$4yy' + x = 0$

$4y\left(\frac{-x}{4y}\right) + x = 0 \Rightarrow -x + x = 0 \Rightarrow 0 = 0$

Hence, the given fn is the solution of corresponding D.E.

(2nd part)

put $x = 2$ and $y = 1$.

$x^2 + 4y^2 = c$

$(2)^2 + 4(1)^2 = c \Rightarrow 4 + 4 = c \Rightarrow \boxed{c = 8}$

x ——— x ——— x

Lecture : 03 — Mon, 1st Sept, 2008

Exercise 1.3

QNo1 — QNo6 ⇒ variable separable and QNo11.

QNo7 — QNo10 → Reduce to Variable Separ

QNo2) $yy' + 25x = 0$.

Solution:

$yy' = -25x$

$y\frac{dy}{dx} = -25x$ (variable sep)

$y\,dy = -25x\,dx$

Integration,

$\int y \cdot dy = -25\int x \cdot dx$

$\frac{y^2}{2} + c_1 = -25\frac{x^2}{2} + c_2$

$y^2/2 = -25x^2/2 + C_1$

$y^2 = -25x^2 + 2C_1$

$\boxed{y^2 = -25x^2 + C}$ $\because 2C_1 = C$

③ $y' = 1 + 0.0$

Sol:

$\frac{dy}{dx} = 1$

(V.S) ⇒ (

$\int \frac{1}{1+0.0}$

$\int \frac{100}{100+y}$

$100\int \frac{1}{(10)^2}$

⑤ $y' =$

Sol:

$2\frac{d}{d}$

$\frac{dy}{dx}$

$\frac{1}{y}d$

$\int \frac{1}{y}d$

$\ln y +$

$\ln y$

⑦ xy

Solu

$\int$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1} x/a$$

④ $y' = 1+0.01y^2$

Sol:

$$\frac{dy}{dx} = 1+0.01y^2$$

$$(v.s) \Rightarrow \left(\frac{1}{1+0.01y^2}\right) dy = dx$$

$$\int \frac{1}{1+0.01y^2}\,dy = \int dx$$

$$\int \frac{100}{100+y^2}\,dy = \int (1)\,dx$$

$$100\int \frac{1}{(10)^2+(y)^2}\,dy = \int (1)\,dx$$

$$100\left(\frac{1}{10}\tan^{-1} y/10\right) = x+C_1$$

$$\tan^{-1} y/10 = 0.01x + 0.01C$$

$$y/10 = \tan(0.01x+C)$$

$$y = 10\tan(0.01x+C)$$

⑤ $y' = \frac{xy}{2}$

Sol:

$$\frac{dy}{dx} = \frac{xy}{2}$$

$$\frac{dy}{dx} = \frac{1}{2}(xy)$$

$$\frac{1}{y}dy = \frac{1}{2}x\,dx$$

$$\int \frac{1}{y}dy = \frac{1}{2}\int x\,dx$$

$$\ln y + C_1 = \frac{1}{2}\cdot\frac{x^2}{2} + C_2$$

$$\ln y = \frac{1}{4}x^2 + C \Rightarrow y = e^{x^2/4 + C} = e^{x^2/4}\cdot e^{C_1}$$

$$\Rightarrow y = ce^{x^2/4} \qquad \because e^{C_1} = C$$

⑦ $xy' = y^2 + y$; $(y/x = u)$

Solution:

By v.s:

$$x\frac{dy}{dx} = y^2 + y$$

$$\int \frac{1}{y^2+y}\,dy = \int \frac{1}{x}\,dx \Rightarrow$$ integration of $\frac{1}{y^2+y}$ is very difficult that

By reduceable variable form

① $y' = y^2/x + y/x$ —(i)

Let $y/x = u$

$y = xu$ & ÷ by x → eq (i)

$y' = \frac{y^2}{x \cdot x} \times x + y/x$

$= \frac{y^2}{x^2} \cdot x + y/x \Rightarrow y' = u^2x + u$ —(ii)

put $y/x = u$.

$y = ux$
diff w.r.t x.

$y' = u + u'x$.

put $y' = u + u'x$ in (ii)

$y' = u^2x + u$.

