

## Subarrays

- Continuous part of an array
- single elements / complete array are subarrays
- empty  $[]$  is not a subarray.

Ex

	0	1	2	3	4	5	6
	3	4	5	6	-2	8	10

indices	}	
2 3 4		✓
0 1 3 4		✗
6		✓
0, 6		✗
0, 1, 2, 3, 4, 5, 6	✓	

# of subarrays.

arr[7] = 

	0	1	2	3	4	5	6
	4	2	10	3	12	-2	15

Subarrays starting from index 0<sup>th</sup> =  $[0,0] [0,1] [0,2] \dots [0,6]$   
= 7.

Subarrays starting from index 1<sup>st</sup> =  $[1,1] [1,2] [1,3] \dots [1,6]$   
= 6

Subarrays starting from index 2<sup>nd</sup> = [2,2] [2,3] [2,4] ... [2,6]  
= 5.

N size array

Start at 0<sup>th</sup> = (0,0) (0,1) (0,2) ... (0,N-1) N  
Start at 1<sup>st</sup> = (1,1) (1,2) ... (1,N-1) N-1  
2<sup>nd</sup> = N-2  
⋮  
⋮  
⋮  
(N-1)<sup>th</sup> = 1

$$\begin{aligned} \text{Total subarrays} &= 1 + 2 + 3 + \dots + (N-1) + N \\ &= \frac{N(N+1)}{2} \end{aligned}$$

Q. Print all value of given subarray.

```
void printsub ( int arr[], int s, int e)
{
    for ( i=s; i<=e; i++)
    {
        print ( A[i])
    }
}
```

$O(e-s+1)$

TC:  $O(N)$

SC:  $O(1)$

Q. Given subarray, cal sum of all ele in the subarray.

```
int sumsub ( int arr[], int s, int e)
{
    sum = 0
    for ( i = s ; i <= e ; i++)
    {
        sum = sum + arr[i]
    }
    return sum,
}
```

$O(e-s+1)$   
TC:  $O(N)$   
SC:  $O(1)$

Q. Print all the subarrays.

A = <sup>0 1 2</sup>  
[ 2, 8, 9 ]

[0,0]      ⇒      2  
[0,1]      2      8  
[0,2]      2      8      9  
[1,1]      8  
[1,2]      8      9  
[2,2]      9

```

for ( i = 0 ; i < n ; i++ )
{
    // i is starting point
    for ( j = i ; j < n ; j++ )
    {
        // i is starting & j is ending
        printsub ( arr, i, j )
    }
}

```

TC:  $O(N^3)$   
SC:  $O(1)$

```

0 0
0 1
0 2
0 3
0 N-1
1 1
1 2
1 3
1 N-1
2
⋮
/
/
/
/

```

Q. Print sum of every subarray.

$A = \begin{matrix} & 0 & 1 & 2 \\ & 3 & -2 & 4 \end{matrix}$

$\begin{matrix} 0-0 & = & 3 \\ 0-1 & = & 1 \\ 0-2 & = & 5 \\ 1-1 & = & -2 \\ 1-2 & = & 2 \\ 2-2 & = & 4 \end{matrix}$

```
for ( i = 0 ; i < n ; i++ )  
    // i is starting point  
    for ( j = i ; j < n ; j++ )  
        // i is starting & j is ending  
        {  
            sum = sumSub ( arr, i, j )  
            print ( sum )  
        }
```

TC:  $O(N^3)$   
SC:  $O(1)$

$A = \begin{matrix} & 0 & 1 & 2 \\ & 3 & -2 & 4 \end{matrix}$   
P.F:  $\begin{matrix} & 3 & 1 & 5 \end{matrix}$

// PF array

```
for ( i = 0 ; i < n ; i++ )  
    // i is starting point  
    for ( j = i ; j < n ; j++ )  
        // i is starting & j is ending  
        {  
            sum = pref[j] - pref[i-1] ← i > 0  
            sum = pref[j] ← i == 0  
        }
```

