Theory of Automata

(CSAL3253)

Assignment no 4

Submitted to

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Submitted by

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Section: F3

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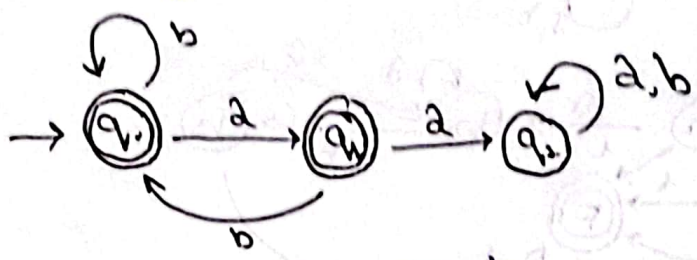
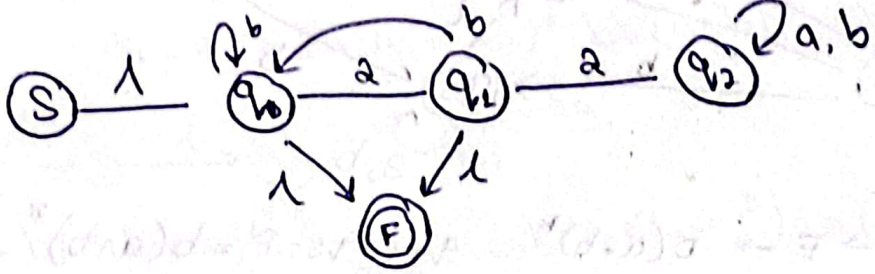
em 1:

Convert the following NFA, DFA to regular expression using state elimination method.

①



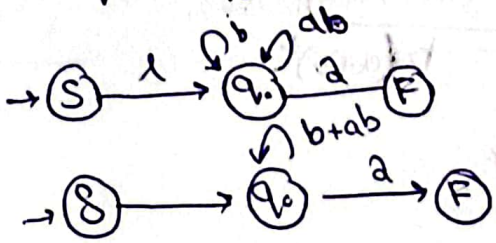
⇒



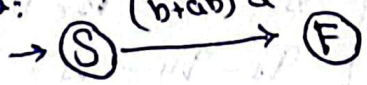
- Ignore trap state (q_2)

Remove q_1:

$q_0 \to q_1 \to F = 1a = a$, $q_0 \to q_1 \to q_0 = ab$

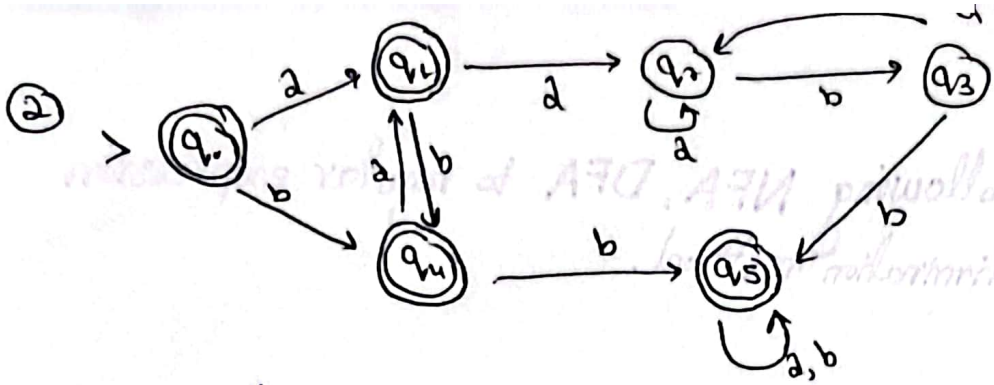


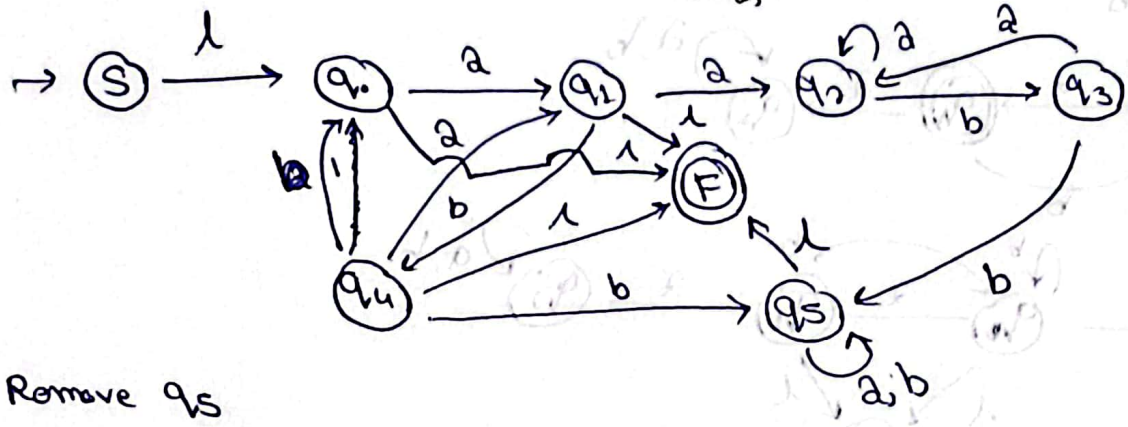
Remove q_0:



So

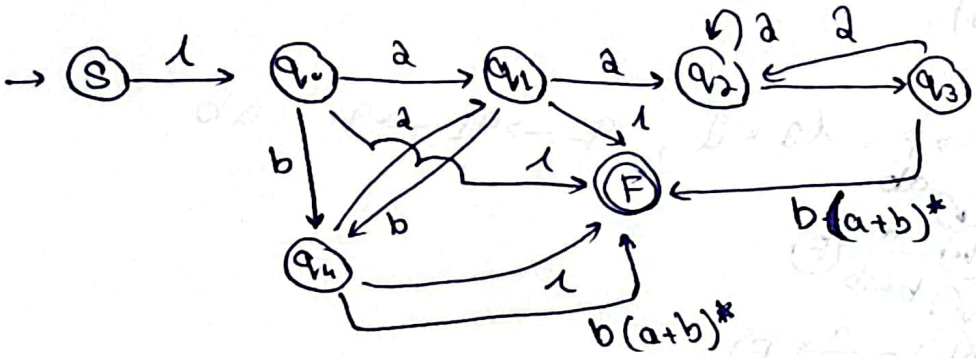
RE: $(b+ab)^*a$.





