EXAMPLE 6 Consider the parallelepiped with a vertex at the origin and edges $\mathbf{u} = \mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$, $\mathbf{v} = \mathbf{i} + 3\mathbf{j} + \mathbf{k}$, and $\mathbf{w} = 2\mathbf{i} + \mathbf{j} + 2\mathbf{k}$. Then

$$\mathbf{v} \times \mathbf{w} = 5\mathbf{i} - 5\mathbf{k}.$$

Hence $\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w}) = -10$. Thus the volume V is given by (8) as

$$V = |\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w})| = |-10| = 10.$$

We can also compute the volume by Equation (9) as

$$V = \left|\det\left(\begin{bmatrix} 1 & -2 & 3 \\ 1 & 3 & 1 \\ 2 & 1 & 2 \end{bmatrix}\right)\right| = |-10| = 10.$$

■

Key Terms

Cross product
Jacobi identity

5.1 Exercises

In Exercises 1 and 2, compute $\mathbf{u} \times \mathbf{v}$.

1. (a) $\mathbf{u} = 2\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}$, $\mathbf{v} = -\mathbf{i} + 3\mathbf{j} - \mathbf{k}$
   (b) $\mathbf{u} = (1, 0, 1)$, $\mathbf{v} = (2, 3, -1)$
   (c) $\mathbf{u} = \mathbf{i} - \mathbf{j} + 2\mathbf{k}$, $\mathbf{v} = 3\mathbf{i} - 4\mathbf{j} + \mathbf{k}$
   (d) $\mathbf{u} = (2, -1, 1)$, $\mathbf{v} = -2\mathbf{u}$
2. (a) $\mathbf{u} = (1, -1, 2)$, $\mathbf{v} = (3, 1, 2)$
   (b) $\mathbf{u} = 2\mathbf{i} + \mathbf{j} - 2\mathbf{k}$, $\mathbf{v} = \mathbf{i} + 3\mathbf{k}$
   (c) $\mathbf{u} = 2\mathbf{j} + \mathbf{k}$, $\mathbf{v} = 3\mathbf{u}$
   (d) $\mathbf{u} = (4, 0, -2)$, $\mathbf{v} = (0, 2, -1)$
3. Let $\mathbf{u} = \mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$, $\mathbf{v} = 2\mathbf{i} + 3\mathbf{j} + \mathbf{k}$, $\mathbf{w} = 2\mathbf{i} - \mathbf{j} + 2\mathbf{k}$, and $c = -3$. Verify properties (a) through (d) of Theorem 5.1.
4. Let $\mathbf{u} = 2\mathbf{i} - \mathbf{j} + 3\mathbf{k}$, $\mathbf{v} = 3\mathbf{i} + \mathbf{j} - \mathbf{k}$, and $\mathbf{w} = 3\mathbf{i} + \mathbf{j} + 2\mathbf{k}$.
   (a) Verify Equation (3).
   (b) Verify Equation (4).
5. Let $\mathbf{u} = \mathbf{i} - \mathbf{j} + 2\mathbf{k}$, $\mathbf{v} = 2\mathbf{i} + 2\mathbf{j} - \mathbf{k}$, and $\mathbf{w} = \mathbf{i} + \mathbf{j} - \mathbf{k}$.
   (a) Verify Equation (3).
   (b) Verify Equation (4).
6. Verify that each of the cross products $\mathbf{u} \times \mathbf{v}$ in Exercise 1 is orthogonal to both $\mathbf{u}$ and $\mathbf{v}$.
7. Verify that each of the cross products $\mathbf{u} \times \mathbf{v}$ in Exercise 2 is orthogonal to both $\mathbf{u}$ and $\mathbf{v}$.
8. Verify Equation (7) for the pairs of vectors in Exercise 1.
9. Find the area of the triangle with vertices $P_1(1, -2, 3)$, $P_2(-3, 1, 4)$, $P_3(0, 4, 3)$.
10. Find the area of the triangle with vertices P_1, P_2, and P_3, where $\overrightarrow{P_1P_2} = 2\mathbf{i} + 3\mathbf{j} - \mathbf{k}$ and $\overrightarrow{P_1P_3} = \mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$.
11. Find the area of the parallelogram with adjacent sides $\mathbf{u} = \mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$ and $\mathbf{v} = 3\mathbf{i} - \mathbf{j} - \mathbf{k}$.
12. Find the volume of the parallelepiped with a vertex at the origin and edges $\mathbf{u} = 2\mathbf{i} - \mathbf{j}$, $\mathbf{v} = \mathbf{i} - 2\mathbf{j} - 2\mathbf{k}$, and $\mathbf{w} = 3\mathbf{i} - \mathbf{j} + \mathbf{k}$.
13. Repeat Exercise 12 for $\mathbf{u} = \mathbf{i} - 2\mathbf{j} + 4\mathbf{k}$, $\mathbf{v} = 3\mathbf{i} + 4\mathbf{j} + \mathbf{k}$, and $\mathbf{w} = -\mathbf{i} + \mathbf{j} + \mathbf{k}$.

