Square of Summation

Ever stuck by summation operator inside a square form? Or got the need to send the square inside the summation? This is the mathematical derivation for the same.

$$\left(\sum_{i=1}^{n} a_i\right)^2 = (a_1 + a_2 + a_3 + \dots + a_{n-1} + a_n)^2$$

$$= (a_1 + a_2 + a_3 + \dots + a_{n-1} + a_n)(a_1 + a_2 + a_3 + \dots + a_{n-1} + a_n)$$

$$= a_1(a_1 + a_2 + a_3 + \dots + a_{n-1} + a_n) + a_2(a_1 + a_2 + a_3 + \dots + a_{n-1} + a_n) +$$

$$a_3(a_1 + a_2 + a_3 + \dots + a_{n-1} + a_n) + \dots +$$

$$a_{n-1}(a_1 + a_2 + a_3 + \dots + a_{n-1} + a_n) + a_n(a_1 + a_2 + a_3 + \dots + a_{n-1} + a_n)$$

$$= a_1^2 + a_1a_2 + a_1a_3 + \dots + a_1a_{n-1} + a_1a_n +$$

$$a_2a_1 + a_2^2 + a_2a_3 + \dots + a_2a_{n-1} + a_2a_n +$$

$$a_3a_1 + a_3a_2 + a_3^2 + \dots + a_3a_{n-1} + a_3a_n +$$

$$\vdots$$

$$a_{n-1}a_1 + a_{n-1}a_2 + a_{n-1}a_3 + \dots + a_{n-1}^2 + a_{n-1}a_n +$$

$$a_na_1 + a_na_2 + a_na_3 + \dots + a_na_{n-1} + a_n^2$$

If you observe, the diagonal acts as a mirror. The terms above it are equal to the terms below it. Indirectly, we are adding them twice.

$$\left(\sum_{i=1}^{n} a_i\right)^2 = \left(a_1^2 + a_2^2 + a_3^2 + \cdots + a_{n-1}^2 + a_n^2\right) + 2a_1\left(a_2 + a_3 + \cdots + a_{n-1} + a_n\right)$$

$$+ 2a_2\left(a_3 + \cdots + a_{n-1} + a_n\right)$$

$$\vdots$$

$$+ 2a_{n-1}\left(a_n\right)$$

$$= \sum_{i=1}^{n} a_i^2 + 2a_1 \sum_{j=2}^{n} a_j + 2a_2 \sum_{j=3}^{n} a_j + \cdots + 2a_{n-1} \sum_{j=n}^{n} a_j$$

$$\left(\sum_{i=1}^{n} a_i\right)^2 = \sum_{i=1}^{n} a_i^2 + 2 \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} a_i a_j$$