Q#1

$L1 = \{ a^n b a^{2n} \mid n \geq 0 \}$

Let L1 be infinite regular language has DFA contains finite states.

$$\text{Let } w = a^P b a^{2P} \in L1$$

Decompose w in xyz

$$x = a^\alpha \qquad \alpha \geq 0$$
$$y = a^B \qquad B \geq 1$$
$$z = a^{P-\alpha-B}\, b\, a^{2P}$$

Choose i for which $xyz \notin L1$

$$xy^iz = a^\alpha \cdot a^{Bi} \cdot a^{P-\alpha-B}\, b a^{2P}$$
$$= a^{P+Bi-B}\, b\, a^{2P}$$

$$P + B(i-1) = 2P$$

$B = 1$

$$P + (i-1) = 2P$$

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Q # 02

$$L_2 = \{ a^i b^j a^k \mid k > i + j \}$$

Let L_2 is regular infinite language has DFA contains p finite states.

$$w = a^p b^p a^{2p+1} \in L_1$$

Decompose

$$x = a^\alpha \qquad \alpha \geq 0$$

$$y = a^\beta \qquad \beta \geq 1$$

$$z = a^{p-\alpha-\beta} b^p a^{2p+1}$$

choose i for which xy^iz does not belongs to L_2

$$xy^iz = a^{p+\beta(i-1)} b^p a^{2p+1}$$

Check condition

$$k > i + j$$

$\beta \geq 1$

$$\cancel{2p} + 1 > \cancel{p} + \beta(i-1) + \cancel{p}$$

$$1 > \beta(i-1)$$

only when $i \leq 1$

let $i = 2$

its not regular for $i > 1$

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Q#03

$$L_3 = \{ a^i b^j \mid j = i \text{ or } j = 2 \}$$

let L3 is regular finite language contains DFA with P finite states

$$w = a^P b^{2P} \in L_3$$

Decompose

$x = a^{\alpha}$ $\quad \alpha \geq 0$

$y = a^{B}$ $\quad B \geq 1$

$$z = a^{P-\alpha-B} b^{2P}$$

Choose i for which xy^iz does not belongs to L3

$$xy^iz = a^{\alpha} \cdot a^{Bi} \cdot a^{P-\alpha-B} b^{2P}$$

$$xy^iz = a^{P+B(i-1)} b^{2P}$$

$$P + B(i-1) = 2P$$

For $i = 1$ it will have proof but when $i > 1$ it become disproof

$$P+1 \neq 2P$$

L3 is not regular.

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Q4

$L_4 = \{ a^i b^j \mid j \text{ is multiple of } i \}$

Let L_4 is infinite regular language

Let

$W = a^P b^{KP}$ where K is any multiple

Consider $K = P$

$a^P b^{P^2}$

$x = a^\alpha$

$y = a^B$

$z = a^{P-\alpha-B} \; b^{P^2}$

choose i for which $xy^iz \notin L_4$

$xy^iz = a^{P+B(i-1)} \; P^2$

$B = 1$

$P^2 = P + B(i-1)$

when $i = 1$

$P^2 = P + (1-1)$

$P^2 = P$ Prove

when $i > 1$

$i = 2$

$P^2 \neq P + 1$

So it fails when $i > 1$

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Q # 05

$L_2 = \{ x \in \{a, b\}^* \; n_a(x) < 2n_b(x) \}$

Let L5 is regular finite language

$$w = a^p b^p \in L5$$

Decompose

$$x = a^\alpha \qquad \alpha \geq 0$$
$$y = a^\beta \qquad \beta \geq 1$$

$$z = a^{p-\alpha-\beta} b^p$$

Choose i for which $xy^iz \notin L5$

$$xy^iz = a^{p-\alpha-\beta} \cdot a^\alpha \cdot a^{\beta i} b^p$$
$$= a^{p+\beta(i-1)} b^p$$

Condition

$$p + \beta(i-1) < 2p$$

$\beta = 1$

only when $i = +1$

for $i > 1$

$$p + (1-1) \nless 2p$$

So it is not regular.

