

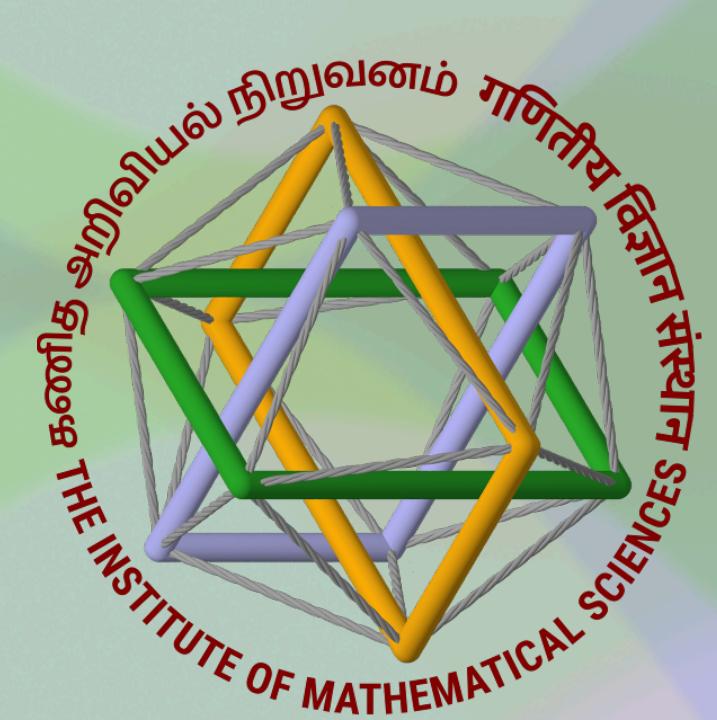
Talk slides will be available at

<https://navdeep-dhindsa.github.io/>

navdeep.s.dhindsa@gmail.com

Navdeep Singh Dhindsa 15/07/2024

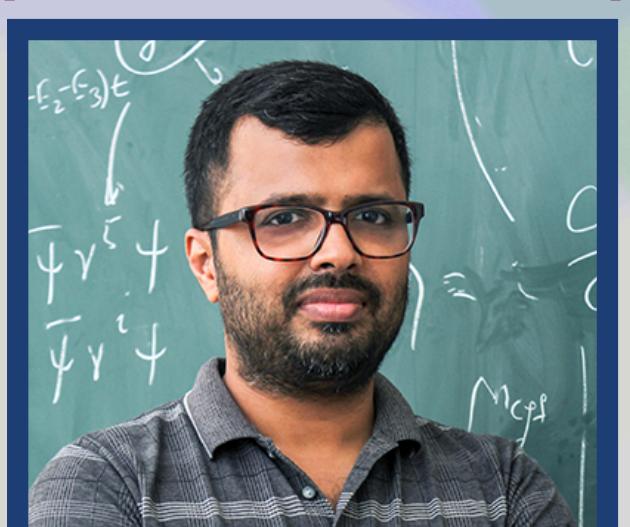
Insights into Dibaryon Interactions in the Heavy Quark Sector

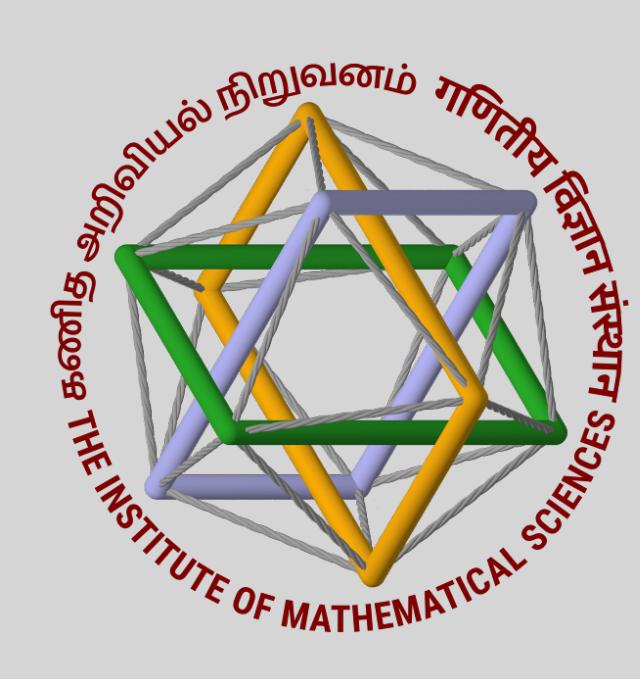


Funding resources



Work in collaboration with
M. Padmanath (IMSc Chennai) and Nilmani Mathur (TIFR Mumbai)





Deuteron

- Deuterium destroyed in interior of stars faster than it is produced.
- All deuterium found in nature has origin from Big Bang nucleosynthesis.
(Temperature hot enough to produce it but not hot enough to produce byproduct or get destroyed)
- Deuterium bottleneck after big bang and then only 20 minutes for nucleosynthesis.

Based on theory of strong interactions, we cannot rule out more dibaryons in nature.

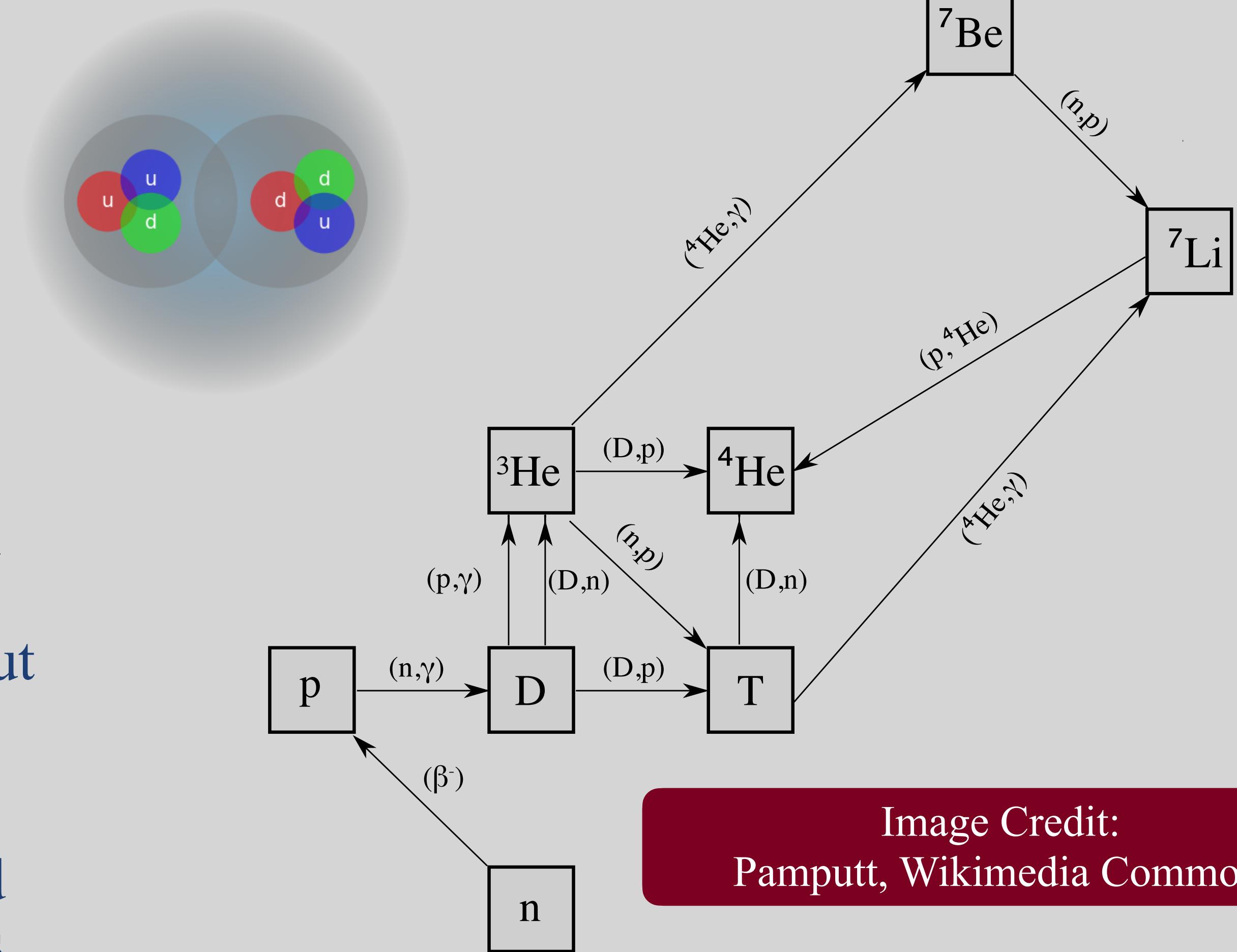
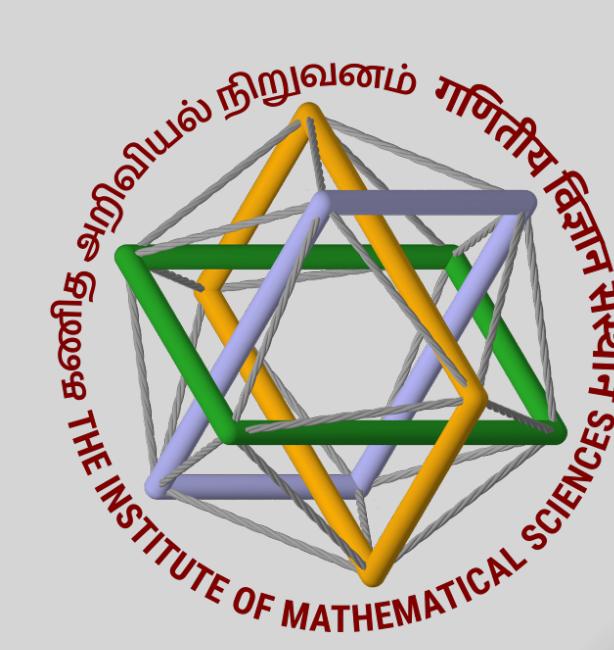
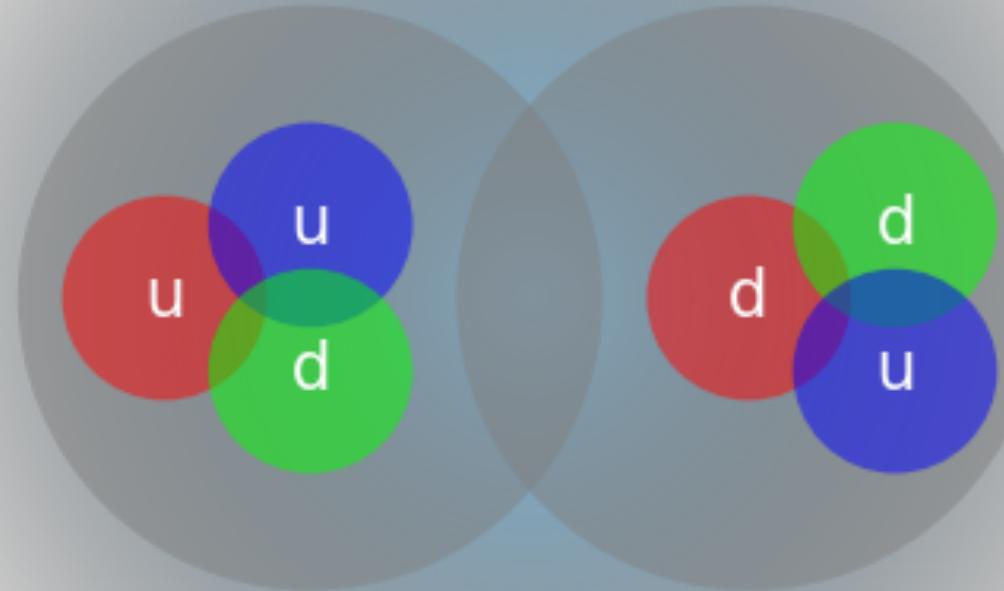


Image Credit:
Pamputt, Wikimedia Commons

Dineutron? Diproton?
NN scattering experiments indicates absence of bound state



Deuteron Dibaryon



- Many predictions of various dibaryon states but failed experimental checks
- Recent renewal in interest due to discoveries of complex quark systems (not just baryons/mesons)
- Experimental evidence of existence of d^*

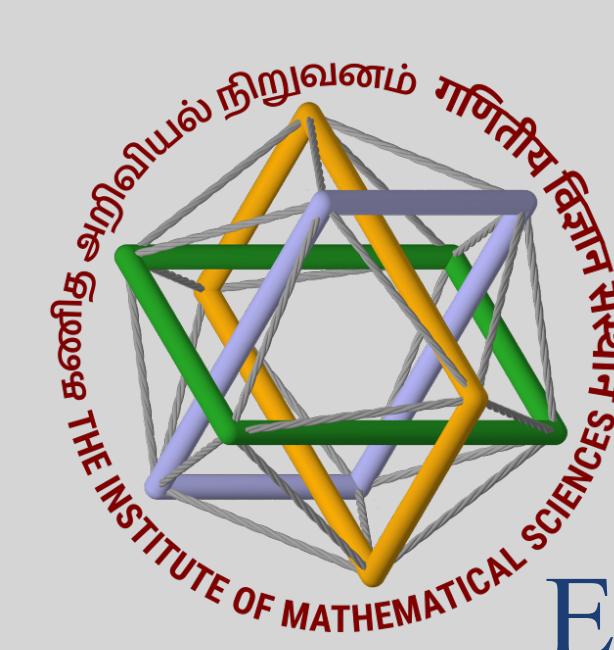
1932

1950's

2010's

- Discovered around a century back
- Proton (uud) - Neutron (udd) bound state
- Binding Energy = 2.2 MeV

Clement
PPNP (2016)



Hyperon-Nucleon

Experimental results indicate:

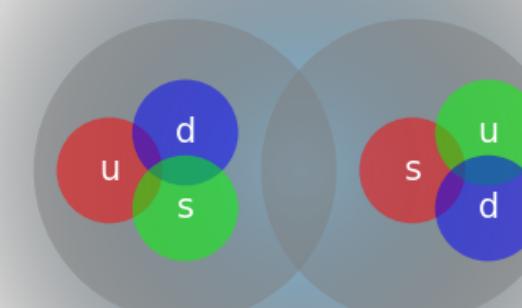
- ΛN interaction is attractive, though less than NN , there is no strange deuteron.
- ΣN interaction even weaker than ΛN .
- $\Lambda\Lambda$ does not rule out bound system.

* Jaffe prediction of dihyperons

Jaffe
PRL 138 (1977) 195

* NAGARA event - constraint on binding energy

* Dedicated experiments for H dibaryon indicates existence unlikely but its existence not ruled out yet.



* Theoretical calculations vary from very deep bound (even more than Jaffe's prediction) to unbound.

* Lattice QCD - gives bound result (8 MeV) - Large pion mass used (discussed later).

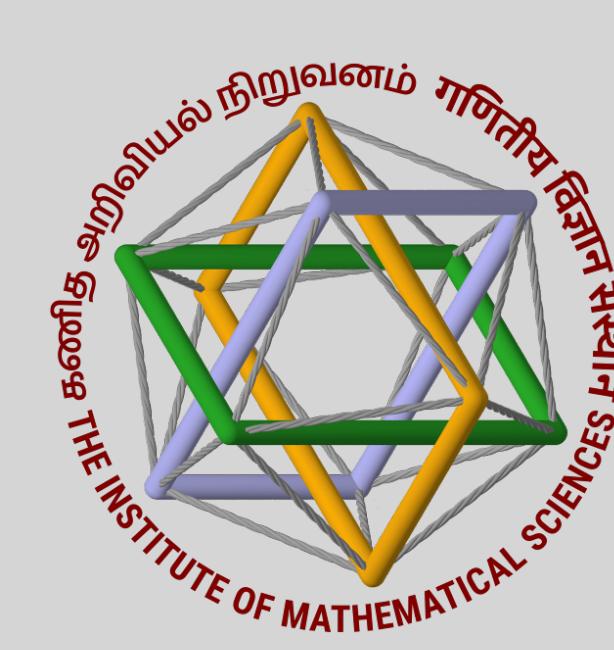
Beane et al.
PRL 106(2011) 162001

* Chiral effective field theory calculation - smaller binding energy.

Talk by Green
Santa Fe Workshop 2023

Inoue et al.
PRL 106(2011) 162002

Haidenbauer and Meissner
PLB 206(2011) 100



$d^*(2380)$

Dyson and Xuong
PRL 13 (1964) 815

WASA @ COSY collaboration
SAID Data Analysis Center
PRL 112 (2014) 202301

1964 - Prediction of possible bound states

They predicted mass of D_{03} (close to d^* which was found later)

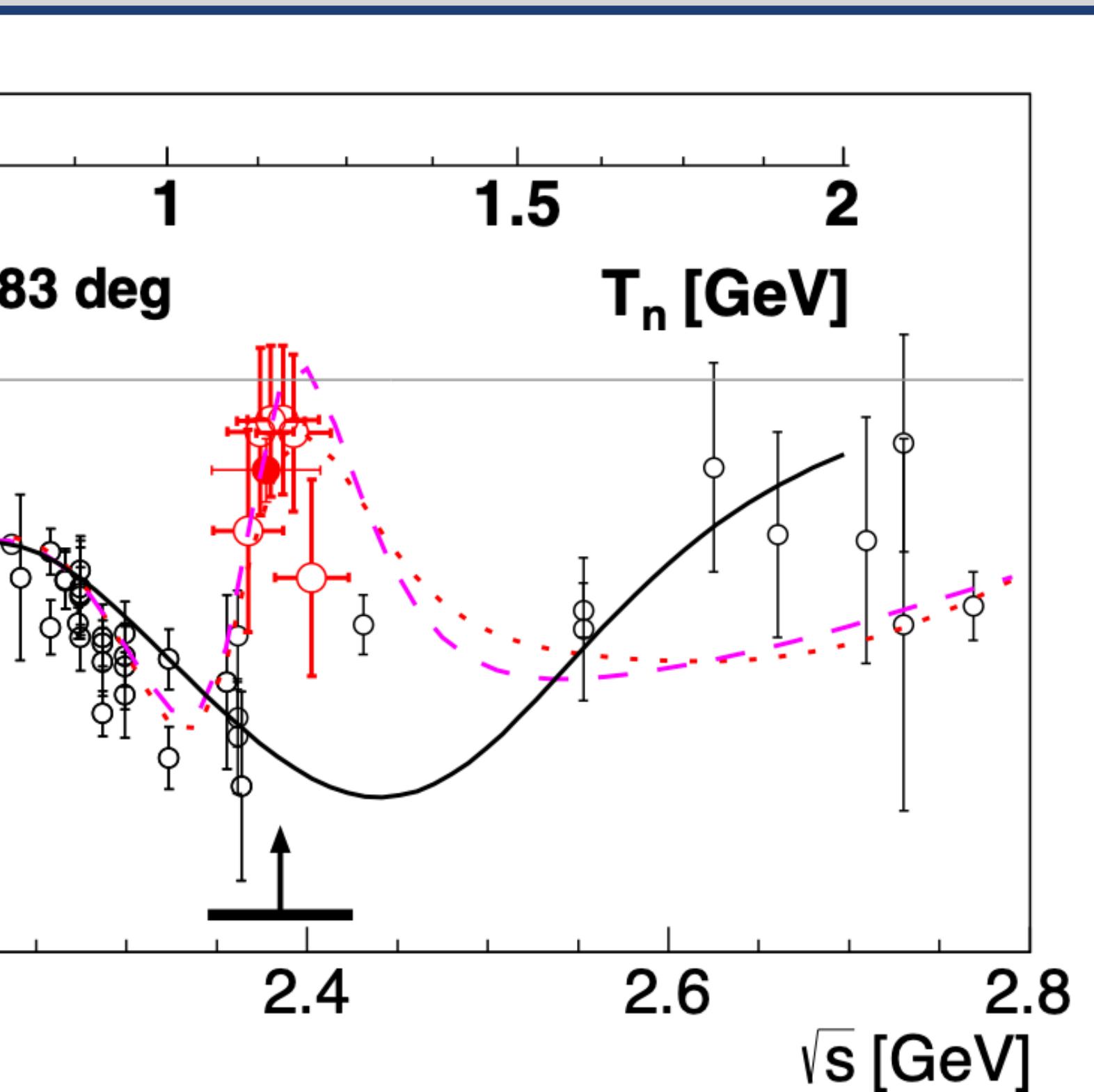
Bremsstrahlung measurements

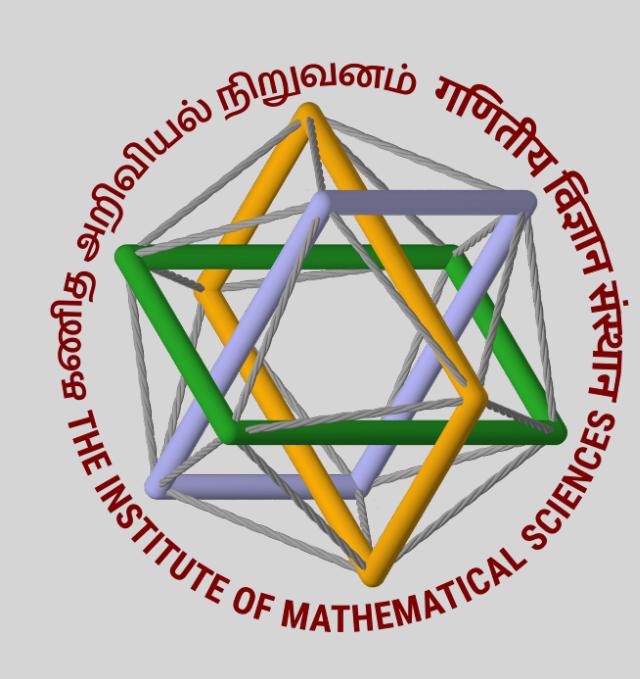
Kamae and Fujita
PRL 38 (1977) 471

“An inevitable non-strange dibaryon”