Remove q_5

$q_3 \to q_5 \to F \to b(a+b)^*$ $\quad q_4 \to q_5 - F = b(a+b)^*$



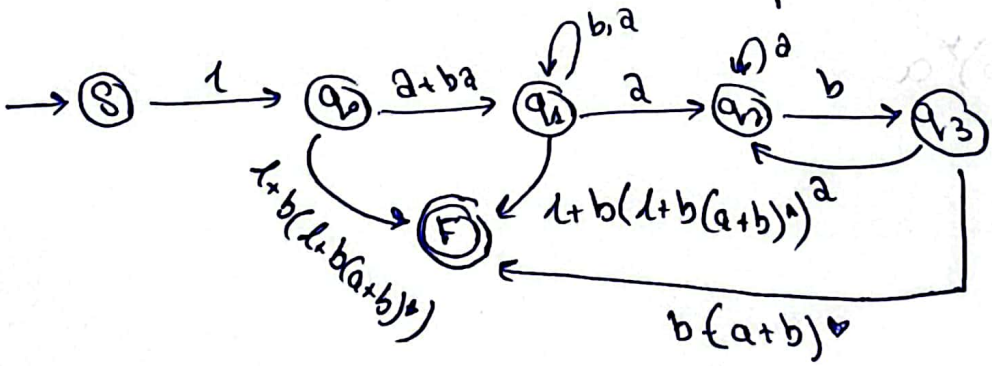
Remove q_4

$q_0 \to q_4 - F = b(\lambda + b(a+b)^*)$

$q_0 \to 4 \to q_1 = ba$

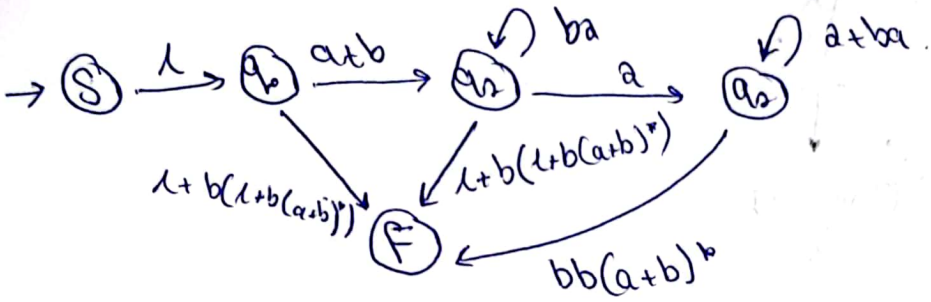
$q_1 - q_4 - F = b(\lambda + b(a+b)^*)$

$q_1 - q_4 - q_1 = ba$



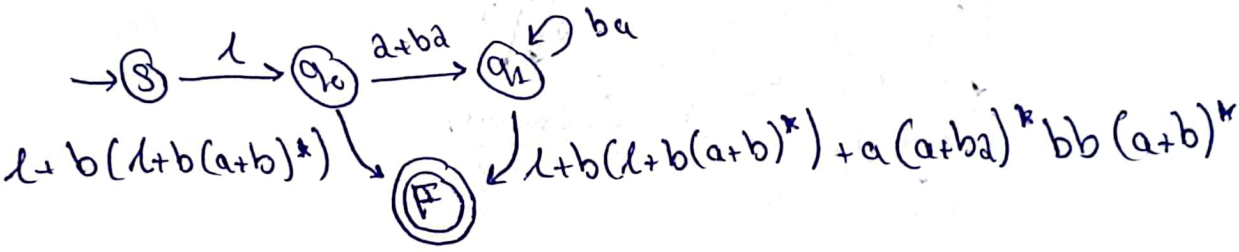
Remove q_3

$q_2 - q_3 - F = bb(a+b)^*$, $q_2 - q_3 - q_2 = ba$

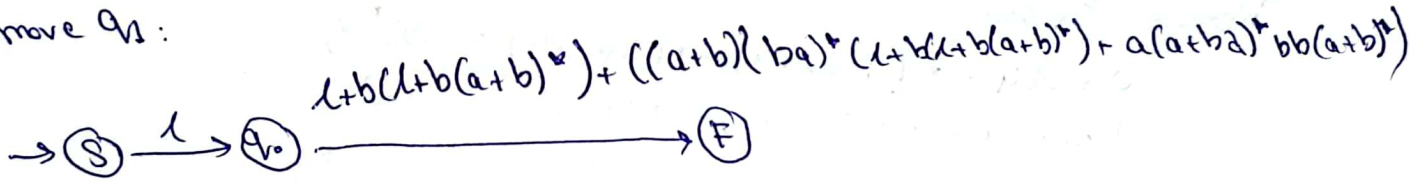


Remove q_2:

$q_1 - q_2 - F = a(a+ba)^* bb(a+b)^*$

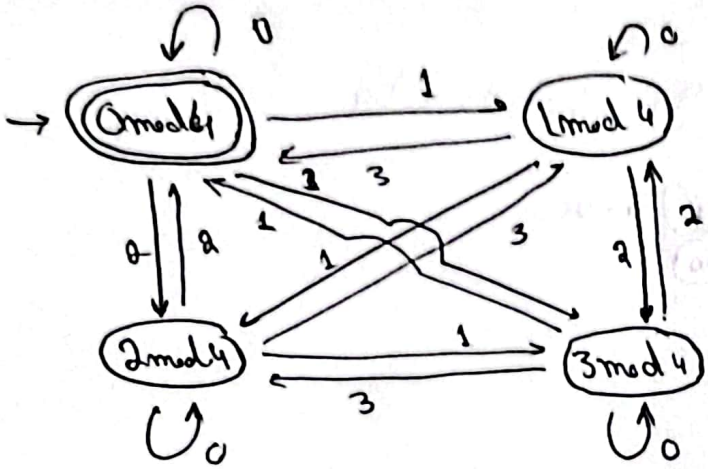


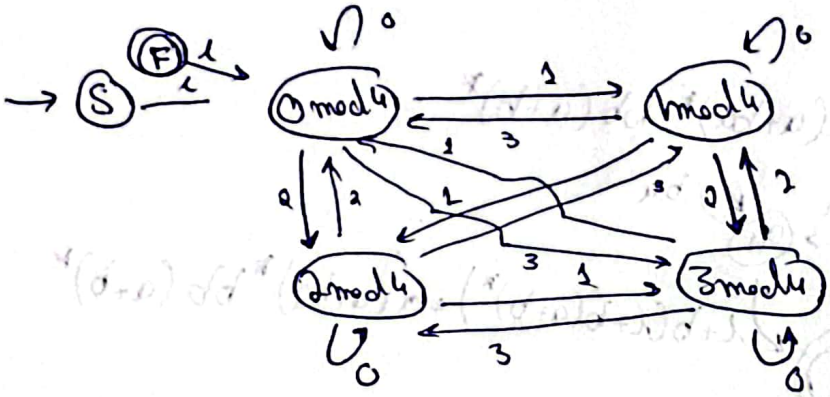
Remove q_1:



RE:

$$\lambda+b(\lambda+b(a+b)^*)+((a+ba)(ba)^*(\lambda+b(\lambda+b(a+b)^*)+a(a+ba)^* bb(a+b)^*))$$





Remove 3mod4:

$0 \bmod 4 \to (3 \bmod 4) \to (2 \bmod 4) = 30^*3$

$1 \bmod 4 \to (3 \bmod 4) \to (1 \bmod 4) = 20^*2$

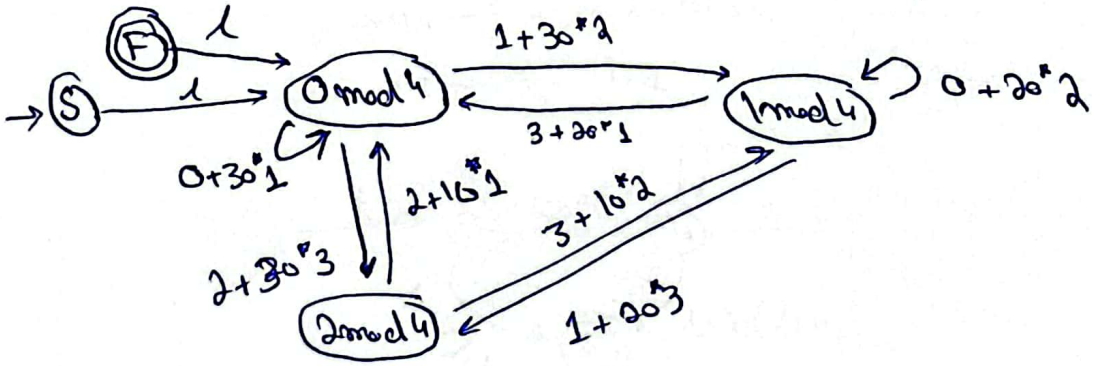
$0 \bmod 4 \to (3 \bmod 4) \to (0 \bmod 4) = 30^*1$

$2 \bmod 4 \to (3 \bmod 4) \to (2 \bmod 4) = 10^*3$

$0 \bmod 4 \to (3 \bmod 4) \to (1 \bmod 4) = 30^*2$

$2 - 3 - 0 = 10^*1$

$2 - 3 - 1 = 10^*2$



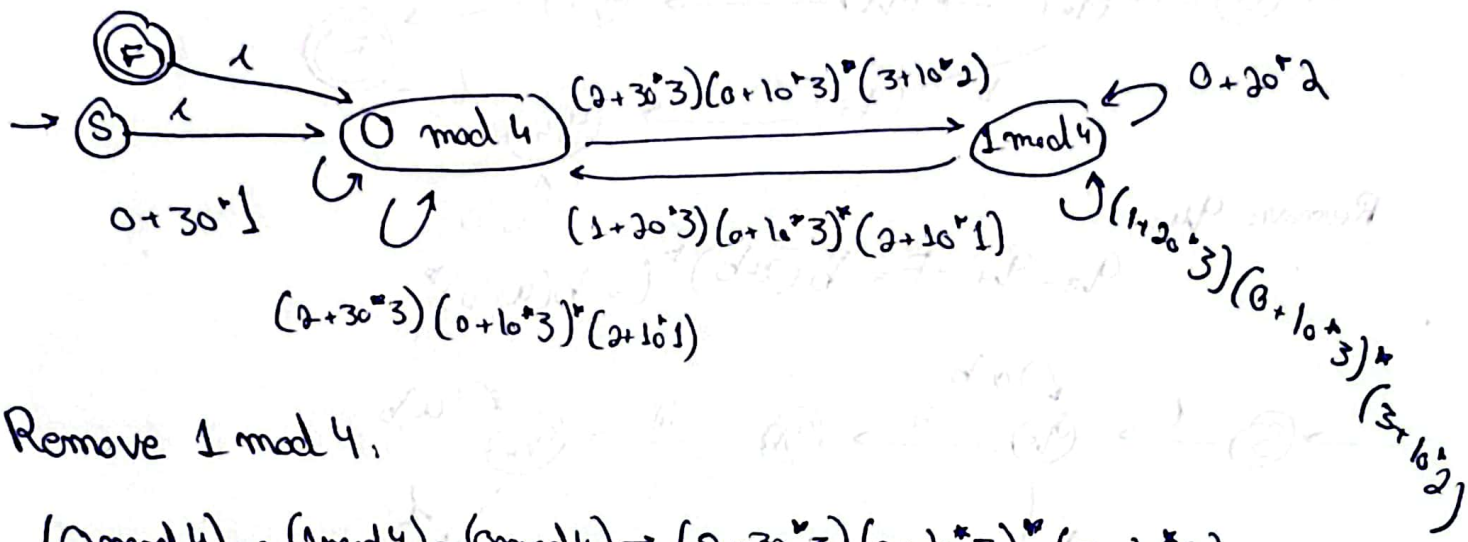
2 mod 4:

0 mod 4 — (2 mod 4) → (0 mod 4) = $(2+30^*3)(0+10^*3)^*(2+10^*1)$

0 mod 4 → (2 mod 4) → (1 mod 4) = $(2+30^*3)(0+10^*3)^*(3+10^*2)$

1 mod 4 → (2 mod 4) → (1 mod 4) = $(1+20^*3)(0+10^*3)^*(3+10^*2)$

1 mod 4 → (2 mod 4) → (0 mod 4) = $(1+20^*3)(0+10^*3)(2+10^*1)$



Remove 1 mod 4:

