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A REGIONAL GENETIC ALGORITHM FOR THE DISCRETE OPTIMAL DESIGN OF TRUSS STRUCTURES

A. A. GROENWOLD, N. STANDER* AND J. A. SNYMAN

Structural and Multidisciplinary Optimization Research Group, Department of Mechanical Engineering, University of Pretoria, Pretoria 0002, South Africa

SUMMARY

A regional genetic algorithm (R-GA) is used for the discrete optimal design of truss structures. The chromosomes are selected from a sub-region centred on the continuous optimum. This approach replaces genetic rebirth as previously proposed by other authors, thereby significantly reducing computational costs. As a pure discrete method, the R-GA method does not require heuristic arguments or approximations. This makes the algorithm highly effective when buckling and slenderness constraints with scatter in the data are introduced. A large set of numerical test examples is used to illustrate the capabilities of the method. The algorithm is shown to be effective and robust, making it suitable for the optimal design of very large truss structures.

KEY WORDS: structural optimization; truss; discrete optimization; genetic algorithm

INTRODUCTION

Discrete optimization is an active research topic. Many natural phenomena are mathematically modelled in terms of discrete variables. The optimization of these models is difficult in the sense that multiple local minima may occur. Numerous methods do, however, exist for solving such problems, and a recent overview of methods specifically aimed at structural optimization is presented by Arora *et al.*[1] The most general methods are Branch and Bound (B&B), Simulated Annealing (SA) and the Genetic Algorithm (GA).[2] However, they are also the most time consuming.

In this study the optimal design of truss structures with discrete variables is considered. The least expensive method of obtaining a discrete solution is through rounding of the continuous solution. Rounding of the continuous solution, however, frequently yields non-optimal discrete solutions, while feasibility cannot be guaranteed. Ringertz[3] proposed the use of dynamic rounding for the optimal discrete design of truss structures, which yielded mostly feasible, if not optimal, results. A *selective* dynamic rounding method was proposed by Groenwold *et al.*,[4] which gave a significantly improved discrete approximation to the optimum.

While highly efficient in terms of computational effort, the dynamic rounding methods of Ringertz[3] and Groenwold *et al.*[4] lack robustness. The algorithms also become complex when buckling and slenderness constraints are introduced, since curve fitting of scattered data has to be

* Correspondence to: N. Stander, c/o Livermore Software Technology Corporation, 97 Rickenbacker Circle, Livermore, CA 94550, U.S.A. E-mail: nielen@lstc.com

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performed to approximate the design functions[5] during the intermediate continuous programming problems. Furthermore, convergence criteria are not available. Notwithstanding the good numerical behaviour demonstrated in Reference 4, convergence to a feasible solution cannot be guaranteed. This is particularly true for sparsely populated prescribed discrete sets. Nevertheless, the dynamic rounding methods are attractive due to their low computational costs.

Pure discrete methods, e.g. Branch and Bound (B&B), Simulated Annealing (SA) and the Genetic Algorithm (GA) are more robust. Furthermore, these methods do not require gradient or Hessian information. These methods are, however, extremely expensive,[4] which severely limits their application to the optimal design of large structures with many design variables. For the general non-convex programming problem, absolute optimality conditions are not available.

Genetic algorithms are stochastic implicit enumeration methods based on Darwinism and in particular, on the natural theory of survival of the fittest. In brief, genetic algorithms attempt to improve the fitness of designs (expressed in terms of a scalar objective function) in consecutive generations. The initial generation is populated in a random fashion with chromosomes representing possible discrete designs. The genetic operators of selection, cross-over and mutation are then used in a controlled random manner to ensure that fit parents have a high probability of passing fit genetic material to their off-spring.

Even considering their high computational costs, GA's are attractive to engineers, their construction being simple and transparent. As a result, a number of papers considering the optimal design of truss structures using GA's have appeared (e.g. References 6 and 7). GA's are also suitable for implementation on massively parallel processing machines and lend themselves to be tailored according to the behaviour of the objective function under consideration.

This paper attempts to combine the efficiency of rounding methods and the robustness of GA's for the optimal design of truss structures through exploitation of knowledge of the structural behaviour. The development of the paper is as follows: firstly, the general discrete programming problem for truss structures is presented, together with observations with respect to the behaviour of the objective function. Secondly, a chromosome selection rule is proposed based on selection from a sub-region centred at the continuous optimum. Thirdly, the construction of a basic efficient regional genetic algorithm (R-GA) suitable for the optimal design of truss structures is presented. Finally, the R-GA results for a large number of numerical test examples are presented and compared with results reported for other methods. This also allows for future comparison of the new algorithm with other algorithms.

PROBLEM STATEMENT

Consider the continuous optimization problem

$$(\mathrm{P_c}) \qquad \begin{aligned} &\text{minimize} \quad f(\mathbf{x}); \quad \mathbf{x} \in \mathbb{R}^n \\ &\text{subject to} \quad g_i(\mathbf{x}) \leqslant 0; \quad i = 1, 2, \ldots, m \\ &x_j \geqslant x_{\mathrm{L}j}; \quad j = 1, 2, \ldots, n \end{aligned} \tag{1}$$

where f and g_i are continuous real functions of n real variables and m indicates the number of inequality constraints. The starting vector is denoted by $\mathbf{x}^0$, while the solution is denoted by $\mathbf{x}^*$, with corresponding function value f^*. $x_{\mathrm{L}j}$ denotes the lower bound of variable x_j. In the discrete problem the components of the discrete solution $\bar{\mathbf{x}}$ are restricted to elements of prescribed sets of discrete real values, i.e. the second constraint above becomes

$$x_j \geqslant x_{\mathrm{L}j}; \quad x_j \in A_j = \{a_{1_j}, a_{2_j}, \ldots, a_{l_j}\}, \quad j = 1, 2, \ldots, n \tag{2}$$

For the sake of simplicity, the assumption is made that the subset A_j is identical for all j and denoted by $A = (a_1, a_2, \ldots, a_l)$ and it is assumed that $a_1 < a_2 < \cdots < a_l$ with $x_{\mathrm{L}j} = x_{\mathrm{L}} = a_1$ for all j. This also implies the existence of a discrete upper bound a_l. The discrete optimum is denoted by $\bar{\mathbf{x}}$, with corresponding function value $\bar{f}$, i.e. $\bar{\mathbf{x}} \leftrightarrow \bar{f}$. It is desired to find $\bar{\mathbf{x}}$, or at least, a near optimal discrete solution $\hat{\mathbf{x}} \leftrightarrow \hat{f}$. An indication of the quality of $\hat{f}$ can be obtained through a comparison with f^*.