Theoretical Exercises

T.1. Prove Theorem 5.1.

T.2. Show that $(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{w} = \mathbf{u} \cdot (\mathbf{v} \times \mathbf{w})$.

T.3. Show that $\mathbf{j} \times \mathbf{i} = -\mathbf{k}$, $\mathbf{k} \times \mathbf{j} = -\mathbf{i}$, $\mathbf{i} \times \mathbf{k} = -\mathbf{j}$.

T.4. Show that

$$(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{w} = \begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}.$$

T.5. Show that $\mathbf{u}$ and $\mathbf{v}$ are parallel if and only if $\mathbf{u} \times \mathbf{v} = \mathbf{0}$.

T.6. Show that $\|\mathbf{u} \times \mathbf{v}\|^2 + (\mathbf{u} \cdot \mathbf{v})^2 = \|\mathbf{u}\|^2\|\mathbf{v}\|^2$.

T.7. Prove the **Jacobi identity**:

$$(\mathbf{u} \times \mathbf{v}) \times \mathbf{w} + (\mathbf{v} \times \mathbf{w}) \times \mathbf{u} + (\mathbf{w} \times \mathbf{u}) \times \mathbf{v} = \mathbf{0}.$$

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Ex 5.1

In Exercises 1 and 2 compute U×V.

Q1 (a) $U = 2i + 3j + 4k$, $V = -i + 3j - k$.

$$U \times V = \begin{vmatrix} i & j & k \\ 2 & 3 & 4 \\ -1 & 3 & -1 \end{vmatrix}$$

$$i\begin{vmatrix} 3 & 4 \\ 3 & -1 \end{vmatrix} - j\begin{vmatrix} 2 & 4 \\ -1 & -1 \end{vmatrix} + k\begin{vmatrix} 2 & 3 \\ -1 & 3 \end{vmatrix}$$

$i(-3-12) - j(-2+4) + k(6+3)$

$i(-15) - j(2) + k(9)$

$-15i - 2j + 9k.$

(b) $U = (1, 0, 1)$, $V = (2, 3-1)$

$\vec{U} = i + 0j + k \quad \vec{V} = 2i + 3j - k$

$$U \times V = \begin{vmatrix} i & j & k \\ 1 & 0 & 1 \\ 2 & 3 & -1 \end{vmatrix} = i(0-3) - j(-1-2) + k(3)$$

$= -3i + 3j + 3k.$

(c) Same as Part (a)

(d) $U = (2, -1, 1)$, $V = -2U = -2(2, -1, 1) = (-4, 2, -2)$

$\vec{U} = 2i - j + k$ & $\vec{V} = -4i + 2j - 2k$

$$\vec{U} \times \vec{V} = \begin{vmatrix} i & j & k \\ 2 & -1 & 1 \\ -4 & 2 & -2 \end{vmatrix} = i(2-2) - j(-4+4) + k(4-4)$$

$= 0\tilde{i} + 0j + 0\vec{k}$.

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Q2 is same as Q1.

