Mesh Parameterization

Motivation

- Corresponds to finding a mapping from discrete surface patch to an isomorphic planar mesh.
- Each mesh node in surface will be assigned to coordinates (u,v) in planar region.
- This allows for mesh processing like texture mapping, remeshing etc. in the flat parametric space.

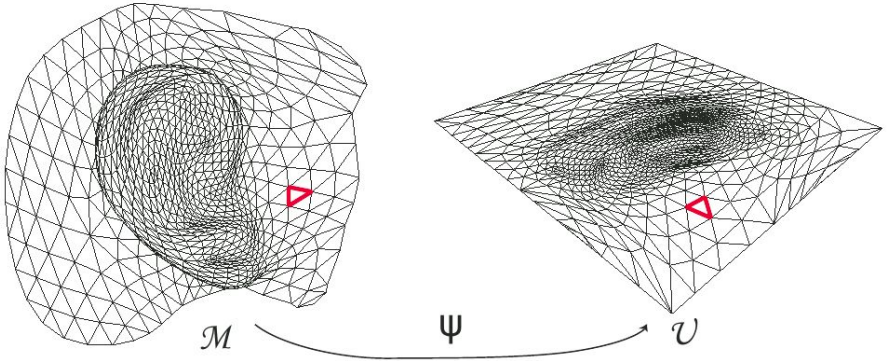


Figure 1: *A piecewise linear mapping between a 3D mesh* M *and an isomorphic flat mesh* U*, where a triangle on the mesh is mapped to a triangle in the parameterization.*

Conformal mapping

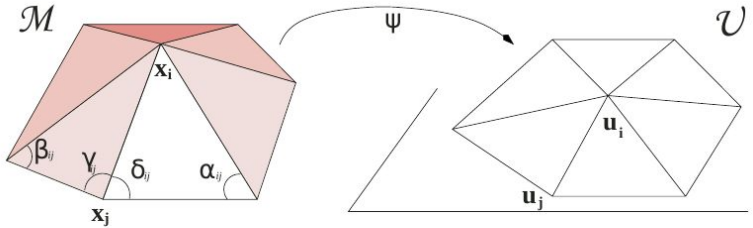


Figure 4: *A 3D 1-ring, and its associated flattened version.*

- We look for “smooth parameterizations” that minimize distortion
- Let energy E(M, U) be distortion measure, then simple criterion for energy to be minimum over M ~ U is

$$\frac{\partial E}{\partial \mathbf{u}_i} = 0$$

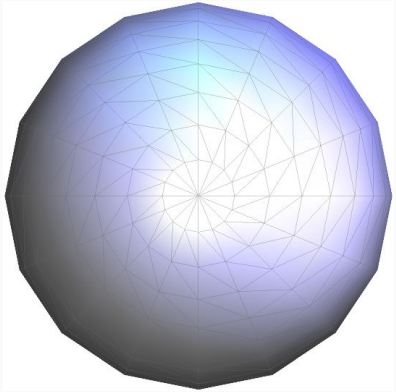
- Discrete conformal mapping is angle preserving $\frac{\partial E_A}{\partial \mathbf{u}_i} = \sum_{j \in N(i)} (\cot \alpha_{ij} + \cot \beta_{ij})(\mathbf{u}_i - \mathbf{u}_j) = 0$
-
- Solve sparse linear system as $M^A U = 0$

  with boundary conditions $U^{boundary} = X^{boundary}$

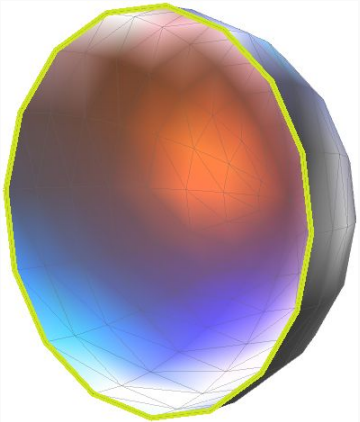
$$\mathbf{M}_{ij}^A = \begin{cases} \cot(\alpha_{ij}) + \cot(\beta_{ij}) & \text{if } j \in N(i) \\ -\sum_{k \in N(i)} \mathbf{M}_{ik}^A & \text{if } i = j \\ 0 & \text{otherwise,} \end{cases}$$