Based on the experience of the authors with the optimal design of truss structures,[4, 5] it is assumed that in general, truss structures belong to the class of problems for which the following may be postulated:

1. the continuous global optimum $\mathbf{x}^*$ exists and can be determined,
2. the global discrete optimum $\bar{\mathbf{x}}$ (or a near-optimal discrete point $\hat{\mathbf{x}}$) exists, within a small discrete sub-space surrounding $\mathbf{x}^*$,
3. components of $\mathbf{x}^*$ which are equal to the discrete lower bound x_{L} are assumed to correspond to the optimal components of $\bar{\mathbf{x}}$ and may effectively be excluded from the discrete optimization process.

The following remarks should be made with regard to postulates 1 and 2 above. For non-convex problems it cannot be proved that the continuous solution $\mathbf{x}^*$ is indeed global. However, numerical experience has shown that for truss structural problems the obtained optimum $\mathbf{x}^*$ frequently corresponds to the global solution. Furthermore, the global optimum, or a very good approximation to the global optimum can usually be found by re-solving the continuous optimization problem a number of times with different (random) starting designs $\mathbf{x}_0$. Also of importance is to realize that in general the existence of a continuous local optimum cannot be established *a priori*. However, if a $\mathbf{x}^*$ does not exist then neither will the discrete solution $\bar{\mathbf{x}}$ exist. Under these conditions the optimization problem is allowed to become that of minimization of the constraint violations or a minimization of a weighted average of the objective function and constraint violations. Finally, the existence of a near optimal $\hat{\mathbf{x}}$ is also not self-evident but depends on the prescribed discrete set A. However, if $\hat{\mathbf{x}}$ does not exist then a similar approach to when $\mathbf{x}^*$ does not exist may be adopted.

CHROMOSOME SELECTION RULE

In this study, it is proposed that a two-step strategy be used to determine a discrete near-optimal solution $\hat{\mathbf{x}}$: Firstly, Problem (1) is solved to determine $\mathbf{x}^*$ (continuous phase). Secondly, the genetic algorithm is used to determine the discrete approximation $\hat{\mathbf{x}}$ in a sub-region about $\mathbf{x}^*$ (discrete phase). Based on postulates 1 and 2, the initial genetic population is generated from the prescribed discrete values $a_j \in A$ by selecting the genes x_i, $i = 1, 2, \ldots, n$ of a chromosome according to the following rule: given a prescribed integer $r \geqslant 1$ set

$$x_i := a_j \in A \quad \text{and} \quad j \in (u_i - r, u_i - r + 1, \ldots, u_i, \ldots, u_i + r - 1) \tag{3}$$

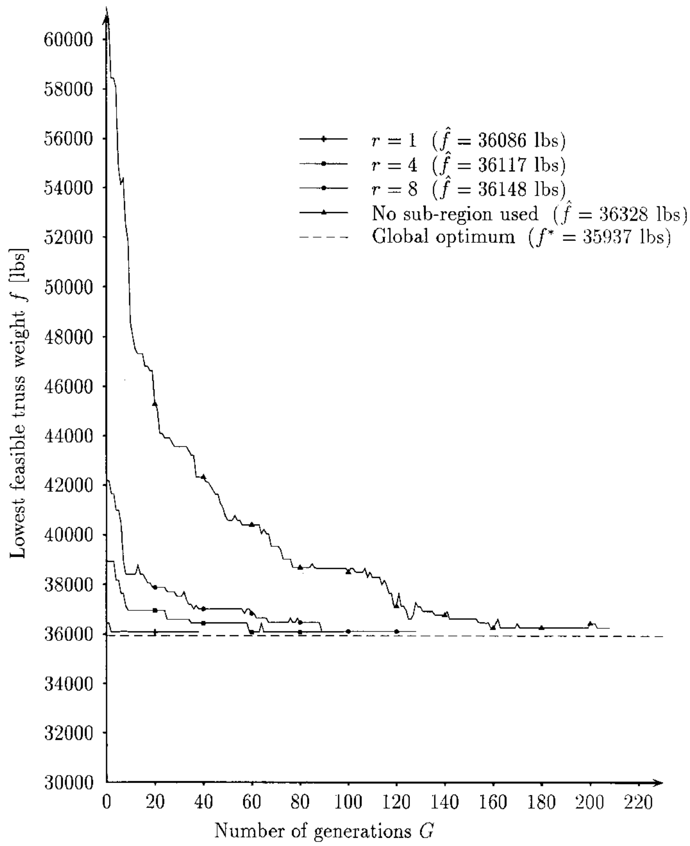


Figure 1. 36-bar truss: convergence characteristics as a function of r in Rule (3)

where u_i is chosen such that

$$a_{u_i} - x_i^* = \min_j (a_j - x_i^*) \quad \text{subject to } (a_j - x_i^*) \geqslant 0 \tag{4}$$

for $j = 1, 2, \ldots, l$. If no such integer u_i exists, then set $x_i := a_l$.

Clearly if u_i exists then a_{u_i} corresponds to the discrete upper neighbour of x_i^*. It is emphasized that the continuous phase serves to determine the centre of the discrete design space such that the initial genetic population generated in this region will have a high probability of being near-optimal.

Extensive experience with the optimal design of truss structures has shown that it is reasonable to expect that $\bar{\mathbf{x}}$ or at least a very good near-optimal solution $\hat{\mathbf{x}}$ exists within the subregion defined by Rule (3). Even if $\bar{\mathbf{x}}$ does not lie within this initial sub-region then, due to mutations occurring in the application of the genetic algorithm, convergence is still possible to $\bar{\mathbf{x}}$. This possibility is further enhanced by additionally specifying that a fraction p_0 of the population p be generated from the full prescribed set A.

Selection by Rule (3) has some similarity with genetic rebirth (e.g. see Reference 6) where the GA is restarted with a random population selected from a new subset, the centre of which corresponds to the best (current) discrete optimum obtained after exhaustion of the previous subset. Rebirth has proved to be effective in improving the quality of the discrete approximation $\hat{\mathbf{x}}$. It is, however, clear that the implementation of Rule (3) should significantly reduce the computational effort since the GA starts sampling in a promising small subset of the full prescribed design space.

The effect of selected values of r in Rule (3) on the convergence characteristics of the R-GA is depicted in Figure 1 for the 36-bar truss problem. This problem is presented in more detail in the section Numerical Results and is included here to illustrate the current approach. For each value of r, the best results selected from 10 independent runs of the R-GA are presented. The GA parameters, i.e. population size, probability of mutation and cross-over, and the stopping criteria are identical for each value of r and not necessarily optimal, but serve to illustrate the current implementation.

