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Emergent Properties in Reactive Systems *

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Abstract

Reactive systems are often described by interconnecting sub-components along architectural connectors defining communication policies. Generally, such global systems may exhibit properties, often called emergent properties, that cannot be anticipated just from a complete knowledge of components. These emergent properties are twofold: (1) the global system can question properties attached to components ; (2) some global properties cannot be inferred only from a complete knowledge of components, but for being inferred, need the knowledge of cooperation mechanisms between components. In practice, properties of the second form combine knowledge inherited from components. Thus, they are often defined in a richer language than the ones associated to each component and the presence of such emergent properties is quite natural. In this paper, we restrict ourselves to reactive systems described by means of transition systems as components and of the usual synchronous product as architectural connector and whose behavior is expressed by logical properties over a modal first-order logic. In this framework, we propose to study complexity of reactive systems through this notion of emergent properties and we will give some conditions to guarantee when a system has not emergent properties of the first form.

Keywords: transition systems, synchronous product, modal first-order logic, emergent properties, property preservation

1. Introduction

Reactive systems are often described by interconnecting sub-components along architectural connectors defining communication policies. Generally, such global systems may exhibit properties, often called *emergent properties*, that cannot be anticipated just from

a complete knowledge of components. The presence of such emergent properties makes systems *complex*, that is they cannot be reduced to simple rules of property inferences. These emergent properties are twofold:

1. the global system questions properties attached to components;
2. some global properties cannot be inferred only from a complete knowledge of components. They need for being inferred, of the knowledge of cooperation mechanisms between components.

In practice, properties of the second form, that we will call in this paper *true emergent properties*, combine knowledge inherited from components. Thus, they are defined in a richer language than the ones associated to each component, and the presence of such emergent properties is quite natural. Conversely, properties of the first form, that we will call in this paper *non-conformity properties*, are often the consequence of bad interactions between components. They characterize properties that are satisfied (resp. not satisfied) by the component over which they rest on when considered in isolation, but are not satisfied (resp. satisfied) by the global system within which the component is plugged in. Such properties have been already observed in many situations but outside component-based systems. We can cite the way modular systems have been defined in the algebraic specification community (see [11] for a precursor work in the framework of algebraic specifications and [15] for a state-of-the-art on the modular approach in the formal software engineering framework), or the more recent problem of feature interactions that have been firstly studied for telecommunication systems but since have been extended to many other systems such as embedded systems [3, 8, 17]. Let us exemplify this notion of emergent property in the field of feature design dedicated to telecommunication systems. In this field, features (so-called services) are optional functionalities to which telephone users may subscribe. When several services are integrated together on a given software telecommunication system, it induces mutual in-

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fluences which may perturb the behavior of the system in an unexpected way. The term of "feature interactions" regroups all the phenomena emerging from multiple joint integration of features, that is, those reflecting the appearance of new properties, or on the contrary those reflecting the disappearance of existing properties. For example, when considering the two following features, known under the names "Call Forward Unconditionally" or CFU for short (all calls to the subscriber's phone are diverted to another phone) and "Terminating Call Screening" or TCS for short (all calls from a number on the screening list are rejected), an interaction necessarily occurs when a user A is calling a user B subscribing to the service CFU by diverting all the entering calls toward a third user C who has subscribed to TCS with A in its black list. According to the way the two services are integrated together, either A can join C, and this represents what we call a "non-conformity property", or A cannot join C because the service TCS is activated. This last property is an example of a "true emergent property" since it involves subscriptions of both services. Even if service-oriented systems represent typical systems with numerous emergent properties, they cannot be considered as systems built over components interconnected through architectural connectors. Indeed, service integration is often designed by modifying the descriptions of low level abstraction service (implementation, architecture [4], executable specifications [2, 16]). Conversely, component-based systems postulate that a component is reusable from its specification made of its interface and of its behavioral requirements, and then that systems can be designed by assembling existing components to fit in the desired global requirements. Different techniques have been explored to circumvent as much as possible non-conformity properties and to take advantage of true emergent properties when designing component-based systems: coordination models aiming at managing interactions between concurrent programs [12]; adaptation techniques for reusing existing software entities in accordance with new requirements and for predicting some extent properties of the composed system from partial knowledge on components [5], "design by contract"-based approaches expliciting obligations (e.g. pre and post conditions) on components and their environments [13, 14]. Such contributions represent pragmatic, and often efficient computer-aided methodologies devoted to the design of complex component-based systems while avoiding bad non-conformity properties issued from component interactions. The problem is in the identification and the prediction of emergent properties before handling them either for enhancing appropriate true emergent ones at the global level or for controlling

undesirable non-conformity properties. This partly originates in the absence of a clear and general definition of emergent properties.

In this paper, we will first give a rigorous and formal definition of emergent properties in the framework of reactive system modeling. We restrict ourselves to reactive systems described by means of the usual synchronous product of transition systems, and whose behavior is expressed by logical properties over a modal first-order logic. The reason is this is sufficient for the purpose of the study, and the results given in this paper could easily be adapted to temporal logics more classically used to reason on reactive systems and other composition connector whose the greatest number are based-on transition system product. In our setting, we will study some conditions under which non-conformity properties do not occur. The interest is this provides guidance in the design process. Indeed, the apparition of non-conformity properties leads to make a posteriori verification of the global system without benefiting from the decomposition of the system into components.

The paper is structured as follows: In Section 2, we recall basic definitions and notations about the many sorted first-order logic and the conditional equational logic. In the second logic, we recall two important results dealing respectively, with free extension by addition of conditional equations and the preservation of conditional equations along cartesian product of algebras. These two results will be used afterwards to establish our results of non-conformity property preservation. In Section 3, we introduce transition systems and their semantics, and define the synchronous product as means to compose them. In Section 4, we present the modal first-order logic over which properties of transition systems will be expressed. In Section 5, the notion of emergent property is introduced. This notion allows to classify transition systems resulting from the synchronous product of other transition systems as complex. Finally, Section 6 presents results ensuring the non-existence of non-conformity properties along synchronous product.