Methodology

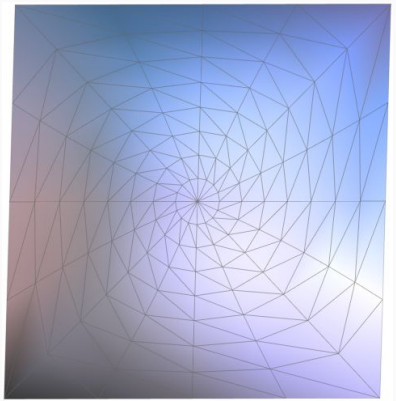
1. Get boundary loop of the triangulated patch
2. Fix $U^{boundary}$ in the planar region while preserving length between boundary points
3. Solve sparse system $MU = 0$ to find $U^{internal}$
4. Visualize the flattened mesh using normals of the original 3d mesh



3d mesh of hemisphere

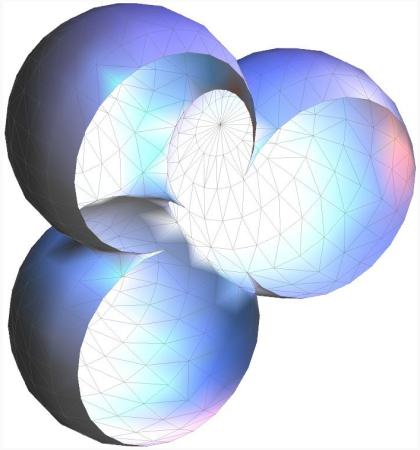


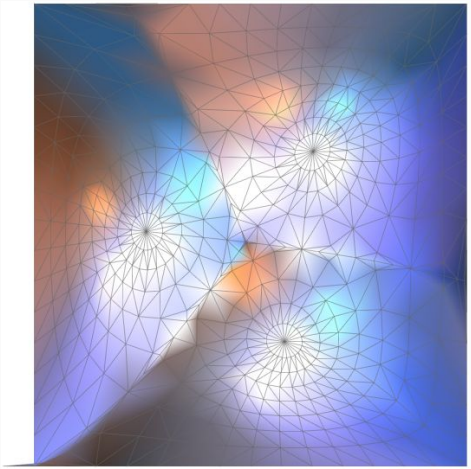
Boundary loops of the mesh

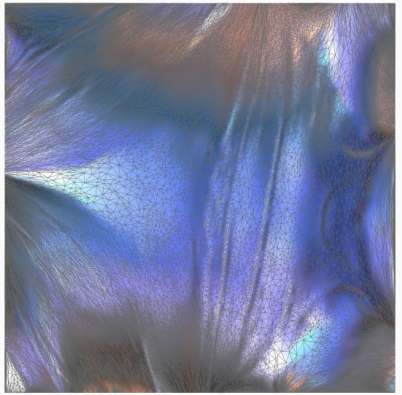


UV coordinates after flattening

Some more results

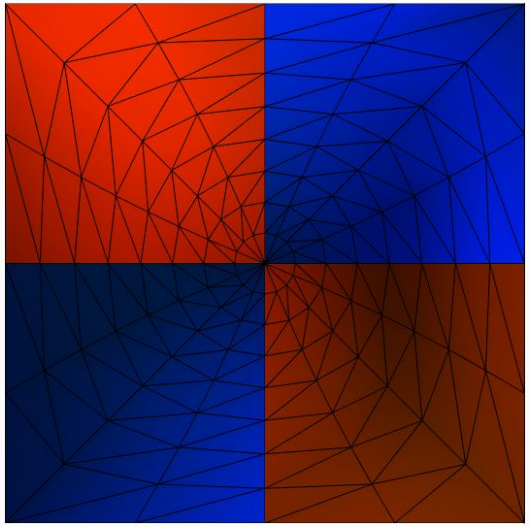


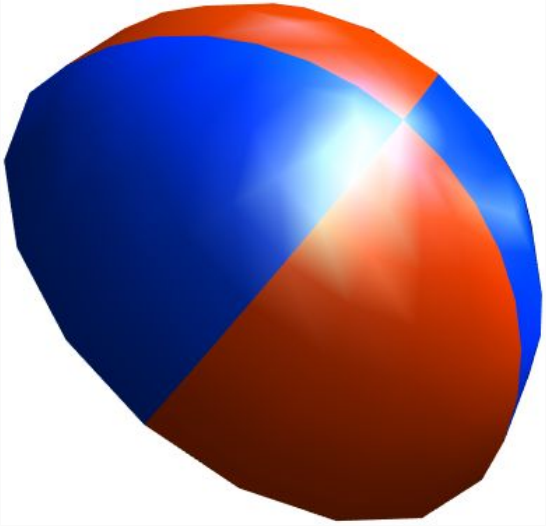




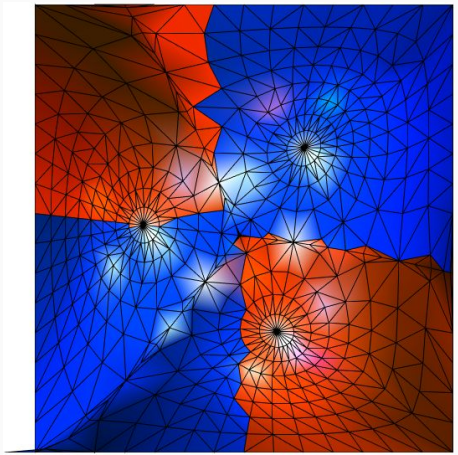
Texture mapping

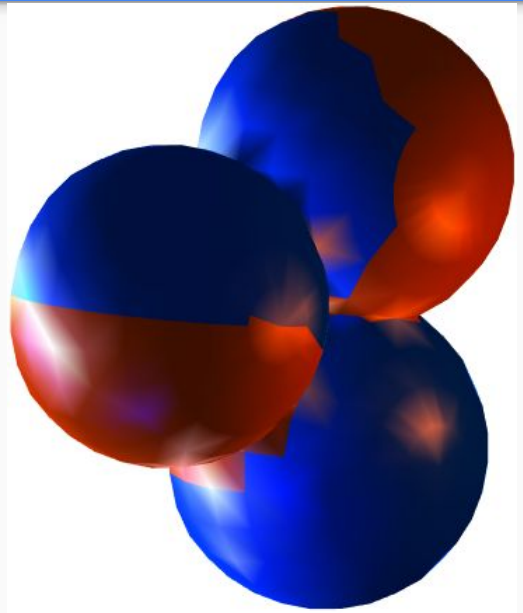