(0 mod 4) → (1 mod 4) - (0 mod 4) = $(2+30^*3)(0+10^*3)^*(3+10^*2)$

$(0+20^*2)+(1+20^*3)(0+10^*3)^*(3+10^*2))^*$

$(1+20^*3)(0+10^*3)^*(2+10^*1)$

→ (S) —1→ (0 mod 4) —1→ (F)

$(2+30^*3)(0+10^*3)^*(3+10^*2)(0+20^*2)+(1+20^*3)(0+10^*3)^*(2+10^*2))^*$

$(1+20^*3)(0+10^*3)^*(2+10^*1)+(2+30^*3)(0+10^*3)^*(2+10^*1)$

Remove 0 mod 4

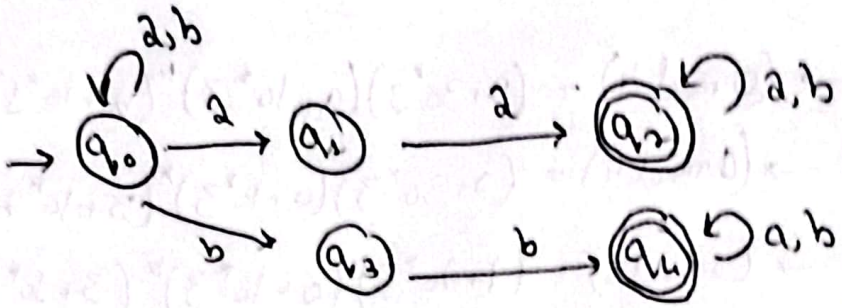
RE = $((0+30^*1)+((2+30^*3)(0+10^*3)^*(2+10^*1))+((2+30^*3)(0+10^*3)^*$

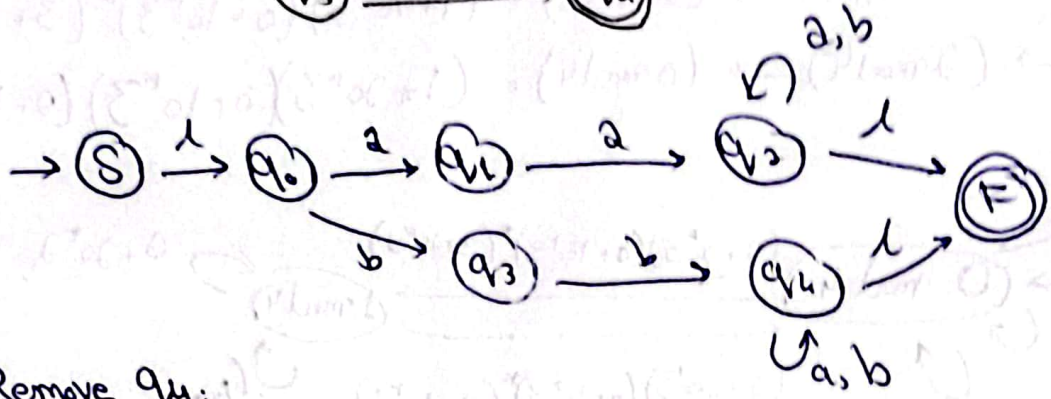
$(3+10^*2)((0+20^*2)+(1+20^*3)(0+10^*3)^*(3+10^*2))^*(1+20^*3)$

$(0+10^*3)^*(2+10^*1))^*$

→ (S) → (F)

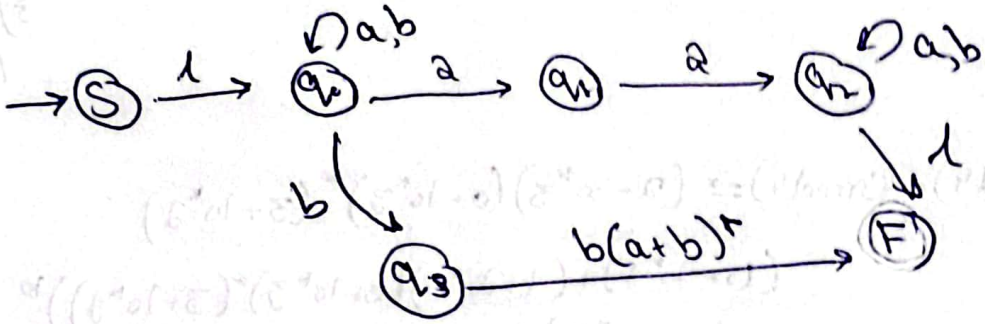
RE:





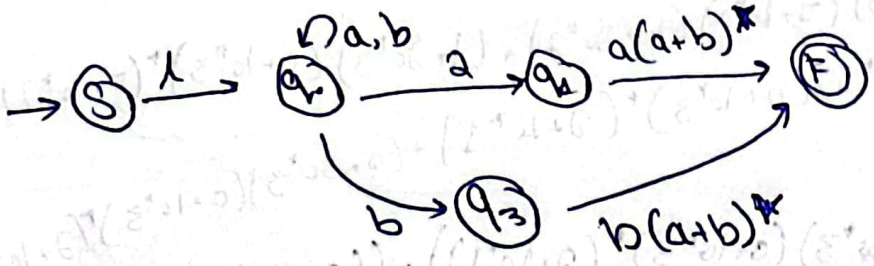
Remove q_4:

