Roll No. [ ] [ ] [ ] [ ] [ ] [ ] [ ] [ ] [ ] [ ] [ ] [ ] **Total No. of Pages : 03**

Total No. of Questions : 18

B.Tech. Only for CSE/IT (2018 Batch) (Sem.–1)

MATHEMATICS-I

Subject Code : BTAM-104-18

Paper ID : [75362]

Time : 3 Hrs. **Max. Marks : 60**

INSTRUCTIONS TO CANDIDATES :

1. **SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.**
2. **SECTION - B & C have FOUR questions each.**
3. **Attempt any FIVE questions from SECTION B & C carrying EIGHT marks each.**
4. **Select atleast TWO questions from SECTION - B & C.**

SECTION-A

1. Verify Rolle's theorem for $f(x) = x(2-x)e^{\frac{3x}{4}}$ in (0,2)

2. Define Beta function and show that it is symmetric.

3. Obtain first three terms of Taylor's series of cosx about $x = \frac{\pi}{4}$.

4. If $A+B = \begin{bmatrix} 1 & -1 \\ 3 & 0 \end{bmatrix}$ and $A-B = \begin{bmatrix} 3 & 1 \\ 1 & 4 \end{bmatrix}$ find AB.

5. Find rank of the matrix $\begin{bmatrix} 1 & 6 & -2 \\ 2 & 2 & 1 \\ 3 & 8 & -1 \end{bmatrix}$.

6. State rank-nullity theorem.

7. Define range of a linear transformation.

8. Define symmetric matrix, also give suitable example.

9. Show that, If zero is an Eigen value of a matrix then it is singular.

10. In an n dimensional space every set of 'n + 1' vectors is linearly dependent or independent. Justify your answer.

SECTION-B

11. a) Suppose that a function f is differentiate on $[0,1]$ and that its derivative is never zero. Using mean value theorem, Show that $f(0) \neq f(1)$

b) Evaluate the limit $\lim_{x \to \frac{\pi}{2}} \left(\frac{1 - sinx}{sinx + \cos 2x} \right)$

12. a) Evaluate the integral $\int_2^{\infty} \frac{2dx}{x^2 - x}$, if it exists.

b) Find the area of the surface generated by revolving the curve $y = 2\sqrt{x}$ $1 \leq x \leq 2$ about the x-axis

13. a) Find the inverse of the matrix $\begin{bmatrix} 2 & 1 & -1 \\ 0 & 2 & 1 \\ 5 & 2 & -3 \end{bmatrix}$ using Gauss Jordan method.

b) Find the rank of the matrix $\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$

14. Solve the following system of equations by Cramer's rule

$$2x - 2y + z = 1, x + 2y + 2z = 2, 2x + y - 2z = 7$$

SECTION-C

15. a) By giving proper reasoning determine whether S forms a subspace of Vector space V.

Operations vector addition '+' and scalar multiplications '.' are usual addition and scalar multiplication defined on set of polynomials of degrees less than or equal to $3(P_3)$ and 3-tuple space (V_3).

If (i) $S = \{p \in P_3 \mid deg(p) = 3\}, V = P_3$

(ii) $S = \{(x, y, z) \mid x = 3y\}, V = V_3$

b) Determine whether the following are Linearly dependent or not?

$$x_1 = (1,2,1), x_2 = (2,1,4), x_3 = (1,8,-3)$$

16. a) Let V = P_4, vector space formed by polynomials of degrees less than or equal to 4 under usual addition and scalar multiplication of polynomials. Find the dimension of subspace U of V, where U is

$$S = \{p \in P_4 \mid p(1) = 0, p'(0) = 0\}$$

b) Check whether the transformation $T : V_3 \rightarrow V_2$ defined by $T(x,y,z) = (x+z, x+y)$ represent a Linear transformation or not?

17. Find the Eigen values and Eigen vectors for the matrix.

$$\begin{bmatrix} 2 & -2 & 3 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$$

18. a) If A is an orthogonal matrix prove that $|A| = \pm 1$

b) Define similar matrices and prove that similar matrices have same eigen values.

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B.Tech.(CSE/IT) (2018 & Onwards)/(Civil Engg.)/(Computer Engg.)
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SECTION-A

1. Calculate $\Gamma\left(\frac{1}{2}\right)$.

2. Show that beta function is symmetric.

3. Compute $\lim\limits_{x \to 0} \frac{\log x}{\cot x}$.

4. If $A=\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B=\begin{bmatrix} 1 & 3 \\ 0 & 5 \end{bmatrix}$. Compute AB.