2. Preliminaries

The data part addresses the functional issues of transition systems. It will be described with the many-sorted first-order logic with equalities **FOL**. **FOL**-Signatures are triples (S, F, P) where S is a set of sorts, and F and P are sets of function and predicate names respectively, both with arities in $S^* \times S$ and S^+ respectively.¹ Signature morphisms $\sigma : (S, F, P) \rightarrow (S', F', P')$ consist

¹ S^+ is the set of all non-empty sequences of elements in S and $S^* = S^+ \cup \{\epsilon\}$ where ϵ denotes the empty sequence.

of three functions between sets of sorts, sets of functions and sets of predicates respectively, the last two preserving arities. In the sequel, given two **FOL**-signatures $\Sigma = (S, F, P)$ and $\Sigma' = (S', F', P')$ such that $S \subseteq S'$, $F \subseteq F'$ and $P \subseteq P'$, we will note $\Sigma \hookrightarrow \Sigma'$ the underlying inclusion morphism.

Given a **FOL**-signature $\Sigma = (S, F, P)$, the Σ -atoms are of two possible forms: equations $t_1 = t_2$ where $t_1, t_2 \in T_F(X)_s$ ($s \in S$), and predicates $p(t_1, \dots, t_n)$ where $p : s_1 \times \dots \times s_n \in P$ and $t_i \in T_F(X)_{s_i}$ ($1 \leq i \leq n, s_i \in S$). The set of Σ -sentences is the least set of formulas built over the set of Σ -atoms by finitely applying Boolean connectives in $\{\neg, \vee, \wedge, \Rightarrow\}$ and quantifiers \forall and \exists .

Given a **FOL**-signature $\Sigma = (S, F, P)$, a Σ -model \mathcal{M} is a S -indexed set $M = (M_s)_{s \in S}$ of sets (one for every $s \in S$), each one equipped with a function $f^{\mathcal{M}} : M_{s_1} \times \dots \times M_{s_n} \rightarrow M_s$ for every $f : s_1 \times \dots \times s_n \rightarrow s \in F$ and with a n -ary relation $p^{\mathcal{M}} \subseteq M_{s_1} \times \dots \times M_{s_n}$ for every $p : s_1 \times \dots \times s_n \in P$. A Σ -morphism μ from a Σ -model \mathcal{M} to a Σ -model \mathcal{M}' is a mapping from M to M' respecting sorts, and compatible with functions and predicates. Given a signature morphism $\sigma : \Sigma = (S, F, P) \rightarrow \Sigma' = (S', F', P')$ and a Σ' -model \mathcal{M}' , $Mod(\sigma)(\mathcal{M}')$ is the Σ -model \mathcal{M} defined for every $s \in S$ by $M_s = M'_{\sigma(s)}$, and for every function name $f \in F$ and predicate name $p \in P$, by $f^{\mathcal{M}} = \sigma(f)^{\mathcal{M}'}$ and $p^{\mathcal{M}} = \sigma(p)^{\mathcal{M}'}$. Let us note $Mod(\Sigma)$ the category of Σ -models. Therefore, given a signature morphism $\sigma : \Sigma \rightarrow \Sigma'$, $Mod(\sigma) : Mod(\Sigma') \rightarrow Mod(\Sigma)$ is the functor classically called *forgetful functor*. Finally, satisfaction is the usual first-order satisfaction.

FOL satisfies the following property, called the *satisfaction condition*:³ $\forall \sigma : \Sigma \rightarrow \Sigma', \forall \mathcal{M}' \in |Mod(\Sigma')|, \forall \varphi \in Sen(\Sigma),$

$$\mathcal{M}' \models_{\Sigma'} Sen(\sigma)(\varphi) \Leftrightarrow Mod(\sigma)(\mathcal{M}') \models_{\Sigma} \varphi$$

By this property, if for a given set of Σ -formulas Γ , we note $Mod(\Gamma)$ the sub-category of $Mod(\Sigma)$ whose models satisfy all the properties of Γ , then we can restrict the forgetful functor $Mod(\sigma)$ from $Mod(\Sigma')$ to $Mod(\Gamma)$ for every signature morphism $\sigma : \Sigma \rightarrow \Sigma'$, and every set of formulas Γ and Γ' over respectively, Σ and Σ' such that $\sigma(\Gamma) \subseteq \Gamma'$.

An important logic that can be obtained as a **FOL** restriction is the conditional equational Logic **CEL**. A **CEL**-signature $\Sigma = (S, F)$ simply is a **FOL** signature without predicate symbols. Then, the logic **CEL** is the

² $T_F(X)_s$ is the term algebra of sort s built over F with sorted variables in a given set X . T_{F_s} is then the ground term algebra of sort s .

³Hence, **FOL** is an institution [9].

sub-logic of **FOL** whose signatures are **CEL**-signatures, models are algebras over **CEL**-signatures, and formulas are sentences of the form $\Gamma \Rightarrow \alpha$ where Γ is a finite conjunction of equations and α is an equation.

CEL has important results that we will use in Section 6. These results deal respectively, with free extension by addition of conditional equations and the preservation of conditional equations along cartesian product of algebras.⁴ In the sequel, we recall these two results.

2.1 Free extension

Let $\Sigma = (S, F)$ and $\Sigma' = (S', F')$ be two **CEL**-signatures such that $\Sigma \hookrightarrow \Sigma'$. Let Γ and Γ' be two sets of conditional equations over, respectively, Σ and Σ' such that $\Gamma \subseteq \Gamma'$. We can build a functor $T_{\Gamma'/\Gamma} : \mathcal{A} \mapsto T_{\Gamma'/\Gamma}(\mathcal{A})$, from the category of Γ -algebras to the category of Γ' -algebras. Let \mathcal{A} be a Γ -algebra. $T_{\Gamma'/\Gamma}(\mathcal{A})$ is the quotient of $T_F(\mathcal{A})$ by the congruence generated by the kernel of the Σ -morphism $T_F(\mathcal{A})$ in \mathcal{A} extending the identity on X .⁵ This algebra satisfies the following universal property: for every Γ' -algebra \mathcal{B} and every Σ -morphism $\mu : \mathcal{A} \rightarrow Mod(\Sigma \hookrightarrow \Sigma')(\mathcal{B})$, there exists a unique Σ' -morphism $\eta_{\mathcal{B}} : T_{\Gamma'/\Gamma}(\mathcal{A}) \rightarrow \mathcal{B}$ such that for every $a \in A$, $\eta_{\mathcal{B}}(a) = \mu(a)$. This universal property directly shows that the functor $T_{\Gamma'/\Gamma}$ is left-adjunct to $Mod(\Sigma \hookrightarrow \Sigma')$, i.e., for every Γ -algebra \mathcal{A} there exists a universal morphism $\mu_{\mathcal{A}} : \mathcal{A} \rightarrow Mod(\Sigma \hookrightarrow \Sigma')(T_{\Gamma'/\Gamma}(\mathcal{A}))$. $\mu_{\mathcal{A}}$ is called the *adjunct morphism* for \mathcal{A} .