$$q_3 - q_4 - F = b(a+b)^* \lambda \cdot b(a+b)^*$$



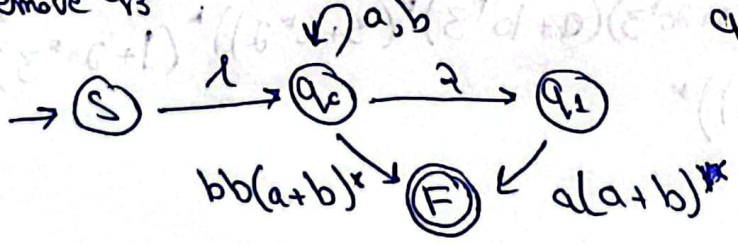
Remove q_2

$$q_1 - q_2 - F = a(a+b)^*$$



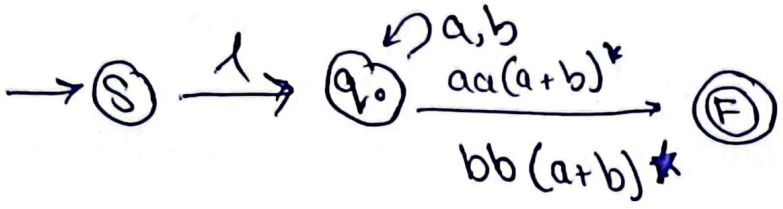
Remove q_3

$$q_0 \to q_3 - F = bb(a+b)^*$$



Remove q_1

$q_0 \rightarrow q_1 - F = aa(a+b)^*$



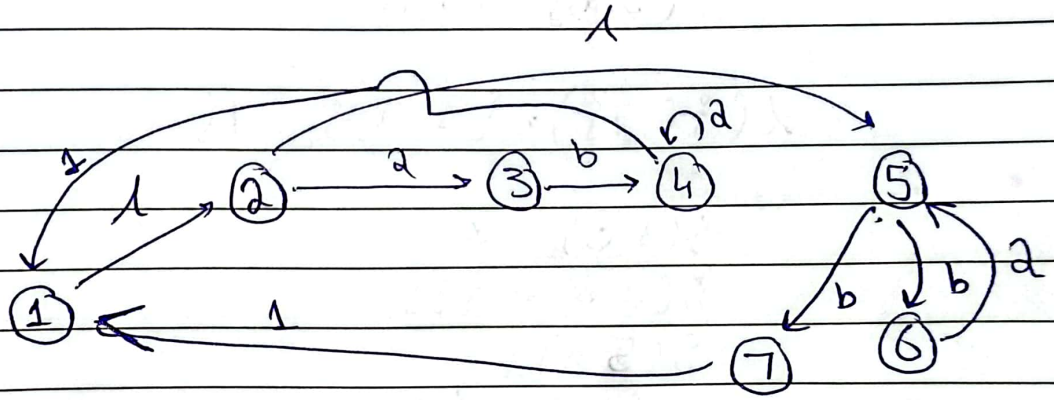
Problem 2

Consider the following transition table, draw the diagram and answer the following question:

q	δ(q, a)	δ(q, b)	ℓ(q, λ)
1	(1)	(2)	(2)
2	(3)	∅	(5)
3	∅	(4)	∅
4	(4)	∅	(1)
5	∅	(6, 7)	∅
6	(5)	∅	∅
7	∅	∅	(1)

Find:

a A({2,3})



Find:

a A({2,3})

From 2 → {2,5}

3 → {3}

λ({2,3}) = {2,3,5}

b) λ({1})

λ({1}) = {1,2,5}

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c $\lambda(\{3,4\})$

$\lambda(\{3,4\}) = \{1,2,3,4,5\}$

d $\delta^*(1, ba)$

$\lambda(\{1\}) = \{1,2,5\}$

input b

$\delta(1,b) = \emptyset$

$\delta(2,b) = \emptyset$

$\delta(5,b) = \{6,7\}$

$\lambda(\{6,7\}) = \{1,2,5,6,7\}$

$\delta(1,a) = \emptyset$

$\delta(2,a) = \{3\}$

$\delta(5,a) = \emptyset$

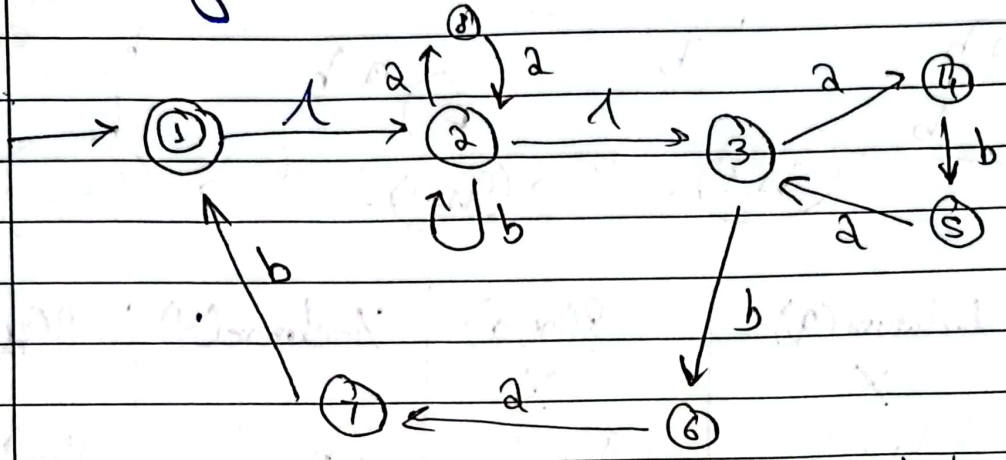
$\delta(6,a) = \{5\}$

$\delta(7,a) = \emptyset$

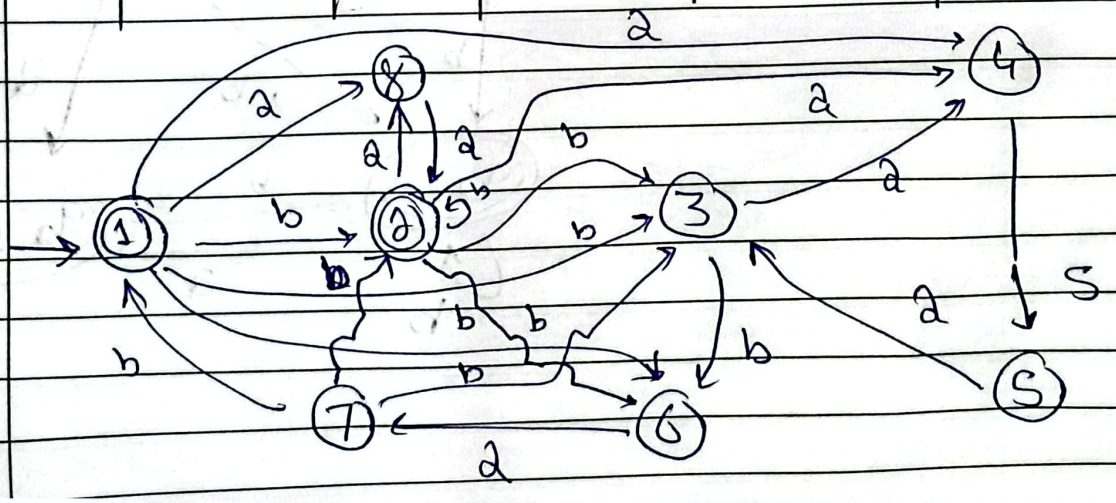
$\delta(1,ba) = \{3,5\}^*$

Problem 5

Convert following NFA-λ to NFA by removing λ transition.



