Circuits and Systems 1 - Week 11

Recap of Thevenin Theorem

In Thevenin analysis of circuit, we need to compute the following three terms

- Open-circuit voltage denoted by $\boldsymbol{v_{oc}}$ (computed by disconnecting the load)
- Thevenin resistance denoted by $\boldsymbol{R_{th}}$ or $\boldsymbol{R_t}$ (obtained by deactivating voltage source by replacing it with short circuit and deactivate a current source by replacing it with open circuit)
- Short-circuit current denoted by $\boldsymbol{i_{sc}}$ to verify the correctness of circuit (otherwise it is not needed)

Norton Theorem - Basics

In this theorem, we obtain an equivalent circuit having current source in parallel with Norton equivalent resistance.

Using source transformation, we can easily convert a Thevenin equivalent circuit to Norton equivalent circuit.

Let us proceed to an example on next slide.

Norton Theorem - Example 5.5-1 on page 182

Determine the Norton equivalent circuit for this circuit

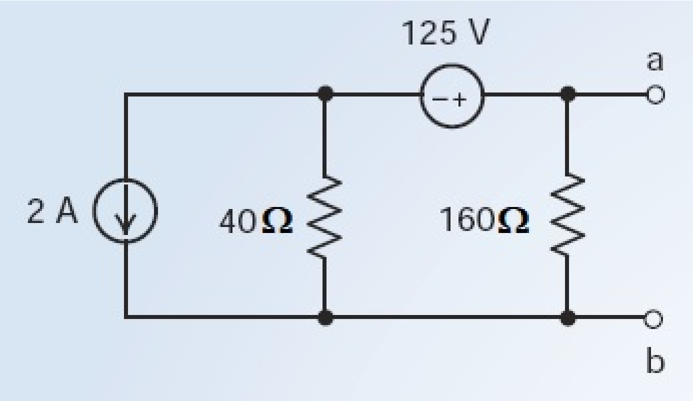

Figure: Norton equivalent circuit example

Norton Theorem - Example 5.5-1 on page 182 - Solution

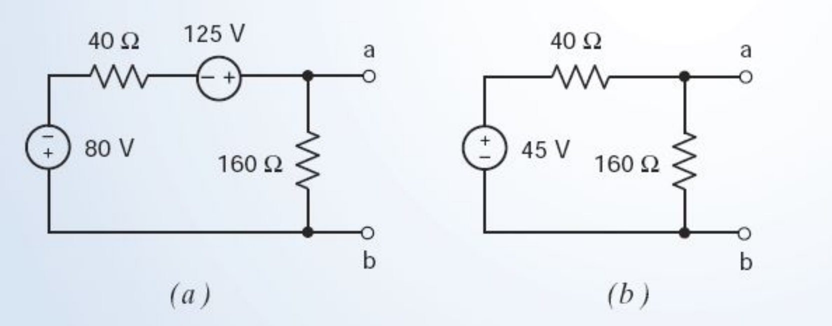

Figure: Norton equivalent circuit example - Part a

Norton Theorem - Example 5.5-1 on page 182 - Solution

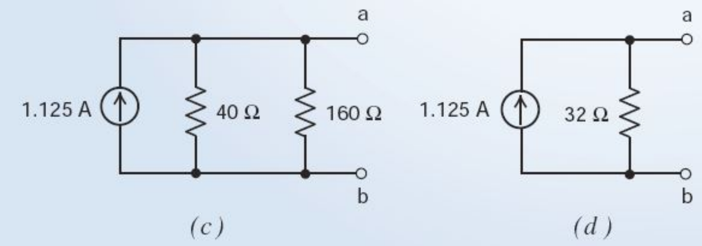

Figure: Norton equivalent circuit example - Part b

Norton Theorem Steps

The steps of Norton theorem are similar to Thevenin theorem.

The values of $\boldsymbol{v_{oc}}$, $\boldsymbol{i_{sc}}$ and $\boldsymbol{R_t}$ are determined in the same way as Thevenin Theorem

If $\boldsymbol{R_n}$ represent the Norton equivalent resistance, then we can state the following:

$$\boldsymbol{R_{th} = R_n}$$

Maximum Power Transfer

In most of engineering applications, when we connect the load resistor so we want maximum power to be transferred to the load resistor.

