CS 508 Assignment 6

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1. What is the clock cycle time for a CPU with a clock rate of 2.5 GHz?

$$\begin{aligned}\text{clock cycle time} &= \frac{1}{\text{clock frequency}} \\ &= \frac{1}{2.5 \times 10^9 \text{ Hz}} \\ &= 4.0 \times 10^{-10} \text{ sec} \\ &= 400 \text{ picoseconds}\end{aligned}$$

2. How long does it take to execute a program that requires 2.5 million clock cycles if the CPU clock rate is 2.5 GHz?

$$\begin{aligned}\text{CPU time} &= \frac{\text{CPU clock cycles}}{\text{clock rate}} \\ &= \frac{2.5 \times 10^6 \text{ cycles}}{2.5 \times 10^9 \text{ Hz}} \\ &= 1.0 \times 10^{-3} \text{seconds} \\ &= 1.0 \text{ millisecond}\end{aligned}$$

3. A program required 40 microseconds to execute on a CPU with a clock rate of 3 GHz. If the average CPI of the processor is 2.0, how many instructions were executed?

$$\begin{aligned}\text{clock cycle time} &= \frac{1}{\text{clock frequency}} \\ \text{CPU time} &= \text{instruction count} \times \text{CPI} \times \text{clock cycle time} \\ \text{instruction count} &= \frac{\text{CPU time}}{\text{CPI} \times \text{clock cycle time}} \\ &= \frac{\text{CPU time} \times \text{clock frequency}}{\text{CPI}} \\ &= \frac{40 \times 10^{-6} \text{seconds} \times (3 \times 10^9 \text{ Hz})}{2.0 \text{ cpi}} \\ &= 60,000 \text{ instructions}\end{aligned}$$

4. We know that the clock rate (cycles/second) and CPI (cycles/instruction) are two components of computer performance calculations. Another measure of performance is IPS (instructions/second). Derive the equation for calculating IPS using clock rate and CPI.

   *using dimensional analysis, IPS can be derived from clock rate and CPI.

$$\begin{aligned}\frac{\text{instructions}}{\text{second}} &= \frac{\frac{\text{cycles}}{\text{second}}}{\frac{\text{cycles}}{\text{instruction}}} = \frac{\text{cycles}}{\text{second}} \cdot \frac{\text{instruction}}{\text{cycles}} \\ &= \frac{\text{clock rate}}{\text{CPI}}\end{aligned}$$

5. Consider three different processors P1, P2, and P3 executing the same instruction set. The clock rates and CPIs for each processor are stated in the following table. For each processor, calculate the IPS for each processor and indicate which one has the highest performance.

CPU	Clock Rate	CPI
P1	3.0 GHz	1.5
P2	2.5 GHz	1.0
P3	4.0 GHz	2.5

$$\text{IPS} = \frac{\text{clock rate}}{\text{CPI}}$$

$$\text{IPS P1} = \frac{3.0 \times 10^9}{1.5} = 2.0 \times 10^9 \text{ instructions per sec}$$

$$\text{IPS P2} = \frac{2.5 \times 10^9}{1.0} = 2.5 \times 10^9 \text{ instructions per sec}$$

$$\text{IPS P3} = \frac{4.0 \times 10^9}{2.5} = 1.6 \times 10^9 \text{ instructions per sec}$$

P2 has the highest performance.

6. All three processors in question 5 execute a program in 10 seconds.

   a. Calculate the number of cycles for the program for each processor.

$$\text{cycles} = \text{clock rate} \times \text{runtime}$$

$$\text{P1: cycles} = 3.0 \times 10^9 \text{Hz} \times 10\text{sec} = 3.0 \times 10^{10} \text{ cycles} = 30 \text{ billion cycles}$$

$$\text{P2: cycles} = 2.5 \times 10^9 \text{Hz} \times 10\text{sec} = 2.5 \times 10^{10} \text{ cycles} = 25 \text{ billion cycles}$$

$$\text{P3: cycles} = 4.0 \times 10^9 \text{Hz} \times 10\text{sec} = 4.0 \times 10^{10} \text{ cycles} = 40 \text{ billion cycles}$$

   b. Calculate the number of instructions executed for each processor.

$$\text{CPU clock cycles} = \text{number of instructions} \times \text{CPI}$$

$$\text{number of instructions} = \frac{\text{CPU clock cycles}}{\text{CPI}}$$

$$\text{P1: number of instructions} = \frac{3.0 \times 10^{10}}{1.5} = 2.0 \times 10^{10} \text{instructions} = 20 \text{ billion instructions}$$

$$\text{P2: number of instructions} = \frac{2.5 \times 10^{10}}{1.0} = 2.5 \times 10^{10} \text{instructions} = 25 \text{ billion instructions}$$

$$\text{P3: number of instructions} = \frac{4.0 \times 10^{10}}{2.5} = 1.6 \times 10^{10} \text{instructions} = 16 \text{ billion instructions}$$

7. For the processors in question 5, we want to reduce the execution time by 30%, but this leads to an increase in the CPI by 20% for all three processors. For each of the processors, calculate the clock rate required to get this time reduction.

   New execution time for all CPUs: $10 \cdot (1 - 0.3) = 7$ sec

   New CPI for each processor:

$$\begin{aligned} &\text{P1: } 1.5 \times 1.2 = 1.8 \\ &\text{P2: } 1.0 \times 1.2 = 1.2 \\ &\text{P3: } 2.5 \times 1.2 = 3.0 \end{aligned}$$

$$\begin{aligned} \text{clock rate} &= \frac{\text{instruction count} \times \text{CPI}}{\text{CPU time}} \\ \text{P1 clock rate} &= \frac{2.0 \times 10^{10} \text{ instructions} \cdot 1.8}{7 \text{ secs}} = 5.14 \text{ MHz} \\ \text{P2 clock rate} &= \frac{2.5 \times 10^{10} \text{ instructions} \cdot 1.2}{7 \text{ secs}} = 4.29 \text{ MHz} \\ \text{P3 clock rate} &= \frac{1.6 \times 10^{10} \text{ instructions} \cdot 3.0}{7 \text{ secs}} = 6.86 \text{ MHz} \end{aligned}$$