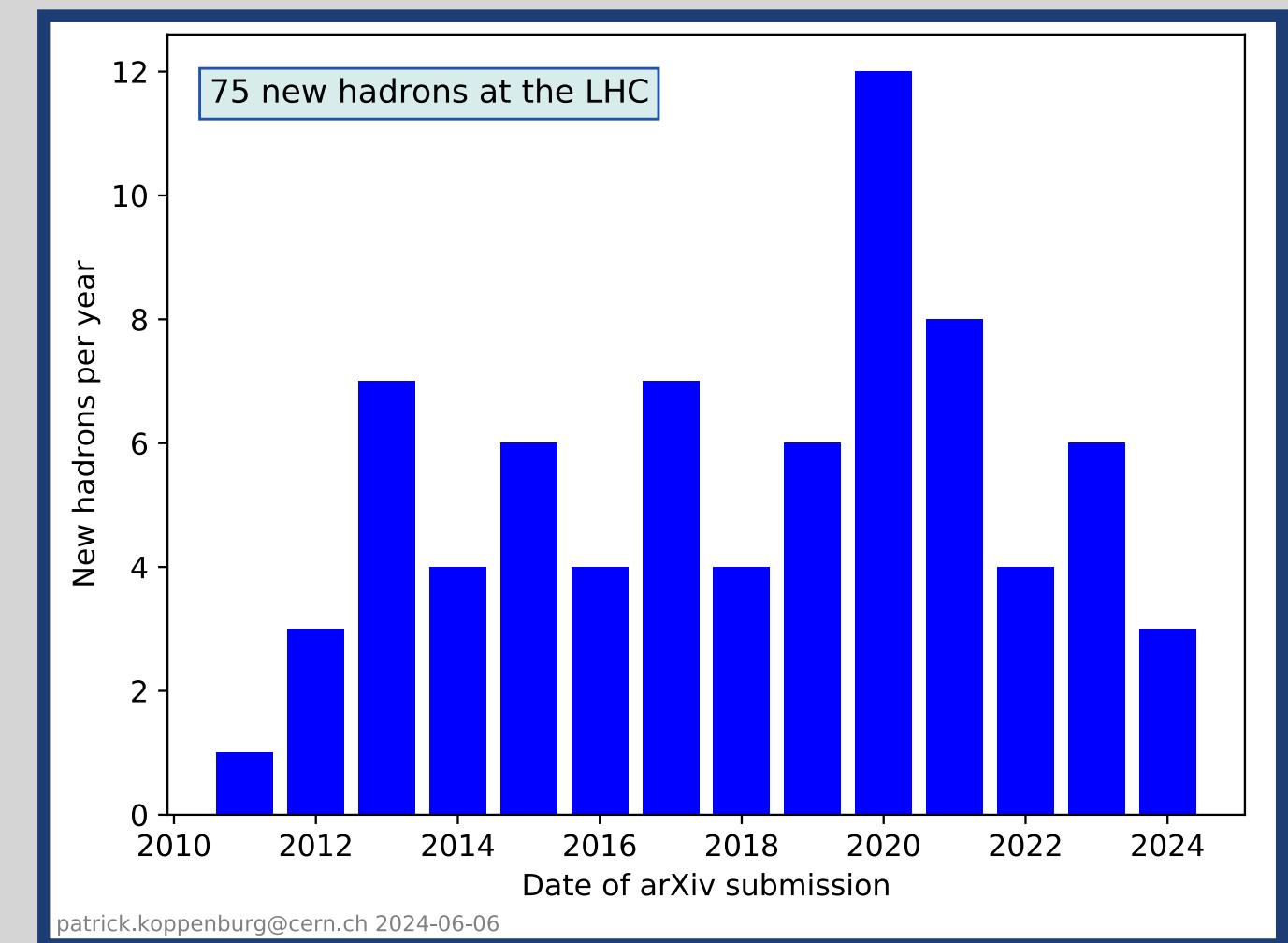
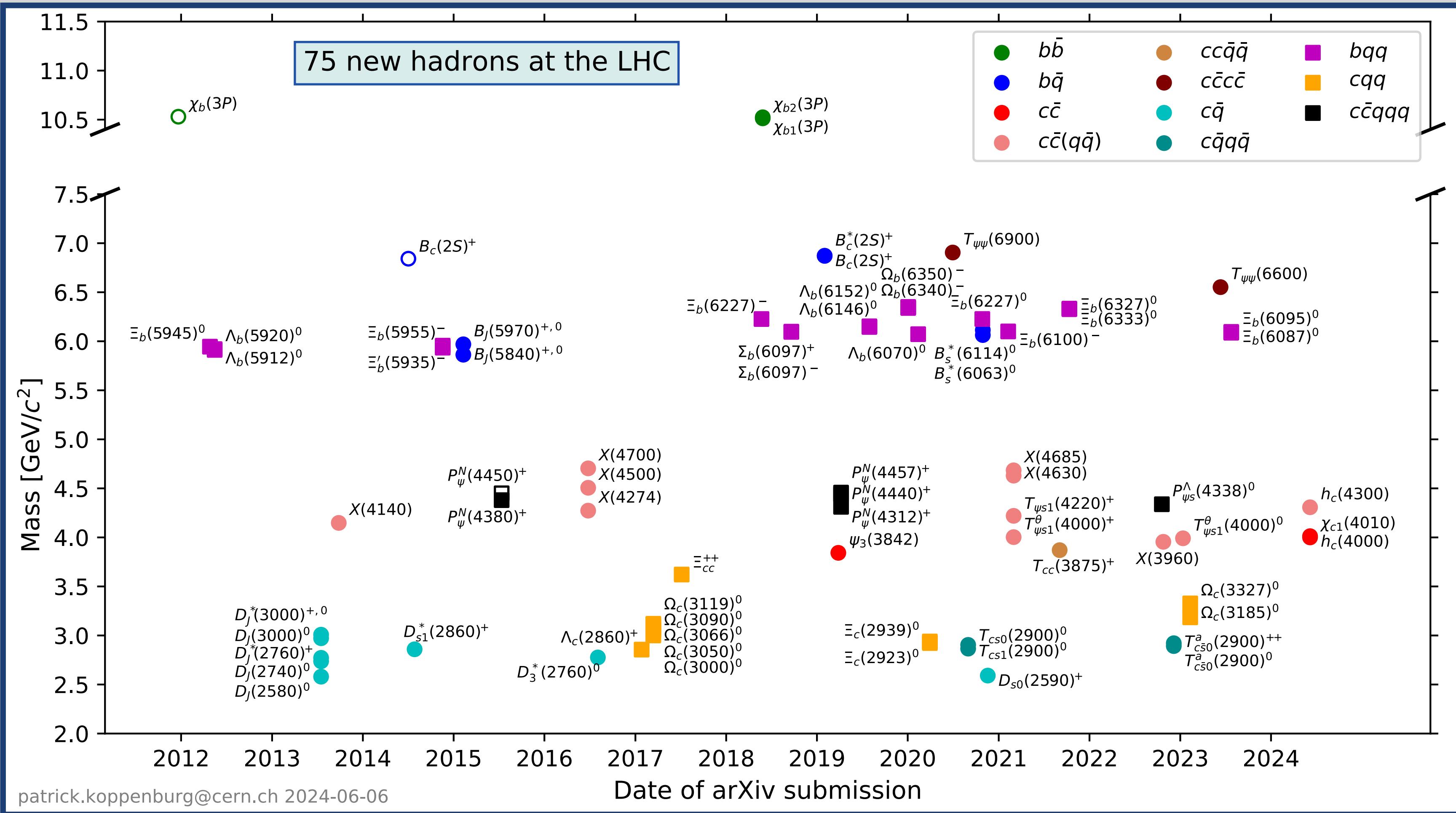
Goldman et al.
PRC 39 (1989) 1889

s-channel resonance
Pole in ${}^3D_3 - {}^3G_3$

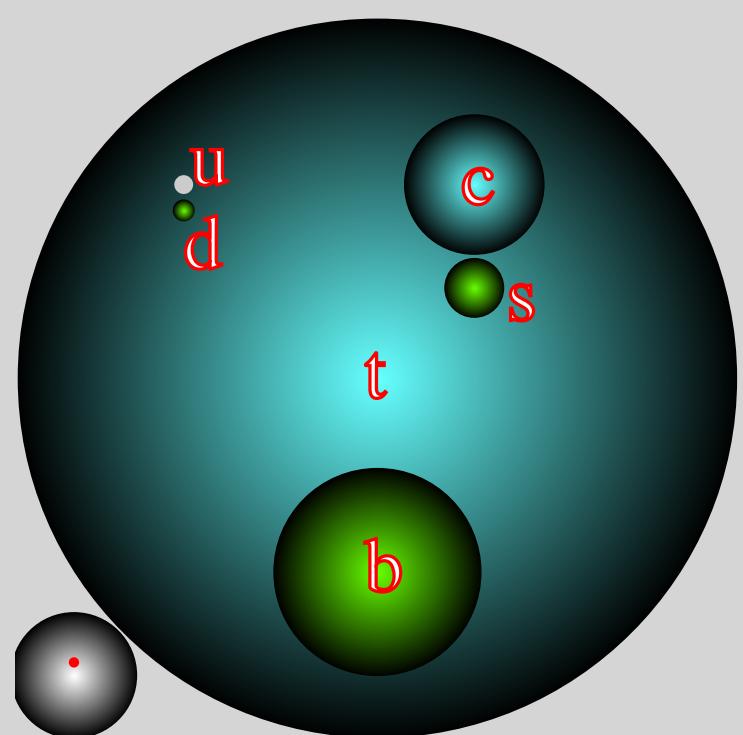




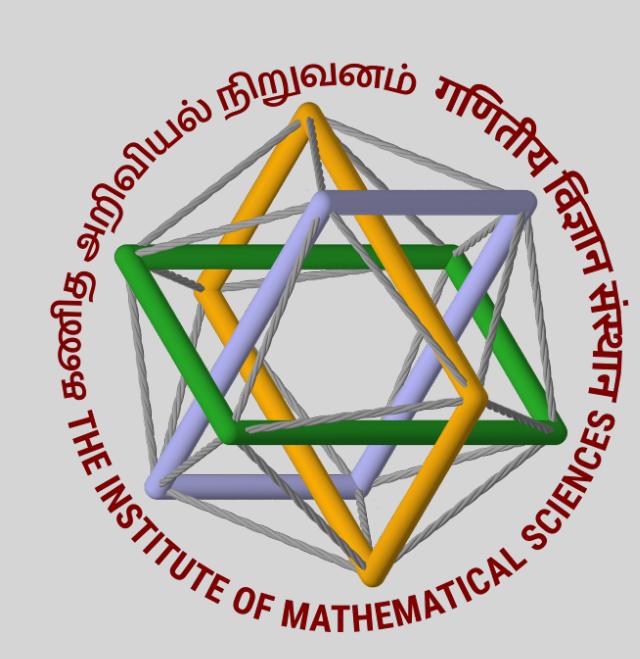
What's up at LHC??



- More interest around heavier hadrons

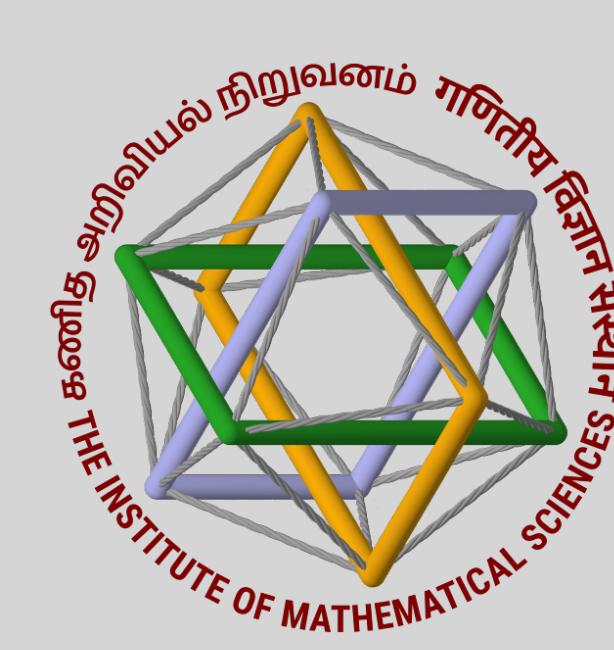


Picture from wiki

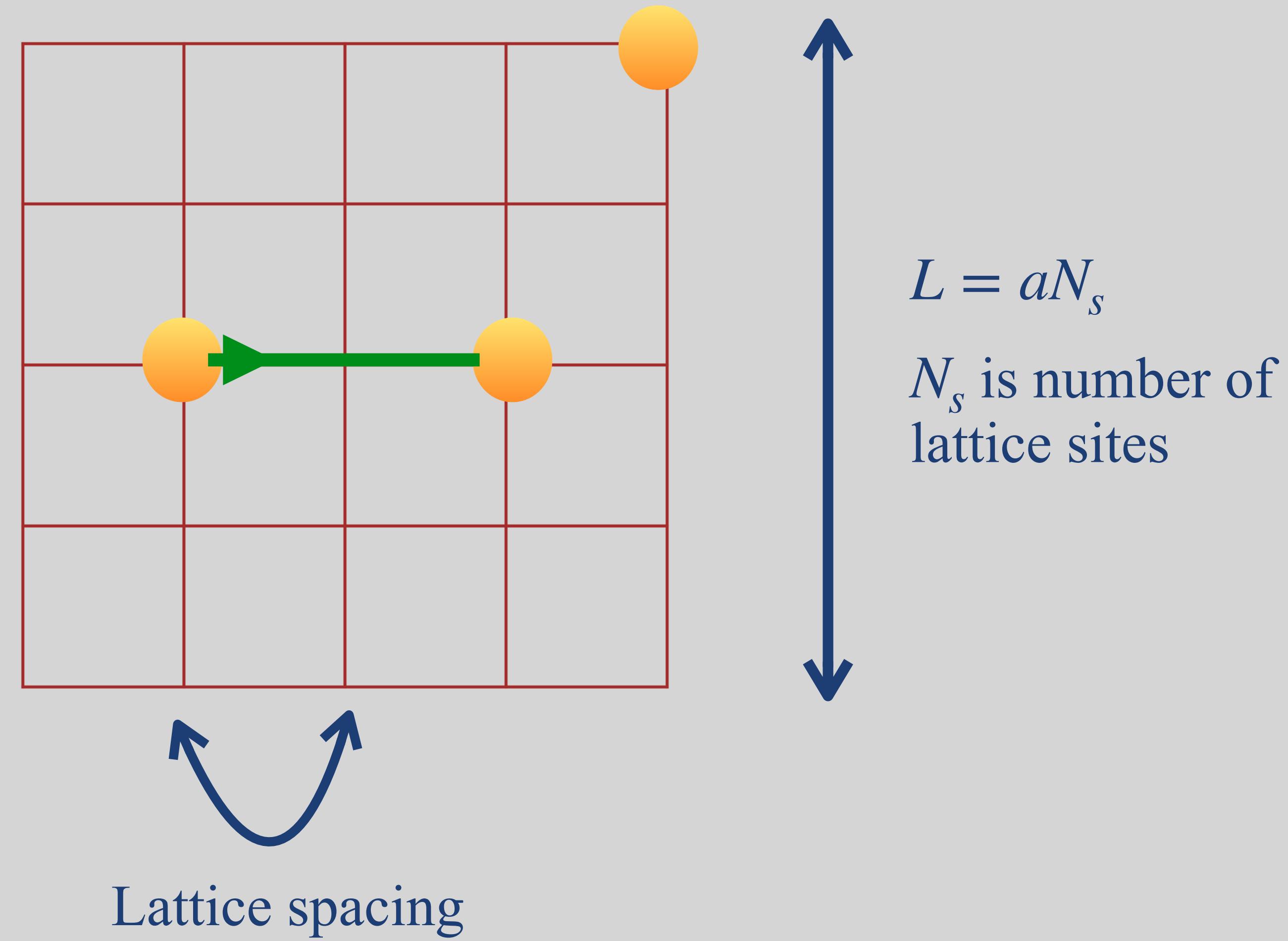


Today's discussion

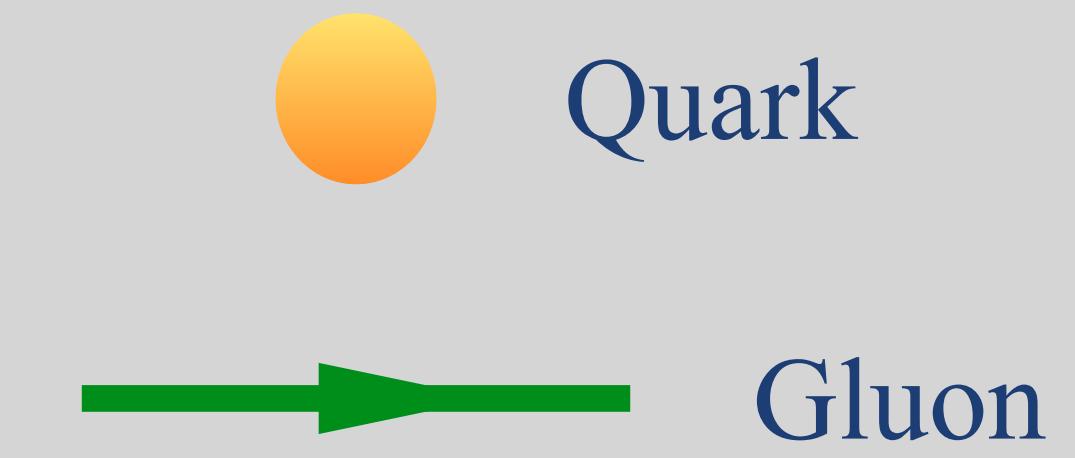
- Baryons consisting of either strange or charm
- Dibaryons consisting of either strange or charm

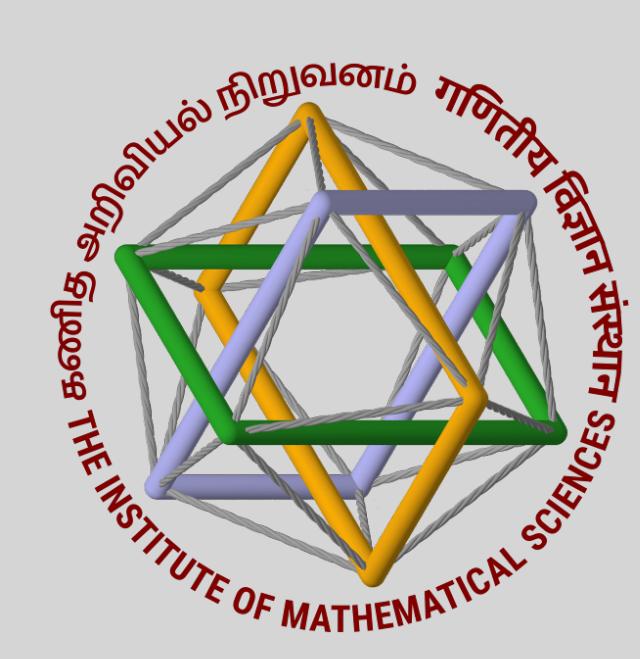


Lattice QCD



- First principles calculation
- Use MCMC to create ensembles of QCD configurations
- Most computational resources spent on generating gauge configurations and evaluating quark propagators





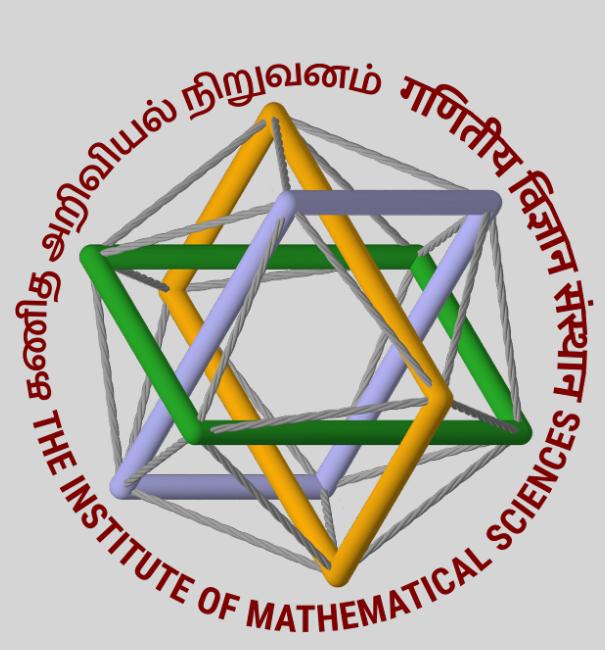
Hadrons



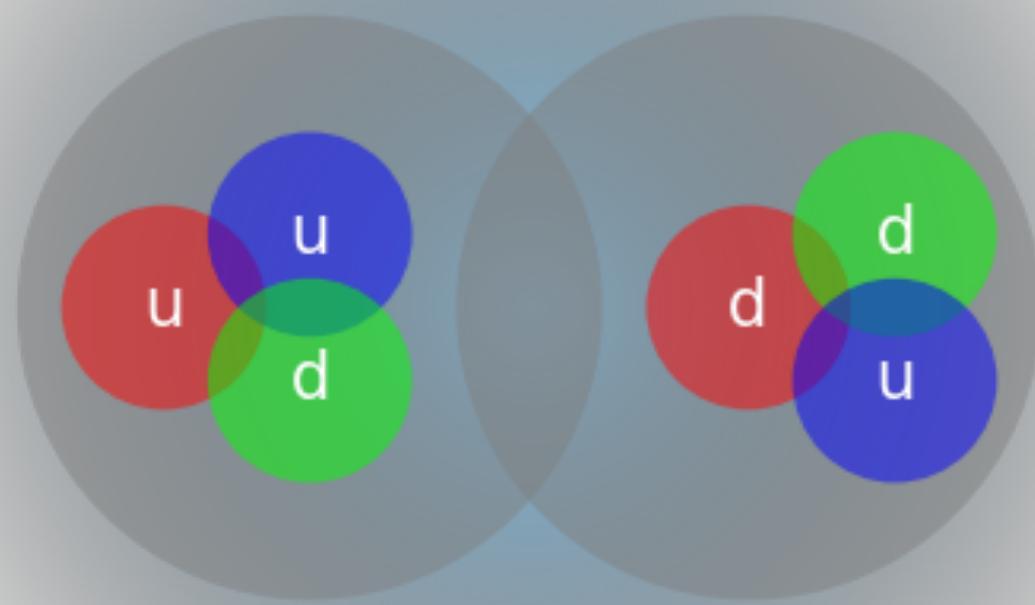
Announcement
Talk Series
August
By Sasa and Feng-Kun

- * Desire to understand nature through fundamental forces and interactions.
- * Baryons and Mesons — Wide range of energy scale
- * Recent experiments reveal missing baryon states, exotic tetraquark, and pentaquark hadrons.
- * Lattice hadron spectroscopy predicted numerous bound states, including exotic hadrons.
- * More progress for bound states stable under strong decay i.e. below threshold or for states closer to threshold





Deuteron



- Lattice calculations use $M_\pi \geq 300 \text{ MeV}$

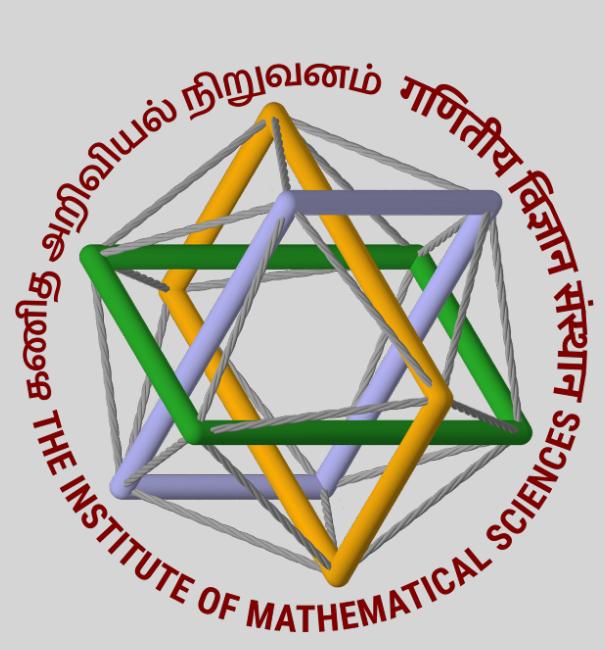
HAL QCD, JHEP 1903 (2019) - Unbound Deuteron, Dineutron
NPLQCD, PRD 107 (2023) - Bound Deuteron, Dineutron

$$M_\pi \approx 800 \text{ MeV}$$

- Discovered around a century back
- Proton (uud) - Neutron (udd) bound state
- Binding Energy = 2.2 MeV

Why different observations from different Lattice studies?
Will this discrepancy be the case for heavier quarks?





