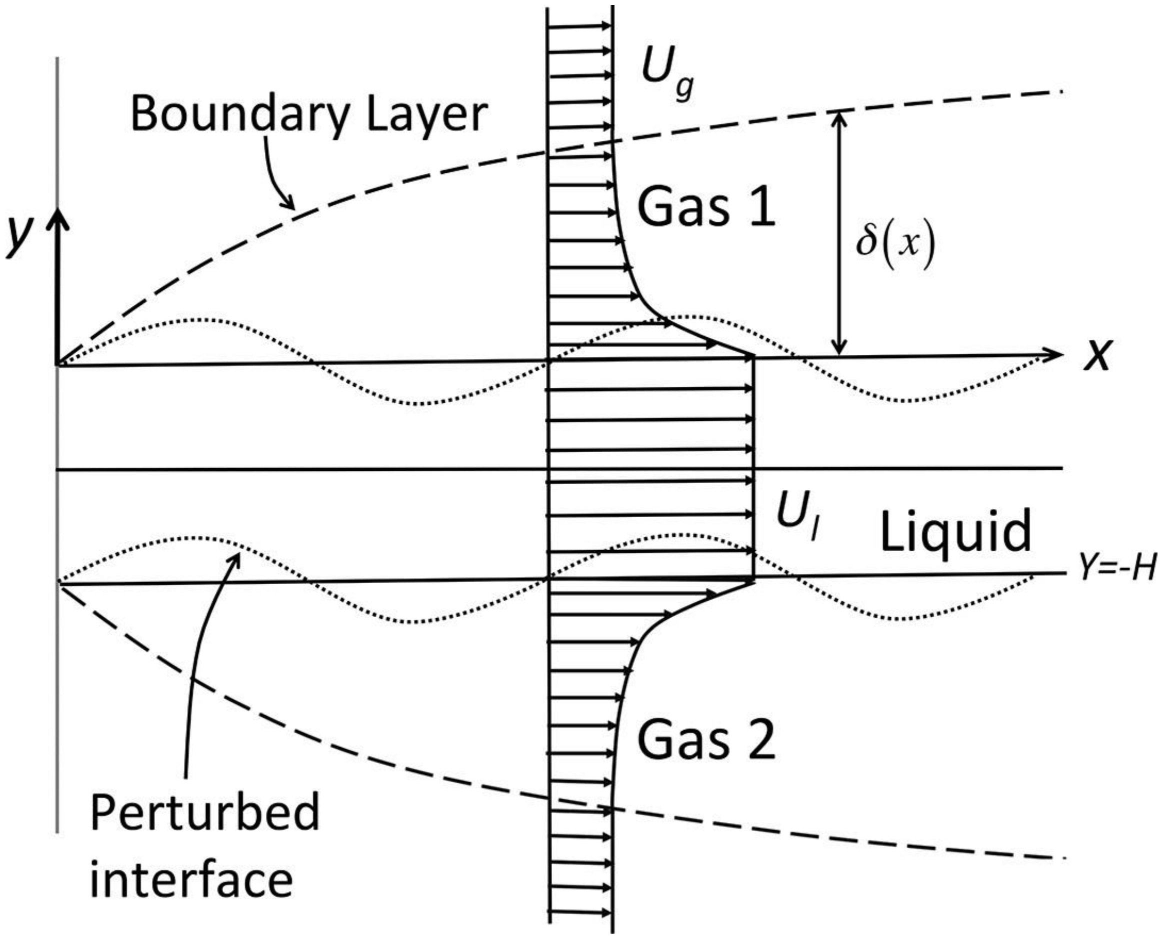

Gas Boundary Layer Effects on Liquid Sheet Stability

SUPERVISED LEARNING PROJECT

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Abstract

This project investigates the stability of a moving planar liquid sheet in the presence of a gas boundary layer, focusing on computational analysis to reproduce graphs comparing theoretical predictions with experimental results from the study by Tammisola et al. The primary task was solving the third-order differential equation for the sheet-normal disturbance velocity to evaluate the impact of the gas boundary layer.

The results confirmed that including the gas boundary layer significantly dampens the growth rates of instabilities compared to inviscid Kelvin-Helmholtz instability predictions, aligning closely with experimental observations. The findings highlight the critical role of viscous effects in the gas phase on the dynamics of the liquid sheet, demonstrating the importance of considering these effects in stability analyses for industrial applications like atomization. This work contributes to better designing and optimizing industrial spraying systems by providing more accurate predictions under real-world conditions. Future work will aim to incorporate nonlinear effects and more complex flow conditions.