The benefits of decreasing the value of r in Rule (3) in order to reduce the computational effort are obvious. Furthermore, the quality of the approximate solution $\hat{f}$ is also improved since the initial populations contain a high percentage of highly fit genetic material. In Figure 1, the number of function and gradient evaluations associated with obtaining the continuous optimum f^* is comparable to the number of function evaluations during four discrete generations. Similar behaviour is observed for the other test problems presented in the section Numerical Results, with a reduction in computational effort of up to an order of magnitude for $r=1$ as compared to a 'standard' GA.

ALGORITHM CONSTRUCTION

The implementation of a simple regional genetic algorithm based on the postulates presented above is now described. Consider a truss structure consisting of e members, with n design variables and m constraints. The genetic scalar objective function F may conveniently be expressed by

$$F = f + \sum_{i=1}^{m} \lambda_i g_i^2 \tag{5}$$

where f is the structural weight and λ_i represents the penalty associated with constraint g_i. For truss structures, the penalty factors are chosen such that a 5–10 per cent constraint violation results in a 10 per cent increase in F.[6] In the current implementation, a simple integer alphabet (of size l) as opposed to a binary representation was used to represent the prescribed discrete sections. Provision was made for a two-point cross-over, and an elitist implementation (e.g. the best optimum is kept unchanged). The integer alphabet need not be optimal (e.g. see Reference 8). However, since the GA is used only in a neighbourhood defined by the continuous optimum and as a replacement for rebirth, this approach suffices.

Chromosome fitness is expressed as the relative rank with respect to the genetic objective F. After ranking in descending order, the fitness of member i is expressed as

$$\frac{2(p+1-i)^c}{p^2+p} \tag{6}$$

with p the population size, and c a constant (usually unity). The sum of all fitnesses is unity, and selection is performed through the generation of a random real number v with $0 \leqslant v \leqslant 1$. The selected chromosome k is obtained when fitness $(k) < v \leqslant$ fitness$(k+1)$. The criterion above was chosen for its simplicity, and may not be optimal for truss structures.

Cross-over is the most important operator in the genetic process. It is applied at high probability, i.e. relatively few parents are copied unchanged into future generations. In combination with ranking and selection, cross-over is used to obtain new generations in such a manner that future generations contain previously tested and 'fit' genetic material:

$$
\begin{array}{ll}
\text{Selected parent 1} & \langle 23\,|3\ 42\ 9\ 19|\ 1\ 18\ 21\ 41\ 12\rangle \\
\text{Selected parent 2} & \langle 26\,|12\ 8\ 14\ 7|\ 32\ 2\ 16\ 13\ 35\rangle \\
\text{Child 1} & \langle 23\,|12\ 8\ 14\ 7|\ 1\ 18\ 21\ 41\ 12\rangle \\
\text{Child 2} & \langle 26\,|3\ 42\ 9\ 19|\ 32\ 2\ 16\ 13\ 35\rangle
\end{array}
$$

In this study a two point cross-over was chosen, which may not be optimal.

After cross-over, mutation is performed. The stochastic operator mutation is used to protect against loss of genetic diversity. It is applied with a low probability. Mutation is applied depending on the value of a random number. A second number randomly selects the position or 'bit' to be mutated. In standard implementations of the GA, the new value of the bit is chosen with equal probability out of the full prescribed discrete set, e.g.

$$
\begin{array}{ll}
\text{Before mutation} & \langle 23\ 12\ \underline{8}\ 14\ 7\ 1\ 18\ 21\ 41\ 12\rangle \\
\text{After mutation} & \langle 23\ 12\ 4\ 14\ 7\ 1\ 18\ 21\ 41\ 12\rangle
\end{array}
$$

Galante[6] observed that the contribution of mutation in improving the discrete design of trusses was not of great importance. However, he used relatively large populations ($p = 200$ for a problem with $n = 10$), which implies that there is significant genetic diversity due to the initial generation. When using smaller populations, as is the aim of this paper, mutation is required to protect against loss of diversity.

Based on the postulates above, biased mutation promises to be effective. This may be implemented such that mutation to close neighbours has a high probability. This strategy is used in this study, and is denoted *manipulated mutation*. Manipulated mutation does not replace standard mutation, but merely complements it. Standard mutation is retained to allow convergence to regions outside the bounds defined by Rule (3). Furthermore, variables on the lower bound x_L are protected against mutation.

The R-GA is stopped after s generations without improvement, although a stopping criterion based on the saturation of successive generations may be more suitable to protect against both over and under sampling of the reduced design space. Inversion and permutation are not suitable for the design of truss structures, since little benefit can be expected through interchanging genetic information of different design variables.

Finally, the algorithm performance may be further improved through utilization of a pseudo-random initial generation, where a random member of the population is substituted with the rounded-off vector of the continuous optimum. When the R-GA is re-run a predetermined number of times, this vector may be included randomly based on a low probability (per run) of say 0·2.

NUMERICAL RESULTS

In this section numerical results for a number of test problems are presented. The solution obtained with the genetic algorithm is compared with the solutions obtained with a number of recent

Table I. Discrete problems: size and nature

No.	Problem name	Problem nature	Bucking	References	n	m	x_i^0
1	10-bar	Convex	No	4, 3	10	32	20·0
2	10-bar	Non-convex	No	4, 3, 9	10	34	20·0
3	10-bar (AISC)	Non-convex	Yes and No	5, 6	10	44	20·0
4	25-bar	Non-convex	No	4, 3, 10	8	84	6·0
5	36-bar	Convex	No	4, 3	21	95	30·0
6	160-bar	Non-convex	Yes	5, 11	38	518	5·0
7	200-bar	Non-convex	No	4, 9, 12	29	435	20·0

algorithms taken from the open literature. Unless otherwise stated, the B&B, dynamic rounding and generalized Lagrangian results included for comparison were taken from Ringertz.[3] In each table, the best result obtained is underlined, the structural weight is indicated by f and 'NI' indicates the Normalized Infeasibility, defined such that a 100 per cent violation corresponds to unity. Infeasible weights appear in parentheses.

An overview of the size and nature of the respective problems is presented in Table I. This table also gives the starting vectors used in this study, as well as selected references which may be consulted for problem specific details, i.e. the geometry, nodal co-ordinates and design variable linkage of the respective problems.

To illustrate the robustness of the current method, no problem specific optimal values were used for the genetic parameters. Instead, unless otherwise stated, the following applies:

1. population size $p = 20$,
2. fraction of the population generated in A, $p_0 = 0{\cdot}15$,
3. number of generations encountered without improvement before stopping, $s = 20$,
4. probability of manipulated mutation (upwards), $P(m_{\mathrm{u}}) = 0{\cdot}015$,
5. probability of manipulated mutation (downwards), $P(m_{\mathrm{d}}) = 0{\cdot}015$,
6. probability of mutation $P(m) = 0{\cdot}005$,
7. probability of cross-over $P(c) = 0{\cdot}8$,
8. number of cross-over points, $n_{\mathrm{c}} = 2$.