2.2 Cartesian product and preservation results

Let Σ be a signature, let I be a set and let $(\mathcal{A}_i)_{i \in I}$ be a I -indexed family of Σ -algebras. Let us note $\prod_{i \in I} \mathcal{A}_i$ the Σ -algebra defined as follow:

- for every $s \in S$, its carrier of sort s is $\prod_{i \in I} (A_i)_s$,
- for every $f : s_1 \times \dots \times s_n \rightarrow s \in F$, $f^{\prod_{i \in I} \mathcal{A}_i}$ is the mapping that to every $(a_1, \dots, a_n) \in \prod_{i \in I} (A_i)_{s_1} \times \dots \times \prod_{i \in I} (A_i)_{s_n}$, associates $(f^{\mathcal{A}_i}(a_1^i, \dots, a_n^i))_{i \in I}$ where given $a \in \prod_{i \in I} (A_i)_s$, a^i is the i -th coordinate of a .

⁴This last result also holds for the sub-logic of **FOL** whose formulas are restricted to universal Horn formulas.

⁵ $T_F(\mathcal{A})$ is the term algebra built over F with sorted variables in the carrier A of the Γ -algebra \mathcal{A} .

By construction, we can notice that:

$$\prod_{i \in I} \mathcal{A}_i \models_{\iota} \varphi \iff \forall i \in I, \mathcal{A}_i \models_{\iota^i} \varphi$$

where for every interpretation ι , ι^i is the interpretation defined by $x \mapsto a^i$ if $\iota(x) = a$. It is well-known that conditional equations are preserved by Cartesian product of algebras, that is, if for every $i \in I$, $\mathcal{A}_i \models \Gamma \Rightarrow \alpha$, then $\prod_{i \in I} \mathcal{A}_i \models \Gamma \Rightarrow \alpha$.

3 Transition systems

3.1 Syntax

As usual when considering automata, transition systems describe possible evolutions of system states. Elementary evolutions are represented by a transition relation between states. Each transition between two states is labeled by three elements: actions of the system, guards expressed here by formulas of **FOL** presented in Section 2, and side-effects on states defined by pairs of ground terms or of the form $(p(t_1, \dots, t_n), b)$ where $p(t_1, \dots, t_n)$ is a ground atom and b is equal to *true* or *false*. As usual, we start by defining the language, so-called signature, over which transition systems are built:

Definition 3.1 (Signature) A signature is a triple $\mathcal{L} = (\Sigma, V, A)$ where: Σ is a **FOL**-signature, V is a set of variables over Σ and A is a set whose elements are called actions.

Definition 3.2 (Side-effect) Given a signature $\mathcal{L} = (\Sigma, V, A)$ where $\Sigma = (S, F, P)$, a side-effect over \mathcal{L} is a pair of ground terms over Σ (t, t') of the same sort (i.e. $\exists s \in S, t, t' \in T_{F_s}$) or a couple $(p(t_1, \dots, t_n), b)$ where $p(t_1, \dots, t_n)$ is a ground Σ -atom (i.e. each t_i is a ground term) and b is equal to *true* or *false*. In the sequel, a side-effect (t, t') will be noted $t \mapsto t'$. We note $\mathcal{SE}(\mathcal{L})$ the set of side-effects over \mathcal{L} .

A transition system is then defined as follows:

Definition 3.3 (Transition system) Given a signature $\mathcal{L} = (\Sigma, V, A)$, a transition system is a couple (Q, \mathbb{T}) where:

- Q is a set of states, and
- $\mathbb{T} \subseteq Q \times A \times \mathcal{SE}(\Sigma) \times 2^{\mathcal{SE}(\mathcal{L})} \times Q$.

Example 3.1 Let us consider the example of a cash dispenser. Its informal specification is the following. A user inserts his card and keys his code. If it is wrong, the user

has to key his code again, except if it is the third time that the code is wrong. In this case, the user does not get his card back, and the dispenser is reset. If the code is valid, the user keys the amount he wants to withdraw. Then the dispenser gives an authorization depending on the card number and the asked amount. According to this authorization, the dispenser will give or not his card back to the user, and will give or not his money. In all these cases, when the operation is finished, the dispenser is reset.

A transition system modeling such a cash dispenser is shown on figure 1. It is built over the following signature \mathcal{L} :⁶

$$\Sigma = \begin{cases} S = \{\text{nat}\}, \\ F = \{a, \text{count} : \rightarrow \text{nat}, \text{authorize} : \text{nat} \times \text{nat} \rightarrow \text{nat}\}, \\ P = \{\text{is_valid} : \text{nat} \times \text{nat}\} \end{cases}$$

$$A = \{\text{card}?n, \text{card}!n, \text{Coode}?n, \text{Money}!n \mid n \in \mathbb{N}\}$$

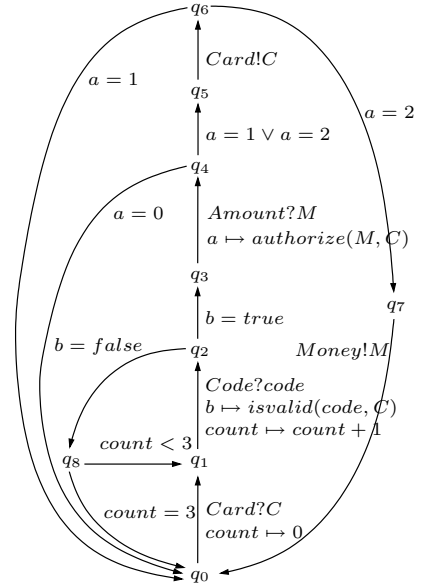


Figure 1. A cash dispenser

The predicate *isvalid* checks the validity of the code. The function *authorize* gives an authorization (0, 1 or 2) according to the card number and the asked amount.

3.2 Semantics

Semantics of transition systems are defined by Kripke frames themselves defined as follows:

⁶In this paper, actions are just names. This is why, we consider an infinite set of actions, each one indexed by positive integers. Of course, a better approach when we are interested by specifying real systems, is to consider actions as first-order terms. However, this was not very interesting for the purpose that we aimed at in this paper except for making heavy the presentation.