Q3 Let $u = i+2j-3k$, $v = 2i+3j+k$, $w = 2i-j+2k$, & $c = -3$

(i) $u \times v = -(v \times u)$.

$$u \times v = \begin{vmatrix} i & j & k \\ 1 & 2 & -3 \\ 2 & 3 & 1 \end{vmatrix}$$

$$= 11i - 7j - k \quad \text{--- (1)}$$

$$-(v \times u) = -\begin{vmatrix} i & j & k \\ 2 & 3 & 1 \\ 1 & 2 & -3 \end{vmatrix}$$

$$= -(-11i + 7j + k) = 11i - 7j - k \quad \text{--- (2)}$$

From eq (1) & eq (2) we have.

$u \times v = -(v \times u)$ Proved.

Part b, c, d same as Part (a)

Ex 5.1

Q4 $U = 2i - j + 3k$

$V = 3i + j - k$

$W = 3i + j + 2k$.

(a) verify Equation 3 $(U \times V) \cdot W = U \cdot (V \times W)$.

$$U \times V = \begin{vmatrix} i & j & k \\ 2 & -1 & 3 \\ 3 & 1 & -1 \end{vmatrix}$$

$$= -2i + 11j + 5k$$

$$(U \times V) \cdot W = (-2i + 11j + 5k) \cdot (3i + j + 2k)$$

$$= -6 + 11 + 10 = 15 \quad \text{--- (1)}$$

Now

$$V \times W = \begin{vmatrix} i & j & k \\ 3 & 1 & -1 \\ 3 & 1 & +2 \end{vmatrix} = 3i - 9j + 0k$$

$$U \cdot (V \times W) = (2i - j + 3k) \cdot (3i - 9j + 0k)$$

$$= 6 + 9 + 0 = 15 \quad \text{--- (2)}$$

From eq (1) & eq (2) we have

$(U \times V) \cdot W = U \cdot (V \times W)$ Proved.

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Part (b) is same as Part (a).

Q5 is same as Q4.

<u>orthogonal</u>:- If $U \times V$ is orthogonal to both U & V i.e

$$(U \times V) \cdot U = 0.$$
$$\& \quad (U \times V) \cdot V = 0.$$

<u>Q6</u> Exercis a — to — (d)

(a) $\vec{U} = 2i + 3j + 4k$ & $\vec{V} = -i + 3j - k$

$$U \times V = \begin{vmatrix} i & j & k \\ 2 & 3 & 4 \\ -1 & 3 & -1 \end{vmatrix}$$

$$= -15i - 2j + 9k$$

Now $(U \times V) \cdot U = (-15i - 2j + 9k) \cdot (2i + 3j + 4k)$

$$= -30 - 6 + 36$$
$$= -\cancel{36} + \cancel{36} = 0$$

$\therefore (U \times V) \cdot U = 0$

Also $(U \times V) \cdot V = (-15i - 2j + 9k) \cdot (-i + 3j - k)$

$$= 15 - 6 - 9 \Rightarrow \cancel{15} - \cancel{15} = 0$$

$(U \times V) \cdot V = 0$

which is orthogonal.

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Ex 5.1

Q7 is same as Q 6.

Q9 Area of triangle $= \frac{1}{2}|\vec{U}\times\vec{V}|$ —— ①

let $\vec{U} = P_1P_2 = (-3-1,\ 1-(-2),\ 4-3)$

$= (-4, 3, 1)$

& $\vec{V} = P_1P_3 = (0-1,\ 4-(-2),\ 3-3)$

$= (-1, 6, 0)$

$$\vec{U}\times\vec{V} = \begin{vmatrix} i & j & k \\ -4 & 3 & 1 \\ -1 & 6 & 0 \end{vmatrix}$$

$\vec{U}\times\vec{V} = -6i - j - 21k$

Now $|\vec{U}\times\vec{V}| = \sqrt{(-6)^2+(-1)^2+(-21)^2} = \sqrt{478}$

eq ① => Area of triangle $= \frac{1}{2}|\vec{U}\times\vec{V}|$

$= \frac{1}{2}\sqrt{478}\ (\text{units})^2$.

Q10 i same as Q9.