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Q # 06

Already solved in question 1

Q # 07

Let L_7 belongs to regular language

$w = a^p b a^p b$

Decompose

$x = a^\alpha \quad a \geq 0$

$y = a^B \quad B \geq 1$

$z = a^{p-\alpha-B} \, b \, a^p b$

choose xy^iz ~~$\in$~~ L_7

$xy^iz = a^{p+B(i-1)} \, b \, a^p b$

$p + B(i-1) = p$

$B = 1$

$p + (i-1) = p$

Regular for $i = 1$

For $i > 1$ $(i = 2)$

$p+1 \neq p$ so it is not regular.

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Q # 08

$L8 = \{ xy \quad |x| = |y| \}$

Let L8 is infinite regular language

$w = a^P b^P \in L8$

Decompose

$x = a^\alpha \quad x \geq 0$

$y = a^B$

$z = a^{P-\alpha-B} b^P$

Choose i ~~to~~ xy^iz L8

$xy^iz = a^{P+B(i-1)} b^P$

$\cancel{P} = \cancel{P} + B(i-1)$

$0 = B(i-1)$

$0 = (i-1)$

if $i > 1$ false

it become irregular.

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Q # 09 $\{x \in (a,b)^* \mid n_a(x) = n_b(x)\}$

Let L_9

$$w = a^P b^P \in L_9$$

Decompose in xyz

$x = a^{\alpha}$ $\quad \alpha \geq 0$

$y = a^{B}$ $\quad B \geq 1$

$z = a^{P-\alpha-B}$

Choose i for which $xy^iz \notin L_9$

$$xy^iz = a^{P+B(i-1)} b^P$$

$$P = P + B(i-1)$$

$$0 = B(i-1)$$

for $i = 1$ true

$i > 1$ false

becomes irregular.

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Q# 10

$$\{ w \in \{a,b\}^* \mid w\#w \}$$

Let L10 is infinite regular Langu-
-age $w = a^p b \# a^p b$

Decompose

$a^\alpha = x \qquad \alpha \geq 0$

$y = a^B \qquad B \geq 1$

$z = a^{p-\alpha-B}$

Choose xy^iz belongs to L10

$xy^iz = a^{p+B(i-1)} b\#a^p b$

$\cancel{p} = \cancel{p} + B(i-1)$

$0 = B(i-1)$

for $i = 1$ true
for $i > 1$ false becomes
irregular.

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Q # 11

$L_{11} = \{ xcx^R \mid x \in \{a,b\}^* \}$

Let L_{11} be finite regular language

$$w = a^p b c b a^p \; L_{11}$$

$x = a^\alpha$ $\quad \alpha \geq 0$

$y = a^B$ $\quad B \geq 1$

$z = a^{p-\alpha-B}$

Choose xy^iz belongs to L_{11}

$$xy^iz = a^{p+B(i-1)} bc \underline{b a^p}$$

$B = 1$

$$\cancel{p} = \cancel{p} + B(i-1)$$

$$0 = B(i-1)$$

for $i = 1$ true

for $i > 1$ false becomes irregular.

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Q#12

$L_{12} = \{ w w^R \mid w \in \{a,b\}^* \}$

Let L_{12} be regular finite language

$$w = a^p b\, b a^p$$

$x = a^\alpha \quad \alpha \geq 0$

$y = a^B \quad B \geq 1$

$z = a^{p-\alpha-B}$

Choose $xy^i z \in L_{11}$

$$xy^i z = a^{p+B(i-1)} b\, b\, a^p$$

$$\cancel{p} = \cancel{p} + B(i-1)$$

$$0 = B(i-1) \qquad B = 1$$

for $i = 1$ true

for $i > 1$ false

↳ Non regular.

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