The rounded-off vector was included in the first pseudo-random generation. For problems with buckling and slenderness constraints, an approximate interpolation between member area and least radius of gyration as presented in Reference 5 was performed. For these problems, $r = 2$ was used in Rule (3) to accommodate scatter in the data. Otherwise, $r = 1$ was used. To facilitate a fair comparison with the results in the literature, infeasible designs were penalized to the extent that the probability of them propagating into future generations was negligible. For each problem, the best values obtained after 10 independent runs of the algorithm are reported.

The SAM (Spherical Approximation Method) algorithm of Snyman and Stander[13] was used to solve the continuous programming problems.

Convex 10-bar truss

This problem (Figure 2) illustrates weight minimization subject to a symmetric displacement constraint for which the global optimum can be found using the B&B method.[14]

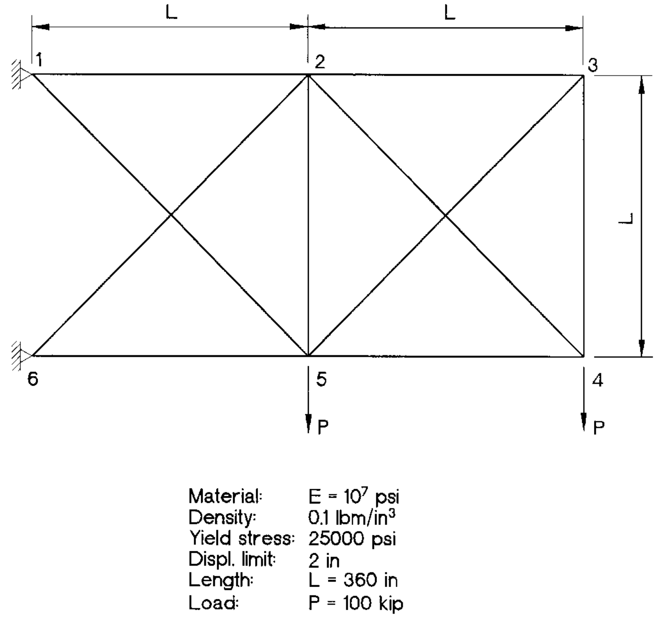


Figure 2. 10-bar truss

Table II. Convex 10-bar truss, $A = (0{\cdot}5, 1{\cdot}0, 1{\cdot}5, \ldots)$

	Continuous optimum	Rounding	Global optimum	B&B	Generalized Lagrangian	Dynamic rounding	SDR[4]	This study
f (lb)	4193·2	4229·4	4193·4	<u>4193·4</u>	4196·5	4221·4	<u>4193·4</u>	<u>4193·4</u>
NI	0·0	0·0	0·0	0·0	0·0	0·0	0·0	0·0
x_1 (in^2)	28·433	28·5	28·5	28·5	29·0	28·0	28·5	28·5
x_2	0·500	0·5	0·5	0·5	0·5	0·5	0·5	0·5
x_3	14·743	15·0	14·5	14·5	15·0	15·0	14·5	14·5
x_4	14·087	14·5	14·0	14·0	14·5	14·5	14·0	14·0
x_5	0·500	0·5	0·5	0·5	0·5	0·5	0·5	0·5
x_6	0·500	0·5	0·5	0·5	0·5	0·5	0·5	0·5
x_7	0·500	0·5	0·5	0·5	0·5	0·5	0·5	0·5
x_8	19·887	20·0	20·0	20·0	19·0	20·0	20·0	20·0
x_9	19·923	20·0	20·0	20·0	20·0	20·0	20·0	20·0
x_{10}	0·500	0·5	0·5	0·5	0·5	0·5	0·5	0·5

Prescribed discrete sets of $A = (0{\cdot}5, 1{\cdot}0, 1{\cdot}5, \ldots)$ and $A = (5{\cdot}0, 10{\cdot}0, 15{\cdot}0, \ldots)$ are considered. Results are presented in Tables II and III, respectively. The results reveal that the B&B method, the SDR methods and the R-GA yield the global optima for both prescribed discrete sets. The result presented for the first set is slightly infeasible ($\mathrm{NI} = 1{\cdot}38 \times 10^{-5}$). However, such results are usually accepted as 'feasible' in the literature.

Table III. Convex 10-bar truss, $A = (5{\cdot}0, 10{\cdot}0, 15{\cdot}0, \ldots)$

	Continuous optimum	Rounding	Global optimum	B&B	Generalized Lagrangian	Dynamic rounding	SDR[4]	This study
f (lb)	4607·1	5425·6	4736·5	4736·5	4736·5	4991·0	4736·5	4736·5
NI	0·0	0·0	0·0	0·0	0·0	0·0	0·0	0·0
x_1 (in^2)	25·358	30·0	25·0	25·0	30·0	25·0	25·0	25·0
x_2	5·000	5·0	5·0	5·0	5·0	5·0	5·0	5·0
x_3	17·839	20·0	25·0	25·0	20·0	20·0	20·0	20·0
x_4	11·238	15·0	10·0	10·0	10·0	15·0	15·0	15·0
x_5	5·000	5·0	5·0	5·0	5·0	5·0	5·0	5·0
x_6	5·000	5·0	5·0	5·0	5·0	5·0	5·0	5·0
x_7	5·000	5·0	5·0	5·0	5·0	5·0	5·0	5·0
x_8	15·501	20·0	15·0	15·0	15·0	20·0	15·0	15·0
x_9	15·893	20·0	15·0	15·0	15·0	15·0	15·0	15·0
x_{10}	5·000	5·0	5·0	5·0	5·0	5·0	5·0	5·0

Table IV. Non-convex 10-bar truss, $A = (0{\cdot}1, 0{\cdot}5, 1{\cdot}0, 1{\cdot}5, \ldots)$

	Continuous optimum	Rounding	B&B	Generalized Lagrangian	Dynamic rounding	SDR[4]	This study
f (lb)	5060·85	(5164·8) $2{\cdot}72 \times 10^{-3}$	(5059·9) $8{\cdot}86 \times 10^{-4}$	5067·3	5077·9	5077·9	5067·3
NI	0·0			0·0	0·0	0·0	0·0
x_1 (in^2)	30·522	31·0	30·5	30·0	30·0	31·0	31·0
x_2	0·100	0·1	0·1	0·1	0·1	0·1	0·1
x_3	23·200	23·5	23·0	23·5	23·5	23·0	23·0
x_4	15·223	15·5	15·5	15·0	15·5	15·5	14·5
x_5	0·100	0·1	0·1	0·1	0·1	0·1	0·1
x_6	0·551	1·0	0·5	0·5	1·0	0·5	0·5
x_7	7·457	7·5	7·5	7·5	7·5	7·5	7·5
x_8	21·036	21·5	21·0	21·0	21·0	21·0	21·0
x_9	21·528	22·0	21·5	22·0	21·5	21·5	22·0
x_{10}	0·100	0·1	0·1	0·1	0·1	0·1	0·1

Non-convex 10-bar truss

This is a popular test problem referred to by many authors.[3,10,15] The geometry is similar to that of the convex 10-bar problem presented above (see Figure 2), while the modified loading condition causes the existence of multiple local minima.