Q11 Area of //gm $= |\vec{U} \times \vec{V}|$. —①

Now $\vec{U} \times \vec{V} = \begin{vmatrix} i & j & k \\ 1 & 3 & -2 \\ 3 & -1 & -1 \end{vmatrix}$

$= -5i - 5j - 10k$.

Now $|\vec{U} \times \vec{V}| = \sqrt{(-5)^2 + (-5)^2 + (-10)^2} = \sqrt{150}$

eq① ⟹ Area of //gm $= \sqrt{150}$ (unit)^2.

Q12 $U = 2i - j$, $V = i - 2j - 2k$ & $W = 3i - j + k$.

Volume of //Poid $= \begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}$

$= \begin{vmatrix} 2 & -1 & 0 \\ 1 & -2 & -2 \\ 3 & -1 & 1 \end{vmatrix}$

$= 2(-2-2) + 1(1+6) + 0$

$= 2(-4) + 7$

$= -8 + 1 = |-1|$

V of //P $= 1$ (unit)^3

Q13 is same as Q12

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Two planes are either parallel or they intersect in a straight line. They are parallel if their normals are parallel. In the following example we determine the line of intersection of two planes.

EXAMPLE 10 Find parametric equations of the line of intersection of the planes

$$\pi_1: 2x + 3y - 2z + 4 = 0 \quad \text{and} \quad \pi_2: x - y + 2z + 3 = 0.$$

Solution Solving the linear system consisting of the equations of π_1 and π_2, we obtain (verify)

$$\begin{aligned} x &= -\tfrac{13}{5} - \tfrac{4}{5}t \\ y &= \tfrac{2}{5} + \tfrac{6}{5}t \qquad -\infty < t < \infty \\ z &= 0 + t \end{aligned}$$

as parametric equations of the line L of intersection of the planes (see Figure 5.9). ■

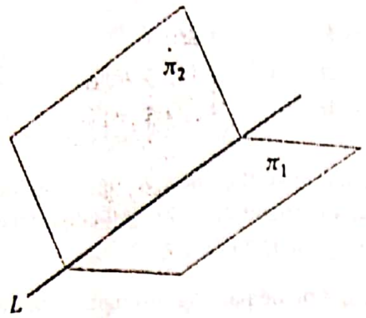

Figure 5.9 ▲

Three planes in R^3 may intersect in a plane, in a line, in a unique point, or have no set of points in common. These possibilities can be detected by solving the linear system consisting of their equations.

Key Terms

Parametric equation(s) of a line
Symmetric form of a line
Normal to a plane
Skew lines

5.2 Exercises

1. In each of the following, find an equation of the line in R^2 determined by the given points.
   (a) $P_1(-2,-3)$, $P_2(3,4)$
   (b) $P_1(2,-5)$, $P_2(-3,4)$
   (c) $P_1(0,0)$, $P_2(-3,5)$
   (d) $P_1(-3,-5)$, $P_2(0,2)$

2. In each of the following, find the equation of the line in R^2 determined by the given points.
   (a) $P_1(1,1)$, $P_2(2,2)$
   (b) $P_1(1,2)$, $P_2(1,3)$
   (c) $P_1(2,-4)$, $P_2(-3,-4)$
   (d) $P_1(2,-3)$, $P_2(3,-2)$

3. State which of the following points are on the line

$$\begin{aligned} x &= 3 + 2t \\ y &= -2 + 3t \qquad -\infty < t < \infty \\ z &= 4 - 3t. \end{aligned}$$

   (a) $(1,1,1)$ (b) $(1,-1,0)$
   (c) $(1,0,-2)$ (d) $(4,-\frac{1}{2},\frac{5}{2})$

4. State which of the following points are on the line

$$\frac{x-4}{-2} = \frac{y+3}{2} = \frac{z-4}{-5}.$$

   (a) $(0,1,-6)$ (b) $(1,2,3)$
   (c) $(4,-3,4)$ (d) $(0,1,-1)$

5. In each of the following, find the parametric equations of the line through the point $P_0(x_0, y_0, z_0)$, which is parallel to the vector $\mathbf{u}$.
   (a) $P_0 = (3,4,-2)$, $\mathbf{u} = (4,-5,2)$
   (b) $P_0 = (3,2,4)$, $\mathbf{u} = (-2,5,1)$
   (c) $P_0 = (0,0,0)$, $\mathbf{u} = (2,2,2)$
   (d) $P_0 = (-2,-3,1)$, $\mathbf{u} = (2,3,4)$