This is an interesting application in circuits and we would study more about its applications in detail today.

Power utility companies

Power utility companies (like PESCO, IESCO) want maximum power to be transferred to load (homes, organizations, industries) by reducing power line losses

It is desired that $\boldsymbol{R_t}$ is reduced (which represent resistance of source plus lines)

Maximum Power Transfer

Communication antennas

In telecom industries, the main research problem is to design antennas so that maximum signal strength is given to load.

For example: in FM radio receiver, we want the received signal to have maximum power.

In mobile and cell-phone communication, we sometimes say signal is poor and sometime the signal is good? What is this phenomenon in terms of signal reception?

Achieving maximum power transfer is not that easy. Lets look at power statistics of Pakistan

Power Stats of Pakistan last year

Daily Power Position
(26/07/2020)

GENERATION			19,989
Province	DISCO	Average Demand (MW)	Average Drawl (MW)
KPK+AJK	PESCO	2,780	1,894
FATA	TESCO	181	138
Total		2,961	2,032
PUNJAB+AJK	IESCO	1,663	1,615
	GEPCO	2,251	2,214
	LESCO	3,636	3,541
	FESCO	2,611	2,587
	MEPCO	3,388	3,280
Total		13,549	13,237
SINDH	SEPCO	1,002	640
	HESCO	1,023	754

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Total		2,025	1,394
BALOCHISTAN	QESCO	1,369	753
Total		1,369	753
Total		19,904	17,416

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Maximum Power Transfer Theorem

Although we do read about Maximum Power Transfer Theorem, but applying it in real-life is not that easy.

The theorem states that maximum power is transferred to load when $R_L = R_t$. The formula for maximum power when $R_L = R_t$ is given as follows:

$$p_{\max} = \frac{v_s^2}{4R_t}$$

$$p_{\max} = \frac{\left(i_s^2 R_t^2\right)}{4R_t} = \frac{i_s^2 R_t}{4}$$

Problem 5.4-1 on page 201

Use source transformation, compute the $\boldsymbol{v_{oc}}$ and $\boldsymbol{R_t}$ in the circuit shown below

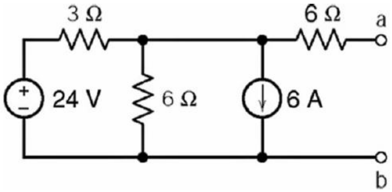

Figure: Problem 5.4-1 on page 201

Let us convert the voltage source into current source using source transformation.

Problem 5.4-1 on page 201 - Solution part a

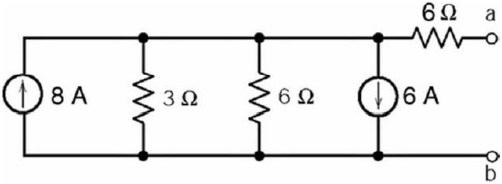

Figure: Problem 5.4-1 on page 201 - Solution

Let us simplify by adding the current sources and resistors.

Problem 5.4-1 on page 201 - Solution part b

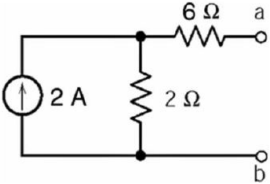

Figure: Problem 5.4-1 on page 201 - Solution

Let us convert back to voltage source using source transformation.

Problem 5.4-1 on page 201 - Solution part c

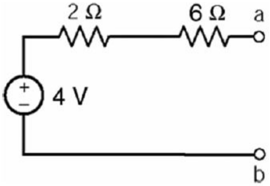

Figure: Problem 5.4-1 on page 201 - Solution

Problem 5.4-1 on page 201 - Solution part d

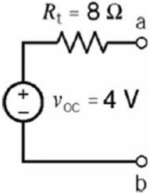

Figure: Problem 5.4-1 on page 201 - Solution

Problem 5.4-3 on page 201

Find the Thevenin equivalent circuit for the circuit shown below:

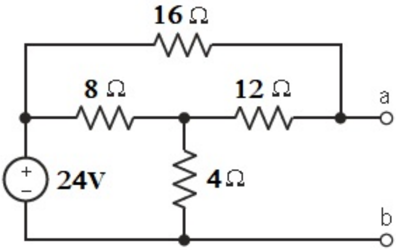

Figure: Problem 5.4-3 on page 201

Let us compute the Thevenin Resistance first

Problem 5.4-3 on page 201 - Solution

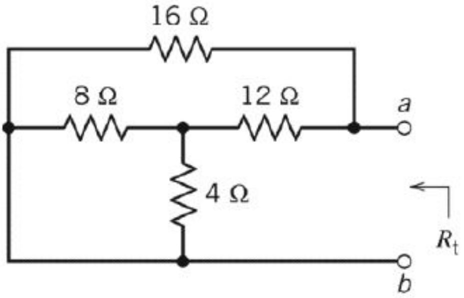

Figure: Problem 5.4-3 on page 201 - Solution

Problem 5.4-3 on page 201 - Solution

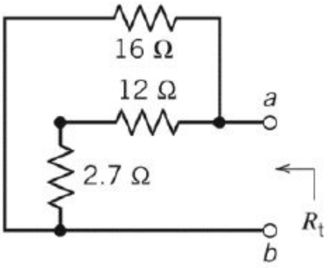

Figure: Problem 5.4-3 on page 201 - Solution

Now $\mathbf{12\Omega}$ and $\mathbf{2.7\Omega}$ are in series, so we obtain $\mathbf{12 + 2.7 = 14.7\Omega}$. Now, $\mathbf{14.7\Omega}$ and $\mathbf{16\Omega}$ are in parallel, so we obtain the following:

$$\boldsymbol{R_t = \frac{14.7 \times 16}{14.7 + 16} = \frac{235.2}{30.7} = 7.67\Omega}$$

Problem 5.4-3 on page 201 - Solution

Let us compute v_{oc} now by using Mesh Analysis

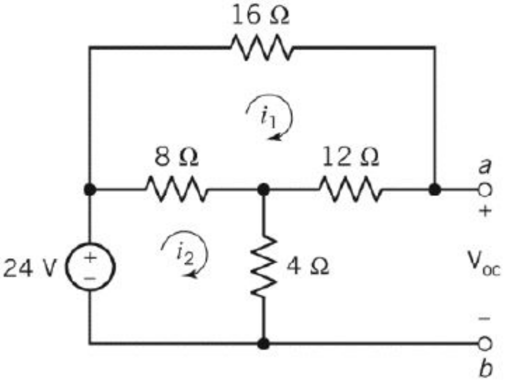

Figure: Problem 5.4-3 on page 201 - Solution

Problem 5.4-3 on page 201 - Solution

$$\mathbf{16}\boldsymbol{i_1} + \mathbf{12}\boldsymbol{i_1} + \mathbf{8}(\boldsymbol{i_1} - \boldsymbol{i_2}) = \mathbf{0} \tag{1}$$

$$\mathbf{8}(\boldsymbol{i_2} - \boldsymbol{i_1}) + \mathbf{4}\boldsymbol{i_2} - \mathbf{24} = \mathbf{0} \tag{2}$$

Using MATLAB, we obtain the following:

$$\boldsymbol{i_1} = \frac{\mathbf{12}}{\mathbf{23}} = \mathbf{0.5217}$$

$$\boldsymbol{i_2} = \frac{\mathbf{54}}{\mathbf{23}} = \mathbf{2.3478}$$

Problem 5.4-3 MATLAB Code

MATLAB code for solving Problem 5.4-3

```
syms i1 i2;

a=solve('16*i1 +12*i1 +8*i1 - 8*i2=0','8*i2-8*i1+4*i2-24=0')

a.i1

a.i2
```

Problem 5.4-3 on page 201 - Solution

Now, a very important point to consider. I state the following:

$$\boldsymbol{v_{oc} = 12i_1 + 4i_2 = 12 \times (\frac{12}{23}) + 4 \times (\frac{54}{23}) = 15.65V}$$

The role of $\mathbf{12\Omega}$ resistor is important (to be explained in class).