Numerical results are presented in Table IV for a prescribed discrete set of $A = (0{\cdot}1, 0{\cdot}5, 1{\cdot}0, 1{\cdot}5, \ldots)$. The R-GA results reflect the lowest known value of 5067·3.

Non-convex 10-bar truss with AISC prescribed discrete set

This example depicted in Figure 2 is used to demonstrate the method. Two different design conditions are considered, namely without and with buckling and slenderness constraints. Results are,

Table V. 10-bar truss with AISC prescribed discrete set (excluding buckling)

	Continuous optimum	Rounding	GA[7]	GA[6]	SDR[4]	This study
f (lb)	5482·834	5846·856	(5613·6) $7{\cdot}52\times10^{-4}$	(5458·3)	5490·738	5490·738
NI	0·0	0·0		0·0123	0·0	0·0
x_1 (in^2)	32·23524	33·50	33·50	33·50	33·50	33·50
x_2	1·62000	1·62	1·62	1·62	1·62	1·62
x_3	23·29631	26·50	22·00	22·00	22·90	22·90
x_4	15·26236	15·50	15·50	14·20	14·20	14·20
x_5	1·62000	1·62	1·62	1·62	1·62	1·62
x_6	1·62000	1·62	1·62	1·62	1·62	1·62
x_7	8·30663	11·50	14·20	7·97	7·97	7·97
x_8	22·68676	22·90	19·90	22·90	22·90	22·90
x_9	21·58424	22·00	19·90	22·00	22·00	22·00
x_{10}	1·62000	1·62	2·62	1·62	1·62	1·62

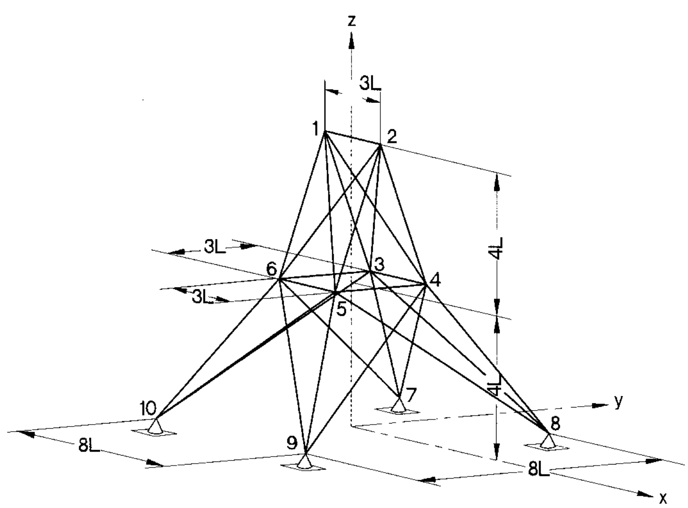
Table VI. 10-bar truss with AISC prescribed discrete set (including buckling)

	Continuous optimum	Rounding	SDR[5]	This study
f (lb)	15774·0	16304·7	14806·5	14806·5
NI	0·0	0·0	0·0	0·0
x_1 (in^2)	7·46363	8·25	8·25	8·25
x_2	6·00644	6·16	4·68	4·68
x_3	23·16550	23·20	23·20	23·20
x_4	13·72781	14·10	14·10	14·10
x_5	6·00643	6·16	4·68	4·68
x_6	6·00644	6·16	4·68	4·68
x_7	10·82307	11·70	10·30	10·30
x_8	24·54129	24·60	23·20	23·20
x_9	10·82310	11·70	9·13	9·13
x_{10}	20·35974	21·10	19·10	19·10

respectively, tabulated in Tables V and VI. For the former condition (no buckling), the prescribed discrete set $A=(1{\cdot}62, 1{\cdot}80, 1{\cdot}99, 2{\cdot}13, 2{\cdot}38, 2{\cdot}62, 2{\cdot}63, 2{\cdot}88, 2{\cdot}93, 3{\cdot}09, 3{\cdot}13, 3{\cdot}38, 3{\cdot}47, 3{\cdot}55, 3{\cdot}63, 3{\cdot}84, 4{\cdot}18, 4{\cdot}22, 4{\cdot}49, 4{\cdot}59, 4{\cdot}80, 4{\cdot}97, 5{\cdot}12, 5{\cdot}74, 7{\cdot}22, 7{\cdot}97, 11{\cdot}5, 13{\cdot}5, 13{\cdot}9, 14{\cdot}2, 15{\cdot}5, 16{\cdot}0, 16{\cdot}9, 18{\cdot}8, 19{\cdot}9, 22{\cdot}0, 22{\cdot}9, 26{\cdot}5, 30{\cdot}0, 33{\cdot}5)$. When buckling is included, the prescribed sections and their associated radii of gyration as tabulated in Table VII are used, with $E=30\times10^6$ psi and $\rho=0{\cdot}28$ lb/in^3. For the continuous optimization phase the relationship between the radius of gyration r_j and the design variable x_j is obtained by linear regression of an intermediate parameter $\zeta_j=\sqrt{r_j^2/x_j}$.[5] For both buckling and non-buckling conditions, the lowest known feasible solutions were found by both the SDR method and the R-GA.

Table VII. Properties of the AISC sections

No.	x_j (in^2)	r_j (in)	$\zeta_j = r_j/\sqrt{x_j}$	No.	x_j (in^2)	r_j (in)	$\zeta_j = r_j/\sqrt{x_j}$
1	1·62	0·793	0·388	17	13·5	1·79	0·237
2	2·13	0·784	0·289	18	14·1	2·08	0·307
3	2·21	0·474	0·102	19	15·4	2·22	0·320
4	2·35	0·427	0·078	20	17·1	2·10	0·258
5	2·65	0·480	0·087	21	19·1	3·02	0·478
6	2·94	0·576	0·113	22	21·1	3·04	0·438
7	3·18	0·537	0·091	23	22·3	2·61	0·305
8	3·83	1·00	0·261	24	23·2	3·05	0·401
9	4·68	1·27	0·345	25	24·6	2·94	0·351
10	5·54	1·28	0·296	26	25·6	3·07	0·368
11	6·16	1·26	0·258	27	26·5	3·70	0·517
12	7·08	1·61	0·366	28	28·2	3·09	0·339
13	8·25	1·62	0·318	29	29·1	3·71	0·473
14	9·13	2·02	0·447	30	30·6	2·91	0·277
15	10·3	2·03	0·400	31	32·0	3·73	0·435
16	11·7	2·04	0·356	32	33·5	2·42	0·175



Material: E = 2.047 x 10^6 kg/cm^2
Density: 0.00785 kg/cm^3
Sress limit: 1500 kg/cm^2
Length: L = 63.625 cm

Figure 3. 25-bar truss

25-bar truss

The geometry, nodal co-ordinates and material parameters of a non-convex space assembly are depicted in Figure 3. Design variable linking is applied.