Introduction

In numerous industrial processes, such as combustion, pharmaceutical manufacturing, and food processing, the atomization of liquids into fine droplets plays a critical role. The ability to control droplet size and distribution directly impacts the efficiency and quality of such processes. Central to understanding atomization is studying the dynamics of liquid sheets—thin liquid films that break into droplets under various conditions. Historically, most analyses of liquid sheet dynamics have relied on simplified assumptions that ignore the complexities introduced by the surrounding gas phase.

Recent advancements in fluid dynamics have highlighted the significant influence of gas boundary layers on the stability of moving liquid sheets. These boundary layers, which form where the liquid and gas phases interact, can dampen or amplify instabilities in the liquid sheet, thereby affecting the breakup process and the resultant droplet size. This project explores the stability of moving planar liquid sheets, explicitly focusing on the effects of the gas boundary layer. By integrating theoretical models with experimental observations, this study seeks to understand fluid dynamics better and refine predictive models used in industrial applications.

This supervised learning project is guided by the foundational work of Professor Mahesh S. Tirumkudulu, whose study on the influence of the gas boundary layer provides a comprehensive framework for analyzing the linear stability of liquid sheets. Building on this groundwork, this report presents a series of computational experiments designed to validate and extend these theoretical insights, offering new perspectives on the interplay between liquid viscosity, gas phase dynamics, and interfacial tensions.

Problem Statement

The primary objective of this project is to reproduce the comparative graphs from the research, which include theoretical predictions aligned with the experimental results conducted by Tammisola et al. These comparisons will also incorporate simulation outcomes from both the Stokes and Sakiadis models alongside the classical inviscid theory proposed by Squire. The specific parameters utilized in these analyses are Ul = U = 8.66 m/s for the liquid velocity, Ug = 0 for the gas velocity, and H = 0.672 mm for the liquid sheet thickness. A vital aspect of this project is adapting the model to consider liquid sheet thicknesses within the **micron range**, which presents unique challenges in capturing the subtle dynamics influenced by the gas boundary layer's effects on stability.

This adaptation will involve recalibrating the model to account for the decreased scale and adjusting for the increased influence of surface tension and other microscale effects on the stability of the liquid sheet. The results will be critical for understanding the stability mechanisms at smaller scales, which are relevant in various applications such as microfluidics and small-scale atomization processes.

Method Utilised

COMPUTATIONAL FRAMEWORK

The stability analysis of a moving planar liquid sheet influenced by a surrounding gas boundary layer was conducted using a computational approach in MATLAB. The domain for the simulation was set to a sufficiently large value to ensure an accurate representation of boundary effects, which is crucial for modeling fluid dynamics in open systems.

PARAMETER DEFINITION

The simulation involved defining several key physical parameters:

- **Velocities:** The liquid velocity (U) was set to 8.66 m/s, normalized to form dimensionless parameters for the gas velocity (Ug) and the liquid velocity (Ul).
- **Physical Properties:** The density and viscosity of the gas (rhog and mug) and liquid (rhol) were specified, along with the surface tension (gamma).
- **Non-dimensional Numbers:** Reynolds (Reg) and Weber (Weg) numbers were calculated based on the liquid sheet thickness (H), which is crucial for characterizing the fluid flow regime and the influence of surface tension.

BLASIUS BOUNDARY LAYER SOLUTION

The boundary layer flow was modeled using the Blasius equation, a fundamental relation describing the velocity profile near a flat plate in laminar flow. This was solved using bvp4c, a MATLAB library for numerical computing, which provided a function handle (ublasius) representing the velocity profile across the boundary layer. Boundary conditions were applied at the domain's beginning (x = 0) and far-field (x = infinity) to simulate realistic flow conditions.

INITIAL STABILITY PREDICTION

An initial estimate of the flow instability was obtained using the Kelvin-Helmholtz inviscid stability theory. This involved solving for the roots of a complex equation that predicts the growth rate of instabilities based on the flow velocities and physical properties of the fluid interface.

ITERATIVE SOLUTION FOR PERTURBATION ANALYSIS

The core of the simulation involved an iterative process where the perturbed state of the velocity field was calculated for different frequencies using a differential operator defined in bvp4c. The process was repeated until the change in computed wave speed fell below a predetermined threshold, ensuring convergence to a stable solution. The boundary conditions were dynamically adjusted during the iterations to reflect changes in the perturbation characteristics.

POST-PROCESSING AND VISUALIZATION

After computing the dispersion relationship for various conditions, the results were plotted to visually assess and compare the growth rates of instabilities under different simulation parameters. This visualization helps in understanding the stability characteristics of the liquid sheet and the influence of the gas boundary layer on these characteristics.