$T.C: O(N^2)$   
 $SC: O(N)$

0 - 0	3
0 - 1	1
0 - 2	5
1 - 1	-2
1 - 2	2
2 - 2	4

$T.C(N^3)$   
 $SC O(1)$

/ PF

↓  
TC:  $O(N^2)$   
SC:  $O(N)$

↓  
TC:  $O(N^2)$   
SC:  $O(1)$

Q. Print sum of all subarrays starting from index 2.

	0	1	2	3	4	5	6
	7	3	2	-1	6	8	2

[2, 2] =	2						
[2, 3] =			5				
[2, 4] =			7	-1			
[2, 5] =			15	6	8		
[2, 6] =			12	11	17	25	

sum = 0

for (i = 2; i < n; i++)

{  
    sum = sum + arr[i]  
    print (sum)  
}

TC:  $O(N)$   
SC:  $O(1)$

Q. Print sum of all subarrays starting from index 3.

```
sum = 0
for ( i = 3 ; i < n ; i++ )
{
    sum = sum + arr[i]
    print (sum)
}
```

TC:  $O(N)$   
SC:  $O(1)$

Q. Print sum of all subarrays starting from index i.

```
sum = 0
for ( j = i ; j < n ; j++ )
{
    sum = sum + arr[j]
    print (sum)
}
```

TC:  $O(N)$   
SC:  $O(1)$



Q print all subarray sum

```
for (i=0; i<n; i++)
```

```
// print all subarrays starting from i
```

```
sum=0
```

```
for (j=i; j<n; j++)
```

```
{  
    sum = sum + arr[j]  
    print (sum)  
}
```

TC:  $O(N^2)$

SC:  $O(1)$

TC  $O(N^3)$  SC  $O(1)$

PF sum

TC:  $O(N^2)$  SC:  $O(N)$

carry forward

TC:  $O(N^2)$  SC:  $O(1)$

Q. Given an array, find sum of all subarray sums.

A =  $\begin{matrix} 0 & 1 & 2 \\ 3 & -1 & 4 \end{matrix}$

$[0,0] = 3$   
 $[0,1] = 2$   
 $[0,2] = 6$   
 $[1,1] = -1$   
 $[1,2] = 3$   
 $[2,2] = 4$

$$3 \times 3 + 4(-1) + 3(4) = 17$$

$$3 + 2 + 6 - 1 + 3 + 4 = 17$$

ans = 0

for (i = 0; i < n; i++)

// print all subarrays starting from i

sum = 0

for (j = i; j < n; j++)

sum = sum + arr[j]

ans = ans + sum

TC:  $O(N^2)$   
SC:  $O(1)$

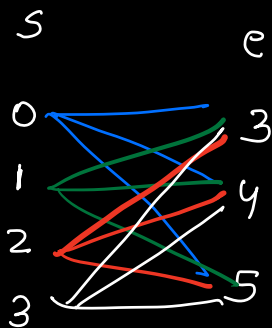
return ans;

Q, In how many subarrays index 3 present

$$\text{arr}[ ] = \overset{0}{3} \overset{1}{-2} \overset{2}{4} \overset{3}{-1} \overset{4}{2} \overset{5}{6} \quad N=6$$

$$(i+1)(N-j)$$

$$4 \times 3$$



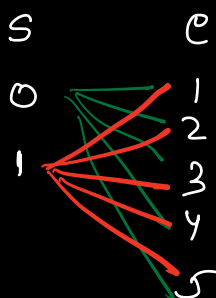
$$\begin{array}{lll} [0,3] & [0,4] & [0,5] \\ [1,3] & [1,4] & [1,5] \\ [2,3] & [2,4] & [2,5] \\ [3,3] & [3,4] & [3,5] \end{array}$$