$\cancel{u} + u'x = u^2x + \cancel{u}$.

$u'\cancel{x} = u^2\cancel{x} \Rightarrow \frac{du}{dx} = u^2$

(V.S)

$u^{-2}du = dx$

$\int u^{-2} du = \int dx$

$\frac{u^{-1}}{-1} + c = x + c \Rightarrow -\frac{1}{u} = x + c$

As $u = y/x$

$-\frac{1}{y/x} = x + c \Rightarrow -\frac{x}{y} = x + c$

$-y/x = \frac{1}{x+c} \Rightarrow \boxed{y = \frac{x}{c-x}}$

(10) $y' = (y+4x)^2 \Rightarrow (y+4x = u)$

Sol:

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$y' = (y+4x)^2 \quad \ldots (i)$

Let $y + 4x = u$

~~$y = cu^2$~~

$y = u - 4x$

diff w.r.t x

$\frac{dy}{dx} = \frac{du}{dx} - 4\frac{d}{dx}(x)$

$y' = u' - 4 \rightarrow$ put in (i)

$u' - 4 = u^2$

$u' = u^2 + 4$

$\frac{du}{dx} = u^2 + 4 \Rightarrow$

$\left(\frac{1}{u^2+4}\right) du = dx$

$\int \frac{1}{(2)^2 + (u)^2} du = \int dx$

$\frac{1}{2} \tan^{-1} u/2 + c = x + c_1$

$\tan^{-1} u/2 = 2x + c$ $\quad \because \int \frac{1}{a^2+x^2} dx = \tan^{-1}\frac{x}{a}$

$u/2 = \tan(2x+c)$

$u = 2\tan(2x+c)$ $\quad \because u = y + 4x$

$y + 4x = 2\tan(2x+c)$

$\boxed{y = -4x + 2\tan(2x+c)}$

(12) $y' = \frac{-x}{y}$, $y(1) = \sqrt{3}$ or $x = 1, y = \sqrt{3}$

Sol:

(particular solution)

$y' = -\frac{x}{y}$

$\frac{dy}{dx} = -\frac{x}{y}$

$y\,dy = -x\,dx$

on Integrate

$\frac{y^2}{2} + c = -\frac{x^2}{2} + c$

$y^2 = -x^2 + C$

$(\sqrt{3})^2 = -(1)^2 + C$

$3 = -1 + C$

$\boxed{C = 4}$

(15) $e^x y' = 2(x+1)y^2$; $y($

$\boxed{c=2}$

$$y = \frac{1}{2+(2x+4)e^x}$$

Sol :-

$$e^x \frac{dy}{dx} = 2(x+1)y^2$$

$$\cancel{e^x \frac{dy}{dx} = 2xy^2 + 2y^2}$$

$$y^{-2}dy = 2(x+1)e^{-x}dx$$

$$\int y^{-2}dy = 2\int (x+1)e^{-x}dx$$

Integration by parts.

$$\frac{y^{-1}}{-1} + c = 2\left((x+1)\frac{e^{-x}}{-1} - \int \frac{e^{-x}}{-1}dx\right)$$

$$\frac{-1}{y} + c = 2\left(-(x+1)e^{-x} + \int e^{-x}dx\right)$$

$$= 2\left(-(x+1)e^{-x} + \frac{e^{-x}}{-1}\right)$$

$$= 2\left(-(x+1)e^{-x} - e^{-x}\right) + c$$

$$= 2\left(-e^{-x}(x+1) - e^{-x}\right) + c$$

$$= 2\left(-xe^{-x} \underbrace{- e^{-x} - e^{-x}}_{-2e^{-x}}\right) + c$$

$$-1/y = -2xe^{-x} - 4e^{-x} + c$$

put $y = \frac{1}{6}$ and $x = 0$

$$\frac{-1}{1/6} = -2(0)e^{-0} - 4e^{-0} + c$$

$$-6 = 0 - 4 + c$$

$$-6 + 4 = c \Rightarrow \boxed{c = -2}$$

(17) $y' \cosh^2 x - \sin^2 y = 0 \quad ; \quad y(0) = \pi/2$

Sol:

$\frac{dy}{dx} \cosh^2 x = \sin^2 y$

$\int \operatorname{cosec}^2 y \, dy = \int \operatorname{sech}^2 x \, dx$

$-\cot y = \tanh x + c \quad \text{—— (1)}$

$-\cot(\pi/2) = \tanh(0) + c \Rightarrow \boxed{c = 0}$

$\boxed{-\cot y = \tanh x}$

(4) $y' + 3x^2 y^2 = 0$

Sol:

$\frac{dy}{dx} + 3x^2 y^2 = 0$

$\frac{dy}{dx} = -3x^2 y^2 \Rightarrow \frac{1}{y^2} dy = -3x^2 \, dx$

Now integrate,

$\int \frac{1}{y^2} dy = -3 \int x^2 \, dx$

$\int y^{-2} dy = -3 \int x^2 \, dx$

$\frac{y^{-1}}{-1} = \frac{-\cancel{3}x^3}{\cancel{3}} + c$

$\boxed{\frac{-1}{y} = -x^3 + c}$

$-y = \frac{1}{-x^3 + c}$

$y = \frac{1}{x^3} + c$

(6) $y' = -ky^2$

Sol:

$\frac{dy}{dx} = -ky^2$

$y^{-2} dy = -k \, dx$

$\int y^{-2} dy = -k \int dx$

$\frac{y^{-1}}{-1} = -kx + c$

$\boxed{\frac{-1}{y} = -kx + c'}$

$\boxed{1/y = kx - c}$

(8) $xy' = x + y \quad ; (y/x = u)$

Sol:

$\frac{dy}{dx} = \frac{x}{x} + y/x$

$\frac{dy}{dx} = 1 + y/x$

$y' = 1 + u$

As, $y/x = u$

$y = ux$

$\frac{dy}{dx} = \frac{d(ux)}{dx}$

Product rule

$y' = u + u'x$

$u + u'x = 1 + u$

$u'x = 1$

$u' = 1/x$

$\frac{du}{dx} = \frac{1}{x}$

$du = \frac{1}{x} \cdot dx$

$\int du = \int \frac{1}{x} \cdot dx$

$u = \ln x + C$

$y/x = \ln x + C$

$\boxed{y = x \ln x + C}$

Sol:

$xyy' = x^2 + y^2$

$yy' = \frac{x^2}{x} + \frac{y^2}{x} = x + y^2/x$

xing & ÷ing by x

$yy' = x + \frac{y^2}{x} \cdot \frac{x}{x} = x + \frac{y^2}{x^2} \cdot x$

put $y/x = u \Rightarrow y = ux$..

$yy' = x + u^2x$

diff $y = ux$ w.r.t x

$y' = \frac{dy}{dx} = \frac{d}{dx}(ux) = u + u'x$..

$yy' = x + u^2x$ $\quad\because y = ux$

$y' = u + u'x$

$ux(u + u'x) = x + u^2x$

$\cancel{u^2x} + uu'x^2 = x + \cancel{u^2x}$

$uu'x^2 = x$

$uu'x = 1 \Rightarrow uu' = 1/x \Rightarrow u\frac{du}{dx} = \frac{1}{x}$

$u \cdot du = \frac{1}{x} dx$

Now integrate, $\int u \cdot du = \int 1/x \, dx$

$\frac{u^2}{2} = \ln x + c$

$u^2 = 2\ln x + c$

As $u = y/x$

$(y/x)^2 = 2\ln x + c$

$y^2/x^2 = 2\ln x + c$

$y^2 = x^2 2\ln x + cx$

$\boxed{y = x\sqrt{2\ln x + c}}$

(11) $y' + \operatorname{cosec} y = 0$

Sol:

$\frac{dy}{dx} + \operatorname{cosec} y = 0$

$\frac{dy}{dx} = -\operatorname{cosec} y \Rightarrow dy = -\operatorname{cosec} y \, dx.$

$\frac{-1}{\operatorname{cosec} y} dy = dx. \Rightarrow -\sin y \, dy = dx.$

$\int -\sin y \, dy = \int (1) \, dx \Rightarrow -\left(\frac{-\cos y}{\cancel{1}}\right) = x + c$

