}$$
$$\frac{1}{2\,y\,(x)} = -\frac{1}{2}\log\left(\left(\sqrt{e^2+1}+e\right)e^{(-1)}\right) - \frac{1}{2}\log\left(\left(\sqrt{e^2+1}-e\right)e^{(-1)}\right) + \frac{1}{2}\log\left(x+1\right) + \frac{1}{2}\log\left(x-1\right) - \frac{1}{2}$$
$$\frac{1}{2\,y\,(x)} = \frac{1}{2}\log\left(x+1\right) + \frac{1}{2}\log\left(x-1\right) - \frac{1}{2}\log\left(-\sqrt{e+1}+1\right) - \frac{1}{2}\log\left(-\sqrt{e+1}-1\right) - \frac{1}{2}$$



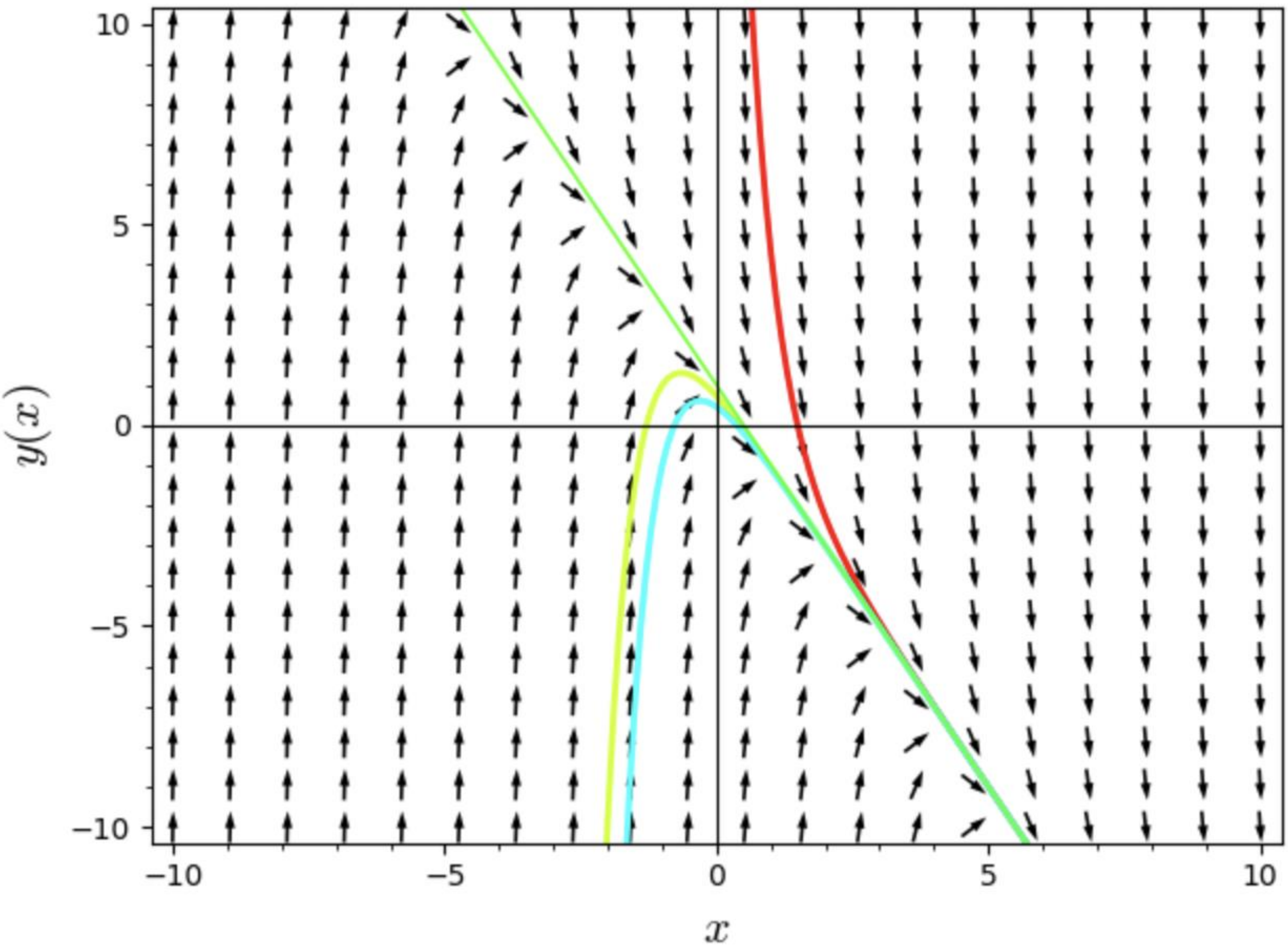
№109

$$Cx = \log\left(\frac{x + y(x)}{x}\right)$$
$$x = \log\left(\frac{x + y(x)}{x}\right)$$
$$-x = \log\left(\frac{x + y(x)}{x}\right)$$
$$x = \log\left(\frac{x + y(x)}{x}\right)$$
$$-x = \log\left(\frac{x + y(x)}{x}\right)$$