- We generate a texture board pattern in flattened mesh thus coloring triangles
- The corresponding faces in the original mesh are visualized using same colors
- Thus, mapping textures onto surface patches is simplified.
- This helps to evaluate the parameterization

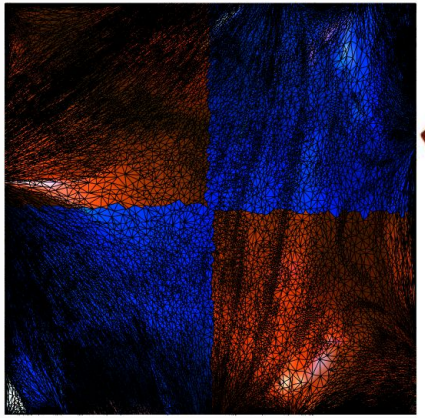




Some more texture maps



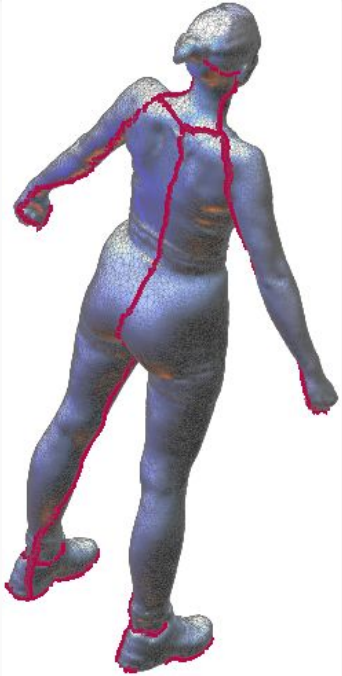




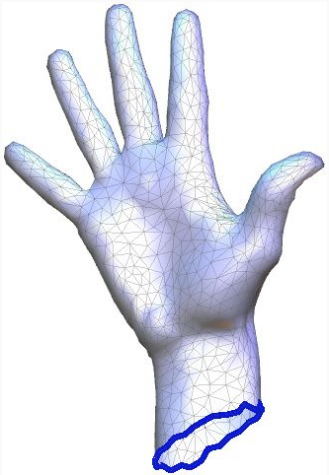
Implementation problems

- Problem - Internal vertices values going outside the boundary
- Remedy - Laplacian matrix was incorrect, row sum need to zero

Implementation problems



- Parameterization not good for some meshes as many triangles are farther away from boundary



Thanks!

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