$\cancel{\cos y = xy}$. $\cos y = x + c \Rightarrow \boxed{y = \cos^{-1}(x+c)}$

(3) $xy' + y = 0$, $y(2) = -2$

Sol:

$x\frac{dy}{dx} + y = 0$

$x\frac{dy}{dx} = -y.$

$x\,dy = -y\,dx.$

$\frac{-1}{y} dy = \frac{1}{x} dx.$

$\ln y = \ln x + c$

$-y = x + \cancel{c}.$

put $x = 2$, $y = -2$

$\cancel{-\ln y = \ln x + c}$

$\cancel{-\ln(-2) = \ln(2)} + c$

$-(-2) = 2 + c$

$2 = 2 + c$

$\boxed{c = 0} \Rightarrow \boxed{-y = x}$ $\boxed{c = -4}$

$\ln y = -\ln x + c$

$= -\ln x \cdot c$

$y = (xc)^{-1}$

$y = \frac{1}{xc}$

$xy = \frac{1}{c}$ $\quad \because \frac{1}{c} = c$

$xy = c$

(4) $y^2 y' + x^3 = 0$, $y(0) = 1$

Sol:

$y^2 y' + x^3 = 0$

$y^2 \frac{dy}{dx} = -x^3$

$y^2 dy = -x^3 dx$

$\frac{y^3}{3} + c = \frac{-x^4}{4} + c$

put $y = 1$ & $x = 0$

$\frac{(1)^3}{3} = -\frac{(0)^4}{4} + c$

$\frac{1}{3} = 0 + c$

$\boxed{c = 1/3}$

$\frac{1}{3} y^3 = \frac{-1}{4} x^4 + \frac{1}{3}.$

$12\left(\frac{1}{3}\right) y^3 = 12\left(\frac{-1}{4} x^4\right) + 12\left(\frac{1}{3}\right)$

$\boxed{4y^3 = -3x^4 + 4.}$

$y^3 = -\frac{3}{4} x^4 + 1$

$y = -\frac{3}{4} x^{4/3}.$

(16) $y' = 1 + 4y^2 , \quad y(0) = 0$

Sol:

$\frac{dy}{dx} = 1 + 4y^2$

$\frac{1}{1+4y^2} \, dy = dx.$

$\int \frac{1}{(1)^2+(2y)^2} \, dy = \int (1) \, dx.$

$\frac{1}{1} \tan^{-1} \frac{2y}{1} = x + c$

$\tan^{-1} 2y = x + c$

put x and y is zero

$\tan^{-1} 2(0) = 0 + c$

$\tan^{-1}(0) = c$

$\boxed{c = 0}$

Hence,

$\boxed{\tan^{-1} 2y = x}$

(18) $\frac{dr}{dt} = -2tr , \quad r(0) = 2.5$

Sol:

$\frac{dr}{dt} = -2tr$

$\frac{1}{r} dr = -2t \, dt$

$\ln r = -\frac{2t^2}{2} + c$

$\ln r = -t^2 + c$

put $t = 0$, $r = 2.5$

$\ln(2.5) = -(0)^2 + c$

$c = 0.3979.$

Hence,

$\boxed{\ln r = -t^2 + 0.8979}$

$\boxed{r = e^{-t^2 + 0.91}}$

(19) $L\left(\frac{di}{dt}\right) + RI = 0 \qquad I(0) = I_0$

Here $i = y$ and $t = x. \Rightarrow i = I_0$ & $t = 0$

$L \frac{di}{dt} = -Ri$

$L\left(\frac{1}{I}\right) di = -R \, dt.$

$L \ln i = -Rt + c$

put $i = I_0$ & $t = 0$

$\ln I_0 = -R(0) + c$

$\boxed{c = L \ln I_0}$

$\ln i = -\frac{Rt}{L} + \ln I_0.$

$I = e^{-Rt/L + \ln I_0}$

$= e^{-Rt/L} \cdot e^{\ln I_0}.$

$= I_0 \, e^{-Rt/L}$

$\boxed{I = I_0 \, e^{-(R/L)t}}$

Hence,

$L \ln i = -Rt + L \ln I_0,$