Why heavier masses ?

$$m_u, m_d \ll m_\pi$$

Light quarks are expensive

$$\text{cost} \propto \left(\frac{1}{m}\right)^{1-2} \left(\frac{1}{a}\right)^{4-6} (L)^{4-5}$$

Lepage, TASI (1989)

Signal (m_N hadron)

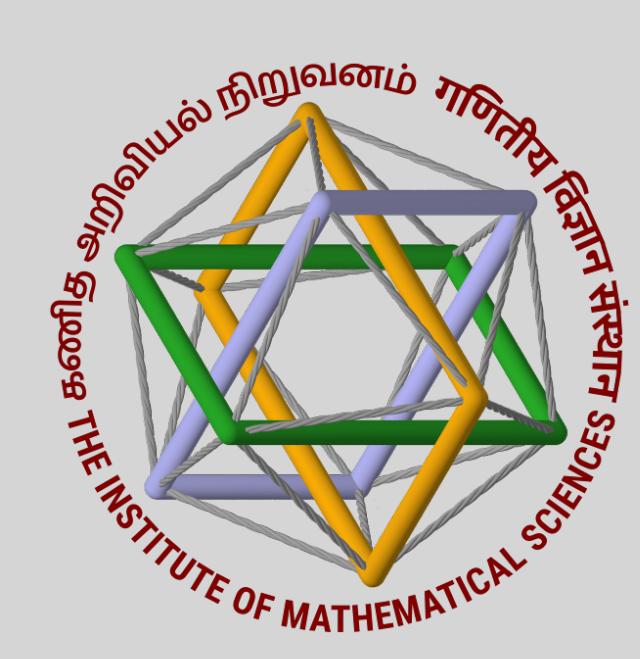
$$\propto e^{-m_N t}$$

Noise

$$\propto e^{-\frac{3}{2}m_\pi t}$$

Error in propagator correlation function
dominated by pions because of virtue of
lower energy states

Signal to Noise ratio exponentially degrades for $m_q \rightarrow 0$



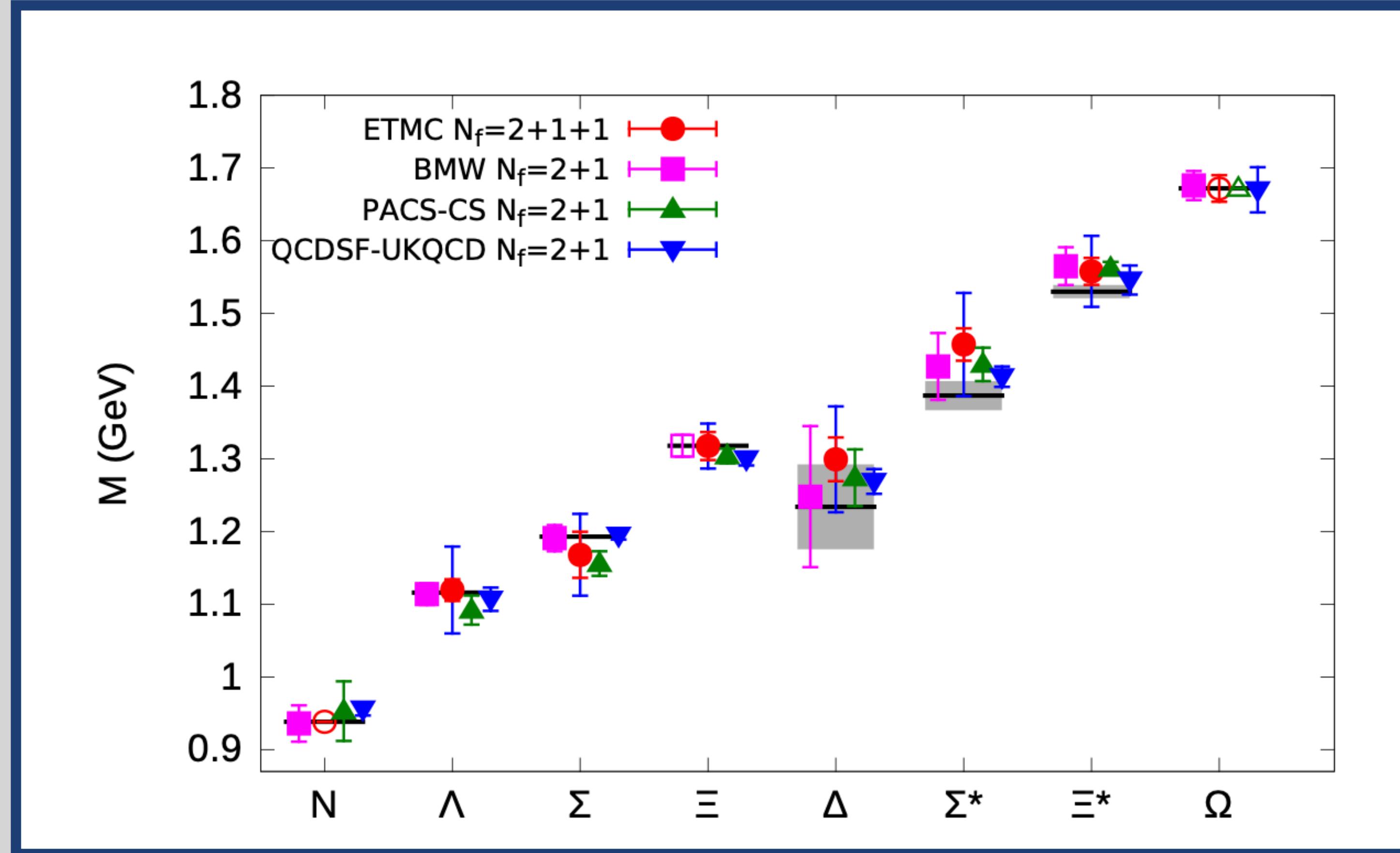
Lattice - More Requirements

Apart from unphysical heavy light quark masses we need:

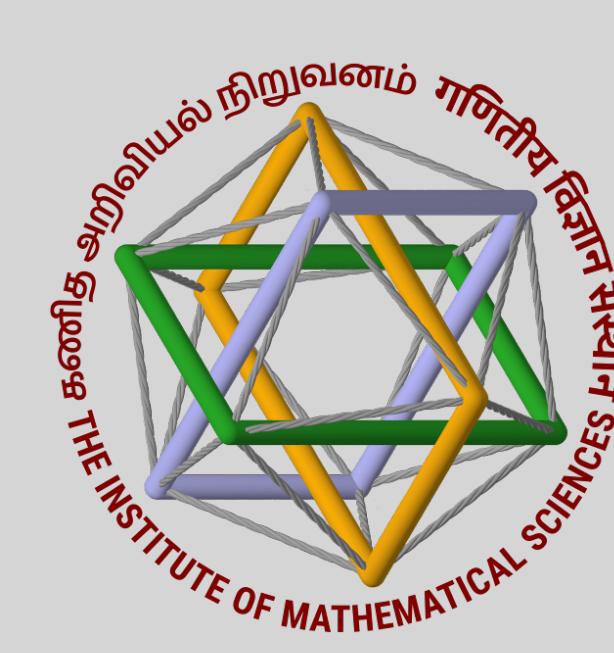
- * Multiple ensembles for continuum limit.
- * Even more computing power in contractions for exotic hadrons.
- * $m_\pi L \geq 4$, to constraint finite volume effects.
- * Finer lattices for lesser discretisation errors.

Interplay between choice of a, L, m_π to have better results

Baryons from Lattice



- * Masses of low lying baryons using LQCD.
- * Results consistent with experiments.
- * Predictions for hadrons not experimentally measured at that time.
- * This calculation with $N_f = 2 + 1 + 1$



Heavier Baryons from Lattice

Observations of heavier baryons improved in Lattice calculations over the years (and experimentally):

* Bigger (and finer) lattice calculations. In our current calculations we have used $64^3 \times 192$ lattice with lattice spacing $a \approx 0.044 \text{ fm}$

* Improved algorithms

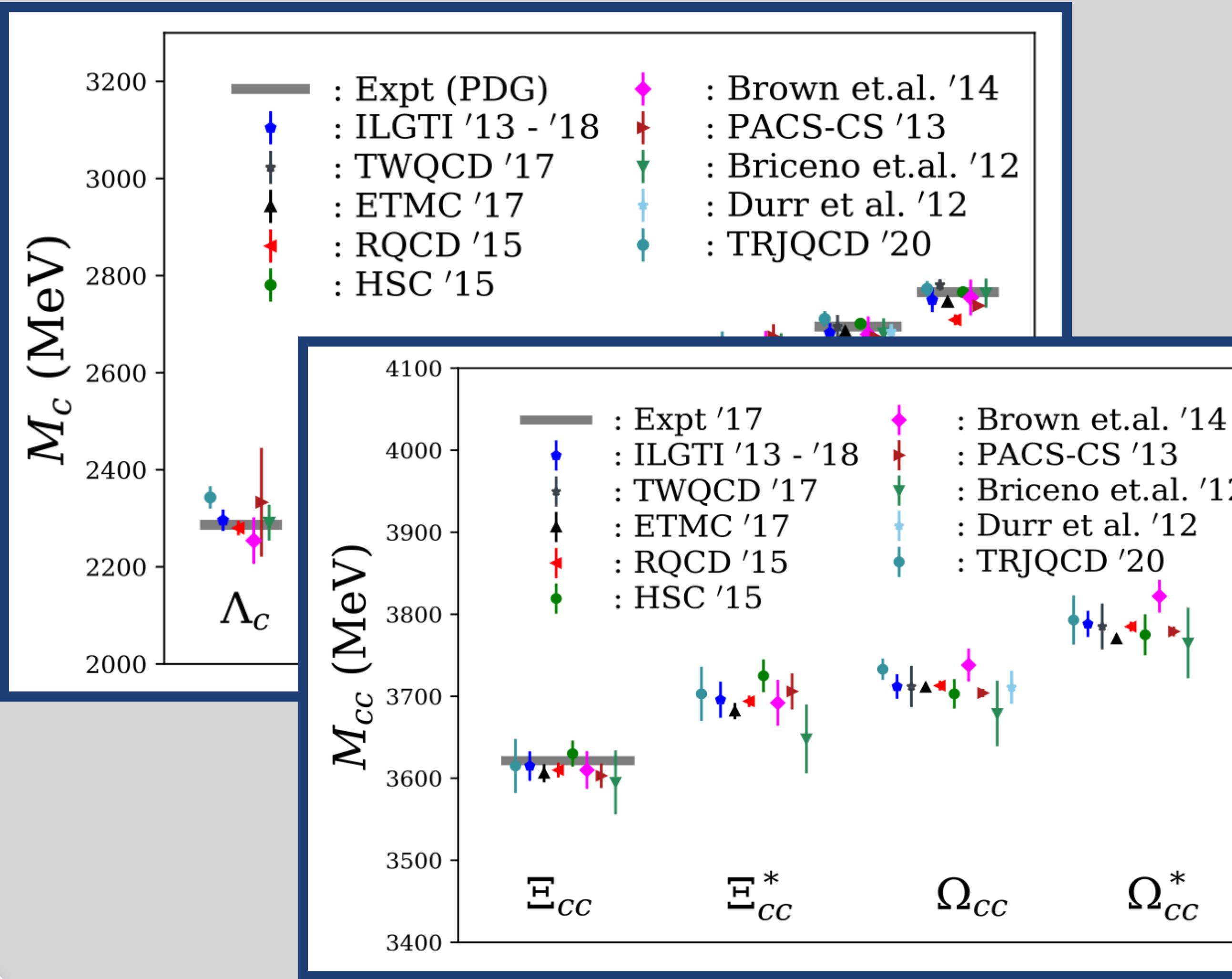
$$N_f = 0 \text{ (quenched)} \rightarrow 2 \text{ } (u, d) \rightarrow 2 + 1 \text{ } (u, d, s) \rightarrow 2 + 1 + 1 \text{ } (u, d, s, c)$$

* Growth of machine power from certain GFLOPS to $\mathcal{O}(10)$ PFLOPS



Heavier Baryons from Lattice

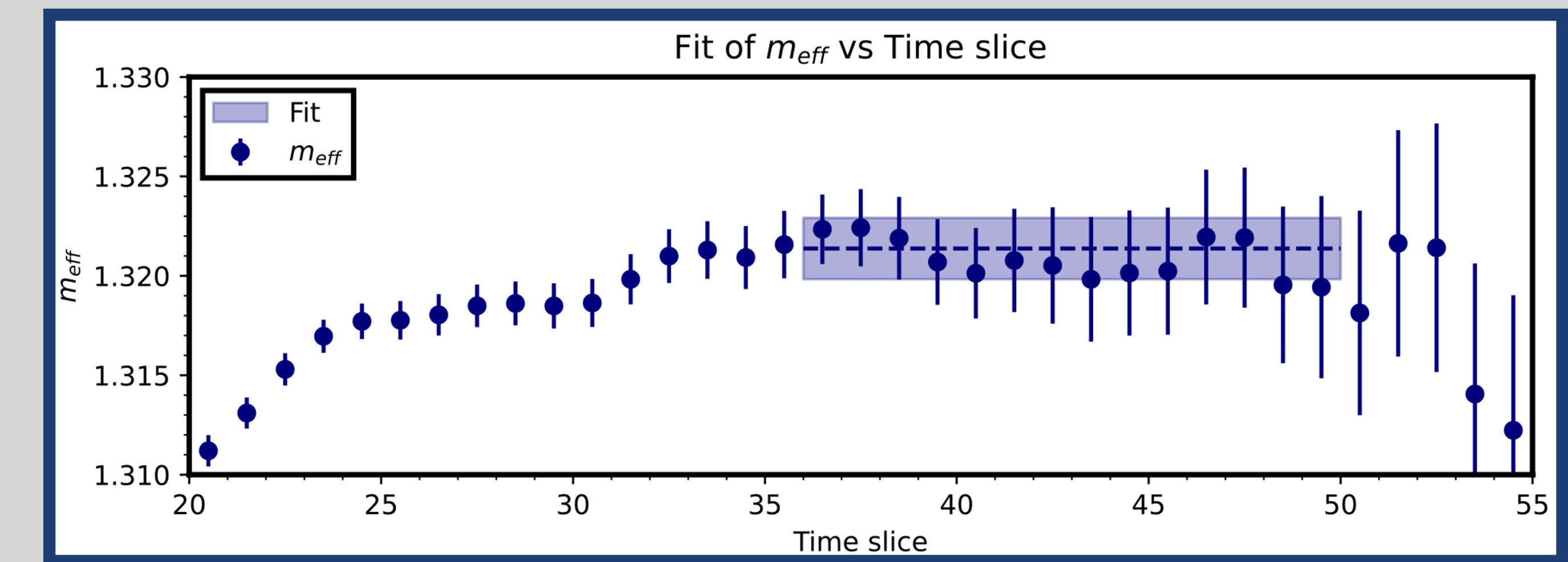
M. Padmanath, CHARM 2020 Talk

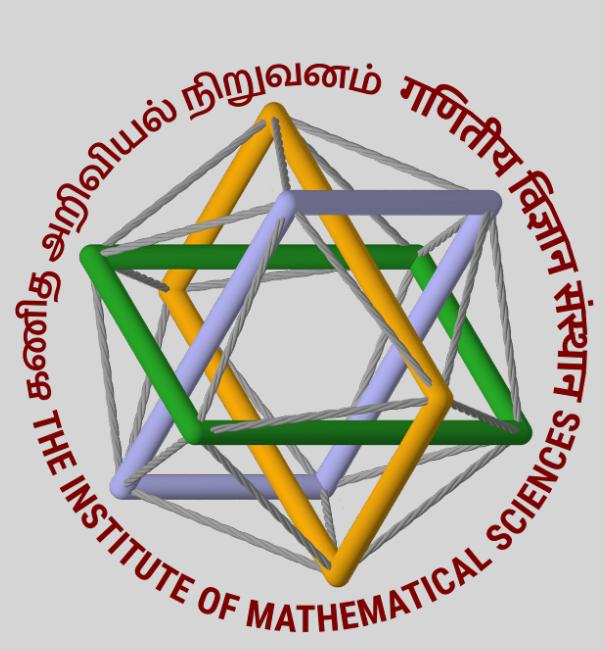


- * Hadrons in charm sector.
- * Benchmarks for lattice calculations.
- * Xi double-charm baryon lattice calculation before experimental prediction.
- * ILGTI calculations matches precisely with other calculations.

Masses from Lattice

- * Euclidean two point correlator as: $C_{ji}(t_f - t_i) = \langle 0 | O_j(t_f) \bar{O}_i(t_i) | 0 \rangle = \sum_n \frac{Z_i^{n*} Z_j^n}{2m_n} e^{-m_n(t_f - t_i)}$
- * $O_j(t_f)$ and $\bar{O}_i(t_i)$ are the desired interpolating operators and $Z_j^n = \langle 0 | O_j | n \rangle$
- * Effective mass = $\log \left[\frac{C(t)}{C(t+1)} \right]$





Operators

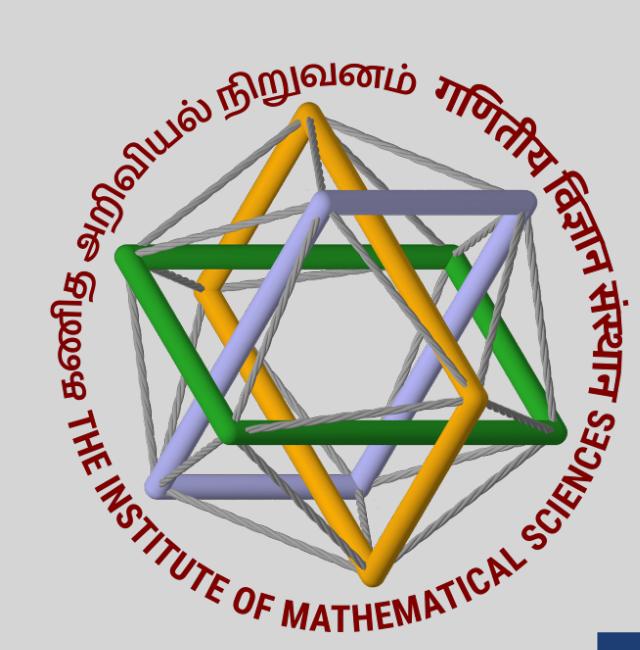
Operator for baryon: $\mathcal{O} = \epsilon_{abc} q_{1,\mu_1}^{a,f_1} q_{2,\mu_2}^{b,f_2} q_{3,\mu_3}^{c,f_3}$

In this work we focus on single flavored baryons

- Total wave function for baryon (fermion) anti symmetric
- Single (symmetric) flavor, color anti symmetric
- Spin must be symmetric - 3/2 - H^+ irrep

H^+ irrep has two embeddings corresponding to non-relativistic and relativistic operators





Operators Contraction

$$\mathcal{O} = \epsilon_{abc} q_{\mu_1}^a q_{\mu_2}^b q_{\mu_3}^c$$

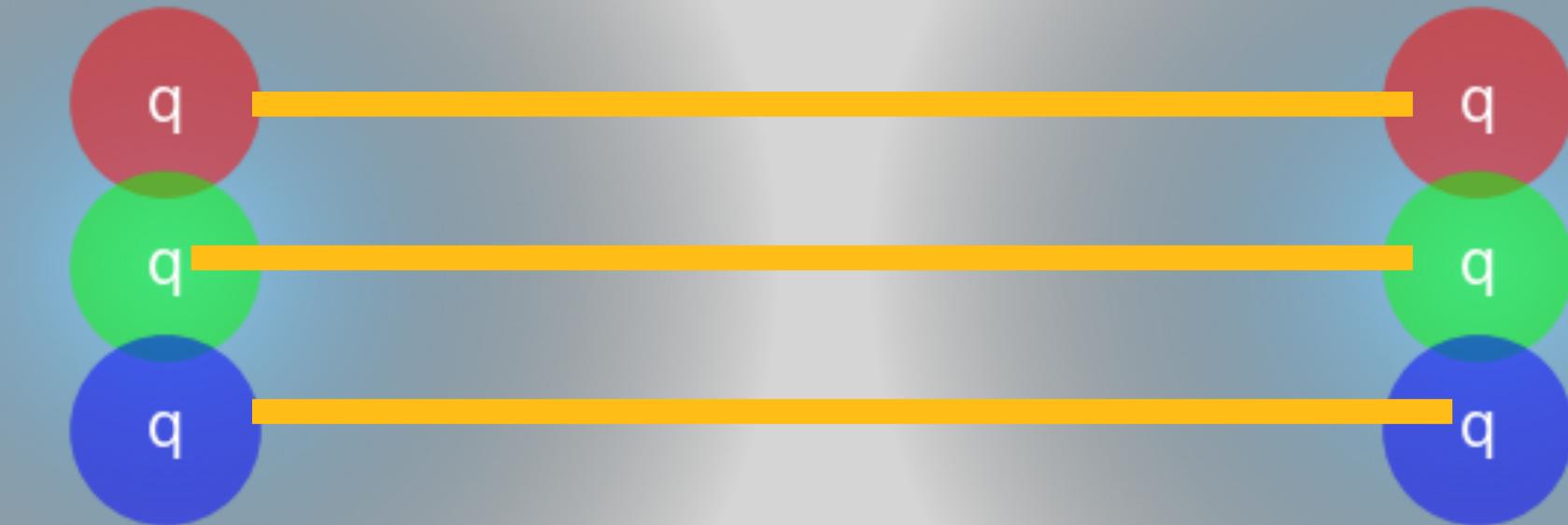
S_z	Operator	State
$3/2$	${}^1H_{3/2}$	111
$1/2$	${}^1H_{1/2}$	112+121+211
$-1/2$	${}^1H_{-1/2}$	122+212+221
$-3/2$	${}^1H_{-3/2}$	222

Non Relativistic Embedding [N]

S_z	Operator	State
$3/2$	${}^2H_{3/2}$	133+313+331
$1/2$	${}^2H_{1/2}$	233+323+332+134+341+413+143+431+314
$-1/2$	${}^2H_{-1/2}$	144+414+441+234+342+423+243+432+324
$-3/2$	${}^2H_{-3/2}$	244+424+442

Relativistic Embedding [R]

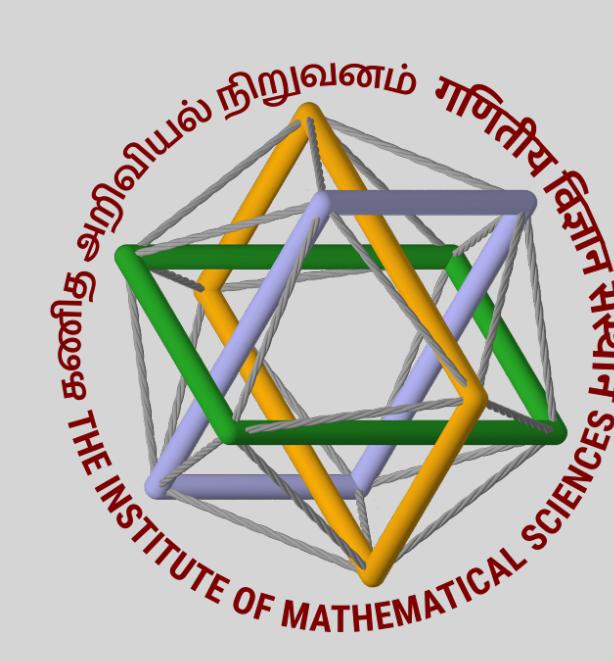
Basak et al., PRD 72, 074501 (2005)



Source slice

Sink slice

- We can choose different embedding for different operators
- Effectively single contraction

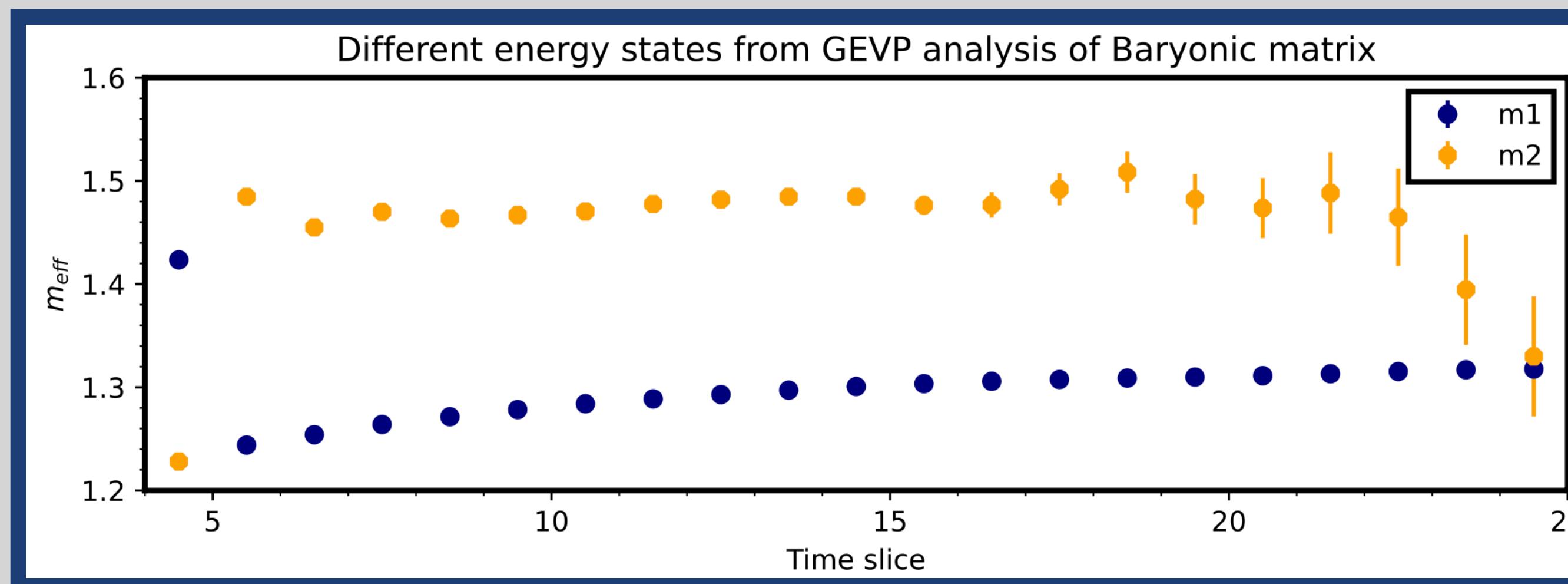


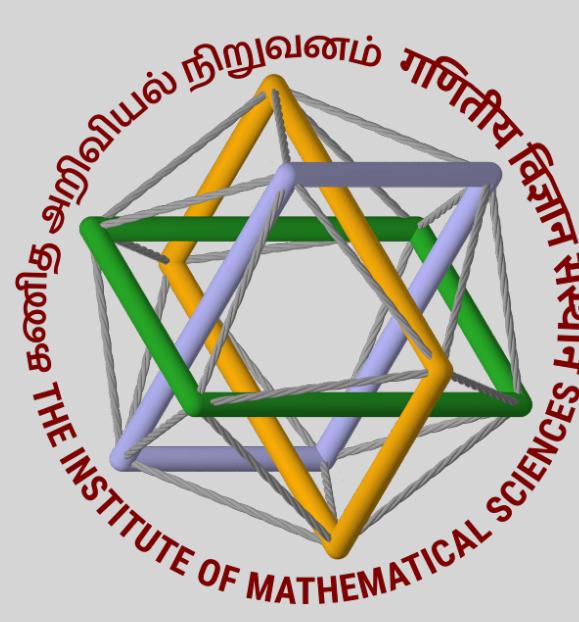
Operators Contraction

N-N	N-R
R-N	R-R

Baryonic matrix constructed out of both non-relativistic and relativistic operators on which we apply GEVP