States q	λ-closure(q) x	$\delta(x,a)$ y	λ-closure(y)	$\delta(x,b)$	λ-closure(z)
1	{1,2,3}	{8,4}	{8,4}	{2,6}	{2,3,6}
2	{2,3}	{8,4}	{8,4}	{2,6}	{2,3,6}
3	{3}	{4}	{4}	{6}	{6}
4	{4}	ϕ	ϕ	{5}	{5}
5	{5}	{3}	{3}	ϕ	ϕ
6	{6}	{7}	{7}	ϕ	ϕ
7	{7}	ϕ	ϕ	{1}	{1,2,3}
8	{8}	{2}	{2,3}	{ }	ϕ



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e $\delta^*(1, ab)$

$\lambda(\{1\}) = \{1, 2, 5\}$

Input a

$\delta(1, a) = \emptyset$

$\delta(2, a) = \{3\}$

$\delta(5, 2) = \emptyset$

$\lambda(\{3\}) = \{3\}$

Input b.

$\delta(3, b) = \{4\}$

$\lambda(\{4\}) = \{1, 2, 4, 5\}$

$\delta(1, ab)^* = \{1, 2, 4, 5\}$,

f $\delta^*(1, ababa)$

State	ε^*	a	ε^*
1	1	$\emptyset$	$\emptyset$
	2	3	3
	5	$\emptyset$	$\emptyset$

State	ε^*	b	ε^*
3	3	4	4
			1
			2
			5

State	ε^*	a	ε^*
1	1	$\emptyset$	
	2	3	3
	5	$\emptyset$	
2	2	$\emptyset$	
	5	$\emptyset$	
4	4	4	1, 2, 4, 5

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	1	ϕ	ϕ
	2	3	3
	5	ϕ	ϕ
5	5	ϕ	ϕ

$\wedge(\delta(1,a)) = 3$

$\wedge(\delta(2,a)) = \phi$

$\wedge(\delta(4,a)) = 1,2,3,4,5$

$\wedge(\delta(5,a)) \neq \phi$

$\wedge(\delta(6,a)) = 5$

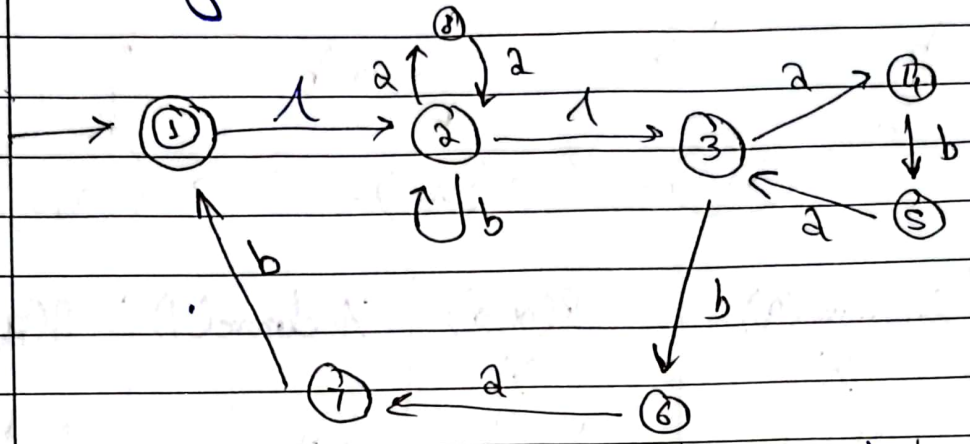
$\wedge(\delta(7,a)) = 3$

$= \{1,2,3,4,5\}$

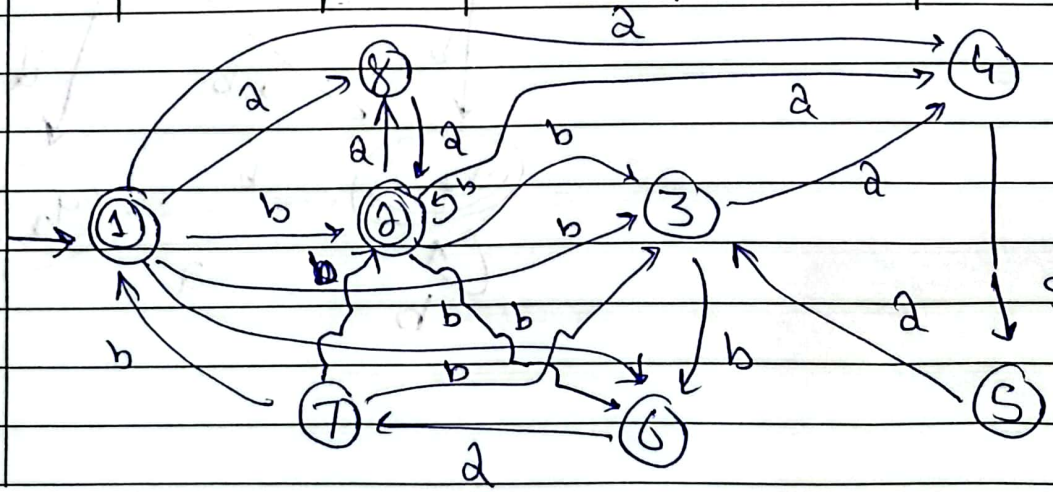
state	ε^k	b	ε^k
1	1	ϕ	
	2	ϕ	
	5	6	6
		7	7
			2
			5
			7
2	2	ϕ	
	5	6	6
		7	1,2,5
3			1,2,4
4	1	ϕ	
	2	ϕ	
	4	ϕ	
	5	6	6
		7	1,2,5
5	5	6	6
		7	1,2,5

Problem 3

Convert following NFA-λ to NFA by removing λ transition.



