

Hazard Rates, Survival Functions, Probability Density Functions, and Expected Lifetimes for Some Common Parametric Distributions

<i>Distribution</i>	<i>Hazard Rate $b(x)$</i>	<i>Survival Function $S(x)$</i>	<i>Probability Density Function $f(x)$</i>	<i>Mean $E(X)$</i>
Exponential $\lambda > 0, x \geq 0$	λ	$\exp[-\lambda x]$	$\lambda \exp(-\lambda x)$	$\frac{1}{\lambda}$
Weibull $\alpha, \lambda > 0, x \geq 0$	$\alpha \lambda x^{\alpha-1}$	$\exp[-\lambda x^\alpha]$	$\alpha \lambda x^{\alpha-1} \exp(-\lambda x^\alpha)$	$\frac{\Gamma(1 + 1/\alpha)}{\lambda^{1/\alpha}}$
Gamma $\beta, \lambda > 0, x \geq 0$	$\frac{f(x)}{S(x)}$	$1 - I(\lambda x, \beta)^*$	$\frac{\lambda^\beta x^{\beta-1} \exp(-\lambda x)}{\Gamma(\beta)}$	$\frac{\beta}{\lambda}$
Log normal $\sigma > 0, x \geq 0$	$\frac{f(x)}{S(x)}$	$1 - \Phi \left[\frac{\ln x - \mu}{\sigma} \right]$	$\frac{\exp \left[-\frac{1}{2} \left(\frac{\ln x - \mu}{\sigma} \right)^2 \right]}{x(2\pi)^{1/2}\sigma}$	$\exp(\mu + 0.5\sigma^2)$
Log logistic $\alpha, \lambda > 0, x \geq 0$	$\frac{\alpha x^{\alpha-1} \lambda}{1 + \lambda x^\alpha}$	$\frac{1}{1 + \lambda x^\alpha}$	$\frac{\alpha x^{\alpha-1} \lambda}{[1 + \lambda x^\alpha]^2}$	$\frac{\pi \text{Csc}(\pi/\alpha)}{\alpha \lambda^{1/\alpha}}$ if $\alpha > 1$
Normal $\sigma > 0, -\infty < x < \infty$	$\frac{f(x)}{S(x)}$	$1 - \Phi \left[\frac{x - \mu}{\sigma} \right]$	$\frac{\exp \left[-\frac{1}{2} \left(\frac{x - \mu}{\sigma} \right)^2 \right]}{(2\pi)^{1/2}\sigma}$	μ
Exponential power $\alpha, \lambda > 0, x \geq 0$	$\alpha \lambda^\alpha x^{\alpha-1} \exp\{[\lambda x]^\alpha\}$	$\exp\{1 - \exp[(\lambda x)^\alpha]\}$	$\alpha e \lambda^\alpha x^{\alpha-1} \exp[(\lambda x)^\alpha] - \exp\{\exp[(\lambda x)^\alpha]\}$	$\int_0^\infty S(x) dx$
Gompertz $\theta, \alpha > 0, x \geq 0$	$\theta e^{\alpha x}$	$\exp \left[\frac{\theta}{\alpha} (1 - e^{\alpha x}) \right]$	$\theta e^{\alpha x} \exp \left[\frac{\theta}{\alpha} (1 - e^{\alpha x}) \right]$	$\int_0^\infty S(x) dx$
Inverse Gaussian $\lambda \geq 0, x \geq 0$	$\frac{f(x)}{S(x)}$	$\Phi \left[\left(\frac{\lambda}{x} \right)^{1/2} \left(1 - \frac{x}{\mu} \right) \right] - e^{2\lambda/\mu} \Phi \left\{ - \left[\frac{\lambda}{x} \right]^{1/2} \left(1 + \frac{x}{\mu} \right) \right\}$	$\left(\frac{\lambda}{2\pi x^3} \right)^{1/2} \exp \left[\frac{\lambda(x - \mu^2)}{2\mu^2 x} \right]$	μ
Pareto $\theta > 0, \lambda > 0, x \geq \lambda$	$\frac{\theta}{x}$	$\frac{\lambda^\theta}{x^\theta}$	$\frac{\theta \lambda^\theta}{x^{\theta+1}}$	$\frac{\theta \lambda}{\theta - 1}$ if $\theta > 1$
Generalized gamma $\lambda > 0, \alpha > 0, \beta > 0, x \geq 0$	$\frac{f(x)}{S(x)}$	$1 - I[\lambda x^\alpha, \beta]$	$\frac{\alpha \lambda^\beta x^{\alpha\beta-1} \exp(-\lambda x^\alpha)}{\Gamma(\beta)}$	$\int_0^\infty S(x) dx$

* $I(t, \beta) = \int_0^t u^{\beta-1} \exp(-u) du / \Gamma(\beta)$.