- Larger overlap if we use just N-N operator combination
- We use full matrix to show that there is no excited state contamination
- Time reversed backward propagator used to double data
- Negative parity calculation also done





Lattice Setup

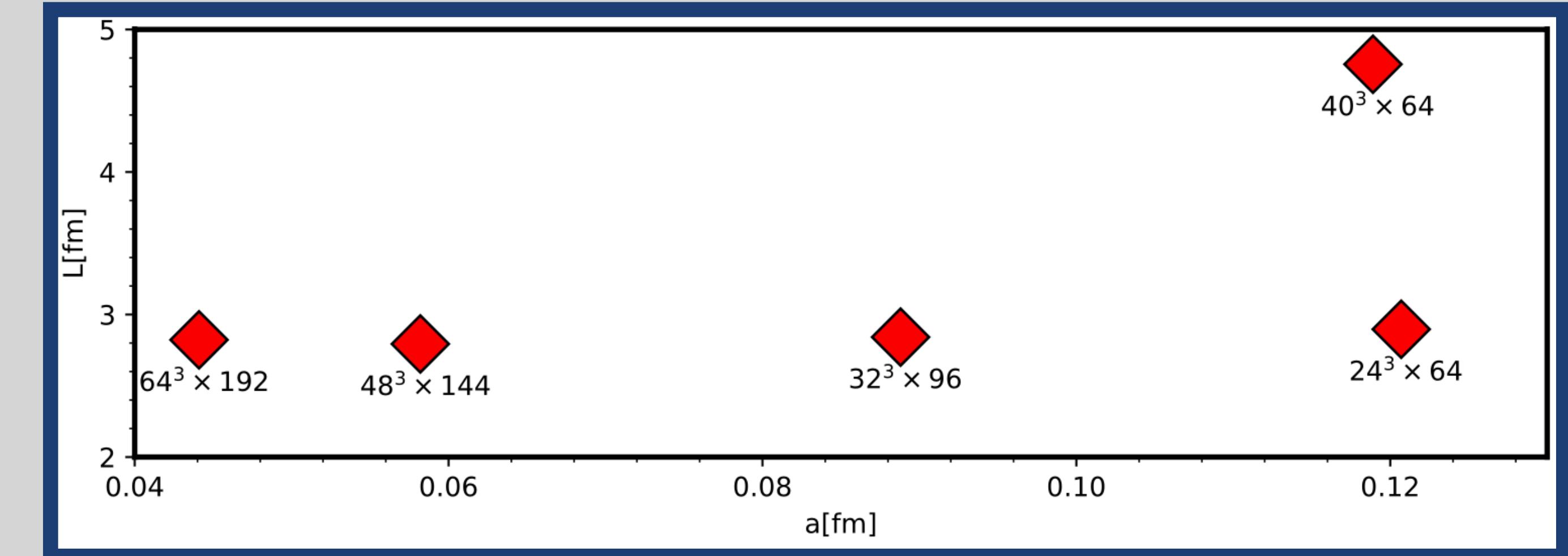
Bazavov et al., PRD 87 (2013) 5, 054505

- Overlap action on background of Highly Improved Staggered Quark (HISQ) gauge configurations.
- Strange and charm masses set at physical values.
- Up and down set as degenerate masses heavier than physical values.
- Finest lattice used $a \approx 0.044 \text{ fm}$ with Volume as $64^3 \times 192$

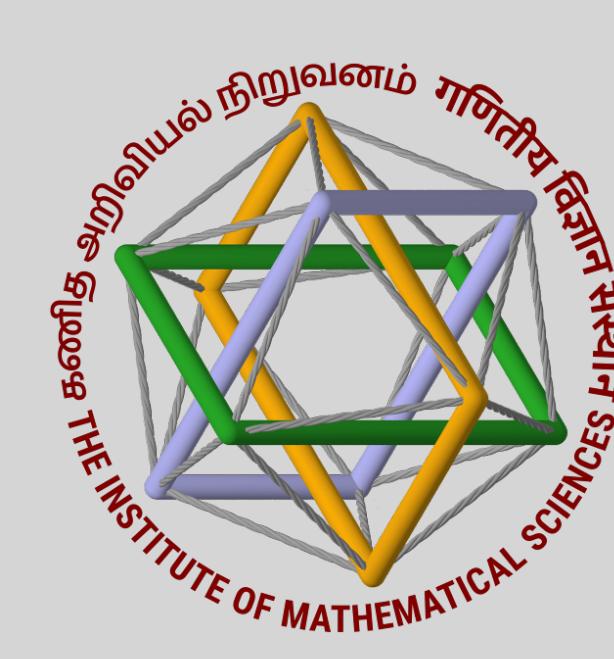
Ensembles

$$\bar{M}_{av} = \frac{1}{4} \left(M_{\eta_c} + 3M_{J/\psi} \right)$$

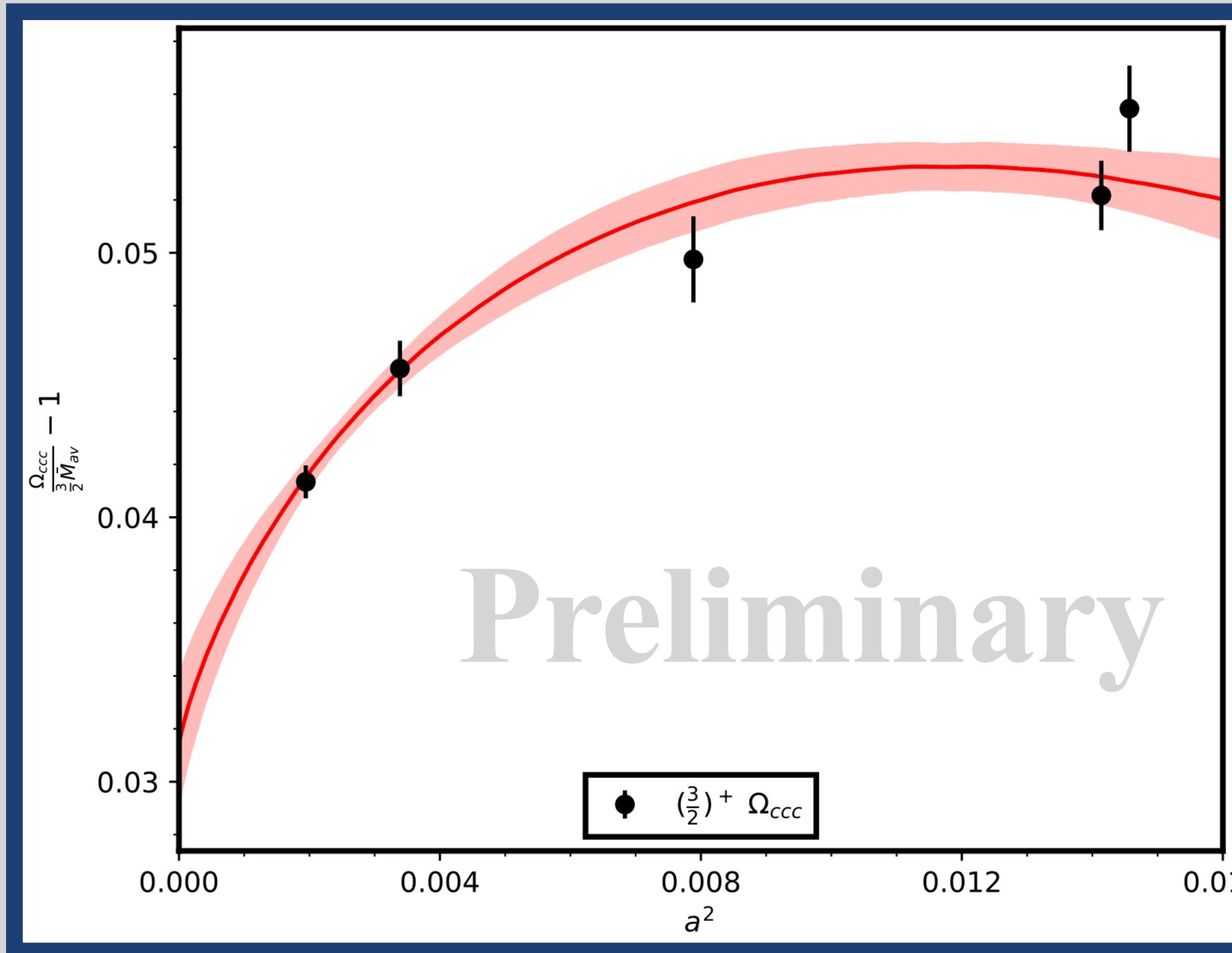
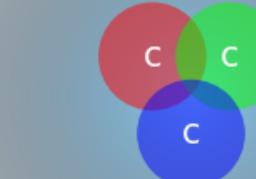
$$M_h - \frac{3}{2} \bar{M}_{av}$$



- Configurations generated by MILC.
- Discretisation effects highest in triply charmed baryons.
- Instead of effective mass in charmed baryons, we will plot effective splittings.

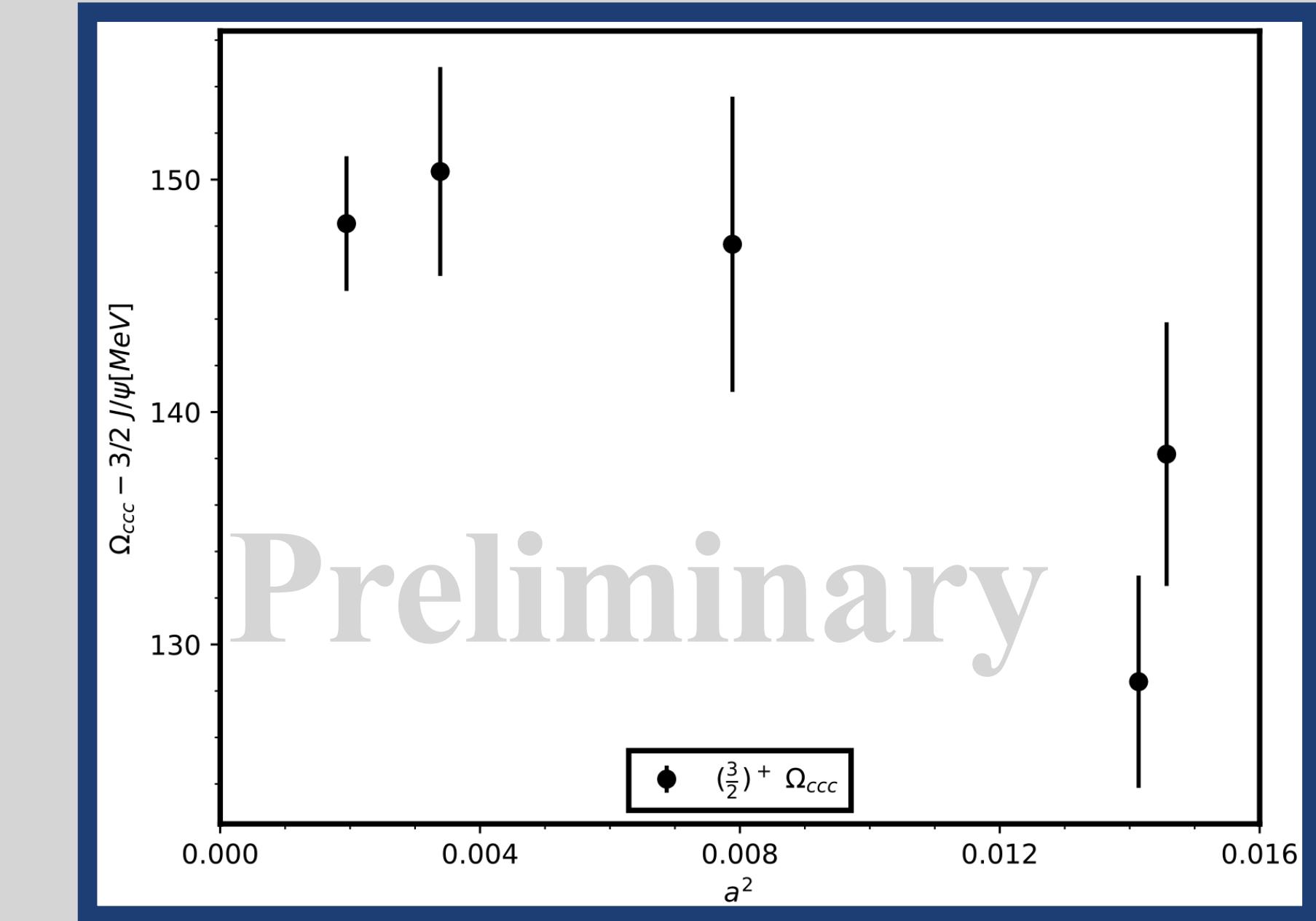


Ω_{ccc} Baryon

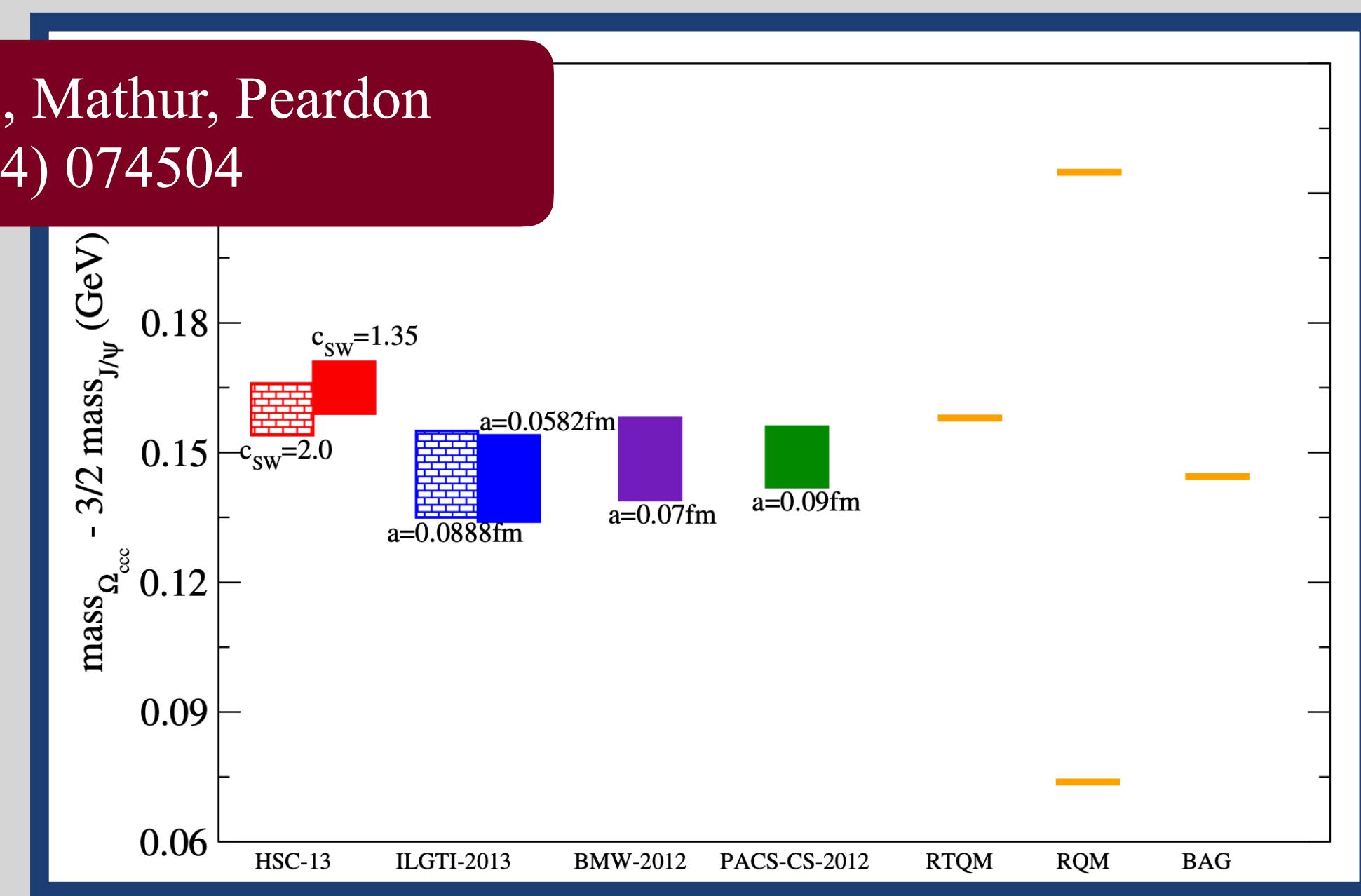


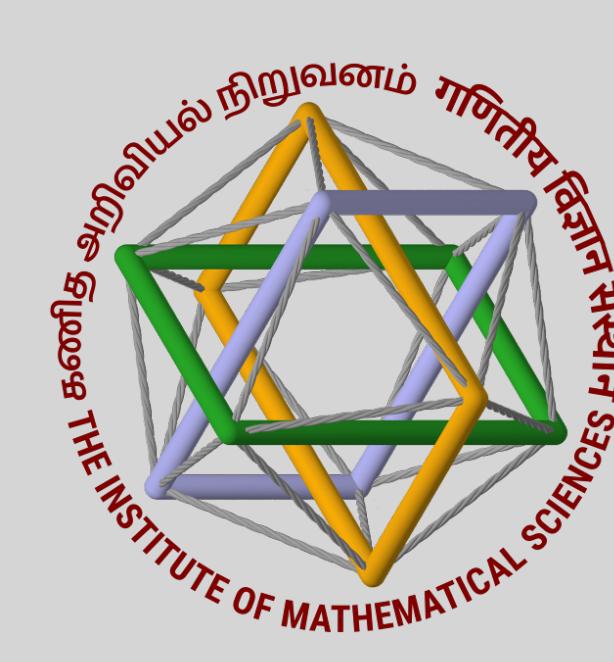
$A + Ba^2 + Ca^2 \ln(a)$ fit

$$\bar{M}_{av} = \frac{1}{4} \left(M_{\eta_c} + 3M_{J/\psi} \right)$$

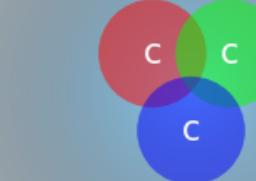


Padmanath, Edwards, Mathur, Peardon
PRD 90 (2014) 074504

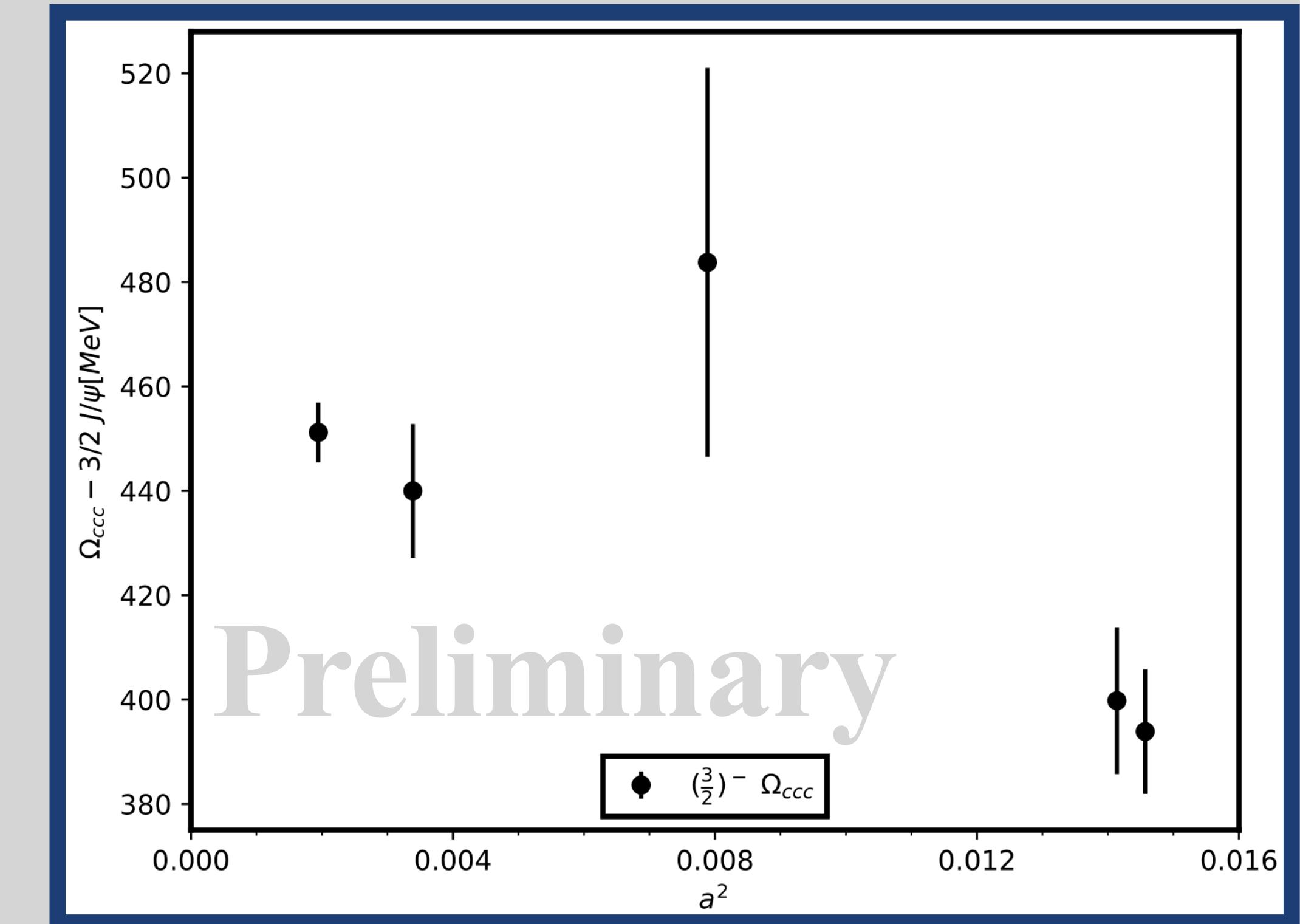
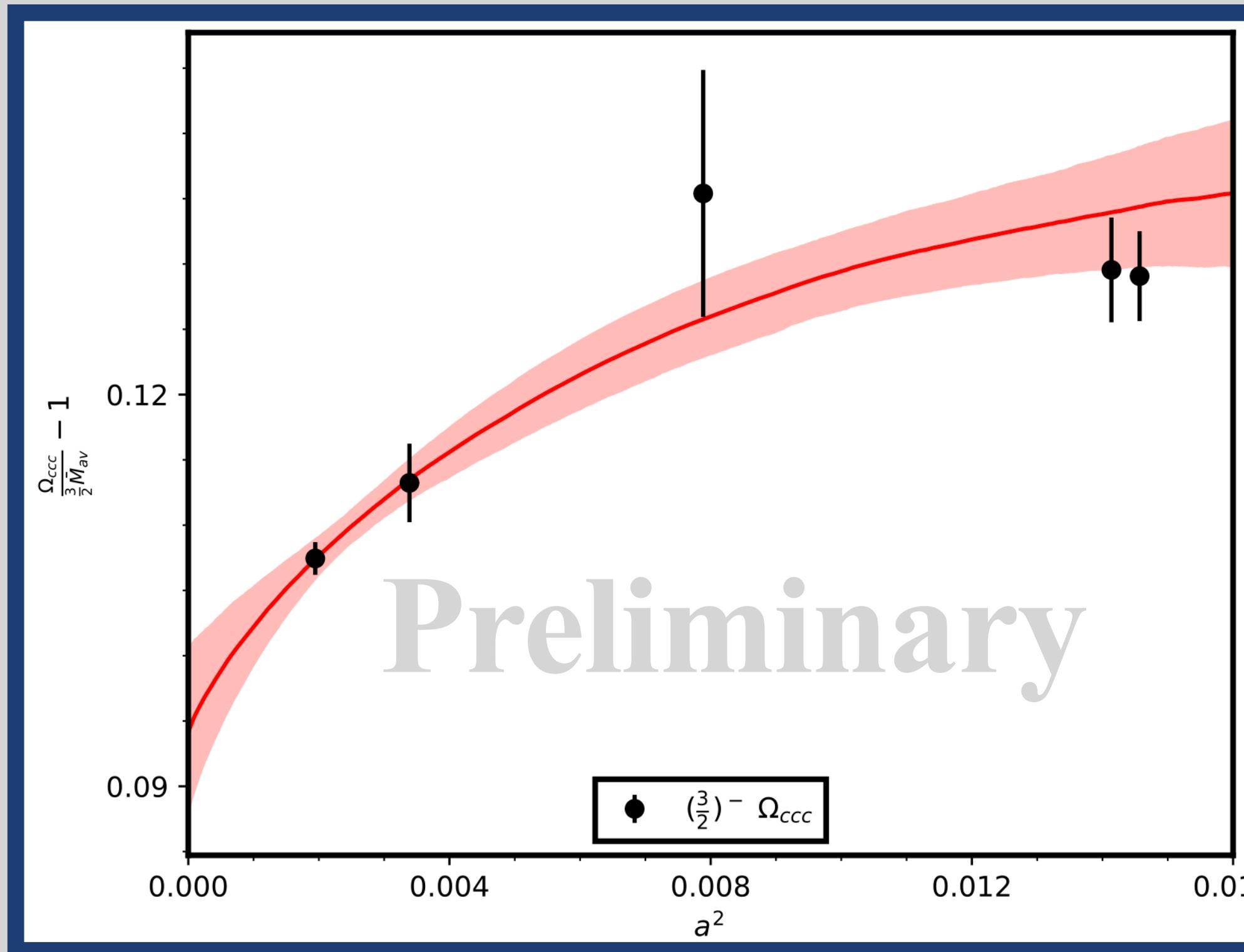