Definition 3.4 (Kripke frame) Given a signature $\mathcal{L} = (\Sigma, V, A)$, an Kripke frame over \mathcal{L} or \mathcal{L} -model, is a couple (\mathcal{W}, R) where:

- \mathcal{W} is a I -indexed set $(\mathcal{W}^i)_{i \in I}$ of Σ -models such that $\mathcal{W}_s^i = \mathcal{W}_s^j$ for every $i, j \in I$ and $s \in S$, and
- R is a A -indexed set of “accessibility” relations $R_a \subseteq I \times I$.

Here, states are defined by Σ -models. Therefore, side-effects will consist on moving from a Σ -model to another one by changing the semantics of functions according the assignments given in the set δ of transitions. Formally, this is defined as follow: if \mathcal{A} is a Σ -model, then $\neg^{\mathcal{A}} : T_F \rightarrow \mathcal{A}$ is the Σ -morphism inductively defined by $f(t_1, \dots, t_n) \mapsto f^{\mathcal{A}}(t_1^{\mathcal{A}}, \dots, t_n^{\mathcal{A}})$

Definition 3.5 (Side-effect semantics) Let

$\mathcal{L} = (\Sigma, V, A)$ be a signature. Let \mathcal{A} and \mathcal{B} be two Σ -models. We note $\mathcal{A} \rightsquigarrow_{\delta} \mathcal{B}$ to mean that the state \mathcal{A} is transformed into the state \mathcal{B} along δ , if and only if \mathcal{B} is defined as \mathcal{A} except that for every $t \mapsto t' \in \delta$ (resp. $p(t_1, \dots, t_n) \mapsto b$), $t^{\mathcal{B}} = t'^{\mathcal{A}}$ (resp. $(t_1^{\mathcal{A}}, \dots, t_n^{\mathcal{A}}) \in p^{\mathcal{B}}$ iff $b = \text{true}$).

Definition 3.6 (Semantics of transition systems)

Given a transition system $\mathcal{S} = (Q, \mathbb{T})$ over a signature \mathcal{L} , the semantics for \mathcal{S} , noted $\text{Real}(\mathcal{S})$, is the set of all the Kripke frames (\mathcal{W}, R) over \mathcal{L} such that the set of indexes $I = Q$, and satisfying both implications:

1. $(q, a, \varphi, \delta, q') \in \mathbb{T} \wedge \mathcal{W}^q \models \varphi \wedge \mathcal{W}^q \rightsquigarrow_{\delta} \mathcal{W}^{q'} \Rightarrow q R_a q'$
2. $q R_a q' \Rightarrow \exists (q, a, \varphi, \delta, q') \in \mathbb{T}, \mathcal{W}^q \models \varphi \wedge \mathcal{W}^q \rightsquigarrow_{\delta} \mathcal{W}^{q'}$

Hence, the way whose dynamic is dealt with in this paper follows the state-as-algebra style [1, 10] where states are Σ -models and state transformations are transitions from a state-model to another state-model.

3.3 Synchronous product

Synchronous product combines two transition systems into a single one by synchronizing transitions. Understandably, executions of synchronous product modelize system behavior as a synchronizing concurrent system. Hence, when an action a is “executed” in the product, then every component with a in its alphabet must execute a transition labeled with a . Formally, the synchronous product of two transition systems is defined as follows:

Definition 3.7 (Synchronous product) Let

$\mathcal{S}_i = (Q_i, \mathbb{T}_i)$ be a transition system over a signature $\mathcal{L}_i = (\Sigma_i, V_i, A_i)$ with $i = 1, 2$ such that:

- for every transition $(q_1, a, \varphi_1, \delta_1, q'_1) \in \mathbb{T}_1$ and every $f(t_1, \dots, t_n) \mapsto t'_1 \in \delta_1$ (resp. $p(t_1, \dots, t_n) \mapsto b \in \delta_1$), there does not exist a transition $(q_2, a, \varphi_2, \delta_2, q'_2) \in \mathbb{T}_2$ and a side-effect $t_2 \mapsto t'_2 \in \delta_2$ with t_2 of the form $f(t'_1, \dots, t'_2)$ (resp. $p(t'_1, \dots, t'_n) \mapsto b' \in \delta_2$),
- and conversely, that is this condition on side-effects has also to be satisfied by replacing \mathbb{T}_1 by \mathbb{T}_2 , δ_1 by δ_2 and δ_2 by δ_1 .

The synchronous product of \mathcal{S}_1 and \mathcal{S}_2 , noted $\mathcal{S}_1 \otimes \mathcal{S}_2$, is the transition system (Q, \mathbb{T}) over $\mathcal{L} = (\Sigma_1 \cup \Sigma_2, V_1 \cup V_2, A_1 \cup A_2)$ defined as follows:

- $Q = Q_1 \times Q_2$
- if $a \in A_1 \cap A_2$, $(q_1, a, \varphi_1, \delta_1, q'_1) \in \mathbb{T}_1$ and $(q_2, a, \varphi_2, \delta_2, q'_2) \in \mathbb{T}_2$ then $((q_1, q_2), a, \varphi_1 \wedge \varphi_2, \delta_1 \cup \delta_2, (q'_1, q'_2)) \in \mathbb{T}$
- if $a \in A_1 \setminus A_2$ and $(q_1, a, \varphi_1, \delta_1, q'_1) \in \mathbb{T}_1$ then for every $q_2 \in Q_2$, $((q_1, q_2), a, \varphi_1, \delta_1, (q'_1, q_2)) \in \mathbb{T}$
- if $a \in A_2 \setminus A_1$ and $(q_2, a, \varphi_2, \delta_2, q'_2) \in \mathbb{T}_2$ then for every $q_1 \in Q_1$, $((q_1, q_2), a, \varphi_2, \delta_2, (q_1, q'_2)) \in \mathbb{T}$

Both conditions on side-effects allow to remove the case where for a same function name f (resp. a predicate p) applied to a same tuple of arguments yields different values, an then makes fail the functionality of f (resp. makes inconsistent the set of side-effects resting on p).

4 A logic for transition systems

In this paper, reactive systems are specified by a transition system on which we can observe properties. In this section, we define a logic powerful enough to express such properties of transition systems. We choose the many-sorted first-order modal logic **MFOL** which extends the Hennessy-Milner logic to data. As properties observed on transition systems will be defined in **MFOL**, signatures in this logic will be those of Definition 3.1.