Tabulated results for the three different prescribed discrete sets $A = (0{\cdot}01, 0{\cdot}1, 0{\cdot}2, 0{\cdot}3, \dots)$, $A = (0{\cdot}01, 0{\cdot}4, 0{\cdot}8, 1{\cdot}2, \dots)$ and $A = (0{\cdot}01, 0{\cdot}8, 1{\cdot}6, 2{\cdot}4, \dots)$ are presented in Tables VIII, IX and X,

Table VIII. 25-bar truss, $A = (0{\cdot}01, 0{\cdot}1, 0{\cdot}2, 0{\cdot}3, \ldots)$

	Continuous optimum	Rounding	B&B	Generalized Lagrangian	Dynamic rounding	Reference 10	SDR[4]	This study
f (lb)	545·04	550·98	547·65	548·96	547·65	553·00	547·04	546·94
NI	0·0	0·0	0·0	0·0	0·0	0·0	0·0	0·0
x_1 (in^2)	0·010	0·01	0·01	0·01	0·01	0·1	0·01	0·01
x_2	2·042	2·0	2·1	1·9	2·1	2·0	2·1	2·0
x_3	3·002	3·0	2·8	3·2	2·8	3·0	3·0	3·2
x_4	0·010	0·01	0·01	0·01	0·01	0·01	0·01	0·01
x_5	0·010	0·01	0·01	0·01	0·01	0·1	0·01	0·01
x_6	0·683	0·7	0·7	0·7	0·7	0·7	0·6	0·7
x_7	1·623	1·7	1·7	1·7	1·7	1·7	1·6	1·6
x_8	2·671	2·7	2·7	2·6	2·7	2·7	2·8	2·6

Table IX. 25-bar truss, $A = (0{\cdot}01, 0{\cdot}4, 0{\cdot}8, 1{\cdot}2, \ldots)$

	Continuous optimum	Rounding	B&B	Generalized Lagrangian	Dynamic rounding	Reference 13	SDR[4]	This study
f (lb)	545·04	593·84	568·69	568·69	572·49	575·41	564·86	560·59
NI	0·0	0·0	0·0	0·0	0·0	0·0	0·0	0·0
x_1 (in^2)	0·010	0·01	0·01	0·01	0·01	0·4	0·01	0·01
x_2	2·043	2·0	1·6	1·6	2·0	2·0	2·0	2·0
x_3	3·003	3·2	3·6	3·6	3·2	3·2	3·2	3·6
x_4	0·010	0·01	0·01	0·01	0·01	0·01	0·01	0·01
x_5	0·010	0·01	0·01	0·01	0·01	0·01	0·01	0·01
x_6	0·683	0·8	0·8	0·8	0·8	0·8	0·8	0·8
x_7	1·623	2·0	2·0	2·0	2·0	2·0	1·6	1·6
x_8	2·671	2·8	2·4	2·4	2·4	2·4	2·8	2·4

Table X. 25-bar truss, $A = (0{\cdot}01, 0{\cdot}8, 1{\cdot}6, 2{\cdot}4, \ldots)$

	Continuous optimum	Rounding	B&B	Generalized Lagrangian	Dynamic rounding	Reference 13	SDR[4]	This study
f (lb)	545·04	665·06	580·59	580·59	630·88	624·87	580·59	580·59
NI	0·0	0·0	0·0	0·0	0·0	0·0	0·0	0·0
x_1 (in^2)	0·000	0·01	0·01	0·01	0·01	0·8	0·01	0·01
x_2	2·043	2·4	1·6	1·6	2·4	2·4	1·6	1·6
x_3	3·003	3·2	3·2	3·2	2·4	3·2	3·2	3·2
x_4	0·010	0·01	0·01	0·01	0·01	0·01	0·01	0·01
x_5	0·010	0·01	0·01	0·01	0·01	0·8	0·01	0·01
x_6	0·683	0·8	0·8	0·8	0·8	0·8	0·8	0·8
x_7	1·623	2·4	2·4	2·4	2·4	1·6	2·4	2·4
x_8	2·671	3·2	2·4	2·4	3·2	3·2	2·4	2·4

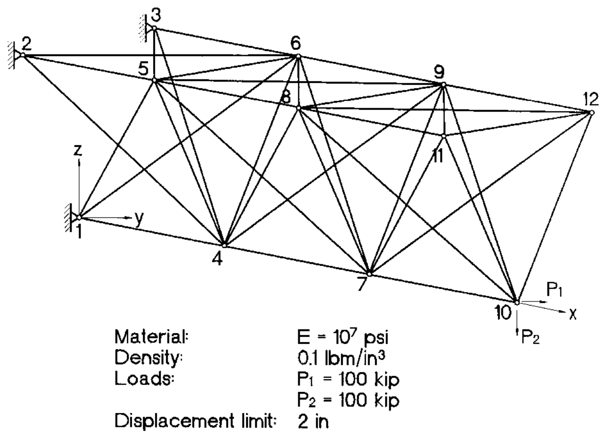


Figure 4. 36-bar truss

Table XI. Convex 36-bar truss, $A = (5{\cdot}0, 10{\cdot}0, 15{\cdot}0, \ldots, 125{\cdot}0)$