6. In each of the following, find the parametric equations of the line through the given points.
   (a) $(2,-3,1)$, $(4,2,5)$ (b) $(-3,-2,-2)$, $(5,5,4)$
   (c) $(-2,3,4)$, $(2,-3,5)$ (d) $(0,0,0)$, $(4,5,2)$

7. For each of the lines in Exercise 6, find the equations in symmetric form.

8. State which of the following points are on the plane $3(x-2) + 2(y+3) - 4(z-4) = 0$.
   (a) $(0,-2,3)$ (b) $(1,-2,3)$
   (c) $(1,-1,3)$ (d) $(0,0,4)$

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9. In each of the following, find an equation of the plane passing through the given point and perpendicular to the given vector $\mathbf{n}$.
   (a) $(0, 2, -3)$, $\mathbf{n} = (3, -2, 4)$
   (b) $(-1, 3, 2)$, $\mathbf{n} = (0, 1, -3)$
   (c) $(-2, 3, 4)$, $\mathbf{n} = (0, 0, -4)$
   (d) $(5, 2, 3)$, $\mathbf{n} = (-1, -2, 4)$

10. In each of the following, find an equation of the plane passing through the given three points.
   (a) $(0, 1, 2)$, $(3, -2, 5)$, $(2, 3, 4)$
   (b) $(2, 3, 4)$, $(-1, -2, 3)$, $(-5, -4, 2)$
   (c) $(1, 2, 3)$, $(0, 0, 0)$, $(-2, 3, 4)$
   (d) $(1, 1, 1)$, $(2, 3, 4)$, $(-5, 3, 2)$

11. In each of the following, find parametric equations of the line of intersection of the given planes.
   (a) $2x + 3y - 4z + 5 = 0$ and $-3x + 2y + 5z + 6 = 0$
   (b) $3x - 2y - 5z + 4 = 0$ and $2x + 3y + 4z + 8 = 0$
   (c) $-x + 2y + z = 0$ and $2x - y + 2z + 8 = 0$

12. In each of the following, find a pair of planes whose intersection is the given line.
   (a) $x = 2 - 3t$, $y = 3 + t$, $z = 2 - 4t$
   (b) $\dfrac{x-2}{-2} = \dfrac{y-3}{4} = \dfrac{z+4}{3}$
   (c) $x = 4t$, $y = 1 + 5t$, $z = 2 - t$

13. Are the points $(2, 3, -2)$, $(4, -2, -3)$, and $(0, 8, -1)$ on the same line?

14. Are the points $(-2, 4, 2)$, $(3, 5, 1)$, and $(4, 2, -1)$ on the same line?

15. Find the point of intersection of the lines

$$x = 2 - 3s,\quad y = 3 + 2s,\quad z = 4 + 2s \qquad \text{and} \qquad x = 5 + 2t,\quad y = 1 - 3t,\quad z = 2 + t.$$

16. Which of the following pairs of lines are perpendicular?
   (a) $x = 2 + 2t$, $y = -3 - 3t$, $z = 4 + 4t$ and $x = 2 + t$, $y = 4 - t$, $z = 5 - t$
   (b) $x = 3 - t$, $y = 4 + t$, $z = 2 + 2t$ and $x = 2t$, $y = 3 - 2t$, $z = 4 + 2t$

17. Show that the following parametric equations define the same line.

$$x = 2 + 3t,\quad y = 3 - 2t,\quad z = -1 + 4t \qquad \text{and} \qquad x = -1 - 9t,\quad y = 5 + 6t,\quad z = -5 - 12t$$

18. Find parametric equations of the line passing through the point $(3, -1, -3)$ and perpendicular to the line passing through the points $(3, -2, 4)$ and $(0, 3, 5)$.