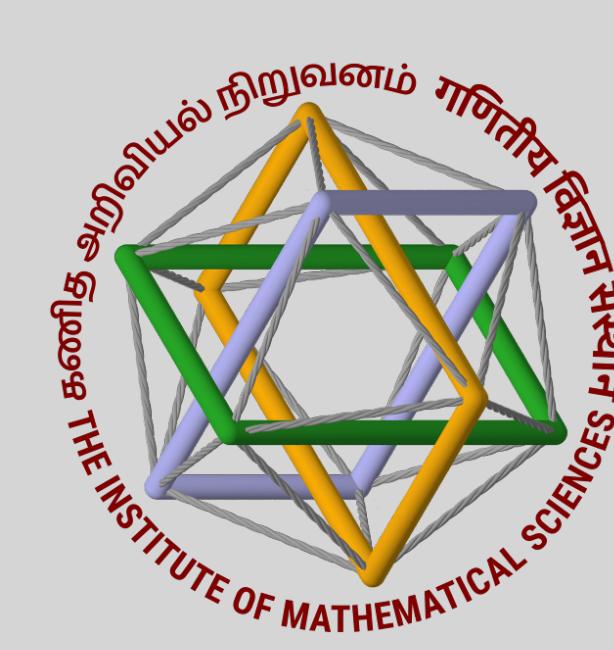
Ω_{ccc} Baryon



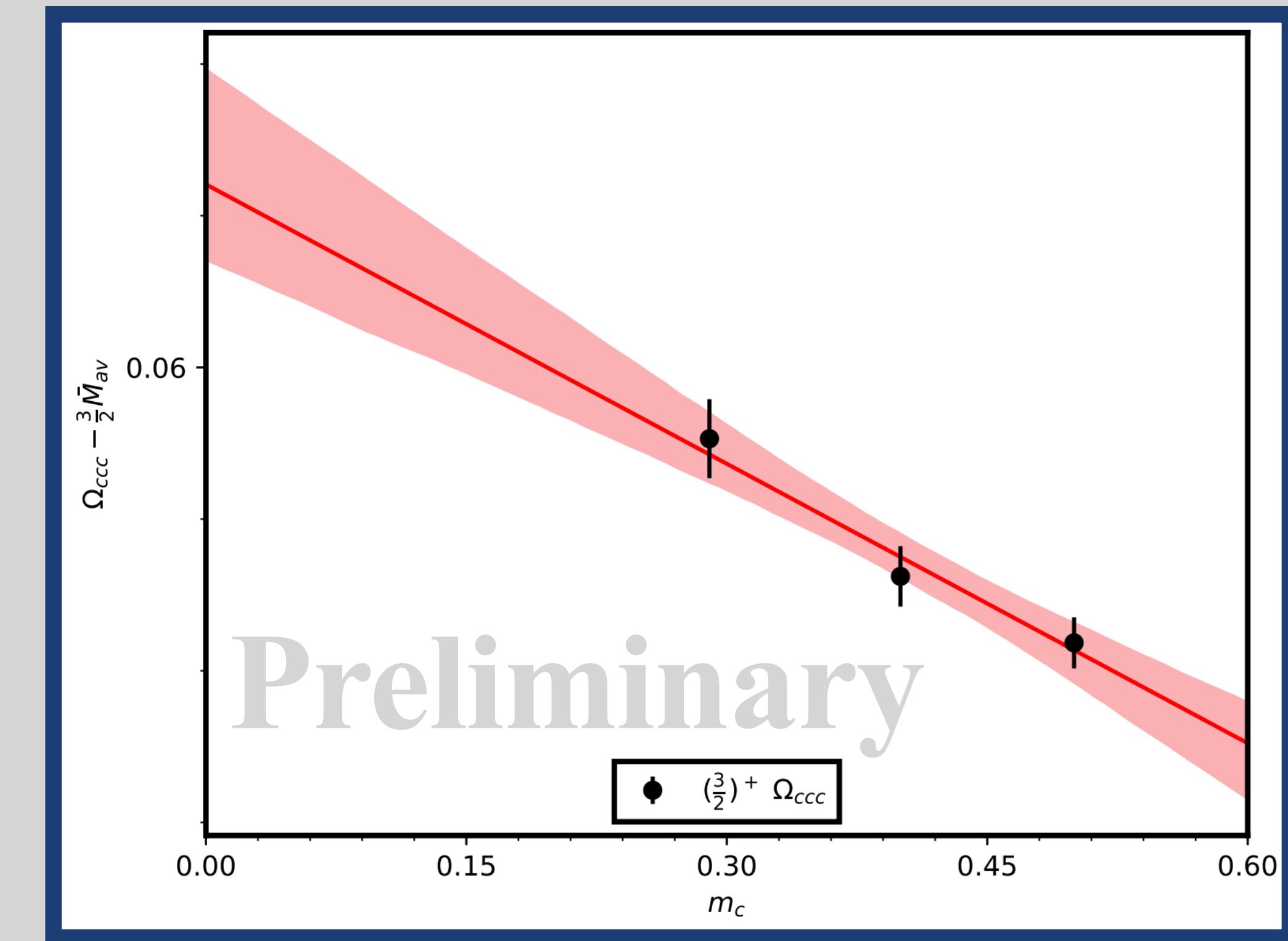
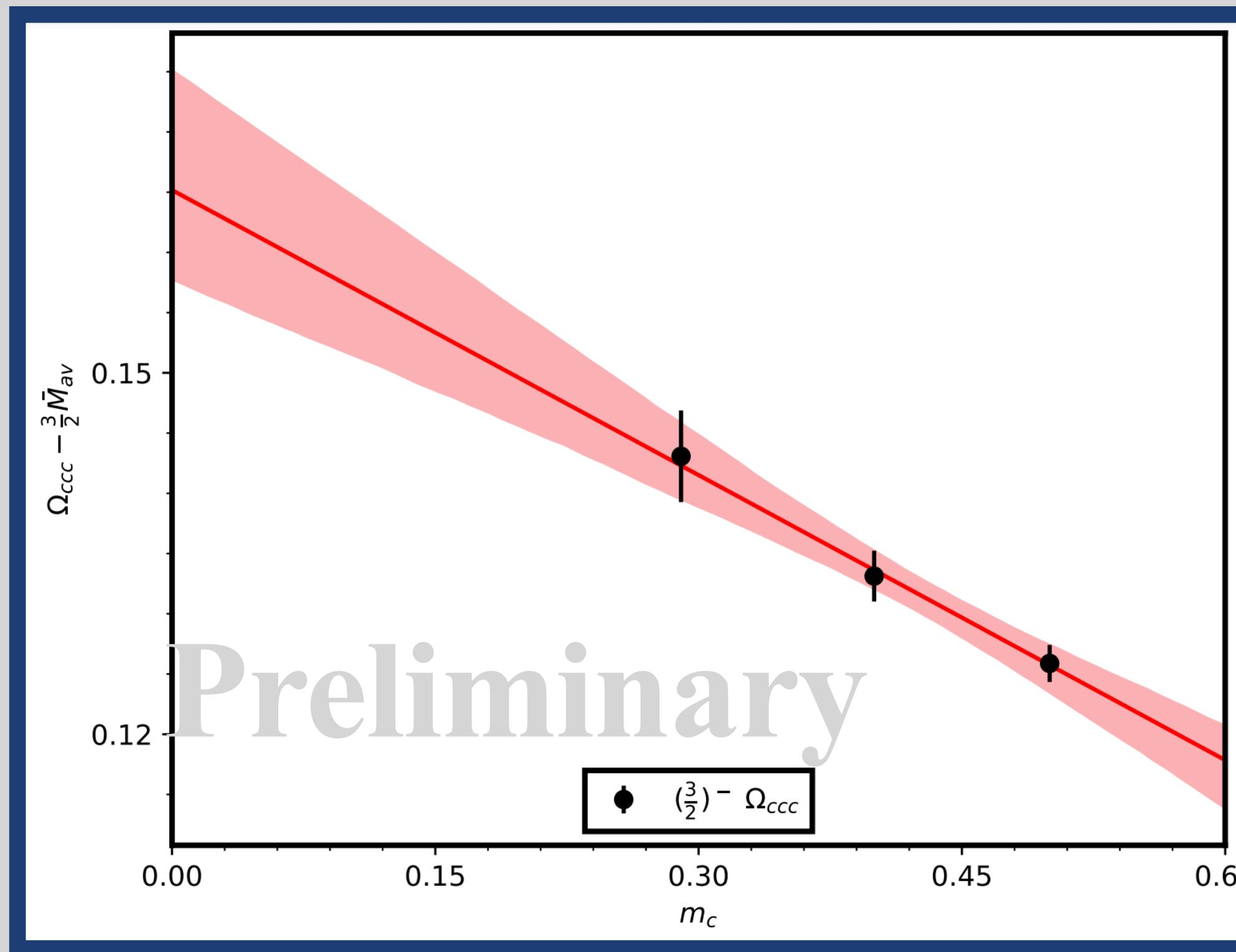
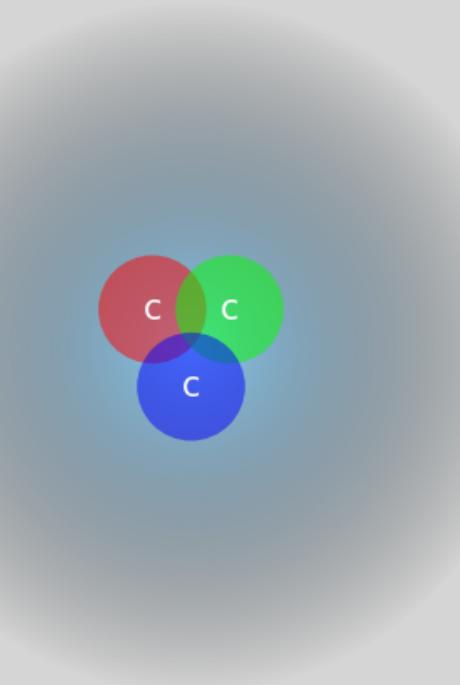
Will we see experimental observation of
 Ω_{cc} and Ω_{ccc} soon?



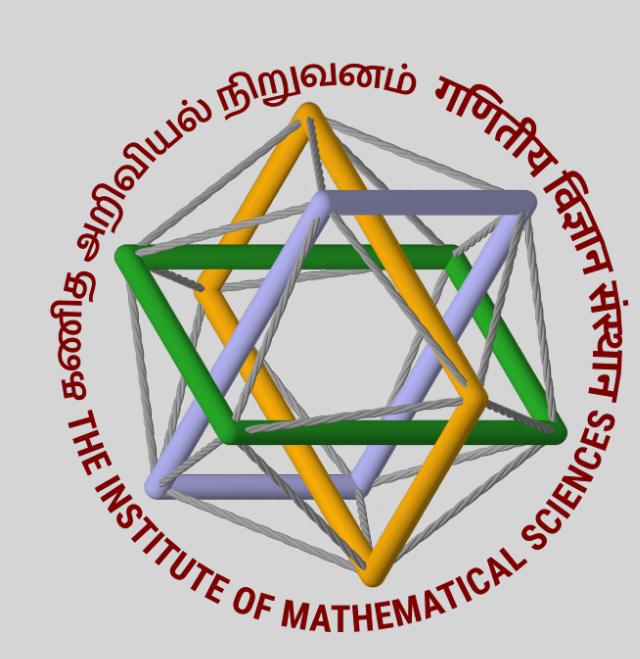
Negative Parity



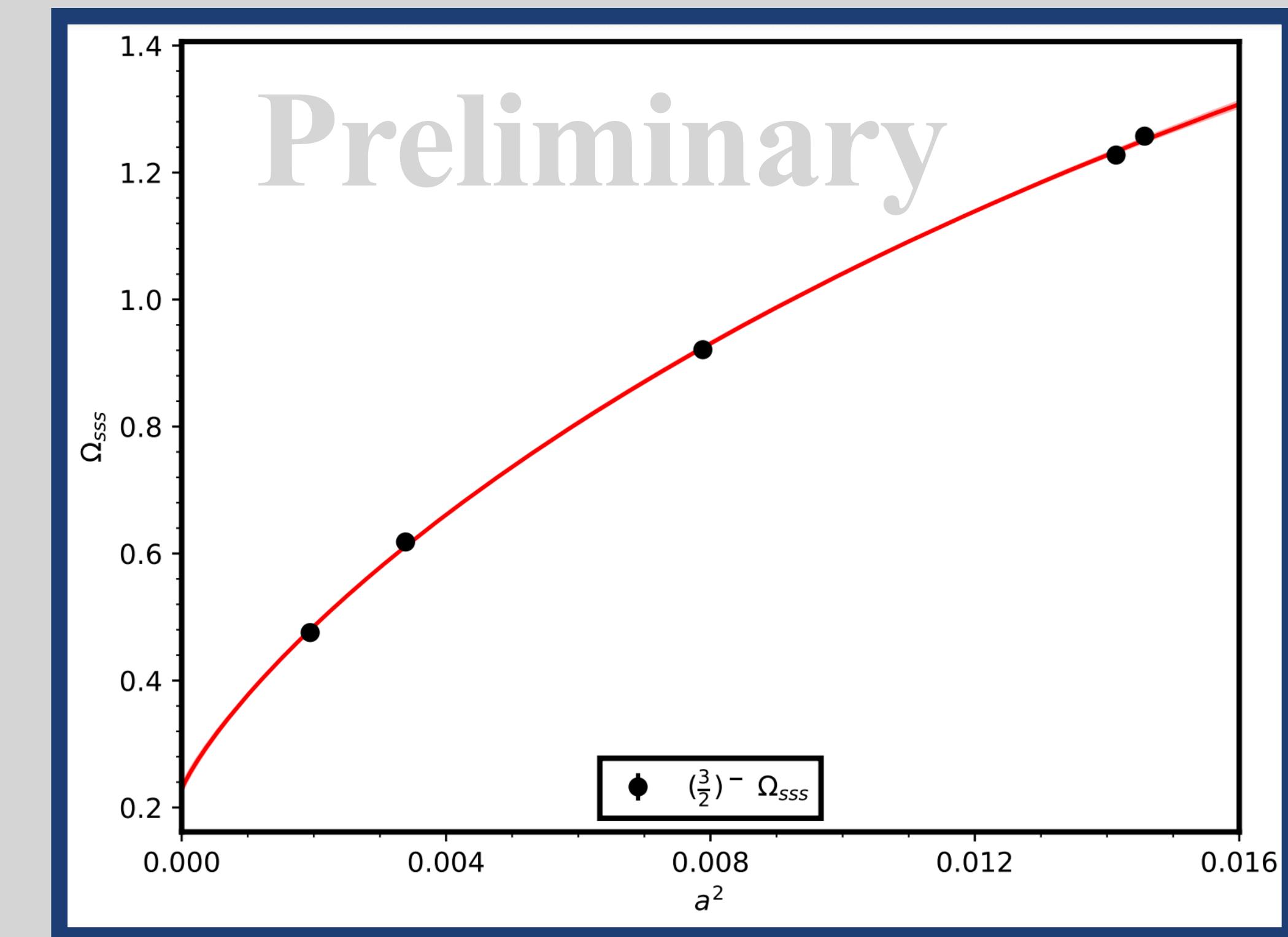
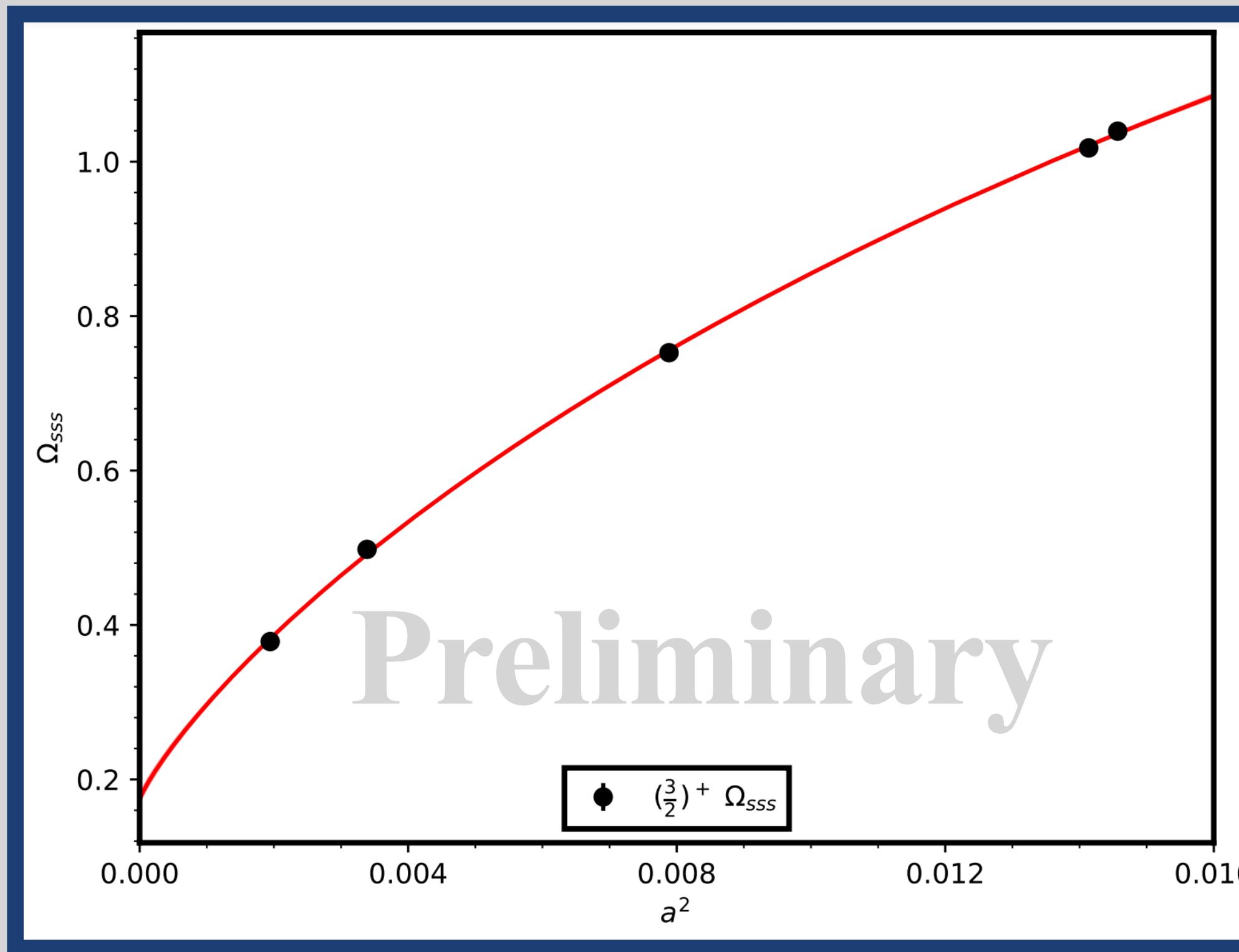
Ω_{ccc} Baryon



Tuning charm mass



Ω Baryon

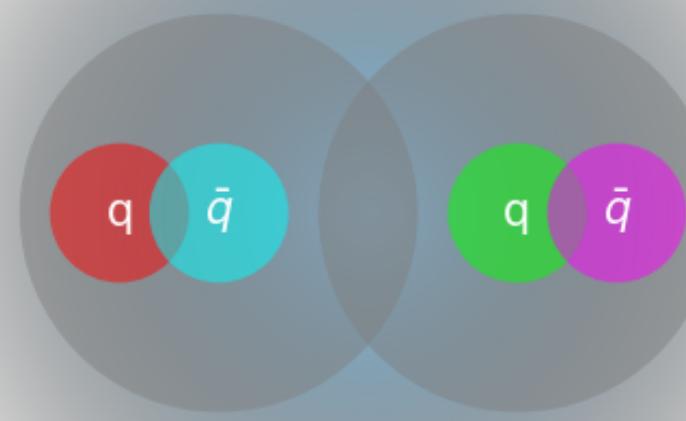


Exotic states

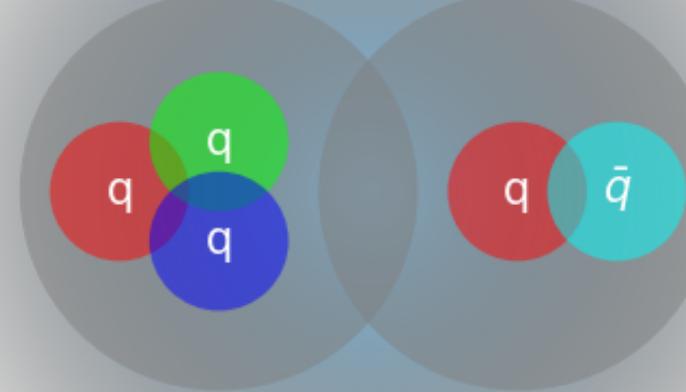
Instead of baryons and mesons we can have more exotic hadrons