№137

$$\left(C+\frac{1}{2\,x+1}+\log\left(2\,x+1\right)\right)(2\,x+1)$$
$$(2\,x+1)\log\left(2\,x+1\right)+1$$
$$-i\,\pi-2i\,\pi x+(2\,x+1)\log\left(2\,x+1\right)+1$$
$$-i\,\pi-2\,(i\,\pi-2)x+(2\,x+1)\log\left(2\,x+1\right)+3$$
$$-\frac{2}{3}\,x(3\,\log\left(3\right)+2)+(2\,x+1)\log\left(2\,x+1\right)-\log\left(3\right)+\frac{1}{3}$$



№549

$$\frac{1}{25}\,(5\,x+2)\cos(x)+(K_2\cos(x)+K_1\sin(x))e^x-\frac{2}{25}\,(5\,x+7)\sin(x)+e^x$$

$$\frac{1}{25}\,(5\,x+2)\cos(x)-\frac{1}{25}\left(\frac{(7\cos(1)e\sin(2)-2\,(12\,e\sin(2)+6\cos(2)-17\sin(2))\sin(1))\cos(x)}{\cos(1)e^2\sin(2)-\cos(2)e^2\sin(1)}+\frac{(24\cos(2)e\sin(1)-(7\cos(2)e-12\cos(2)+34\sin(2))\cos(1))\sin}{\cos(1)e^2\sin(2)-\cos(2)e^2\sin(1)}\right.$$

$$\frac{1}{25}\,(5\,x+2)\cos(x)+\frac{1}{25}\left(\frac{\left(3\cos(1)e^3\sin(2)-2\left(2\,e^3\sin(2)+6\cos(2)-17\sin(2)\right)\sin(1)\right)\cos(x)}{\cos(1)e^2\sin(2)+\cos(2)e^2\sin(1)}+\frac{\left(4\cos(2)e^3\sin(1)-\left(3\cos(2)e^3+12\cos(2)-34\sin(2)\right)\cos(1)\right)}{\cos(1)e^2\sin(2)+\cos(2)e^2\sin(1)}\right.$$

$$\frac{1}{25}\,(5\,x+2)\cos(x)-\frac{1}{25}\left(\frac{\left(8\cos(2)e^2\sin(1)-\left((3\cos(1)-4\sin(1))e-6\,e^2\sin(1)\right)\sin(2)\right)\cos(x)}{\cos(1)\sin(2)-\cos(2)\sin(1)}+\frac{\left(6\cos(1)e^2\sin(2)+\left(8\cos(1)e^2-(3\cos(1)-4\sin(1))e\right)\cos(2)\right.}{\cos(1)\sin(2)-\cos(2)\sin(1)}\right.$$