19. Find an equation of the plane passing through the point $(-2, 3, 4)$ and perpendicular to the line passing through the points $(4, -2, 5)$ and $(0, 2, 4)$.

20. Find the point of intersection of the line

$$x = 2 - 3t,\quad y = 4 + 2t,\quad z = 3 - 5t$$

and the plane $2x + 3y + 4z + 8 = 0$.

21. Find a plane containing the lines

$$x = 3 + 2t,\quad y = 4 - 3t,\quad z = 5 + 4t \qquad \text{and} \qquad x = 1 - 2t,\quad y = 7 + 4t,\quad z = 1 - 3t.$$

22. Find a plane that passes through the point $(2, 4, -3)$ and is parallel to the plane $-2x + 4y - 5z + 6 = 0$.

23. Find a line that passes through the point $(-2, 5, -3)$ and is perpendicular to the plane $2x - 3y + 4z + 7 = 0$.

Theoretical Exercises

T.1. Show that the graph of the equation $ax + by + cz + d = 0$, where a, b, c, and d are constants with a, b, and c not all zero, is a plane with normal $\mathbf{n} = (a, b, c)$.

T.2. Let the lines L_1 and L_2 be given parametrically by

$$L_1: \mathbf{x} = \mathbf{w}_0 + s\mathbf{u} \quad \text{and} \quad L_2: \mathbf{x} = \mathbf{w}_1 + t\mathbf{v}.$$

Show that
   (a) L_1 and L_2 are parallel if and only if $\mathbf{u} = k\mathbf{v}$ for some scalar k.
   (b) L_1 and L_2 are identical if and only if $\mathbf{w}_1 - \mathbf{w}_0$ and $\mathbf{u}$ are both parallel to $\mathbf{v}$.
   (c) L_1 and L_2 are perpendicular if and only if $\mathbf{u} \cdot \mathbf{v} = 0$.
   (d) L_1 and L_2 intersect if and only if $\mathbf{w}_1 - \mathbf{w}_0$ is a linear combination of $\mathbf{u}$ and $\mathbf{v}$.

T.3. The lines L_1 and L_2 in R^3 are said to be skew if they are not parallel and do not intersect. Give an example of skew lines L_1 and L_2.

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Ex 5.2

Q1 (a) let $P_1(x_1, y_1) = (-2, -3)$ & $P_2(x_2, y_2) = (3, 4)$.

Now
$$\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x & y & 1 \\ -2 & -3 & 1 \\ 3 & 4 & 1 \end{vmatrix} = 0$$

$$x(-3-4) - y(-2-3) + 1(-8+9) = 0$$

$$-7x + 5y + 1 = 0$$

This is the required equation of line for the given Points.

Part (b), (c), (d) same as Part (a)

Q2 is same as Q1.

Q3 $x = 3 + 2t \Rightarrow t = \dfrac{x-3}{2}$

$y = -2 + 3t \Rightarrow t = \dfrac{y+2}{3}$

$z = 4 - 3t \Rightarrow t = \dfrac{z-4}{-3}$.

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Now $\frac{x-3}{2} = \frac{y+2}{3} = \frac{z-4}{-3}$ — ①

(a) $(1,1,1)$

eq① $\Rightarrow \frac{1-3}{2} = \frac{1+2}{3} = \frac{1-4}{-3}$

$\frac{-2}{2} = \frac{3}{3} = \frac{-3}{-3}$

$-1 \neq 1 \neq 1$

Hence all the value of t are not same; Therefore pts are not lies on the lines.