	Continuous optimum	Rounding	Global optimum	B&B	Generalized Lagrangian	Dynamic rounding	SDR[4]	This study GA*	This study R-GA†	This study R-GA‡
f (lb)	35726	37496	35937	36117	36328	36415	36117	36328	36086	35937
NI	0·0	0·0	0·0	0·0	0·0	0·0	0·0	0·0	0·0	0·0
x_1 (in^2)	38·715	40·0	40·0	35·0	40·0	40·0	40·0	50·0	40·0	40·0
x_2	24·111	25·0	25·0	25·0	25·0	25·0	25·0	20·0	25·0	25·0
x_3	7·138	10·0	5·0	15·0	10·0	10·0	10·0	5·0	5·0	5·0
x_4	95·047	100·0	95·0	95·0	105·0	90·0	95·0	95·0	95·0	95·0
x_5	59·794	60·0	55·0	60·0	60·0	55·0	60·0	70·0	55·0	55·0
x_6	14·435	15·0	20·0	15·0	15·0	15·0	15·0	15·0	15·0	20·0
x_7	5·000	5·0	5·0	5·0	5·0	5·0	5·0	5·0	5·0	5·0
x_8	5·000	5·0	5·0	5·0	5·0	5·0	5·0	5·0	5·0	5·0
x_9	14·564	15·0	15·0	15·0	15·0	15·0	15·0	15·0	15·0	15·0
x_{10}	5·000	5·0	5·0	5·0	5·0	5·0	5·0	5·0	5·0	5·0
x_{11}	5·000	5·0	5·0	5·0	5·0	5·0	5·0	5·0	5·0	5·0
x_{12}	5·000	5·0	5·0	5·0	5·0	5·0	5·0	5·0	5·0	5·0
x_{13}	28·042	30·0	30·0	30·0	20·0	30·0	30·0	30·0	30·0	30·0
x_{14}	28·042	30·0	25·0	30·0	30·0	30·0	25·0	20·0	30·0	25·0
x_{15}	27·684	30·0	25·0	25·0	30·0	30·0	25·0	25·0	25·0	25·0
x_{16}	27·684	30·0	30·0	25·0	30·0	30·0	30·0	30·0	30·0	30·0
x_{17}	28·653	30·0	30·0	30·0	30·0	30·0	30·0	35·0	30·0	30·0
x_{18}	28·653	30·0	30·0	30·0	25·0	30·0	30·0	25·0	30·0	30·0
x_{19}	5·000	5·0	5·0	5·0	5·0	5·0	5·0	5·0	5·0	5·0
x_{20}	5·000	5·0	5·0	5·0	5·0	5·0	5·0	5·0	5·0	5·0
x_{21}	5·000	5·0	5·0	5·0	5·0	5·0	5·0	5·0	5·0	5·0

*Population size $p = 40$; stopping criterion $s = 40$
†Population size $p = 20$; stopping criterion $s = 20$
‡Population size $p = 40$; stopping criterion $s = 40$

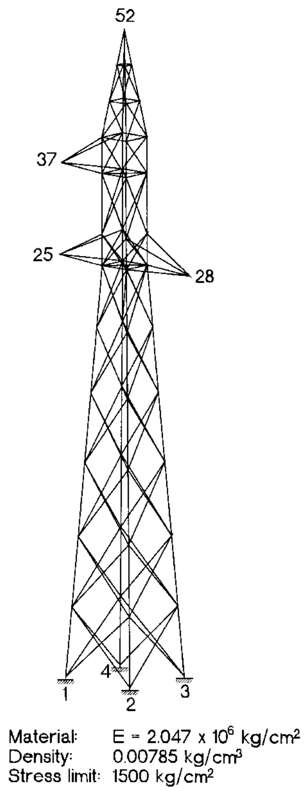


Figure 5. 160-bar truss

respectively. The results obtained with the B&B method, the generalized Lagrangian, the SDR method and the R-GA compare well. However, for the two denser prescribed discrete sets the R-GA converged to *new* feasible optima.

Convex 36-bar truss

Depicted in Figure 4, the nodal co-ordinates and design variable linking of this convex 21 variable space assembly may be found in Reference 3. Table XI presents numerical results for the discrete set $A = (5{\cdot}0, 10{\cdot}0, 15{\cdot}0, \ldots, 125{\cdot}0)$. The R-GA results are superior to the results of the other methods. The result obtained with a population size of $p = 40$ and a stopping criterion of $s = 40$ represents the global discrete optimum.

160-bar truss

To illustrate the effectiveness of the method for large truss structures, the 160-bar truss depicted in Figure 5 was analyzed (Table XII). Buckling was considered using the properties of the IS-808 sections[16] provided in Table XIII. The R-GA result is superior to the SDR result, and almost identical to the continuous optimum. (The latter however does not necessarily reflect the true

Table XII. 160-bar truss: discrete solution

	Continuous optimum	Rounding	SDR[5]	This study
f (kg)	1337·819	(1420·732)	1359·781	<u>1337·442</u>
NI	0·0	0·285	0·0	0·0
x_1 (cm^2)	17·6614	19·03	19·03	19·03
x_2	5·3541	5·75	5·27	5·27
x_3	16·9927	17·03	19·03	19·03
x_4	5·6181	5·75	5·27	5·27
x_5	17·4261	19·03	19·03	19·03
x_6	6·0018	6·25	5·75	5·75
x_7	15·6002	17·03	17·03	15·39
x_8	6·2549	6·84	6·25	5·75
x_9	14·5344	15·39	13·79	13·79
x_{10}	6·1997	6·25	6·25	5·75
x_{11}	6·1763	6·25	5·75	5·75
x_{12}	11·9297	12·21	12·21	13·79
x_{13}	6·7947	6·84	6·84	6·25
x_{14}	6·1157	6·25	5·75	5·75
x_{15}	2·2358	2·26	2·66	2·66
x_{16}	7·4673	8·06	7·44	7·44
x_{17}	1·8400	2·26	1·84	1·84
x_{18}	8·3682	8·66	8·66	8·66
x_{19}	2·2758	2·66	2·66	2·66
x_{20}	2·8172	3·07	3·07	3·07
x_{21}	2·4866	2·66	2·66	2·66
x_{22}	8·4500	8·66	8·06	8·06
x_{23}	5·4960	5·75	5·27	5·27
x_{24}	6·9584	7·44	7·44	6·25
x_{25}	6·4111	6·84	6·25	5·75
x_{26}	1·8400	2·26	1·84	1·84
x_{27}	4·7921	5·27	4·79	4·79
x_{28}	2·2594	2·26	2·66	2·66
x_{29}	3·4421	3·47	3·47	3·47
x_{30}	1·8400	2·26	1·84	1·84
x_{31}	1·8641	2·26	2·26	2·26
x_{32}	4·0711	4·79	3·88	3·88
x_{33}	1·8400	2·26	1·84	1·84
x_{34}	1·8400	2·26	1·84	1·84
x_{35}	4·1017	4·79	3·88	3·88
x_{36}	1·8400	2·26	1·84	1·84
x_{37}	1·8402	2·26	1·84	1·84
x_{38}	4·1059	4·79	3·88	3·88

behaviour of the objective function, since the area vs. least radius of gyration relationships based on regression are used.)