Predicted in quark model

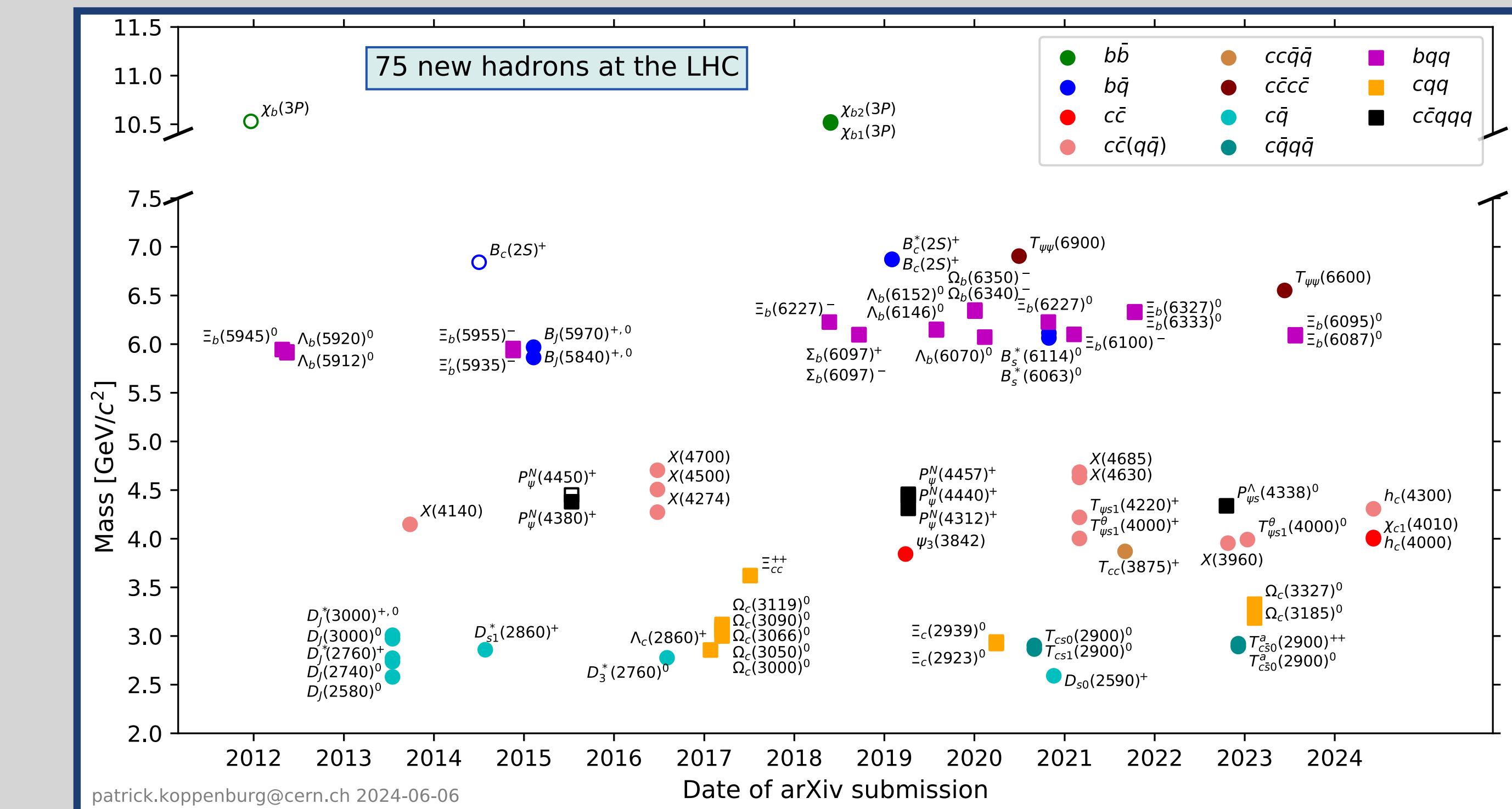
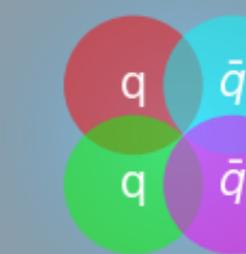
Recent experimental results show their existence

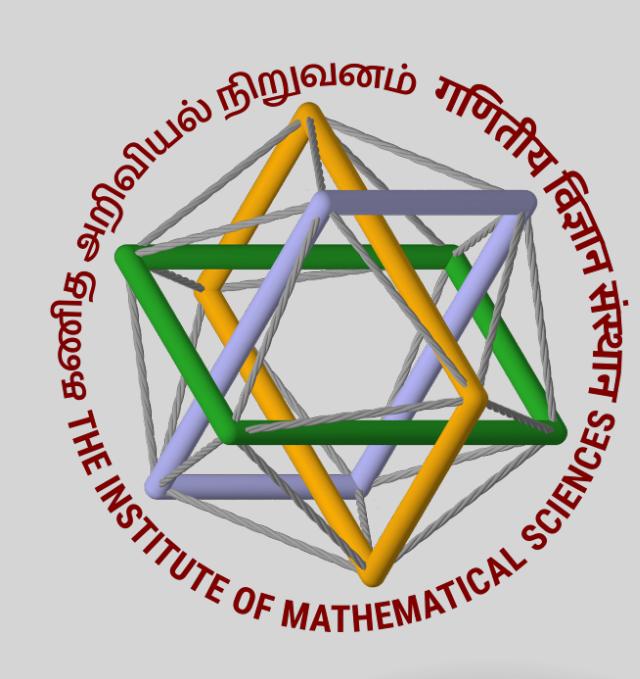


Hadron with 4 quarks



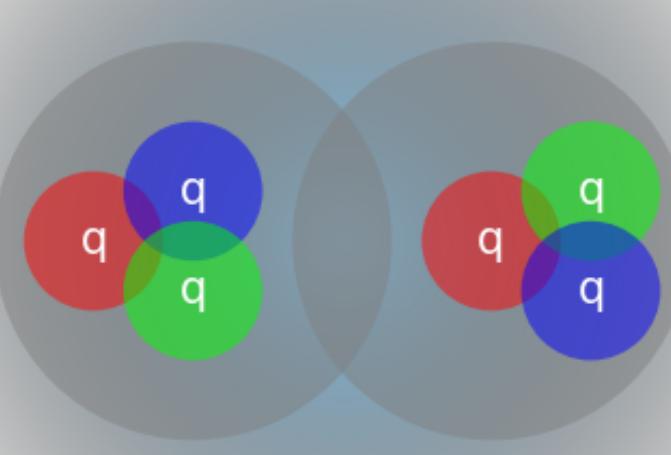
Hadron with 5 quarks





Hexaquark - Dibaryon

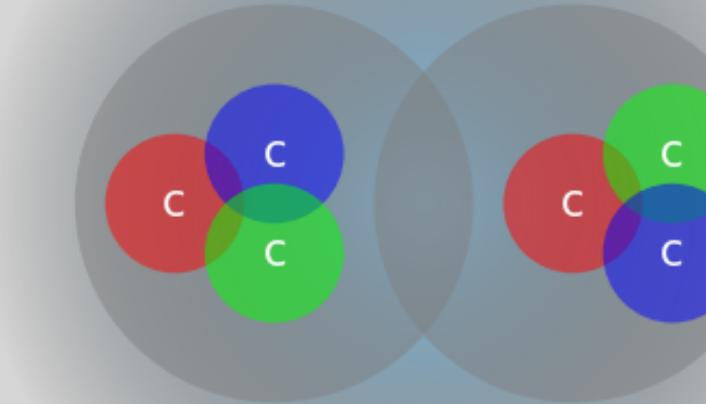
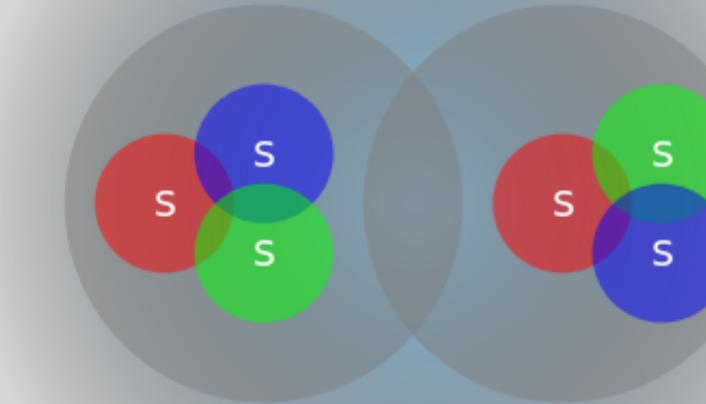
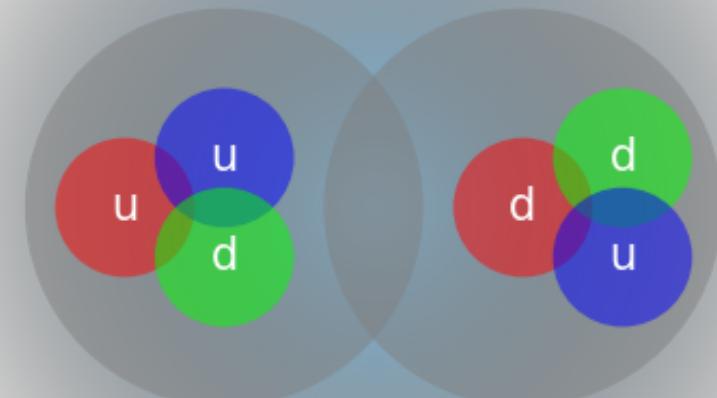
Extensive studies of deuteron like heavy dibaryons using
Lattice QCD



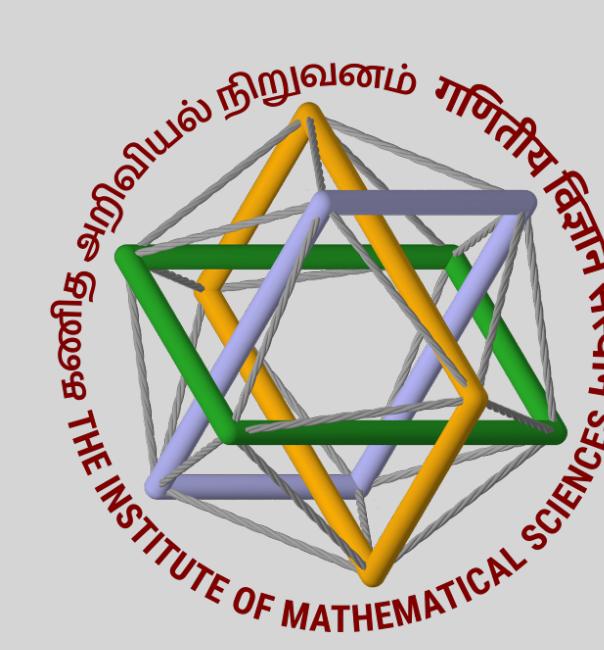
Hadron with 6 quarks



Deuteron



We work with single flavored dibaryons composed of strange
and charm quarks named as \mathcal{D}_{6s} and \mathcal{D}_{6c} respectively



Dibaryons results from Lattice

$\mathcal{D}_{6b}, S = 0$

Mathur, Padmanath, Chakraborty
PRL, 130, 111901 (2023)

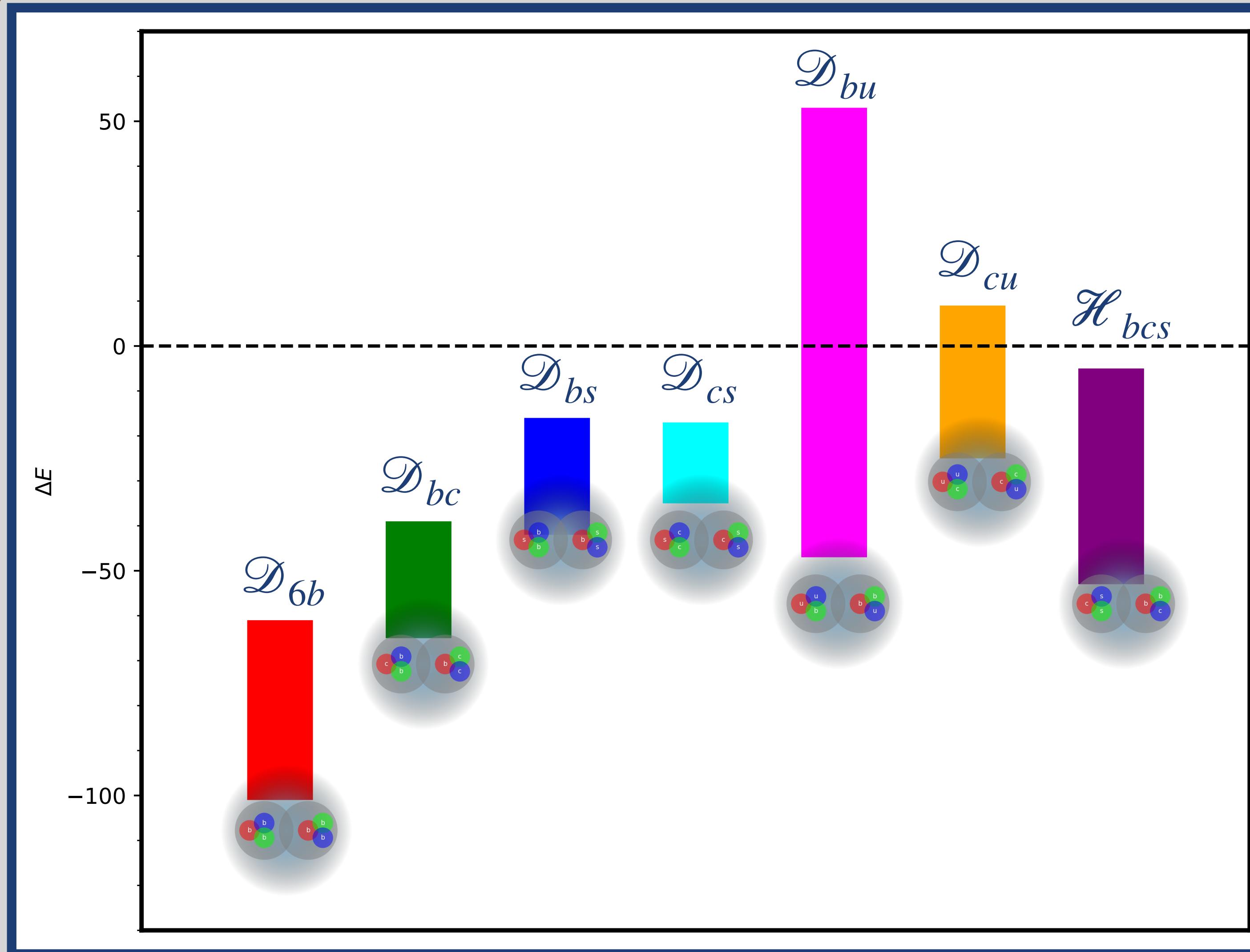
$\mathcal{D}_{bc}, \mathcal{D}_{bs}, \mathcal{D}_{cs}, \mathcal{D}_{bu}, \mathcal{D}_{cu}$
 $S = 1$

Junnarkar, Mathur PRL, 123, 162003 (2019)

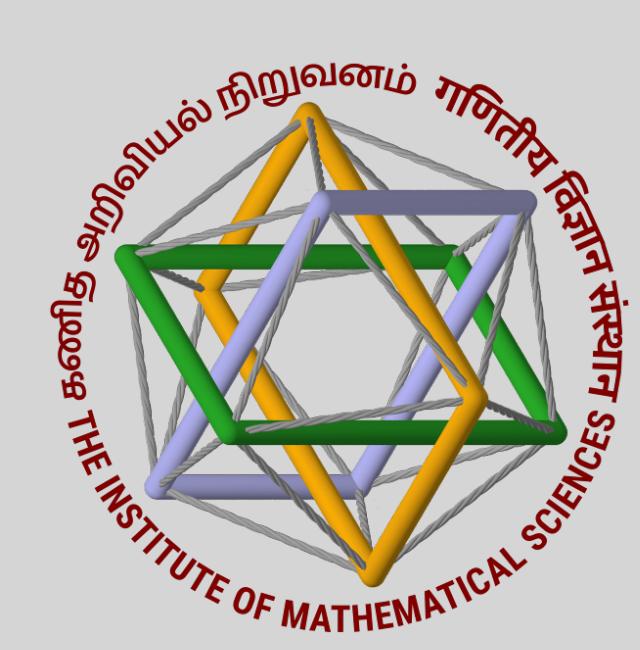
$\mathcal{H}_{bcs}, S = 0$

Junnarkar, Mathur PRD, 106, 054511 (2022)

Where does $\mathcal{D}_{6c}, \mathcal{D}_{6s}$ stand ??



$$\Delta E = E_{\text{dibaryon}} - E_{\text{1st baryon}} - E_{\text{2nd baryon}}$$



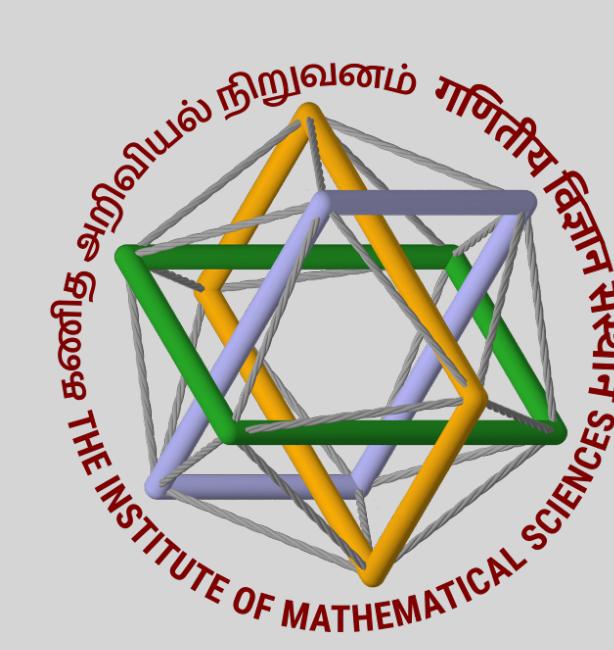
Dibaryon Operators

$$\mathcal{O} = \epsilon_{abc} q_{\mu_1}^a q_{\mu_2}^b q_{\mu_3}^c$$

$$\mathcal{O}_d = \mathcal{O}_1 \cdot CG \cdot \mathcal{O}_2$$

- Total wave function anti-symmetric under exchange of baryons.
- Single flavor baryons (symmetric).
- Assume only s wave interactions (symmetric) in dibaryon system.
- Color singlet baryons (symmetric)
- Hence Spin must be anti-symmetric which is in case of even spin (Spin 0 and 2)

- Use reduction coefficients to project continuum based operators to suitable octahedral group.
- $S = 0$ continuum spin subduces to one dimensional A_1^+ irrep.
- $S = 2$ continuum spin subduces to two dimensional E^+ and three dimensional T_2^+ irrep.



Dibaryon Operators

$$\mathcal{O}_{d,A_1,1}^{[0]} = \frac{1}{2} \left({}^aH_{3/2} {}^bH_{-3/2} - {}^aH_{1/2} {}^bH_{-1/2} + {}^aH_{-1/2} {}^bH_{1/2} - {}^aH_{-3/2} {}^bH_{3/2} \right)$$

For Spin 0, dibaryon operator corresponding to one dimensional A_1^+ irrep.

For Spin 2, 5 such operators corresponding to E^+ and T_2^+ irrep.

S_z	Operator	State
3/2	${}^1H_{3/2}$	111
1/2	${}^1H_{1/2}$	112+121+211
-1/2	${}^1H_{-1/2}$	122+212+221
-3/2	${}^1H_{-3/2}$	222

Non Relativistic Embedding

S_z	Operator	State
3/2	${}^2H_{3/2}$	133+313+331
1/2	${}^2H_{1/2}$	233+323+332+134+341+413+143+431+314
-1/2	${}^2H_{-1/2}$	144+414+441+234+342+423+243+432+324
-3/2	${}^2H_{-3/2}$	244+424+442

Relativistic Embedding

a,b - different embeddings



Operator Contraction

Now we have two baryons at source and two at sink

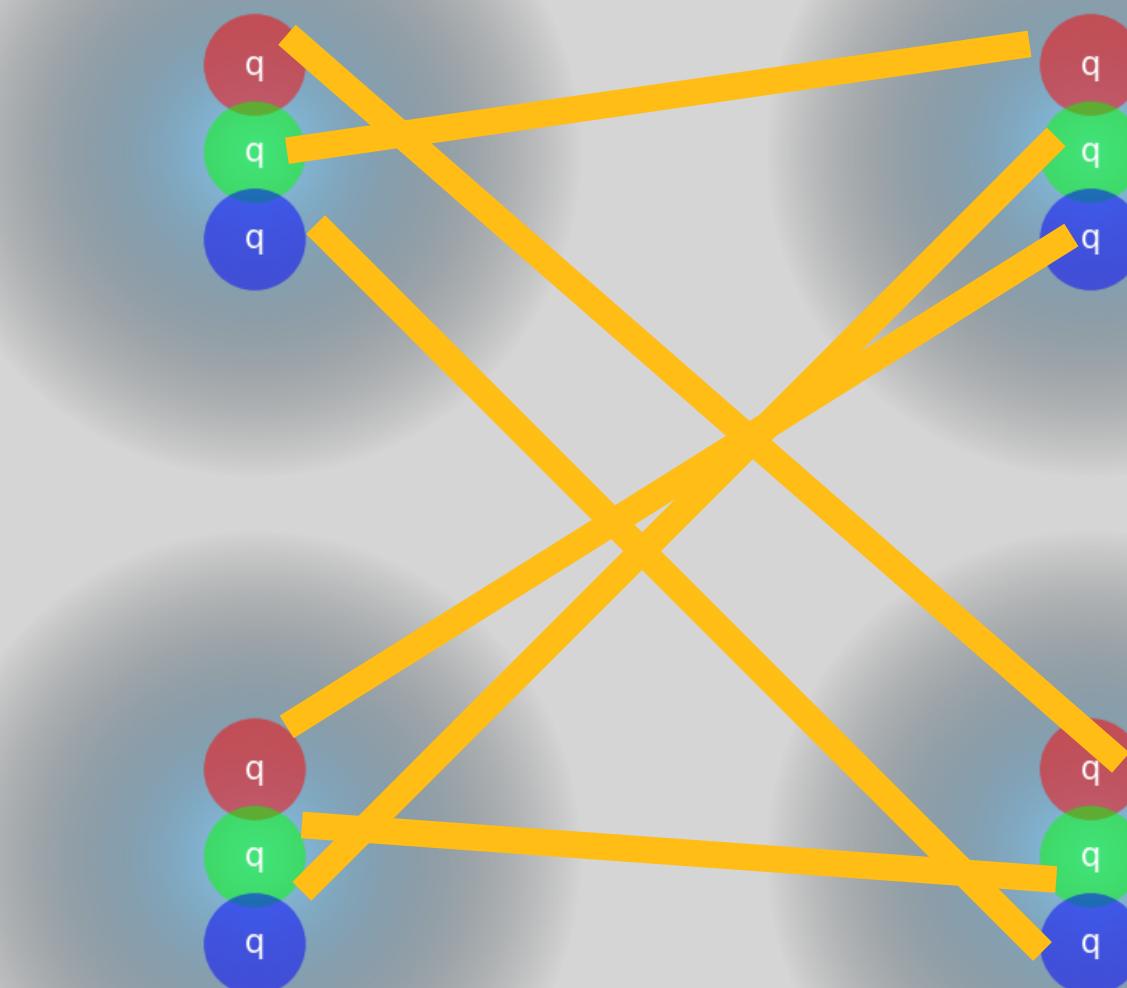


$6! = 720$ contraction possibilities

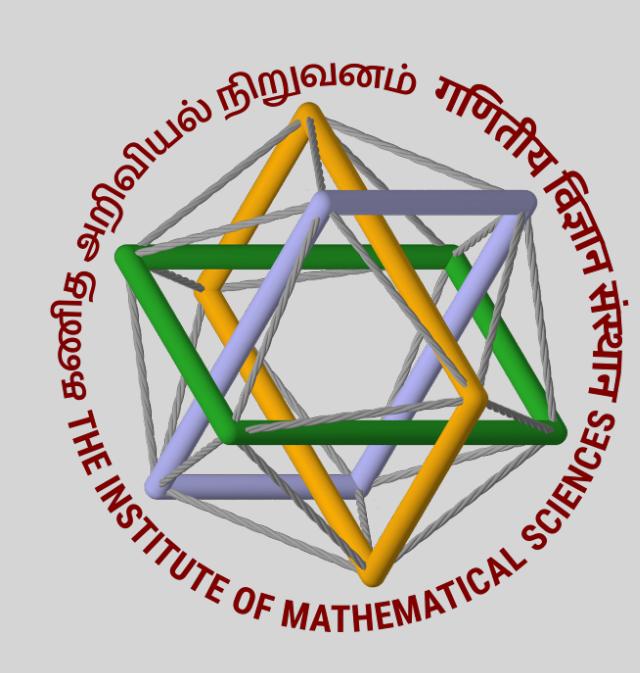


720 contractions can happen in 16 different ways depending on different embeddings