Definition 4.1 (Formulas) Given a signature $\mathcal{L} = (\Sigma, V, A)$ with $\Sigma = (S, F, P)$, \mathcal{L} -atoms are Σ -equations $t = t'$, Σ -atoms $p(t_1, \dots, t_n)$ and the symbol T (for True), and the set of \mathcal{L} -formulas is the least set of formulas built over the set of Σ -atoms by finitely applying Boolean connectives in $\{\neg, \vee, \wedge, \Rightarrow\}$, quantifiers \forall and \exists , and modalities in $\{\Box_a | a \in A\}$.

For every $a \in A$, the intuitive meaning of \Box_a is “always after the action a ”.

The interpretation of **MFOL** formulas is over Kripke frame. It is defined as follows:

Definition 4.2 (Satisfaction) A \mathcal{L} -formula φ is said to be satisfied by a \mathcal{L} -model (\mathcal{W}, R) , noted $(\mathcal{W}, R) \models \varphi$, if for every $i \in I$ and every $\iota : V \rightarrow W^i$, we have $(\mathcal{W}, R) \models_\iota^i \varphi$, where \models_ι^i is inductively defined on the structure of φ as follows:

- for every Σ -formula φ , $(\mathcal{W}, R) \models_\iota^i \varphi$ iff $\mathcal{W}^i \models_\iota \varphi$.
- $(\mathcal{W}, R) \models_\iota^i \Box_a \varphi$ when $(\mathcal{W}, R) \models_\iota^j \varphi$ for every $j \in I$ such that $i R_a j$.
- Boolean connectives and quantifiers are handled as usual.

Definition 4.3 (Logical theory) Given a subset Γ of \mathcal{L} -formulas, let us note $Mod(\Gamma)$ the whole set of \mathcal{L} -models that satisfy all the formulas of Γ . Therefore, let us note $Th(\Gamma)$, called the theory of Γ , the set of \mathcal{L} -formulas defined by:

$$Th(\Gamma) = \{\varphi \mid \forall (\mathcal{W}, R) \in Mod(\Gamma), (\mathcal{W}, R) \models \varphi\}$$

Now, for transition systems we can define their set of semantic consequences that are all the properties they satisfy.

Definition 4.4 (Semantic consequences) Let $\mathcal{S} = (Q, \mathbb{T})$ be a transition system over a signature \mathcal{L} . Let us note \mathcal{S}^\bullet the set of \mathcal{L} -formulas defined by:

$$\mathcal{S}^\bullet = \{\varphi \mid \forall (\mathcal{W}, R) \in Real(\mathcal{S}), (\mathcal{W}, R) \models \varphi\}$$

5 Complexity

Following some works issued from scientific disciplines such as biology, physics, economy or sociology [6, 7], a complex system is characterized by a holistic behavior, i.e. global: we do not consider that its behavior results from the combination of isolated behaviors of some of its components, but instead has to be considered as a whole. This is expressed by the apparition (emergence) of global properties which is very difficult, see impossible, to anticipate just from a complete knowledge of component behaviors. This notion of emergence seems to be the simplest way to define complexity. Succinctly, this could be expressed as follows:

suppose a system XY composed of two sub-systems X and Y . Let us also suppose we have a mathematical function F which gives all the potential richness of XY , X and Y , and an operation ‘+’ to combine potential richness of sub-systems. If we have that $F(XY) = F(X) + F(Y)$ then this means that the system XY integrates in a consistent whole both sub-systems X and Y . Therefore, we can say that the system XY is not complex (i.e. modular). On the contrary, if there exists some $a \in F(X) + F(Y)$ such that $a \notin F(XY)$ or there exists some $a \in F(XY)$ such that $a \notin F(X) + F(Y)$, then there is reconsideration of some potential richness of X or Y in the first case, and apparition of true emergence in the second case. The system XY is then said complex.

Here, the system XY is a transition system \mathcal{S} that has been built from two transition systems $X = \mathcal{S}_1$ and $Y = \mathcal{S}_2$ by synchronous product (i.e. $\mathcal{S} = \mathcal{S}_1 \otimes \mathcal{S}_2$), the function F gives for a transition system \mathcal{S} its set of semantic consequences \mathcal{S}^\bullet , and $F(\mathcal{S}_1) + F(\mathcal{S}_2) = Th(\mathcal{S}_1^\bullet \cup \mathcal{S}_2^\bullet)$. Hence, complexity in reactive systems specified by transition systems is defined as follows:

Definition 5.1 (Complexity) Let $\mathcal{S} = \mathcal{S}_1 \otimes \mathcal{S}_2$ be a transition system obtained by the synchronous product of \mathcal{S}_1 and \mathcal{S}_2 . \mathcal{S} is said complex if, and only if one of the two following properties fails:

1. *Conformity*: for every \mathcal{L}_i -formula φ ($i = 1, 2$)

$$\varphi \in \mathcal{S}_i^\bullet \iff \varphi \in (\mathcal{S}_1 \otimes \mathcal{S}_2)^\bullet$$

2. *Non true emergence*.

$$\forall \varphi \in (\mathcal{S}_1 \otimes \mathcal{S}_2)^\bullet, \varphi \in Th(\mathcal{S}_1^\bullet \cup \mathcal{S}_2^\bullet)$$

A formula φ that makes fail the equivalence of Point 1. and Point 2. is called an emergent property for \mathcal{S} .

6 Results

The synchronous product of two transition systems $\mathcal{S}_1 \otimes \mathcal{S}_2$ have generally true emergent properties. The reason is the set $Mod(Th(\mathcal{S}_1^\bullet \cup \mathcal{S}_2^\bullet))$ of Kripke frames may be greater than $Real(\mathcal{S}_1 \otimes \mathcal{S}_2)$. Indeed, Kripke frames in $Real(\mathcal{S}_1 \otimes \mathcal{S}_2)$ have to preserve the shape of the transition system $\mathcal{S}_1 \otimes \mathcal{S}_2$ unlike Kripke frames in $Mod(Th(\mathcal{S}_1^\bullet \cup \mathcal{S}_2^\bullet))$. Hence, properties in $(\mathcal{S}_1 \otimes \mathcal{S}_2)^\bullet$ may be more numerous than in $Th(\mathcal{S}_1^\bullet \cup \mathcal{S}_2^\bullet)$. However, we can show under some conditions that non-conformity properties cannot occur along synchronous product. More precisely, we are going to show that the “only if” part of the conformity property is satisfied but

the “if” part only holds when formulas that label transitions are conditional equations (i.e. expressed in the logic **CEL**).