200-bar truss

To further illustrate the effectiveness of the method for large truss structures, the 200-bar plane truss depicted in Figure 6 was analysed. A prescribed discrete set of $A=(0{\cdot}1, 0{\cdot}5, 1{\cdot}0, 1{\cdot}5, \ldots, 16{\cdot}0)$

Table XIII. Properties of the IS808 angle sections

No.	x_j (cm^2)	r_j (cm)	$\zeta_j = r_j/\sqrt{x_j}$	No.	x_j (cm^2)	r_j (cm)	$\zeta_j = r_j/\sqrt{x_j}$
1	0·8	0·115	0·017	24	15·39	1·95	0·247
2	1·0	0·144	0·021	25	17·03	1·74	0·178
3	1·2	0·173	0·025	26	19·03	1·94	0·198
4	1·45	0·37	0·094	27	21·12	2·16	0·221
5	1·84	0·47	0·120	28	23·20	2·36	0·240
6	2·26	0·57	0·144	29	25·12	2·57	0·263
7	2·66	0·67	0·169	30	27·50	2·35	0·201
8	3·07	0·77	0·193	31	29·88	2·56	0·219
9	3·47	0·87	0·218	32	32·76	2·14	0·140
10	3·88	0·97	0·242	33	33·90	2·33	0·160
11	4·79	0·97	0·196	34	34·77	2·97	0·254
12	5·27	1·06	0·213	35	39·16	2·54	0·165
13	5·75	1·16	0·234	36	43·00	2·93	0·200
14	6·25	1·26	0·254	37	45·65	2·94	0·189
15	6·84	1·15	0·193	38	46·94	2·94	0·184
16	7·44	1·26	0·213	39	51·00	2·92	0·167
17	8·06	1·36	0·229	40	52·10	3·54	0·241
18	8·66	1·46	0·246	41	61·82	3·96	0·254
19	9·40	1·35	0·194	42	61·90	3·52	0·200
20	10·47	1·36	0·177	43	68·30	3·51	0·180
21	11·38	1·45	0·185	44	76·38	3·93	0·202
22	12·21	1·55	0·197	45	90·60	3·92	0·170
23	13·79	1·75	0·222	46	94·13	3·92	0·163

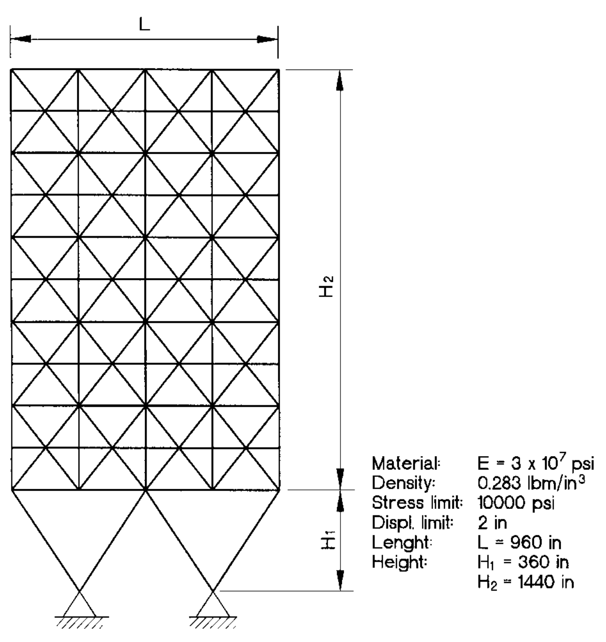


Figure 6. 200-bar truss

Table XIV. 200-bar truss, $A = (0{\cdot}1, 0{\cdot}5, 1{\cdot}0, 1{\cdot}5, \ldots, 16{\cdot}0)$

	Continuous optimum	Rounding	B&B[12]	SDR[4]	This study
f (lb)	25·447	(26·972)	27·413	<u>26·819</u>	<u>26·819</u>
NI	0·0	$8{\cdot}1 \times 10^{-2}$	0·0	0·0	0·0
x_1 (in^2)	13·824	14·0	14·0	14·0	14·0
x_2	10·806	11·0	11·5	11·0	11·0
x_3	6·662	7·0	7·5	8·0	8·0
x_4	0·913	1·0	1·0	1·0	1·0
x_5	11·998	12·0	12·0	12·0	12·0
x_6	0·100	0·1	0·1	0·1	0·1
x_7	10·998	11·0	11·0	11·0	11·0
x_8	0·150	0·5	0·1	0·5	0·5
x_9	0·859	1·0	1·0	1·0	1·0
x_{10}	8·950	9·0	9·0	9·0	9·0
x_{11}	0·100	0·1	0·5	0·1	0·1
x_{12}	7·950	8·0	8·5	8·0	8·0
x_{13}	0·435	0·5	0·5	0·5	0·5
x_{14}	0·526	1·0	1·0	0·5	0·5
x_{15}	6·393	6·5	6·0	6·5	6·5
x_{16}	0·100	0·1	1·0	0·1	0·1
x_{17}	5·393	5·5	5·0	5·5	5·5
x_{18}	0·100	0·1	0·5	0·1	0·1
x_{19}	0·399	0·5	0·5	0·5	0·5
x_{20}	4·123	4·5	4·5	4·5	4·5
x_{21}	0·100	0·1	0·1	0·1	0·1
x_{22}	3·122	3·5	3·0	3·5	3·5
x_{23}	0·100	0·1	0·5	0·1	0·1
x_{24}	0·298	0·5	0·5	0·5	0·5
x_{25}	1·945	2·0	2·0	2·0	2·0
x_{26}	0·100	0·1	0·1	0·1	0·1
x_{27}	0·100	0·1	0·1	0·1	0·1
x_{28}	0·945	1·0	1·0	1·0	1·0
x_{29}	0·148	0·5	0·1	0·5	0·5

is considered. Numerical results are presented in Table XIV. The SDR and R-GA results are identical and superior to the B&B solution presented in Reference 12. Note that the result obtained through rounding of the continuous solution is infeasible.

On running the algorithm 10 times (with $p = 20$ and $s = 50$), the average number of generations required for convergence was 123.

CONCLUSIONS

A Regional Genetic Algorithm (R-GA) for the discrete optimal design of truss structures is presented and implemented in this study. This new approach in the application of the genetic algorithm is based on the selection of chromosomes from a subregion centred on the continuous optimum.

The method is robust and since it is a pure discrete method, it is highly suitable when buckling and slenderness constraints with their associated scatter in the data are introduced. The algorithm is effective in obtaining a good approximation to the global discrete optimum, while at the same time highly efficient in terms of computational effort. It is also simple and easy to implement, although gradient information is required in the continuous phase.

Although more expensive than the Selective Dynamic Rounding (SDR) method previously presented by the authors, the robustness of the R-GA is particularly attractive, and for this reason should be preferred to the SDR method. As a practical tool, the method is highly suitable for the optimal design of large truss structures with many design variables.

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