States q	λ-closure(q) x	$\delta(x,a)$ y	λ-closure(y)	$\delta(x,b)$	λ-closure(z)
1	{1,2,3}	{8,4}	{8,4}	{2,6}	{2,3,6}
2	{2,3}	{8,4}	{8,4}	{2,6}	{2,3,6}
3	{3}	{4}	{4}	{6}	{6}
4	{4}	ϕ	ϕ	{5}	{5}
5	{5}	{3}	{3}	ϕ	ϕ
6	{6}	{7}	{7}	ϕ	ϕ
7	{7}	ϕ	ϕ	{1}	{1,2,3}
8	{8}	{2}	{2,3}	{ }	ϕ



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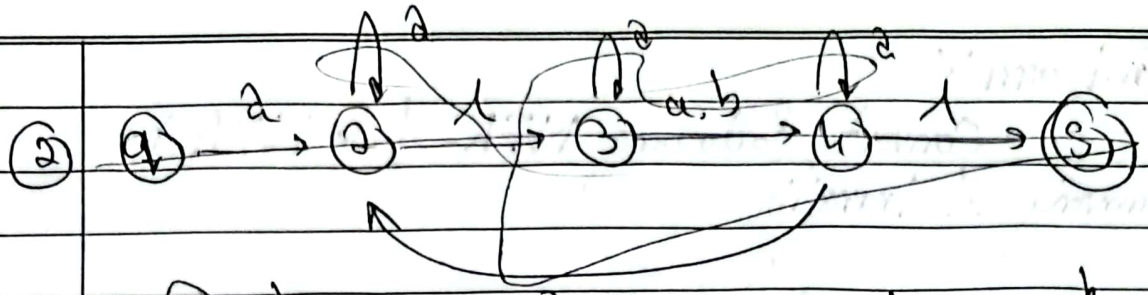
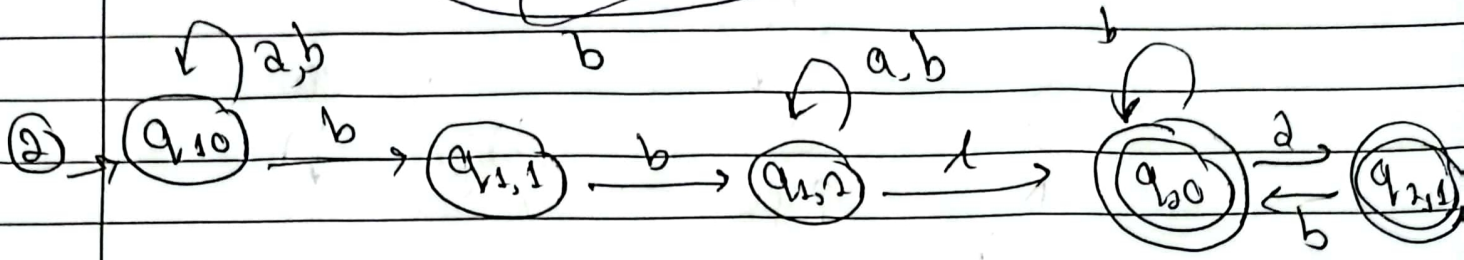
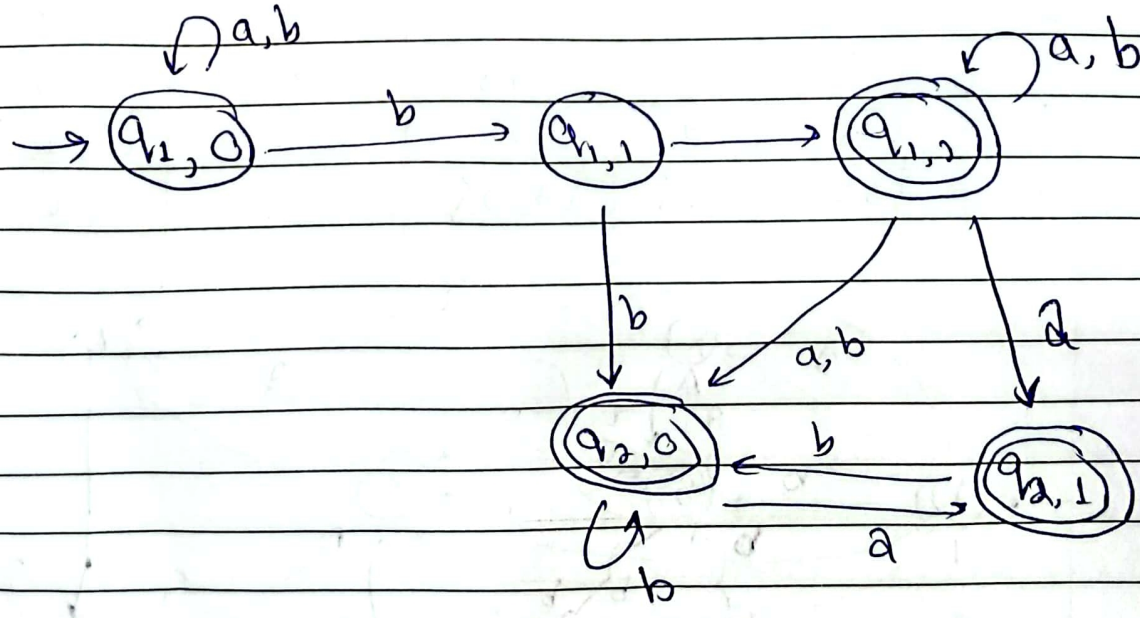




States q	λ-closure(q) X	δ(x, a) Y	λ-closure(Y)	δ(x, b)	λ-cl. z
$q_{1,0}$	$\{q_{1,0}\}$	$\{q_{1,0}\}$	$\{q_{1,0}\}$	$\{q_{1,0}, q_{1,1}\}$	$\{q_{1,0}, q_{1,1}$
$q_{1,1}$	$\{q_{1,1}\}$	ϕ	ϕ	$\{q_{1,2}\}$	$\{q_{1,1}, q_{1,2}$
$q_{1,2}$	$\{q_{1,2}, q_{2,0}\}$	$\{q_{1,2}, q_{2,1}\}$	$\{q_{1,2}, q_{2,0}, q_{2,1}\}$	$\{q_{1,2}, q_{2,0}\}$	$\{q_{1,2}, q_{2,0}$
$q_{2,0}$	$\{q_{2,0}\}$	$\{q_{2,1}\}$	$\{q_{2,1}\}$	$\{q_{2,0}\}$	$\{q_{2,0}$
$q_{2,1}$	$\{q_{2,1}\}$	ϕ	ϕ	$\{q_{2,0}\}$	$\{q_{2,0}\}$



