

$$1 \quad f(x_1, x_2) = (x_1 + 1)^{x_2}$$

Будемо використовувати операції \otimes та \oplus , тому знайдемо для них операторні терми:

$$\oplus \quad f(x_1, x_2) = x_1 + x_2, \quad h(x_1, x_2, x_3):$$

$$\text{OT: } R(I_1^1, S^2(s, I_3^3))$$

$$\otimes \quad f(x_1, x_2) = x_1 x_2, \quad h(x_1, x_2, x_3):$$

$$\text{OT: } R(I_1^1, S^3(\oplus, I_1^3, I_3^3))$$

Щукану g -то $f(x_1, x_2) = (x_1 + 1)^{x_2}$ замінимо через схему рекурсії: $R(g, h)$

$$g(x_1): f(x_1, 0) = (x_1 + 1)^0 = 1, \quad S^2(s, S^2(0, I_1^1))$$

$$h(x_1, x_2, x_3): f(x_1, x_2 + 1) = (x_1 + 1)^{x_2 + 1} = f(x_1, x_2) \cdot (x_2 + 1)$$

$$S^3(\otimes, I_3^3, S^2(s, I_1^3))$$

$$\text{Отже, OT: } R(S^2(s, S^2(0, I_1^1)), S^3(\otimes, I_3^3, S^2(s, I_1^3)))$$

$$2 \quad f(x_1, x_2, x_3) = \lfloor (x_3 + 1) / 3 \rfloor$$

$$x_u = \lfloor (x_3 + 1) / 3 \rfloor$$

$$x_u \leq (x_3 + 1) / 3 < x_u + 1 \quad | \cdot 3$$

$$3x_u \leq x_3 + 1 < 3x_u + 3 \quad | -1$$

$$3x_u - 1 \leq x_3 < 3x_u + 2$$

$$x_3 + 1 \leq (3x_u + 2)$$

Замінимо операторний терм через мінімізацію:

$$\mu_{x_u} ((x_3 + 1) \leq (3x_u + 2)) = \mu_{x_u} (x_3 + 1 - 3x_u + 2 = 0)$$

$$M(S^3(\ominus, S^2(s, I_3^3), S^2/s, S^2(s, S^3(\oplus, I_4^4, S^3(\oplus, I_4^4, I_4^4))))))$$

де \oplus та \otimes були визначені вище, а \ominus : $R(I_1^1, S^2(R(S^2(0, I_1^1), I_1^3)))$

$$f(x_1, x_2) = x_1 \cdot x_2,$$

$$3. f(x, y) = x^2 \cdot (y+1) = x^2 y + x^2$$

$$\Sigma = \{1, \#\}, Ax = \{\#\#\#\}$$

$$f(x+1, y) = (x+1)^2 y + (x+1)^2 = x^2 y + x^2 + 2xy + 2x + y + 1$$

$$f(x, y+1) = x^2(y+1) + x^2 = x^2 y + x^2 + x^2$$

$$P_2 \begin{cases} A\#B\#R \rightarrow A1\#B\#RAA \\ A\#B\#R \rightarrow A\#B1\#RAAAABBB1 \end{cases}$$

$$4. f(x) = \text{sg}(\lfloor x/3 \rfloor)$$

$$q_0 \lambda \rightarrow q_1 R \quad 3C^4(0, \overset{1}{1}, \overset{1}{1}, \overset{1}{1}) + 2 = 3 \cdot 19 + 2 = 59$$

$$q_1 \lambda \rightarrow q_2 R \quad 3C^4(\overset{1}{1}, \overset{1}{1}, \overset{2}{2}, \overset{1}{1}) + 2 = 3 \cdot 376 + 2 = 1130$$

$$q_2 \lambda \rightarrow q_3 L \quad 3C^4(\overset{2}{2}, \overset{1}{1}, \overset{3}{3}, \overset{1}{1}) + 1 = 3 \cdot 2924 + 1 = 8773$$

$$q_3 \lambda \rightarrow q_3 \lambda L \quad 3C^4(\overset{3}{3}, \overset{1}{1}, \overset{3}{3}, \overset{0}{0}) + 1 = 3 \cdot 11324 + 1 = 33973$$

$$q_3 \lambda \rightarrow q^* \lambda \quad 3C^4(\overset{3}{3}, \overset{0}{0}, \overset{4}{4}, \overset{0}{0}) = 3 \cdot 5150 = 15450$$

$$q_0 \lambda \rightarrow q^* \lambda \quad 3C^4(\overset{0}{0}, \overset{0}{0}, \overset{4}{4}, \overset{0}{0}) = 3 \cdot 65 = 195$$

$$q_1 \lambda \rightarrow q_3 \lambda L \quad 3C^4(\overset{1}{1}, \overset{0}{0}, \overset{3}{3}, \overset{0}{0}) + 1 = 3 \cdot 170 + 1 = 510 + 1 = 511$$

$$q_2 \lambda \rightarrow q_3 \lambda L \quad 3C^4(\overset{2}{2}, \overset{0}{0}, \overset{3}{3}, \overset{0}{0}) + 1 = 3 \cdot 902 + 1 = 2707$$

$$Q = \{q_0^0, q_1^1, q_2^2, q_3^3, q_4^4\}, \Sigma = \{a_0, a_1\}$$

$$f(M) = 2^{59} + 2^{1130} + 2^{8773} + 2^{33973} + 2^{15450} + 2^{195} + 2^{511} + 2^{2707}$$