Assignment 1, Math Exercises

CS 543 Machine Learning

Hood College

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2.1 If $f(x) = x^2 - 3x + 1$, find $f(0)$, $f(1)$, and $f(2)$.

$f(0) = 0^2 - 3 * 0 + 1 = 0 - 0 + 1 = \mathbf{1}$

$f(1) = 1^2 - 3 * 1 + 1 = 1 - 3 + 1 = \mathbf{-1}$

$f(2) = 2^2 - 3 * 2 + 1 = 4 - 6 + 1 = \mathbf{-1}$

2.2 If $p(y) = \left[\frac{1}{2}\right]^y$, find $p(0)$, $p(1)$, and $p(4)$.

$p(0) = \left[\frac{1}{2}\right]^0 = \mathbf{1}$

$p(1) = \left[\frac{1}{2}\right]^1 = \mathbf{\frac{1}{2}}$

$p(1) = \left[\frac{1}{2}\right]^4 = \mathbf{\frac{1}{16}}$

2.3 If $f(y) = y^2 + 3$, find $f(0)$ and $f(3)$.

$f(0) = 0^2 + 3 = \mathbf{3}$

$f(3) = 3^2 + 3 = 9 + 3 = \mathbf{12}$

2.4 If $f(y) = y^3$, find $f(1)$, $f(2)$, and $f(3)$.

$f(1) = 1^3 = \mathbf{1}$

$f(2) = 2^3 = \mathbf{8}$

$f(3) = 3^3 = \mathbf{27}$

2.5 If $f(y) = y$, find $f(2)$, $f(-3)$, and $f(-1)$.

$f(2) = \mathbf{2}$

$f(-3) = \mathbf{-3}$

$f(-1) = \mathbf{-1}$

2.6 If $g(y) = (y-1)^2$, find $g(1)$, $g(2)$, and $g(3)$.

$g(1) = (1-1)^2 = \mathbf{0}$

$g(2) = (2-1)^2 = \mathbf{1}$

$g(3) = (3-1)^2 = 2^2 = \mathbf{4}$

2.9 Evaluate the following summations:

a) $\sum_{y=0}^{5}(y-4)$: $(0-4) + (1-4) + (2-4) + (3-4) + (4-4) + (5-4) = -4 - 3 - 2 - 1 - 0 + 1 = \mathbf{-9}$

b) $\sum_{y=2}^{6}(y^2-5)$: $(2^2-5)+(3^2-5)+(4^2-5)+(5^2-5)+(6^2-5) = -1+4+11+20+31 = \mathbf{65}$

c) $\sum_{i=1}^{4}(y_i-2)$: $(y_1-2)+(y_2-2)+(y_3-2)+(y_4-2) = \boldsymbol{y_1+y_2+y_3+y_4-8}$

d) $\sum_{i=1}^{3}(y+2i)$: $(y+2*1)+(y+2*2)+(y+2*3) = \mathbf{3y+12}$

2.12 Consider the set of the opinions of all persons at your college or university regarding the desirability of a tax on gas-guzzling automobiles. Let a person favoring the tax be represented as a 1 and a person opposed as a 0. As an ultimate objective, we wish to sample this set of opinions in order to estimate the proportion of students favoring the tax.

a) Describe the population of interest.

The population of interest is the student body at Hood College (my college).

b) Suppose the first five measurements in the sample are $y_1 = 1$, $y_2 = 1$, $y_3 = 0$, $y_4 = 0$, and $y_5 = 1$. Use summation notation to write the expressions for the sum and the sum of the squares of these five measurements.

b.1) $\sum_{i=1}^{5} y_i$

b.2) $\sum_{i=1}^{5} y_i^2$

c) Find the sum and the sum of the squares of the five measurements.

c.1) $\sum_{i=1}^{5} y_i = 1+1+0+0+1 = \mathbf{3}$

c.2) $\sum_{i=1}^{5} y_i^2 = 1^2+1^2+0^2+0^2+1^2 = \mathbf{3}$

d) Suppose that the complete sample contains $n = 100$ measurements which are represented as y_1, y_2, . . . , y_{99}, y_{100} and that 73 of these favor the tax. Find $\sum_{i=1}^{100} y_i$, $\sum_{i=1}^{100} y_i^2$, and $\left(\sum_{i=1}^{100} y_1\right)^2$.

d.1 $\sum_{i=1}^{100} y_i = \mathbf{73}$

d.2 $\sum_{i=1}^{100} y_i^2 = \mathbf{73}$

d.3 $\left(\sum_{i=1}^{100} y_i\right)^2 = \mathbf{5329}$

2.13 To estimate weekly loss due to theft, a clothing store recorded the total dollar loss over a period of 10 weeks. The losses, recorded to the nearest ten dollars, were $y_1 = 360$, $y_2 = 430$, $y_3 = 210$, $y_4 = 320$, $y_5 = 550$, $y_6 = 170$, $y_7 = 240$, $y_8 = 370$, $y_9 = 280$, $y_{10} = 290$.

a) Describe the population of interest to the store manager.

The population of interest is the losses, in dollars, incurred by the business in the previous 10 weeks.

b) Find $\sum_{i=1}^{10} y_i$: $360 + 430 + 210 + 320 + 550 + 170 + 240 + 370 + 280 + 290 = \mathbf{3220}$

c) Find $\sum_{i=2}^{4} y_i$, $\sum_{i=2}^{4} y_i^2$, and $\left(\sum_{i=1}^{3} y_i\right)^2$.

c.1 $\sum_{i=2}^{4} y_i = 430 + 210 + 320 = \mathbf{960}$

c.2 $\sum_{i=2}^{4} y_i^2 = 430^2 + 210^2 + 320^2 = \mathbf{331400}$

c.3 $\left(\sum_{i=2}^{4} y_i\right)^2 = 960^2 = \mathbf{921600}$

2.14 For $y_i = 2i^2 - 1$, find:

a) $\sum_{i=1}^{5} y_i = (2*1^2-1) + (2*2^2-1) + (2*3^2-1) + (2*4^2-1) + (2*5^2-1) = \mathbf{105}$

b) $\sum_{i=1}^{4} y_i^2 = (2*1^2-1)^2 + (2*2^2-1)^2 + (2*3^2-1)^2 + (2*4^2-1)^2 = \mathbf{1300}$

c) $\left(\sum_{i=1}^{4} y_i\right)^2 = \left[(2*1^2-1) + (2*2^2-1) + (2*3^2-1) + (2*4^2-1)\right]^2 = 56^2 = \mathbf{3136}$

d) $\sum_{i=1}^{5} xy_i = \left[(2*1^2-1) + (2*2^2-1) + (2*3^2-1) + (2*4^2-1) + (2*5^2-1)\right]x = \mathbf{105}\boldsymbol{x}$