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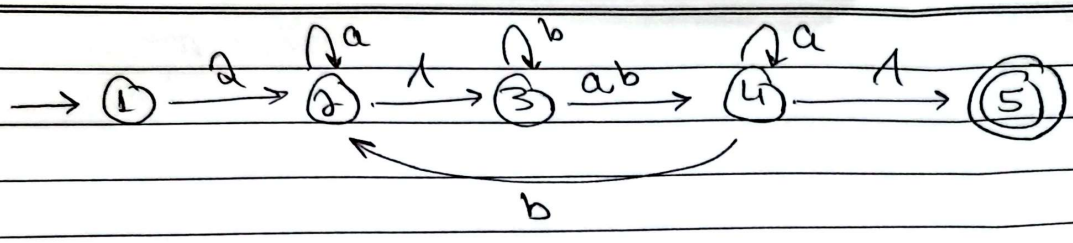
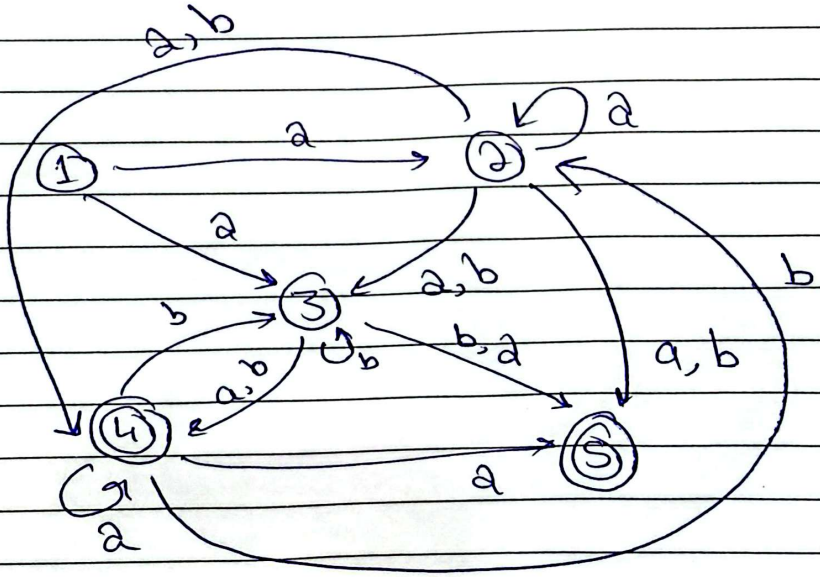


States q	λ-closure(q) x	$\delta(x,a)$ Y	λ-closure(Y)	$\delta(x,b)$ Z	λ-clos (Z)
1	{1}	{2}	{2,3}	ϕ	ϕ
2	{2,3}	{2,4}	{2,3,4,5}	{3,4}	{3,4,5}
3	{3}	{4}	{4,5}	{3,4}	{3,4,5}
4	{4,5}	{4}	{4,5}	{2}	{2,3}
5	{5}	ϕ	ϕ	ϕ	ϕ



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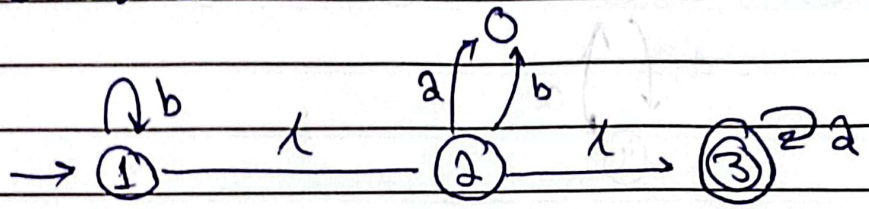
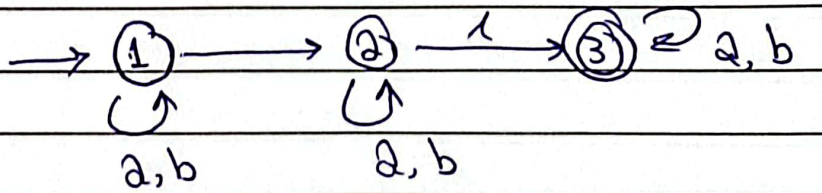
Problem – 4:

Consider following regular expression. Draw NFA-λ diagram

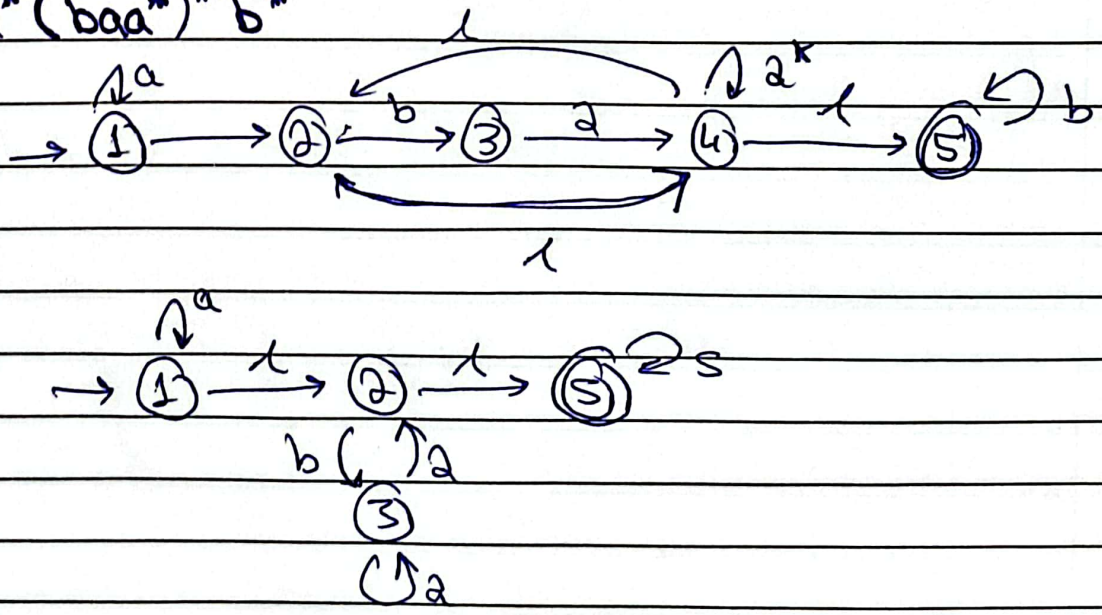
a. $b^*(ab)^*a^*$



b $(a^*+b^*)(a^*+b^*)(a^*+b^*)$



c) $a^*(baa^*)^*b^*$



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d) $b^*(a+ba)^*b^*$