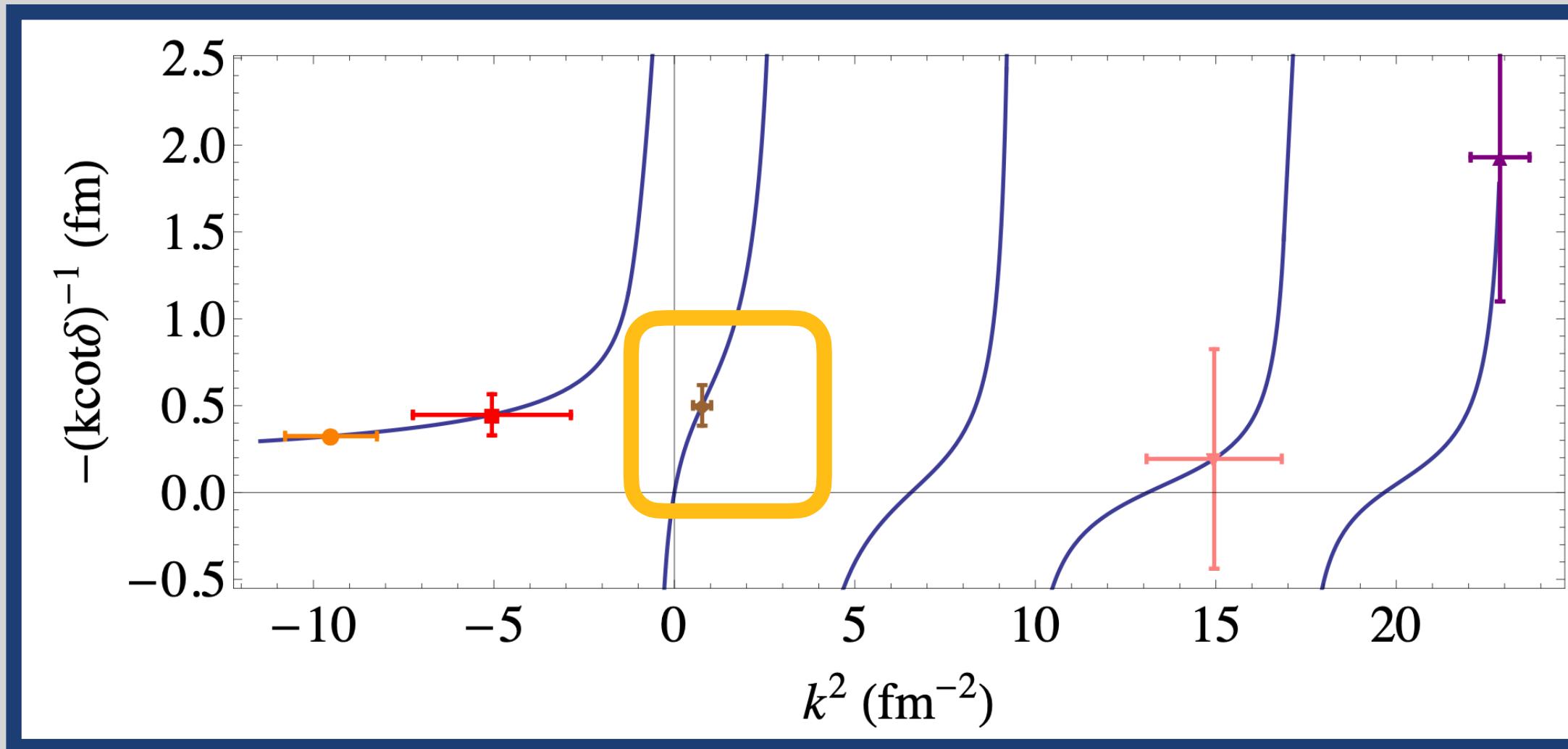
N-N-N-N	N-N-N-R	N-N-R-N	N-N-R-R
N-R-N-N	N-R-N-R	N-R-R-N	N-R-R-R
R-N-N-N	R-N-N-R	R-N-R-N	R-N-R-R
R-R-N-N	R-R-N-R	R-R-R-N	R-R-R-R



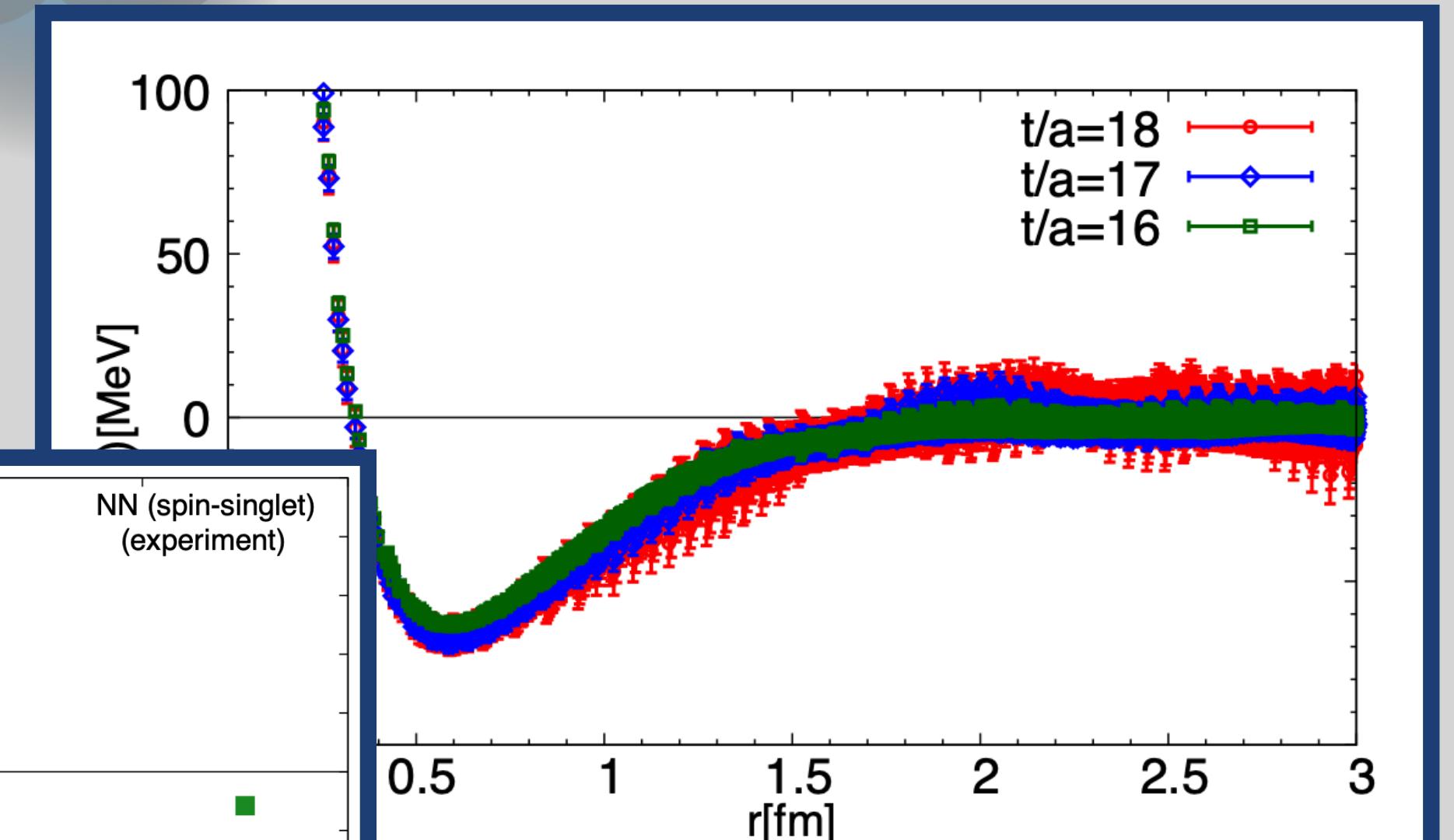
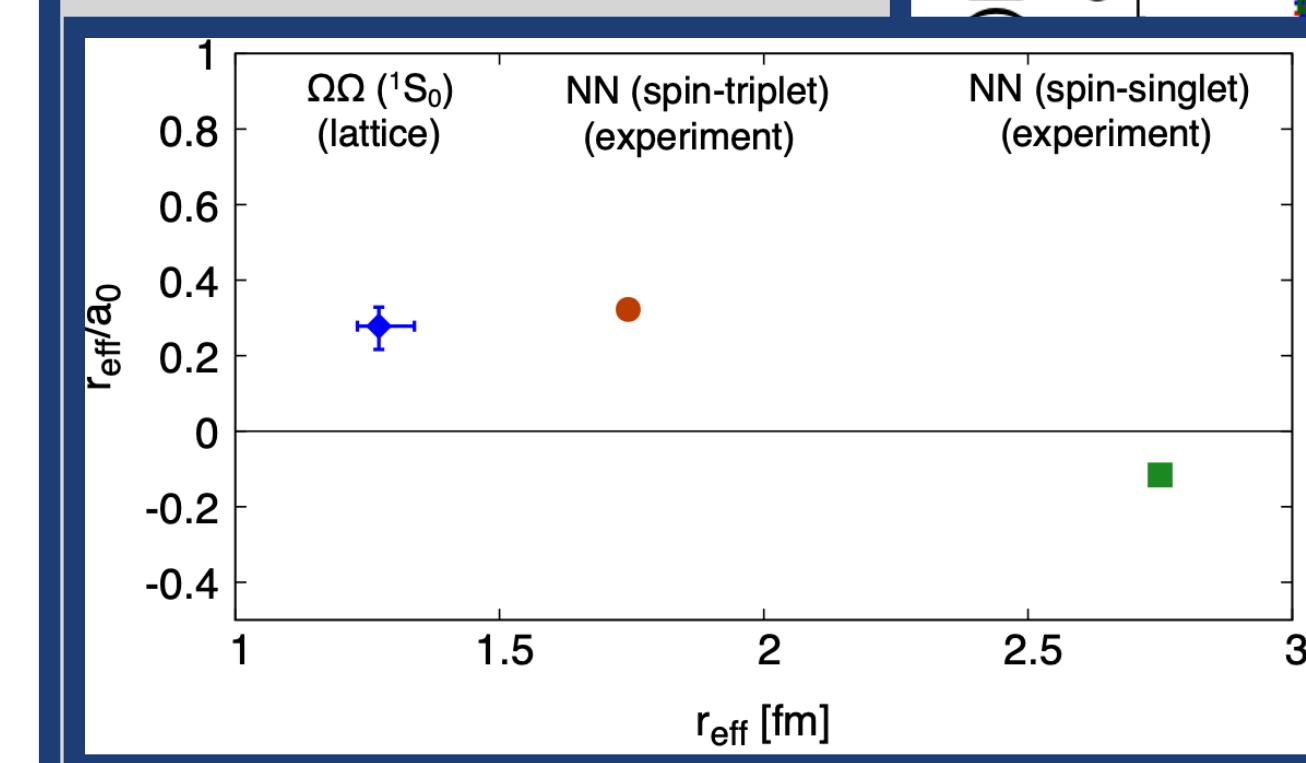
Some of these are degenerate and some of these do not contribute at all for different Spin cases



\mathcal{D}_{6s} Existing Results



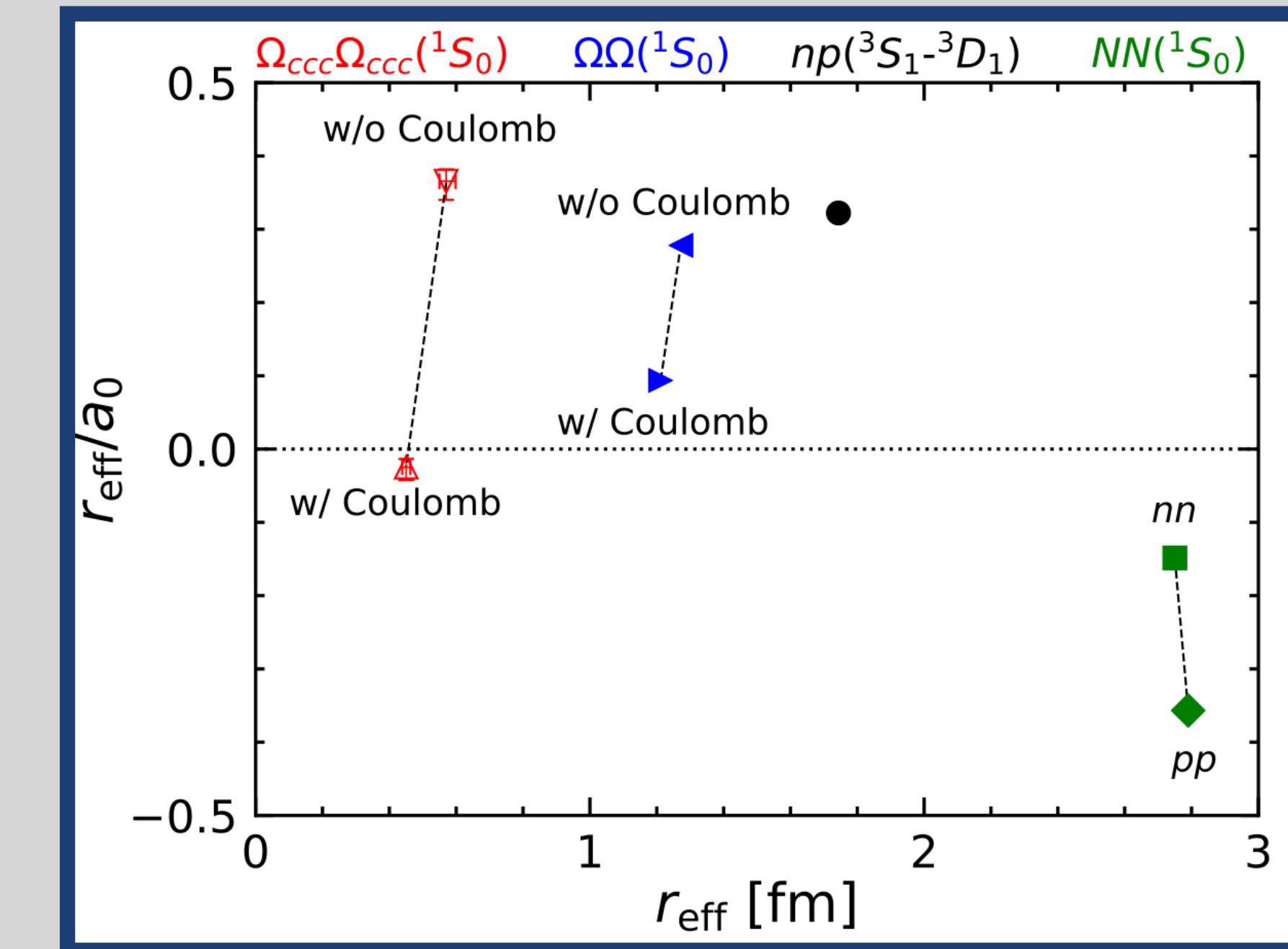
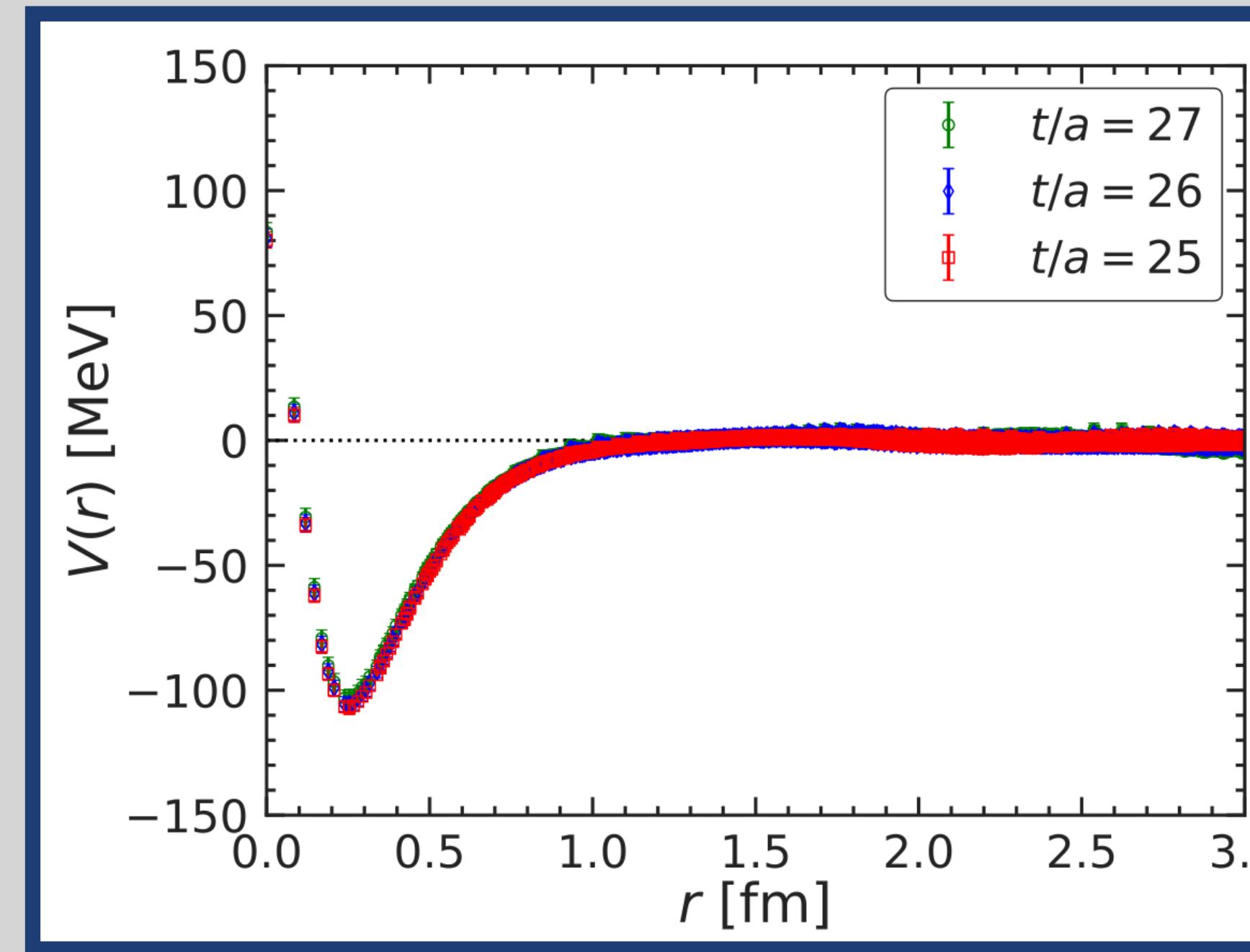
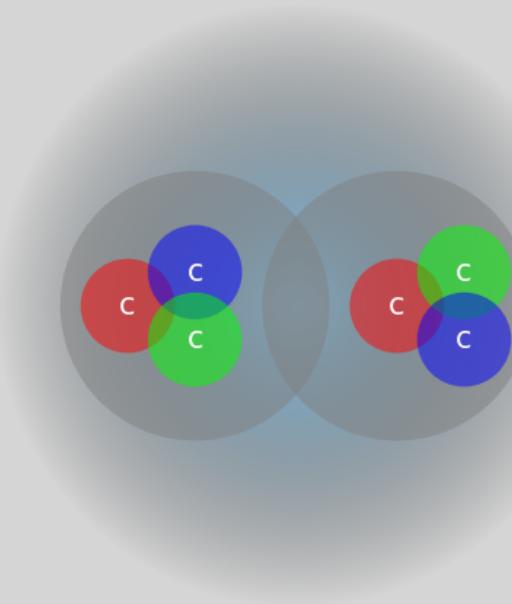
Buchhoff, Luu, Wasem PRD 85, 094511 (2012)



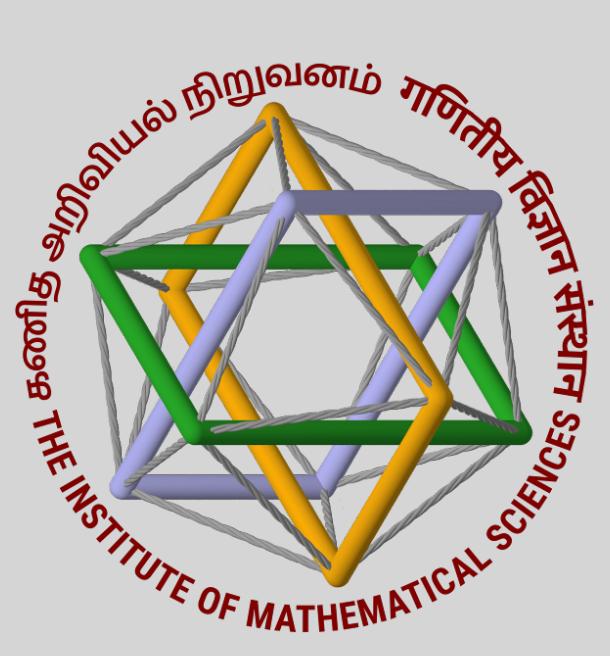
- Weakly attractive in Spin 0, hence bound state
 - “Such a system can be best searched experimentally by the pair-momentum correlation in relativistic heavy-ion collisions.”
 - Weakly repulsive in Spin 0 H^+H^+ irrep, No bound state
 - Attractive in Spin 1,2 $G_1^+H^+$ but only single volume used.

Buchoff, Luu, Wasem
PRD 85, 094511 (2012)

\mathcal{D}_{6c} Existing Results



- Simulation with physical charm mass and near physical light quark mass.
- Dibaryon existence without Coulomb interaction.
- Near unitary region with Coulomb interaction (scattering length less than corresponding strange dibaryon calculation).



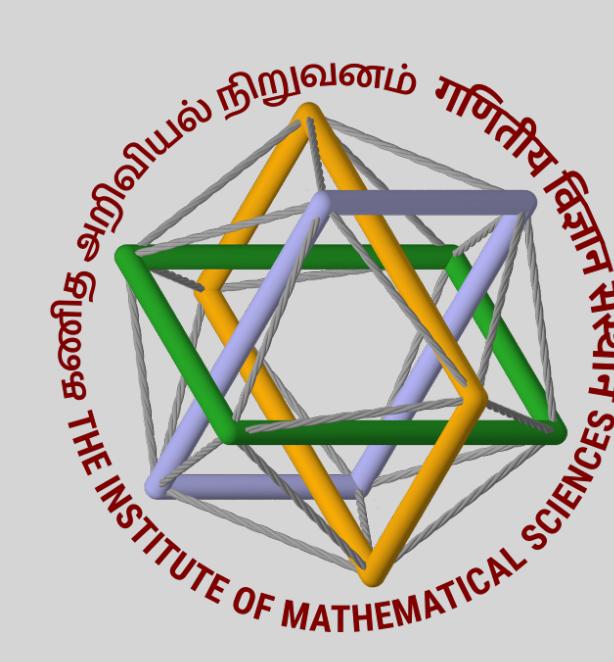
Is this a bound state?

- ★ Quoting bound state and binding energies directly from energy splittings of product hadron and its constituents.
- ★ Extract hadron-hadron interactions by solving QM potentials from Nambu-Bethe-Salpeter wave function.
- ★ Luscher's formalism - relating discrete finite-volume energy spectrum to few-body scattering amplitudes in the infinite limit.