5. Find the eigen values of the matrix $\begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}$.

6. Define symmetric and skew-symmetric matrices.

7. State rank and nullity theorem.

8. Evaluate $\int_1^\infty \frac{dx}{x^2}$.

9. Find the rank of the matrix $\begin{bmatrix} 2 & 1 & -4 \\ 3 & 5 & -7 \\ 4 & -5 & -6 \end{bmatrix}$.

10. State Rolle's theorem.

SECTION-B

11. Find the eigen value and eigen vector of the following matrix $A=\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$

12. Find the maximum and minimum value of $f(x, y) = x^3 + y^3 - 3xy$.

13. Verify Cayley-Hamilton theorem for the matrix $A=\begin{bmatrix} 1 & 2 & 0 \\ -1 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix}$

14. Find the volume generated by revolving the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ about the x-axis.

SECTION-C

15. Solve the following system using Gauss elimination :

$x - 2y + z = 0,\ 2x + y - 3z = 5,\ 4x - 7y + z = -1.$

16. a) Find the volume of the solid generated by the revolution of the cardioids $r = a(1 + \cos\theta)$ about the initial line.

b) Find the volume of the sphere of radius a.

17. a) Use Cramer's rule to solve $2x + 3y - z = 1,\ 4x + y - 3z = 11,\ 3x - 2y + 5z = 21$.

b) Evaluate $\int_0^\infty (x^2+4)e^{-2x^2}dx$ using gamma function.

18. a) Show that the transformation $T : R^3 \to R^3$ define by $T(x, y, z) = (x + y, y + z, z + x)$ is linear.

b) Let $T : R^3 \to R^3$ define by $T(x, y, z) = (x + y + z, 2x + 2y + 2z, 3x + 3y + 3z)$.

Find the associated matrix corresponding to standard basis.

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SECTION-A

1) Can Rolle's theorem be applied to the function $f(x) = 2 + (x-1)^{2/3}$, $x \in [0, 2]$.

2) Define beta function.

3) Evaluate $\lim\limits_{x \to 0} \dfrac{x\cos x - \sin x}{x^2 \sin x}$

4) Find the values of x, y, z, a which satisfy the relation $\begin{bmatrix} x+3 & 2y+x \\ z-1 & 4a-6 \end{bmatrix} = \begin{bmatrix} 0 & -7 \\ 3 & 2a \end{bmatrix}$.

5) Find adjoint of $\begin{bmatrix} 1 & -1 \\ -2 & 0 \end{bmatrix}$

6) Define basis of vector spaces.

7) Give the statement of rank nullity theorem.

8) Give any two properties of Eigen values.

9) Define symmetric matrix with an example.

10) Find sum and product of latent roots of the matrix $\begin{bmatrix} 2 & 1 \\ 2 & 3 \end{bmatrix}$.

SECTION-B

11) a) Expand $f(x) = \sin^{-1}x$ by Maclaurin's theorem.

b) Evaluate $\lim\limits_{x \to a} \dfrac{x^a - a^x}{x^x - a^a}$.

12) a) Evaluate the integral $\int_0^1 \dfrac{1}{\sqrt{1-x^4}}\,dx$ in terms of gamma function.

b) Find maxima of $f(x, y) = 2(x^2 - y^2) - x^4 + y^4$.

13) a) Prove that $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc\left(1 + \dfrac{1}{a} + \dfrac{1}{b} + \dfrac{1}{c}\right)$.

b) Solve the equations $x + y + z = 1$, $x + 2y + 3z = 6$, $x + 3y + 4z = 6$ using Cramer's rule.

14) a) Are the vectors (2, 1, 1), (2, 0, –1), (4, 2, 1) linearly dependent.

b) Find the rank of the matrix : $\begin{bmatrix} 5 & 3 & 7 \\ 3 & 26 & 2 \\ 7 & 2 & 10 \end{bmatrix}$

SECTION-C

15) Show that the matrix $\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$ satisfies the equation $A^3 - 6A^2 + 11A - I = 0$.

16) Let T : $R^3 \to R^2$ be the linear transformation defined by $T\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x+y \\ x-z \end{bmatrix}$, then find the matrix representation of T w.r.t. the ordered basis $X = \{(1, 0, 1), (1, 1, 0), (0, 1, 1)\}^{\mathrm{T}}$ in R^3 and $Y = \{(1, 0), (0, 1)\}^T$ in R^2.

17) a) Is the matrix $\begin{bmatrix} 4 & 2 & 1 \\ 6 & 3 & 4 \\ 2 & 1 & 0 \end{bmatrix}$ orthogonal ?

b) Write the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & -8 & 9 \end{bmatrix}$ as the sum of symmetric and skew symmetric matrices.

18) Reduce the matrix $\begin{bmatrix} -1 & 2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{bmatrix}$ to the diagonal form.

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