

# Navam's Law of Integers

*“The product of the sum of  $x$  &  $y$  and the number of integers in between  $x$  &  $y$  is twice the sum of the integers in between  $x$  &  $y$  (where  $x < y$  and  $x, y \in \mathbb{Z}$ )”*

Or

$$m(x + y) = 2 \Sigma n$$

where  $m \Rightarrow$  number of integers in between  $x$  and  $y$  excluded and

$\Sigma n \Rightarrow$  sum of all the integers in between  $x$  and  $y$  excluded!

## Proof:

$$\Sigma n = n_1 + n_2 + \dots + n_m$$

We know that,  $n_1 = x + 1 \rightarrow a$

$$n_2 = y - 1 \rightarrow l$$

We also know that,  $S_n = \frac{n}{2} (a + l)$

$$\Rightarrow \Sigma n = \frac{m}{2} (x + 1 + y - 1)$$

$$\therefore m(x + y) = 2 \Sigma n$$

## Another Version:

We know that the number of integers in between  $x$  &  $y$  is one less than the difference of  $y$  &  $x$ .

$$\Rightarrow m = (y - x) - 1$$

$$\text{Navam's Law of Integers} \Rightarrow (y - x - 1)(x + y) = 2 \Sigma n$$

$$(y - x)(y + x) - (x + y) = 2 \Sigma n$$

$$y^2 - x^2 - (x + y) = 2 \Sigma n$$

$$\therefore y^2 - x^2 = x + y + 2 \Sigma n$$