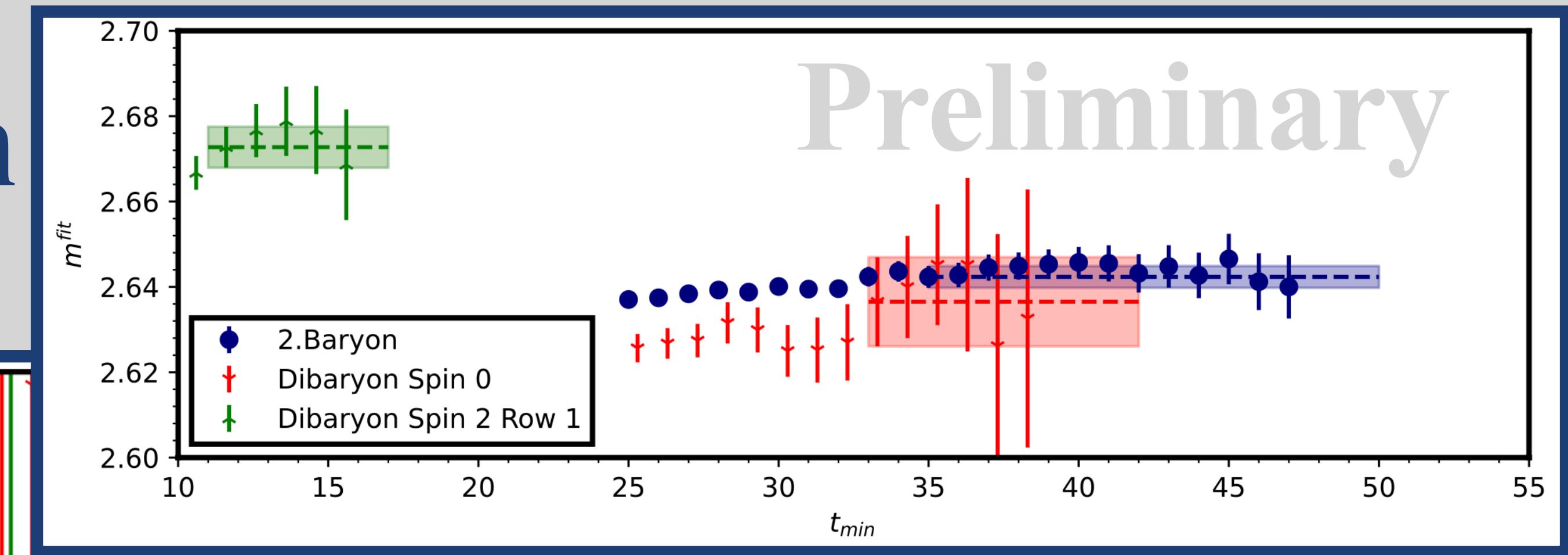
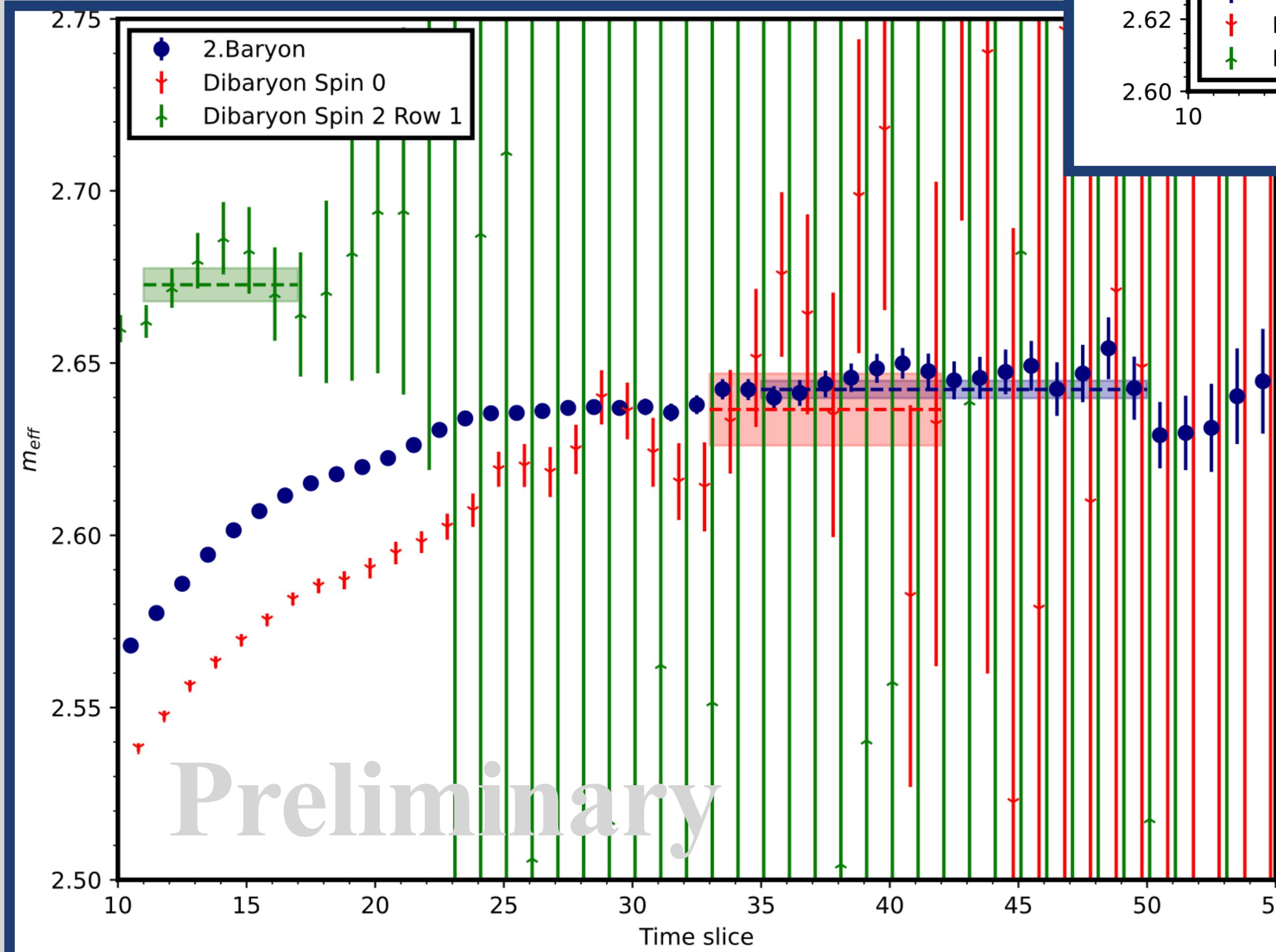
In this talk

Previous slide
HALQCD

Near future

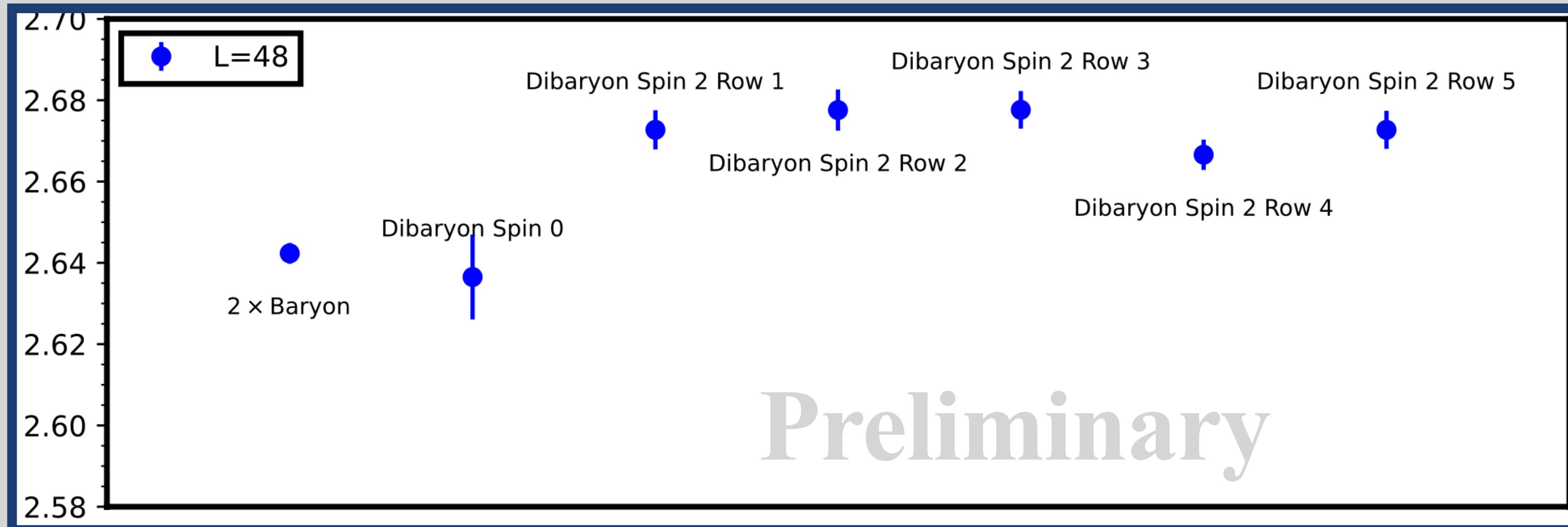
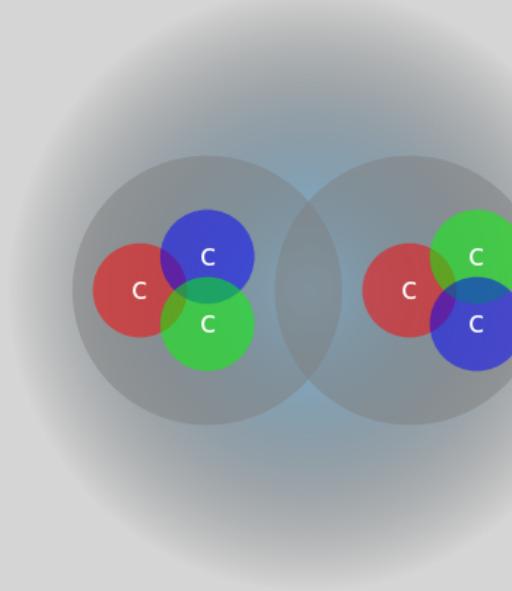


Energy spectrum



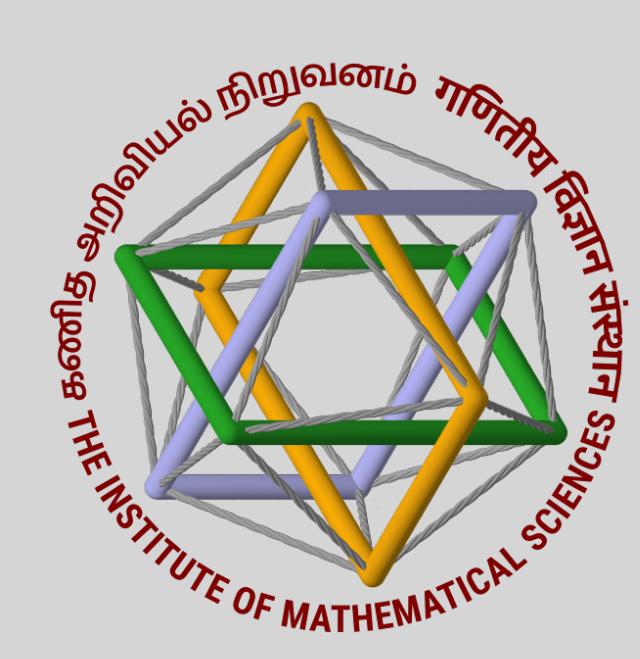
- Dependence of minimum time slice for fitting.
- Effective masses from lattice by using Dibaryon Spin 0 operator, one operator of Spin 2 for Dibaryon.
- Comparison with twice the effective mass from Baryon operator.

\mathcal{D}_{6c} Results

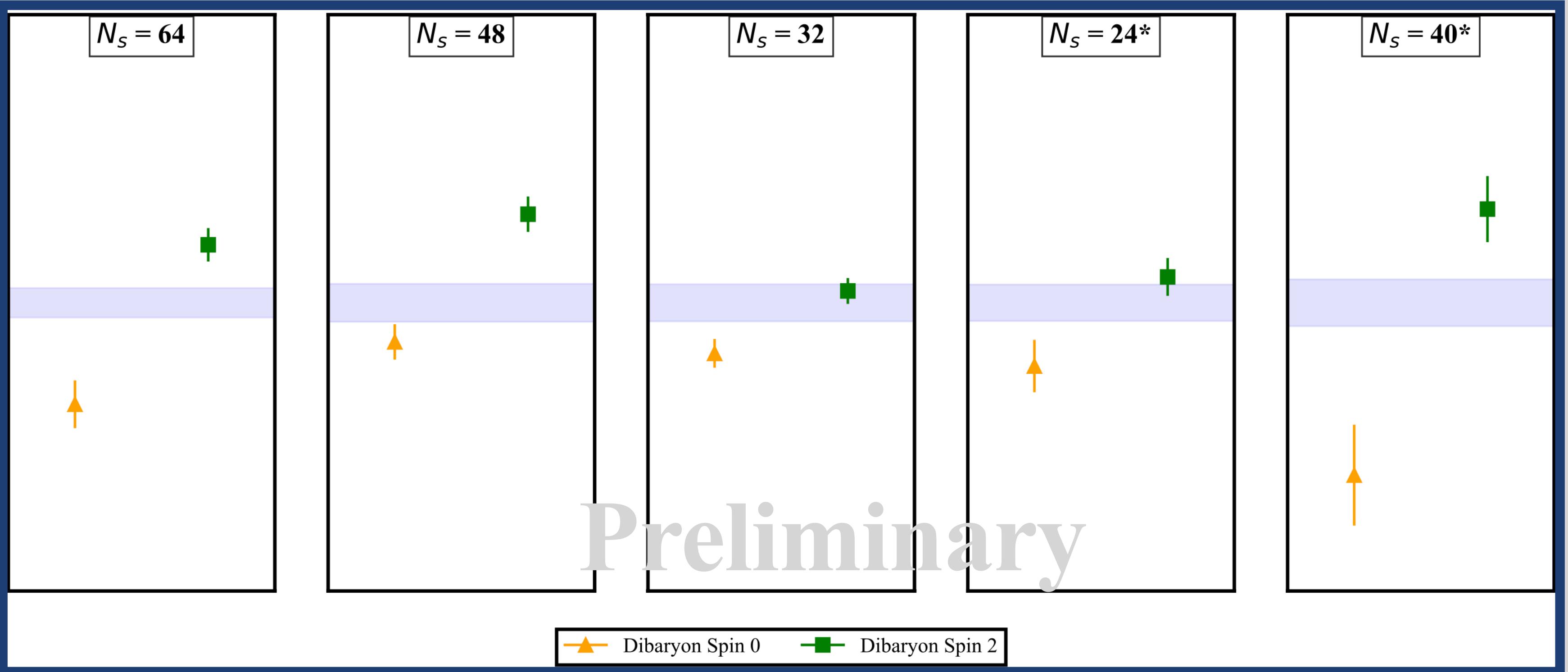
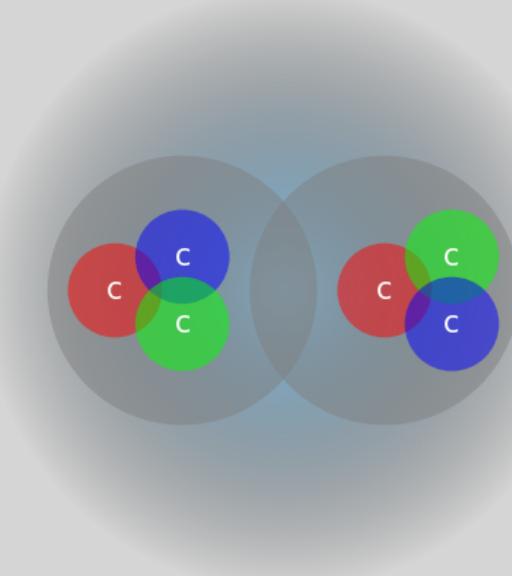


- Bound state, if exist, is shallow
- More probe using Luscher's formalism

- Two lattice volumes, 4 lattice spacings, this plot with $L = 48$
- Spin 2 - repulsive interactions, Spin 0 dibaryon energy same as twice of baryon



\mathcal{D}_{6c} Results

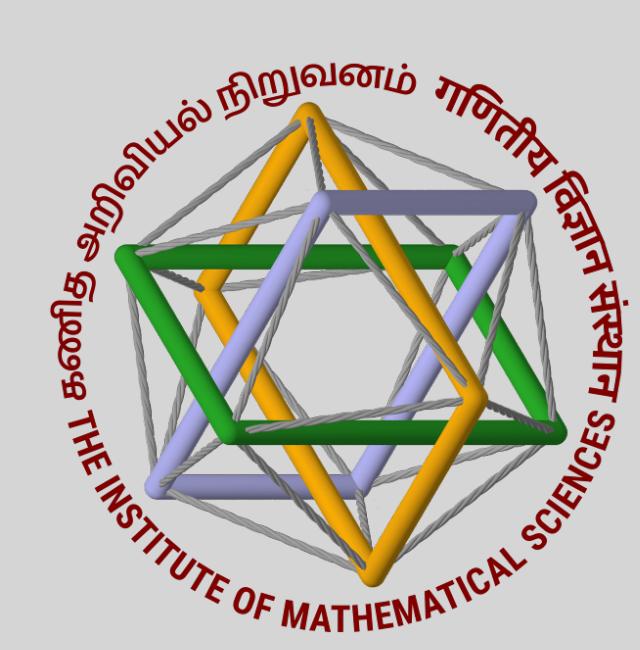


- Bound state, if exist, is shallow
- More probe using Luscher's formalism

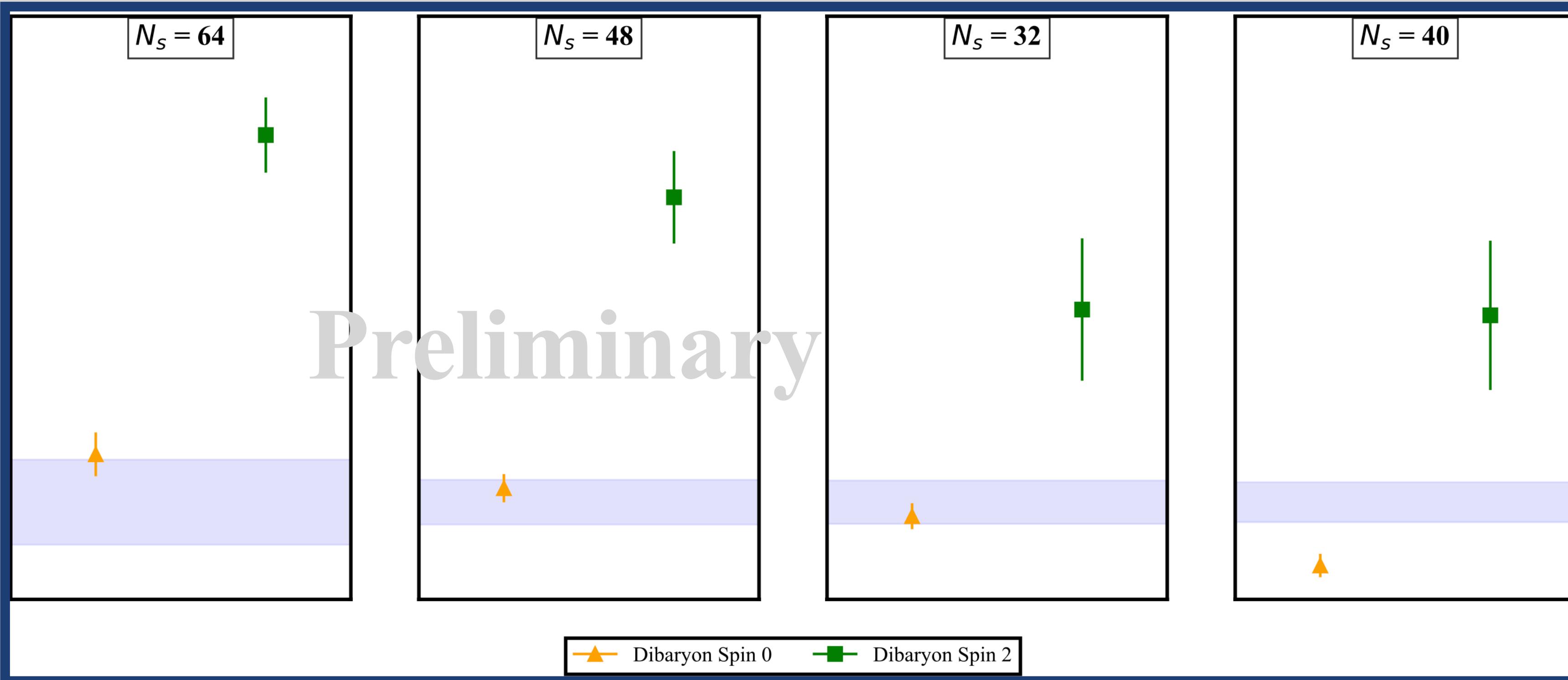
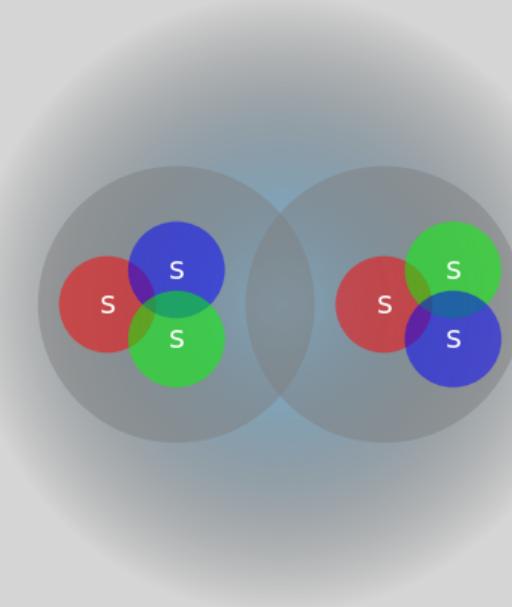
HALQCD
PRL 127, 072003 (2021)

Bound state without Coulomb interaction





\mathcal{D}_{6s} Results



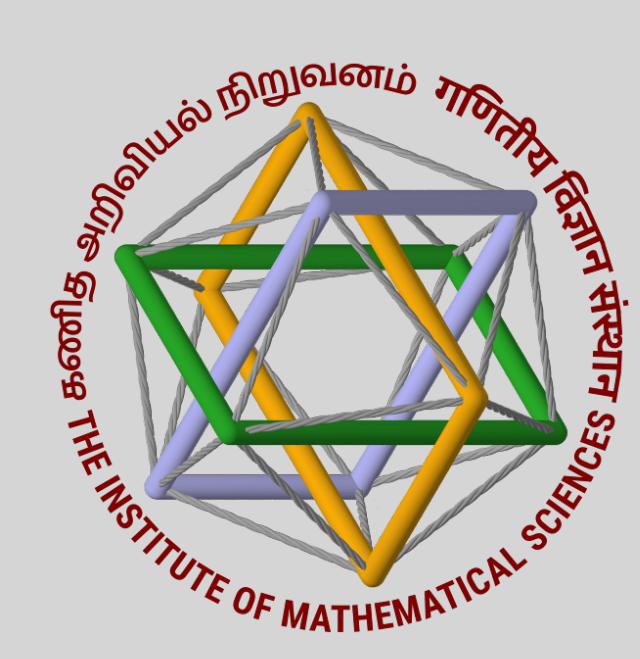
- No bound state
- More probe using Luscher's formalism

Buchoff, Luu, Wasem
PRD 85, 094511 (2012)

No bound state

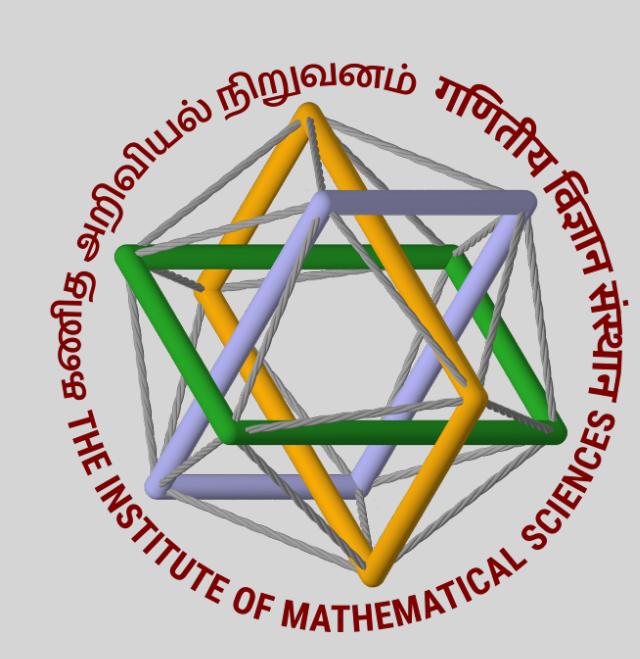
HALQCD
PRL 120, 212001 (2018)

Bound state exists



Conclusions

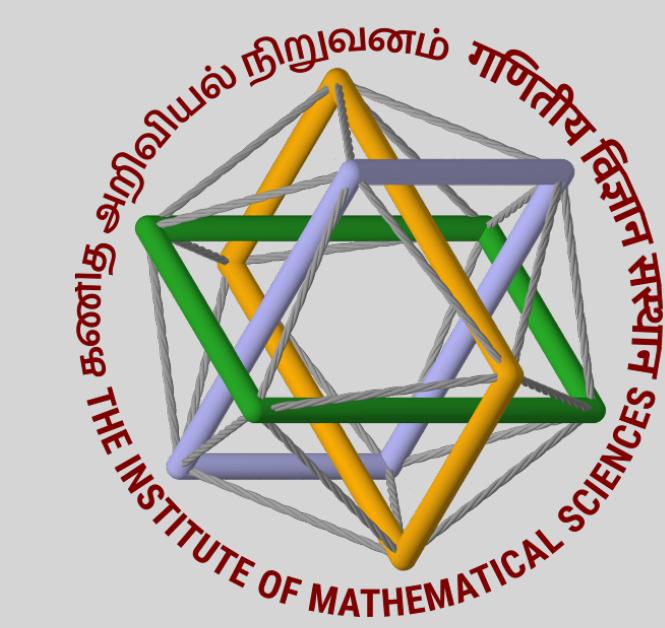
- ★ Simulations with finer and big enough lattices.
- ★ Ω and Ω_{ccc} calculations with more ensembles in the continuum limit.
- ★ Dibaryon investigation in $\Omega - \Omega$ and $\Omega_{ccc} - \Omega_{ccc}$ systems.
- ★ Prediction: Absence of bound state (or very weakly bound state if any)



Future Directions

- ★ Luscher's formalism for dibaryon investigations.
- ★ Initial calculations indicate similar behaviour of E^+ and T_2^+ irrep, more detailed analysis to follow.
- ★ Do the heavier baryons composed of charm and bottom quarks and their corresponding dibaryons (if exist as a bound state) survive with temperature ?
- ★ Lattice estimation of $d^*(2380)\dots$

THANK YOU



Navdeep Singh Dhindsa 15/07/2024