ADDITIONAL COMPUTATIONAL FUNCTIONS

Several auxiliary functions were used to calculate boundary layer thickness and its derivatives and to apply accurate boundary conditions for the perturbation analysis. These functions ensured that the mathematical model accurately represented the physical conditions across the fluid interface and boundary layer.

Results & Discussions

RESULTS

In this project, the stability of a moving planar liquid sheet in the presence of a gas boundary layer was analyzed using a computational approach derived from the linear stability equations that account for the gas boundary layer while ignoring the viscosity in the liquid phase. The primary goal was to reproduce the graphs from the study by Tammisola et al., which involved comparing theoretical predictions with experimental and other simulation results.

Solving Equation 28 and Determining $\hat{\boldsymbol{v}}(\mathbf{0})$

The third-order differential equation (Equation 28 from the referenced paper), which describes the sheet-normal disturbance velocity, was fully solved. The value of (the disturbance velocity at the interface) was computed as a boundary condition. This value was crucial in further calculations, particularly in evaluating the dispersion relations and assessing the stability of the sinuous mode of the liquid sheet.

Computational Findings

The simulation results indicated a significant reduction in the growth rates of instabilities when the gas boundary layer's impact was included, consistent with the findings of Tammisola et al. This demonstrated the damping effect of the gas boundary layer on the instabilities that inviscid models would otherwise predict. Specifically, the growth rates were much lower than predicted by the classical Kelvin-Helmholtz instability theory, emphasizing the importance of considering viscous effects in the gas phase.

DISCUSSION

Interpretation of Results

The analysis showed that the gas boundary layer plays a critical role in stabilizing the liquid sheet by dampening the growth of sinuous instabilities. This effect was quantified by comparing the computational results with the inviscid theory predictions and experimental data. The successful reproduction of the graphs and the close alignment with experimental observations validated the computational model and methods used in this study.

Comparison with Theoretical and Experimental Data

The results were consistent with the experimental measurements of sinuous wave growth rates, which were lower than those predicted by the inviscid theory, aligning with the simulation results for viscous boundary layer calculations. The computational model effectively captured the essential physics of the boundary layer's impact, showing lower growth rates that closely matched the experimental data, confirming the damping effect introduced by the gas viscosity.

Conclusion

In conclusion, the computational analysis effectively mapped the stability landscape of moving planar liquid sheets, revealing critical insights into the role of the gas boundary layer. These insights validate and build upon existing theoretical models, offering a refined tool for engineering applications that demand precise fluid dynamics control. The study paves the way for future research on nonlinear effects and real-world conditions, aiming to bridge the gap between theoretical predictions and experimental realities.

References

1. **Tirumkudulu, M. S.** (2019). Influence of the Gas Boundary Layer on the Stability of a Moving Planar Liquid Sheet. *Industrial & Engineering Chemistry Research, 58*, 7633-7639. https://doi.org/10.1021/acs.iecr.8b02840
2. **Tammisola, O., Sasaki, A., Lundell, F., Matsubara, M., & Soderberg, L. D.** (2011). Stabilizing effect of surrounding gas flow on a plane liquid sheet. *Journal of Fluid Mechanics, 672*, 5-32. https://doi.org/10.1017/S002211201100017X
3. **Squire, H.** (1953). Investigation of the instability of a moving liquid film. *British Journal of Applied Physics, 4*(6), 167-169. https://doi.org/10.1088/0508-3443/4/6/302
4. **Hauke, G., Dopazo, C., Lozano, A., & Barreras, F.** (2001). Linear Stability Analysis of a Viscous Liquid Sheet in a High-Speed Viscous Gas. *Flow, Turbulence and Combustion, 67*, 235-265. https://doi.org/10.1023/A:1016690424895
5. **Lozano, A., Barreras, F., Hauke, G., & Dopazo, C.** (2001). Longitudinal instabilities in an air-blasted liquid sheet. *Journal of Fluid Mechanics, 437*, 143-173. https://doi.org/10.1017/S0022112001003853
6. **Crapper, G., Dombrowski, N., & Jepson, W.** (1975). Wave growth on thin sheets of non-Newtonian liquids. *Proceedings of the Royal Society of London. Series A, Mathematical and Physical Sciences, 342*(1631), 225-236. https://doi.org/10.1098/rspa.1975.0019
7. **Lefebvre, A. H., & McDonell, V. G.** (2017). *Atomization and Sprays* (2nd ed.). CRC Press.