Part (b) & (c) Same as part (a)

(d) $(4, -1/2, 5/2)$

eq① $\Rightarrow \frac{4-3}{2} = \frac{-1/2+2}{3} = \frac{5/2-4}{-3}$

$1/2 = 1/2 = 1/2$

Hence all the value of t are same so it's lie on line

Q4 is same as Q3

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EX 5.2

Q5 (a) $P_0 = (\overset{x_0}{3}, \overset{y_0}{4}, \overset{z_0}{-2})$, $u = (\overset{a}{4}, \overset{b}{-5}, \overset{c}{2})$. $\quad -\infty < t < \infty$

$$x = x_0 + at \Rightarrow x = 3 + 4t$$
$$y = y_0 + bt \Rightarrow y = 4 - 5t$$
$$z = z_0 + ct \Rightarrow z = -2 + 2t.$$

Part b, c, d same as Part (a)

Q6 (a) $P_0 = (2, -3, 1)$ & $P_1 = (4, 2, 5)$

$$\vec{U} = \overrightarrow{P_0 P_1} = (4-2,\ 2+3,\ 5-1) = (2, 5, 4)$$

Now $P_0 = (\overset{x_0}{2}, \overset{y_0}{-3}, \overset{z_0}{1})$ & $U = (\overset{a}{2}, \overset{b}{5}, \overset{c}{4})$

$$x = x_0 + at \Rightarrow x = 2 + 2t$$
$$y = y_0 + bt \Rightarrow y = -3 + 5t$$
$$z = z_0 + ct \Rightarrow z = 1 + 4t.$$

Part (b), (c) & (d) same as part (a).

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Q7(a) $P_0 = (\overset{x_0}{2}, \overset{y_0}{-3}, \overset{z_0}{1})$ & $U = (\overset{a}{2}, \overset{b}{5}, \overset{c}{4})$

For symmetric form

$$\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}$$

$$\frac{x-2}{2} = \frac{y+3}{5} = \frac{z-1}{4}$$

(b), (c), & (d) Part as same as Part (a)

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~o~o~o~o~o

Q8 $3(x-2)+2(y+3)-4(z-4)=0$ —①

(a) $(0, -2, 3)$

eq ① $\Rightarrow 3(0-2)+2(-2+3)-4(3-4) = 0$

$0 = 0$

So it's lie on plane.

(b), (c), & (d) same as Part(a).

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Ex 5.2

Q9 (d) $P_0 = (x_0, y_0, z_0) = (5, 2, 3)$

$n = (a, b, c) = (-1, -2, 4)$

Equation of plane is

$a(x-x_0) + b(y-y_0) + c(z-z_0) = 0$

$-1(x-5) - 2(y-2) + 4(z-3) = 0$

$-x - 2y + 4z - 3 = 0$

Part (a), (b) + (c) same as part (d)

Q10 (b) $P_1(2, 3, 4)$, $P_2(-1, -2, 3)$ $P_3(-5, -4, 2)$

let $U = \overrightarrow{P_1P_2} = (-3, -5, -1)$

$V = \overrightarrow{P_1P_3} = (-7, -7, -2)$

Now $\vec{U} \times \vec{V} = \begin{vmatrix} x & y & z \\ -3 & -5 & -1 \\ -7 & -7 & -2 \end{vmatrix}$

$\Rightarrow x(10-7) - y(6-7) + z(21-35) = 0$

$\Rightarrow 3x + y - 14z = 0$

Which is the required equation of Plane.

others Parts are same as.

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Q11 (h) $3x-2y-5z+4=0$ — (1)

$2x+3y+4z+8=0$ — (2)

Xing eq (1) by "3" & eq (2) by "2"

$9x-\cancel{6y}-15z+12=0$

$4x+\cancel{6y}+8z+16=0$

$13x-7z+28=0$

$\frac{13x+28}{7}=z$ — (3)

Xing eq (1) by '2' & eq (2) by '3' & Subtracting

$\cancel{6x}-4y-10z+8=0$

$\cancel{6x}+9y+12z+24=0$

$-\quad-\quad-\quad-$

$-13y-22z-16=0$

$\frac{-13y-16}{22}=z$ — (4)

comparing eq (3) & eq (4)

$\frac{13x+28}{7}=\frac{-13y-16}{22}=z=t$ (say).

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Now $\frac{13x+28}{7} = t$

$\Rightarrow x = \frac{7t-28}{13} \longrightarrow$ (a)

& $\frac{-13y-16}{22} = t$

$\Rightarrow y = \frac{22t+16}{-13} \longrightarrow$ (b)

Also $z = t \longrightarrow$ (c)

Equation (a), (b) & (c) are the required Parametric equations of the line of intersection of the given planes.