Let us start by showing that the semantic consequences of S_1 and S_2 are preserved by $S_1 \otimes S_2$. Let us suppose a $S_1 \otimes S_2$ -model (\mathcal{W}, R) , and let us define a \mathcal{L}_i -model (\mathcal{W}_i, R_i) for $i = 1, 2$ as follows:

- for every $q \in Q_i$, $\mathcal{W}_i^q = \text{Mod}(\Sigma_i \hookrightarrow \Sigma)(\mathcal{W}^{(q, q')})$ for any $q' \in Q_j$ with $j \neq i \in \{1, 2\}$
- $\forall a \in A_i$, $R_{i_a} = \{(q, q') | \exists \varphi \in \text{Sen}(\Sigma_i), \exists \delta \in \mathcal{SE}(\mathcal{L}), (q, a, \varphi, \delta, q') \in \mathbb{T}_i\}$

Let us note Γ_i for $i = 1, 2$, the set of all these \mathcal{L}_i -models.

Theorem 6.1 *Each $(\mathcal{W}_i, R_i) \in \Gamma_i$ is a S_i -model.*

Proof. *The first condition of Definition 3.6 is obvious. To prove the second condition, let us suppose a transition $(q, a, \varphi, q') \in \mathbb{T}_i$. By construction, there exists a transition $((q, q_j), a, \varphi', \delta', (q', q'_j)) \in \mathbb{T}$ such that either $\varphi' = \varphi$ and $\delta' = \delta$, or $\varphi' = \varphi \wedge \varphi''$ and $\delta' = \delta \cup \delta''$. In both cases, by hypothesis, we have that $(\mathcal{W}^{(q, q_j)}) \models \varphi'$. Therefore, by the satisfaction condition for **FOL** $\mathcal{W}_i^q \models \varphi$. Moreover, by the condition on side-effects in Definition 3.7, we have that $\mathcal{W}_i^q \rightsquigarrow_\delta \mathcal{W}_i^{q'}$*

Proposition 6.1 $\forall \iota : V \rightarrow W$,
 $(\forall (\mathcal{W}_i, R_i) \in \Gamma_i, \forall q \in Q_i, (\mathcal{W}_i, R_i) \models_\iota^q \varphi)$
 $\implies (\forall q_j \in Q_j, (\mathcal{W}, R) \models_\iota^{(q, q_j)} \varphi)$

Proof. *By induction on the structure of φ .*

Basic case. φ is of the form $p(t_1, \dots, t_n)$. Let $q_j \in Q_j$. By definition, there exists $(\mathcal{W}_i, R_i) \in \Gamma_i$ such that $\mathcal{W}_i^q = \text{Mod}(\Sigma_i \hookrightarrow \Sigma)(\mathcal{W}^{(q, q_j)})$. By hypothesis, we have $\mathcal{W}_i^q \models_\iota p(t_1, \dots, t_n)$, and then $\mathcal{W}^{(q, q_j)} \models_\iota p(t_1, \dots, t_n)$.

General case. Let us handle the case where φ is $\Box_a \varphi'$. Then, let us consider (q', q'_j) such that $(q, q_j) R_a (q', q'_j)$. By the hypothesis, we have for every $(\mathcal{W}_i, R_i) \in \Gamma_i$ that $(\mathcal{W}_i, R_i) \models_\iota^q \varphi$. By construction, we also have $q R_{i_a} q'$ for every $(\mathcal{W}_i, R_i) \in \Gamma_i$. Therefore, for every $(\mathcal{W}_i, R_i) \in \Gamma_i$, $(\mathcal{W}_i, R_i) \models_\iota^{q'} \varphi'$, and then by the induction hypothesis, we have $(\mathcal{W}, R) \models_\iota^{(q', q'_j)} \varphi'$, whence we can conclude $(\mathcal{W}, R) \models_\iota^{(q, q_j)} \varphi$.

The cases of Boolean connectives and quantifier are simpler and left to the interested reader.

Theorem 6.2 $S_i^\bullet \subseteq (S_1 \otimes S_2)^\bullet$

Proof. *Let $\varphi \in S_i^\bullet$, and let $(\mathcal{W}, R) \in \text{Real}(S_1 \otimes S_2)$. Let $\iota : V \rightarrow W$ be an interpretation. By Theorem 6.1, for every model $(\mathcal{W}_i, R_i) \in \Gamma_i$, we have $(\mathcal{W}_i, R_i) \models \varphi$, and then by definition, for every $q \in Q_i$ and every ι , we have $(\mathcal{W}_i, R_i) \models_\iota^q \varphi$. Therefore, by Proposition 6.1, we have for every $q_j \in Q_j$ that $(\mathcal{W}, R) \models_\iota^{(q, q_j)} \varphi$, whence we can conclude that $(\mathcal{W}, R) \models \varphi$.*

To show the “if” part of the conformity property, we need to make some restrictions on formulas that label transitions. Hence, we suppose that transition systems are built over the logic **CEL**, and then given a model (\mathcal{W}, R) of transition system \mathcal{S} , for each $q \in Q$, \mathcal{W}^q is now an algebra. Therefore, the logic for transition systems is the modal first-order logic defined as in Definition 4.1 except that now Σ -atoms are restricted to Σ -equations.