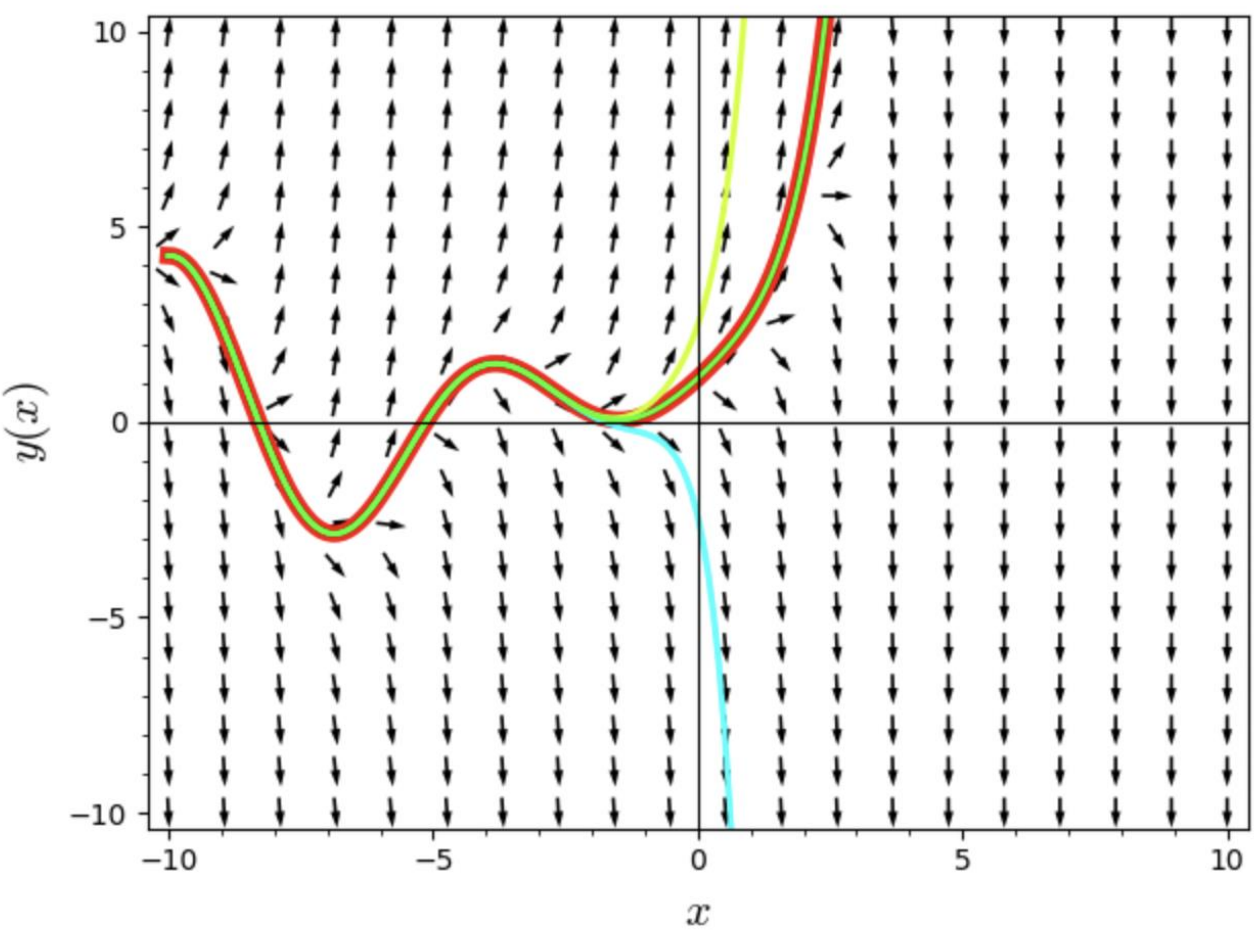
$$\frac{1}{25}\,(5\,x+2)\cos(x)+\frac{1}{50}\left(\frac{\left(\left(3\,e^2-7\right)\cos(1)-4\left(e^2-6\right)\sin(1)\right)\cos(x)e^{(-1)}}{\cos(1)}-\frac{\left(\left(3\,e^2+7\right)\cos(1)-4\left(e^2+6\right)\sin(1)\right)e^{(-1)}\sin(x)}{\sin(1)}\right)e^x-\frac{2}{25}\,(5\,x+7)\sin(x)+e^x$$

$$\left.+\frac{(24\cos(2)e\sin(1)-(7\cos(2)e-12\cos(2)+34\sin(2))\cos(1))\sin(x)}{\cos(1)e^2\sin(2)-\cos(2)e^2\sin(1)}\right)e^x-\frac{2}{25}\,(5\,x+7)\sin(x)+e^x$$

$$\left.+\frac{\left(4\cos(2)e^3\sin(1)-\left(3\cos(2)e^3+12\cos(2)-34\sin(2)\right)\cos(1)\right)\sin(x)}{\cos(1)e^2\sin(2)+\cos(2)e^2\sin(1)}\right)e^x-\frac{2}{25}\,(5\,x+7)\sin(x)+e^x$$

$$\left.+\frac{\left(6\cos(1)e^2\sin(2)+\left(8\cos(1)e^2-(3\cos(1)-4\sin(1))e\right)\cos(2)\right)\sin(x)}{\cos(1)\sin(2)-\cos(2)\sin(1)}\right)e^x-\frac{2}{25}\,(5\,x+7)\sin(x)+e^x$$

$$\left.\frac{4\left(e^2+6\right)\sin(1)\right)e^{(-1)}\sin(x)}{\sin(1)}\right)e^x-\frac{2}{25}\,(5\,x+7)\sin(x)+e^x$$



№855

$$\begin{pmatrix} -(x_1(0) - x_3(0))e^{(-t)} + (x_1(0) - x_2(0))e^t + x_1(0) + x_2(0) - x_3(0) \\ -(x_1(0) - x_3(0))e^{(-t)} + x_1(0) + x_2(0) - x_3(0) \\ -2(x_1(0) - x_3(0))e^{(-t)} + (x_1(0) - x_2(0))e^t + x_1(0) + x_2(0) - x_3(0) \end{pmatrix}$$

$$\begin{pmatrix} -e^{(t+1)} \\ 0 \\ -e^{(t+1)} \end{pmatrix}$$

$$\begin{pmatrix} e^{(-t+1)} + 1 \\ e^{(-t+1)} + 1 \\ 2e^{(-t+1)} + 1 \end{pmatrix}$$

$$\begin{pmatrix} e^{(t-1)} - e^{(-t+1)} \\ -e^{(-t+1)} \\ e^{(t-1)} - 2e^{(-t+1)} \end{pmatrix}$$

$$\begin{pmatrix} e^{(t-1)} \\ 0 \\ e^{(t-1)} \end{pmatrix}$$