Q12 (b) $\frac{x-2}{-2} = \frac{y-3}{4} = \frac{z+4}{3}$

Now $\frac{x-2}{-2} = \frac{y-3}{4}$

$\Rightarrow 4x+2y-14=0$ — (a) which is 1st equation of plane

And $\frac{x-2}{-2} = \frac{z+4}{3}$

$3x+2z+2=0$ which is 2nd equation of plane.

(c) $x = 4t \longrightarrow (a)$

$y = 1+5t \longrightarrow (b)$

$z = 2-t \longrightarrow (c)$

eq (a) $\Rightarrow \frac{x}{4} = t$ —— (i)

eq (b) $\Rightarrow \frac{y-1}{5} = t$ —— (ii)

eq (c) $\Rightarrow 2-z = t$ —— (iii)

From eq (i), (ii) & eq (iii)

$$\frac{x}{4} = \frac{y-1}{5} = 2-z = t$$

Now $x/4 = y-1/5$

$\Rightarrow 5x - 4y + 4 = 0$ —— ① which is 1^{st} eq of Plane.

& $y-1/5 = 2-z$

$y + 5z - 11 = 0$ —— ② which is 2^{nd} eq: of Plane.

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Ex 5.2

Q14 let $P_1(x_1, y_1, z_1) = (-2, 4, 2)$

$P_2(x_2, y_2, z_2) = (3, 5, 1)$

$P_3(x_3, y_3, z_3) = (4, 2, -1)$.

Now
$$\begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix} = 0$$

$$\begin{vmatrix} -2 & 4 & 2 \\ 3 & 5 & 1 \\ 4 & 2 & -1 \end{vmatrix} = 0$$

$$-2(-5-2) - 4(-3-4) + 2(6-20) = 0$$

$$14 \neq 0$$

These Points are not on the same plane.

Q13 is same as Q14.

Q15

$x = 2-3s$ $\quad$ $x = 5+2t$

$y = 3+2s$ and $y = 1-3t$

$z = 4+2s$ $\quad$ $z = 2+t$

$2-3s = 5+2t$ —— (1)

$3+2s = 1-3t$ —— (2)

$4+2s = 2+t$ —— (3)

Subtracting (3) from eq (2), we get

$$\begin{array}{r} 3+2s = 1-3t \\ \underline{4+2s = 2+t} \\ -1 = -1-4t \end{array}$$

$-4t = 0$

$\boxed{t=0}$

eq ① => $2-3s = 5+2(0)$ => $\boxed{s=-1}$

eq ② => $3+2s = 1-3(0)$

$3+2s = 1$ => $2s = -2$ => $\boxed{s=-1}$

$\therefore t = 0, s = 1$

Now

$x = 2-3s$ $\quad$ $x = 5+2t$

$x = 2-3(-1)$ $\quad$ $x = 5+2(0)$

$\boxed{x=5}$ $\quad$ $\boxed{x=5}$

$y = 3+2s$

$y = 3+2(-1)$

$y = 3-2 = 1$

$\boxed{y = 1}$

$z = 4+2s$

$z = 4+2(-1)$

$z = 4-2$

$\boxed{z = 2}$

$y = 1-3t$

$y = 1-3(0)$

$\boxed{y = 1}$

$z = 2+t$

$z = 2+0$

$\boxed{z = 2}$

Therefore the point of intersection of the given line are $P(x,y,z) = P(5,1,2)$.

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Q16 (a) $x = 2+2t$ and $x = 2+t$

$y = -3-3t$ and $y = 4-t$

$z = 4+4t$ and $z = 5-t$.

Let $\vec{u} = (2,-3,4)$

& $\vec{v} = (1,-1,-1)$

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Now $\vec{U} \cdot \vec{V} = (2,-3,4) \cdot (1,-1,-1)$.

$= 2+3-4 = 1$

$\vec{U} \cdot \vec{V} \neq 0$

The given lines are not $\perp$.

$\boxed{\text{Note } \vec{U} \cdot \vec{V} = 0 \text{ then } \perp}$

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