Total subarray

$$4 \times 3 = 12$$

Q, In how many subarrays index 1<sup>st</sup> present

$$\text{arr}[ ] = \overset{0}{3} \overset{1}{-2} \overset{2}{4} \overset{3}{-1} \overset{4}{2} \overset{5}{6}$$



$$2 \times 5 = 10$$

Q. In how many subarrays index  $i^{\text{th}}$  present  
 Array size =  $N$ .

index      0    1    2    3    ...     $j-1$      $i$      $j+1$     ...     $N-1$

$s$	$e$
0	$i$
1	$j+1$
2	$i+2$
3	$\vdots$
$\vdots$	$\vdots$
$i$	$N-1$

$$\# = \binom{\text{count}}{s} \times \binom{\text{count}}{e}$$

$$= (j+1) \times (N-i)$$

$$\cancel{N-i-j+1}$$

$$N-i$$

Q. Given an array, find sum of all subarray sums.

$$A = \begin{matrix} & 0 & 1 & 2 \\ & 3 & -1 & 4 \end{matrix}$$

$$\begin{array}{lcl} [0,0] & = & 3 \\ [0,1] & = & 3 - 1 \\ [0,2] & = & 3 - 1 + 4 \\ [1,1] & = & -1 \\ [1,2] & = & -1 + 4 \\ [2,2] & = & 4 \end{array}$$

$$3 \times 3 + 4(-1) + 3(4) = 17$$

$$3 + 2 + 6 - 1 + 3 + 4 = 17$$

Mela (fair)

Mon 3

Tue 4

Wed 1

Thu 2

Fri 5

15



$P_1 \quad P_2 \quad P_3 \quad P_4 \quad P_5$

$$2 + 3 + 0 + 0 + 0 = 15$$

Total ans  $\approx$  Contribution of 0<sup>th</sup> index + Contribution of 1<sup>st</sup> index + ...

$$= \left( \begin{matrix} \text{No of times} \\ \text{index} \end{matrix} \right)_{0^{\text{th}}} * arr[0] + \left( \begin{matrix} \text{No of times} \\ \text{index} \end{matrix} \right)_{1^{\text{th}}} * arr[1] + \dots$$

```
int ans = 0
```

```
for (i = 0; i < n; i++)
```

```
// add contri of i to ans.
```

no of times =  $(i+1)(n-i)$

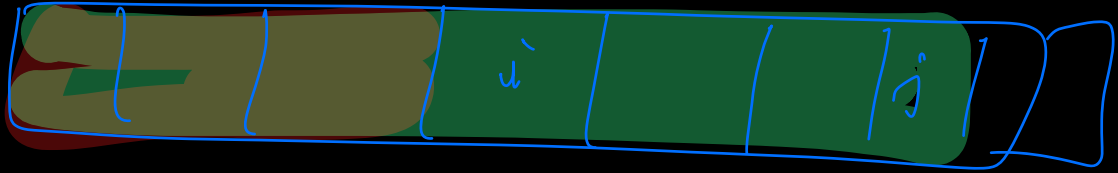
contri of i =  $(\text{no of times}) * arr[i]$

ans = ans + contri of i

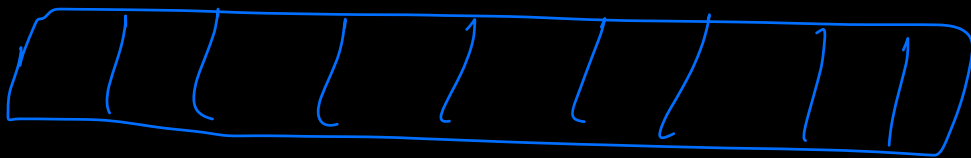
```
return ans;
```

TC:  $O(N)$   
SC:  $O(1)$

Contribution Technique



$$\text{Sum}(i, j) = \text{pref}[j] - \text{pref}[i-1]$$



↔

↔

↔

↔

↔

↔

↔

moving forward

$$\text{ans} = \text{Contrib of } 0^{\text{th}} + \text{Contrib of } 1^{\text{st}} + \dots$$