Given two transition systems \mathcal{S}_1 and \mathcal{S}_2 over the signatures \mathcal{L}_1 and \mathcal{L}_2 , respectively, and satisfying the above restriction, for $i \neq j \in \{1, 2\}$, and for every $(\mathcal{W}_j, R_j) \in \text{Mod}(\mathcal{S}_j)$ we define the mapping $\mathcal{F}_{(\mathcal{W}_j, R_j)} : \text{Mod}(\mathcal{S}_i) \rightarrow \text{Mod}(\mathcal{L})$ where \mathcal{L} is the signature over which the transition system $\mathcal{S}_1 \otimes \mathcal{S}_2$ is built as follows: if we note for a Σ -algebra \mathcal{A} , $\text{th}(\mathcal{A}) = \{\varphi | \varphi : \text{CEL-formula}, \mathcal{A} \models \varphi\}$, then to every (\mathcal{W}_i, R_i) , $\mathcal{F}_{(\mathcal{W}_j, R_j)}((\mathcal{W}_i, R_i)) = (\mathcal{W}, R)$ such that (\mathcal{W}, R) is the \mathcal{L} -model defined by

- $\forall q \in Q_i, \forall q' \in Q_j, \mathcal{W}^{(q, q')} = T_{\Gamma_i/\Gamma}(\mathcal{W}_i^q) \times T_{\Gamma_j/\Gamma}(\mathcal{W}_j^{q'})$
- $R_a = \{((q_1, q'_1), (q_2, q'_2)) | \exists \varphi \in \text{Sen}(\Sigma), \exists \delta \in \mathcal{SE}(\mathcal{L}), ((q_1, q'_1), a, \varphi, (q_2, q'_2)) \in \mathbb{T}\}$

where $\Gamma_i = \text{th}(\mathcal{W}_i^q)$, $\Gamma_j = \text{th}(\mathcal{W}_j^{q'})$, and $\Gamma = \text{th}(\mathcal{W}_i^q) \cup \text{th}(\mathcal{W}_j^{q'})$.

Theorem 6.3 *For every $(\mathcal{W}_j, R_j) \in \text{Mod}(\mathcal{S}_j)$ and every $(\mathcal{W}_i, R_i) \in \text{Mod}(\mathcal{S}_i)$, $\mathcal{F}_{(\mathcal{W}_j, R_j)}((\mathcal{W}_i, R_i))$ is a $S_1 \otimes S_2$ -model.*

Proof. *The first condition of Definition 3.4 is obvious. To prove the second condition, let us suppose a transition $((q_1, q'_1), a, \varphi, \delta, (q_2, q'_2)) \in \mathbb{T}$. By construction, φ and δ are:*

1. *either of the form $\varphi' \wedge \varphi''$ with $\varphi' \in \text{Sen}(\Sigma_i)$ and $\varphi'' \in \text{Sen}(\Sigma_j)$ and $\delta' \cup \delta''$ with $\delta' \in \mathcal{SE}(\mathcal{L}_i)$ and $\delta'' \in \mathcal{SE}(\mathcal{L}_j)$,*
2. *or $\varphi \in \text{Sen}(\Sigma_i) \cup \text{Sen}(\Sigma_j)$ and $\delta \in \mathcal{SE}(\mathcal{L}_i) \cup \mathcal{SE}(\mathcal{L}_j)$.*

This then leads to the two following cases:

1. Suppose that φ is of the form $\varphi' \wedge \varphi''$ and then $\delta = \delta' \cup \delta''$. This means by construction, that $(q_1, a, \varphi', \delta', q'_1) \in \mathbb{T}_i$ and $(q_2, a, \varphi'', \delta'', q'_2) \in \mathbb{T}_j$. By hypothesis, we have $\mathcal{W}_i^{q_1} \models \varphi'$ and $\mathcal{W}_j^{q'_1} \models \varphi''$. Therefore, we have that $T_{\Gamma_i/\Gamma}(\mathcal{W}_i^{q_1}) \models \varphi' \wedge \varphi''$ and $T_{\Gamma_j/\Gamma}(\mathcal{W}_j^{q'_1}) \models \varphi' \wedge \varphi''$, and then so is $T_{\Gamma_i/\Gamma}(\mathcal{W}_i^{q_1}) \times T_{\Gamma_j/\Gamma}(\mathcal{W}_j^{q'_1})$. Moreover, by hypothesis, we also have that $\mathcal{W}_i^{q_1} \rightsquigarrow_{\delta'} \mathcal{W}_i^{q_2}$ and $\mathcal{W}_j^{q'_1} \rightsquigarrow_{\delta''} \mathcal{W}_j^{q'_2}$. By definition, Γ_i (resp. Γ_j) contains the ground equational theory of $\mathcal{W}_i^{q_1}$ (resp. $\mathcal{W}_j^{q'_1}$). If we note $\Gamma'_i = th(\mathcal{W}_i^{q'_1})$, $\Gamma'_j = th(\mathcal{W}_j^{q'_2})$ and $\Gamma' = th(\mathcal{W}_i^{q'_1}) \cup th(\mathcal{W}_j^{q'_2})$, then we have $T_{\Gamma_i/\Gamma}(\mathcal{W}_i^{q_1}) \rightsquigarrow_{\delta} T_{\Gamma'_i/\Gamma'}(\mathcal{W}_i^{q_2})$ and $T_{\Gamma_j/\Gamma}(\mathcal{W}_j^{q'_1}) \rightsquigarrow_{\delta} T_{\Gamma'_j/\Gamma'}(\mathcal{W}_j^{q'_2})$.
2. The case where $\varphi \in Sen(\Sigma_i) \cup Sen(\Sigma_j)$ and $\delta' \in \mathcal{SE}(\mathcal{L}_i)$ and $\delta'' \in \mathcal{SE}(\mathcal{L}_j)$ is noticeably similar to the previous one.

Theorem 6.4 *If for every $(\mathcal{W}_i, R_i) \in Mod(\mathcal{S}_i)$, every $(\mathcal{W}_j, R_j) \in Mod(\mathcal{S}_j)$, and every $q \in Q_i$, the adjunct morphism $\mu_{\mathcal{W}_i^q} : \mathcal{W}_i^q \rightarrow Mod(\Sigma_i \hookrightarrow \Sigma)(T_{\Gamma_i/\Gamma}(\mathcal{W}_i^q))$ is an isomorphism, then $(\mathcal{S}_1 \otimes \mathcal{S}_2)^\bullet \cap Sen(\mathcal{L}_i) \subseteq \mathcal{S}_i^\bullet$.*

Proof. Let $\varphi \in (\mathcal{S}_1 \otimes \mathcal{S}_2)^\bullet \cap Sen(\mathcal{L}_i)$ and let $(\mathcal{W}_i, R_i) \in Mod(\mathcal{S}_i)$. By Theorem 6.3, for every $(\mathcal{W}_j, R_j) \in Mod(\mathcal{S}_j)$, we have that $\mathcal{F}_{(\mathcal{W}_j, R_j)}((\mathcal{W}_i, R_i)) \models \varphi$. As the adjunct morphism $\mu_{\mathcal{W}_i^q}$ is an isomorphism, for every $\iota : V \rightarrow \mathcal{W}_i$ there exists $\iota' : V \rightarrow T_{\Gamma_i/\Gamma}(\mathcal{W}_i) \times T_{\Gamma_j/\Gamma}(\mathcal{W}_j)$ such that $\iota = p_i \circ \iota'$ where p_i is the i -th projection map $p_i : T_{\Gamma_i/\Gamma}(\mathcal{W}_i^q) \times T_{\Gamma_j/\Gamma}(\mathcal{W}_j^{q'}) \rightarrow T_{\Gamma_i/\Gamma}(\mathcal{W}_i^q)$ for $q \in Q_i$ and $q' \in Q_j$. By hypothesis, for every $q \in Q_i$ and every $q' \in Q_j$, $\mathcal{F}_{(\mathcal{W}_j, R_j)}((\mathcal{W}_i, R_i)) \models_{\iota'}^{(q, q')} \varphi$. It is then easy to show by induction on the structure of φ that $(\mathcal{W}_i, R_i) \models_{\iota}^q \varphi$.

Theorem 6.4 extends to the synchronous product of transition systems, the standard condition of modularity based on the notions of hierarchical consistency (the adjunct functor is injective) and sufficient completeness (the adjunct functor is surjective) [11] which has been stated for specification enrichment in the algebraic specification framework (when specifications are expressed in **CEL**).

Example 6.1 *When dealing with formulas expressed in the logic **CEL** to label transitions, we often make restrictions on algebras denoting states. Indeed, to allow inductive proofs or for computability reasons, state-*

algebras are then restricted to reachable⁷ or some quotients of the ground term algebra. Let us suppose for the below counter-example of the conditions given in Theorem 6.4, that we restrict our approach to state-algebras defined by reachable algebras. Let us consider the two following transition systems \mathcal{S}_1 and \mathcal{S}_2 defined respectively over the two following signatures \mathcal{L}_1 and \mathcal{L}_2 :

$$\begin{aligned} \Sigma_1 &= \left\{ \begin{array}{l} S = \{\text{nat}\}, \\ F = \left\{ \begin{array}{l} 0 : \rightarrow \text{nat}; \\ s : \text{nat} \rightarrow \text{nat}, \\ + : \text{nat} \times \text{nat} \rightarrow \text{nat} \end{array} \right\}, \\ P = \emptyset \end{array} \right\}, \\ \Sigma_2 &= \left\{ \begin{array}{l} S = \{\text{nat}\}, \\ F = \{0 : \rightarrow \text{nat}; s, p : \text{nat} \rightarrow \text{nat}\}, \\ P = \emptyset \end{array} \right\}, \\ A_1 = A_2 &= \{a\} \end{aligned}$$

Let us define \mathcal{S}_1 and \mathcal{S}_2 as follows:

- $\mathcal{S}_1 = (\{q_1, q_2\}, \{q_1 \xrightarrow{a, \varphi_1, \delta_1} q_2\})$ where $\varphi_1 = (s(x) = s(y) \Rightarrow x = y) \wedge x + 0 = x \wedge x + s(y) = s(x + y)$ and $\delta_1 = \emptyset$.
- $\mathcal{S}_2 = (\{q'_1, q'_2\}, \{q'_1 \xrightarrow{a, \varphi_2, \delta_2} q'_2\})$ where $\varphi_2 = (s(x) = s(y) \Rightarrow x = y) \wedge s(p(x)) = x \wedge p(s(x)) = x$ and $\delta_2 = \emptyset$.

By definition of \mathcal{S}_1 (resp. \mathcal{S}_2), the unique \mathcal{S}_1 -model (resp. \mathcal{S}_2 -model) is (\mathcal{W}_1, R_1) (resp. (\mathcal{W}_2, R_2)) where $\mathcal{W}_1^{q_1} = \mathcal{W}_1^{q_2} = \mathbb{N}$ (resp. $\mathcal{W}_2^{q'_1} = \mathcal{W}_2^{q'_2} = \mathbb{Z}$). On the contrary, by construction, in $\mathcal{S}_1 \otimes \mathcal{S}_2$, we have the transition $(q_1, q'_1) \xrightarrow{a, \varphi', \delta'} (q_2, q'_2)$ where $\varphi' = \varphi_1 \wedge \varphi_2$ and $\delta' = \emptyset$, and then all the $\mathcal{S}_1 \otimes \mathcal{S}_2$ -models satisfy $\mathcal{W}^{(q_1, q'_1)} = \mathcal{W}^{(q_2, q'_2)} = \mathbb{Z}$. Consequently, the modal formula $\varphi' \Rightarrow \Box_a(\forall x. \exists y. x + y = 0)$ belongs to $(\mathcal{S}_1 \otimes \mathcal{S}_2)^\bullet$ but not to \mathcal{S}_1^\bullet . The reason is $F_{(\mathcal{W}_2, R_2)}(\mathcal{W}_1^{q_1}) = \mathbb{Z} \times \mathbb{Z}$. Therefore, the adjunct functor $\mu_{\mathcal{W}_1^{q_1}}$ is injective but not surjective.

This counter-example is of course a toy example but that lets prefigure the encountered problems with non-conformity properties when dealing with real complex systems.

7 Conclusion

In this paper, our main contributions are twofold. First, we have formally defined the notion of emergent properties for reactive component-based systems

⁷A Σ -algebra is *reachable* if, and only if the unique Σ -morphism $\mu : T_F \rightarrow \mathcal{A}$ is surjective, that is all the values in \mathcal{A} are denoted by the evaluation of a ground term.

described by transition systems and combined together through the synchronous product. Then, we have studied some general conditions that enable us to obtain global systems lacking of non-conformity properties which have been recognized as being the cause of bad interactions between components. These conditions are based on the notion of the category theory of adjunction. Hence, they extend to reactive systems the standard conditions of modularity based on the notions of hierarchical consistency and sufficient completeness that have been established in the algebraic specification setting. We are pursuing this work by studying how to apply algebraic refinement techniques as a basic incremental method dedicated to design and analysis of complex reactive systems. Indeed, one of the main problem when dealing with the design of complex system, is that emergent properties can appear at different levels of design. We then observe that some emergent properties may be inherited from higher to lower levels while some others may disappear. This leads us up to the conclusion that emergent properties should be addressed at the levels they emerge from.

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